PI MU EPSILON JOURNAL

THE OFFICIAL PUBLICATION OF

THE HONORARY MATHEMATICAL FRATERNITY

VOLUME 1 NUMBER 5

CONTENTS

	Page
Some Elementary Cohomology Theory. . . . . Paul Olum	171
On the Equation $\phi(n) = \pi(n)$ . . . . . Leo Moser	177
An Interesting Theorem . . . . . Pedro A. Piza	181
Problem Department.	183
Problems for Solution	183
Solutions	185
The Founder's Pin. . . . . Gertrude G. Campbell	189
General Officers of the Fraternity.	191
Reports of the Chapters	194
Medals. Prizes and Scholarships	200
News and Notices	202
Initiates. Academic Year 1950-1951.	203

NOVEMBER 1951

PI MU EPSILON JOURNAL
THE OFFICIAL PUBLICATION
OF THE HONORARY MATHEMATICAL FRATERNITY

MANUSCRIPTS and EDITORIAL CORRESPONDENCE, including CHAPTER REPORTS, NEWS ITEMS, etc., should be addressed to the Editor, Ruth W. Stokes, 15 Smith College, Syracuse University, Syracuse 10, New York.

PI MU EPSILON JOURNAL is published semi-annually at Syracuse University.

SUBSCRIPTION PRICE: To Individual Members, $1.50 for 2 years; to Non-Members and Libraries, $2.00 for 2 years. Subscriptions, orders for back numbers and correspondence concerning subscriptions and advertising should be addressed to Howard C. Bennett, 15 Smith College, Syracuse University, Syracuse 10, New York.

PI MU EPSILON JOURNAL

THE OFFICIAL PUBLICATION OF
THE HONORARY MATHEMATICAL FRATERNITY

VOLUME 1 NUMBER 5

CONTENTS

	Page
Some Elementary Cohomology Theory. . . . . Paul Olum	171
On the Equation $\phi(n) = \pi(n)$ . . . . . Leo Moser	177
An Interesting Theorem . . . . . Pedro A. Piza	181
Problem Department.	183
Problems for Solution	183
Solutions	185
The Founder's Pin. . . . . Gertrude G. Campbell	189
General Officers of the Fraternity.	191
Reports of the Chapters	194
Medals, Prizes and Scholarships	200
News and Notices	202
Initiates, Academic Year 1950-1951	203

NOVEMBER 1951

SOME ELEMENTARY COHOMOLOGY THEORY

Paul Olum, Cornell University

1. Introduction. There has been a great deal written lately of a general expository character about topology. Our object here, however, will be not at all to give a general discussion of the subject, but rather actually to do some topology. Specifically, we propose to prove a theorem, theorem I below, belonging to what is called "cohomology theory"; this theory is a part of algebraic topology and is one of the most modern and advanced disciplines in mathematics today.

Of course, we shall do only a small bit of cohomology theory and that from quite an elementary point of view. Nevertheless it will contain certain ideas which are basic to the general subject and indeed much of what we shall do here can be generalized quite easily to cover considerably more complicated situations.

As an application of theorem I we shall then use it in proving a theorem on the coloring of maps, theorem II below.

2. Cochains, coboundaries, cocycles. Let S denote the surface of a sphere and let us suppose that it is subdivided into (curvilinear) polygonal regions $p_1, p_2, \ldots, p_n$. Let us denote the edges in this subdivision of S by $e_1, e_2, \ldots, e_m$ and the vertices by $v_1, v_2, \ldots, v_q$.

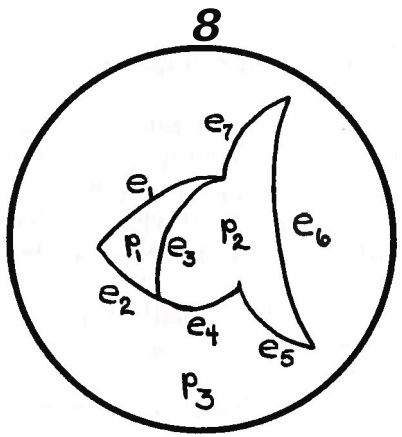
We shall use the integers 0 and 1 here and understand the plus sign throughout to mean the so-called "addition modulo 2", that is,

$$(2.1) \qquad \begin{array}{ll} 0+0=0 & 1+1=0 \\ 0+1=1 & 1+0=1 \end{array}$$

By a one-dimensional cochain (or, for brevity, 1-cochain) in S we shall mean any rule or function whichattaches to each edge e_i either the value 0 or the value 1; we shall denote such a 1-cochain by f^1. A 0-cochain f^0 and a 2-cochain f^2 are defined similarly except that f^0 assigns either 0 or 1 to each vertex v_i, f^2 assigns 0 or 1 to each p_i.

Suppose then f^1 is such a 1-cochain. We can construct from it a 2-cochain, which we shall denote by δf^1, by the following simple process: let δf^1 be the function which assigns to each p_i the sum of the values which are assigned by f^1 to all the edges of this p_i. (Remember that we are using addition modulo 2.) This 2-cochain δf^1 is called the coboundary of f^1.

Thus, for example, in the accompanying figure, if $f^1(e_1) = 0$ and $f^1(e_2) = 0$ and $f^1(e_i) = 1$ for i = 3, 4, 5, 6, 7, then $\delta f^1(p_1) = 1$, $\delta f^1(p_2) = 1$ and $\delta f^1(p_3) = 0$.



Figure

The coboundary δf^0 of a 0-cochain f^0 is defined similarly; it is the 1-cochain which attaches to each e_i the sum of the values attached by f^0 to the two vertices of e_i. The coboundary of a 2-cochain is not defined since there are no 3-cochains.

Finally, we shall call a 1-cochain f^1 a 1-cocycle if $\delta f^1(p_i) = 0$ for all pi. Thus, if f^1 assigns the value 0 to e_1, e_4, e5, e_6 and 1 to e_2, e_3, e_7 then f^1 is a 1-cocycle. Similarly, f^0 is a 0-cocycle if $\delta f^0(e_i) = 0$ for all edges e_i.

3. A theorem about cocycles. We shall prove here a theorem about 1-cocycles in S. First, however, we need some notation and a simple lemma.

Notation. If ℓ is a path in S consisting of a sequence of edges, say, $e_1, e_2, \ldots, e_j$, and f^1 is a 1-cochain, then we shall use $f^1(\ell)$ to denote $f^1(e_1) + f^1(e_2) + \ldots + f^1(e_j)$. (Remember again that addition is always addition modulo 2 here.)

Now let ℓ_c be a simple[1] closed path in S consisting of a sequence of edges. Let us select one of the two regions into which ℓ_c divides S and call it the interior of ℓ_c. This interior will then be a collection of polygonal regions, say, $p_1, p_2, \ldots, p_k$.

Lemma. For any 1-cochain f^1 in S,

$$f^1(\ell_c) = \delta f^1(p_1) + \delta f^1(p_2) + \ldots + \delta f^1(p_k). \tag{3.1}$$

Proof. Each $\delta f^1(p_i)$ is, by definition, simply the sum of the values attached by f^1 to the edges of p_i. On the right of 3.1 these sums are added up for all the p_i's in the interior of ℓ_c. Now each of these edges which is not a part of the path ℓ_c is on two p_i's and consequently (see 2.1) its contribution cancels out in the summation on the right of 3.1. What is left then is just the sum of the values assigned by f^1 to those edges which are part of ℓ_c, and this is precisely what 3.1 asserts.

Remark. The reader may note that 3.1 has a certain formal analogy with Stokes' theorem in the calculus. This is not accidental and the analogy actually goes quite deep.

We can now prove a theorem which states what is, from the topologist's point of view, a fundamental property of the sphere.

Theorem I. In a subdivided sphere S as above, every 1-cocycle is the coboundary of some 0-cochain.

Proof. Let f^1 be a 1-cocycle in S. We begin by constructing a 0-cochain f^0 as follows: We arbitrarily set $f^0(v_1) = 0$; then, for any other vertex v_i, we draw a simple[1] path ℓ_i from v_1 to v_i consisting of a sequence of edges and set $f^0(v_i) = f^1(\ell_i)$, using the notation at the beginning of this section.

We want to show that we get the same value for $f^0(v_i)$ regardless of which path ℓ_i from v_1 to v_i is used. Suppose ℓ_i is another such path. Then ℓ_i and ℓ_i taken together form a closed path ℓ_c in S. If ℓ_i and ℓ_i intersect only at v_1 and v_i, so that ℓ_c is a simple closed path, then 3.1 and the

[1] "Simple" here means one which does not intersect itself.

fact that f^1 is a 1-cocycle give us at once

$$f^1(\ell_i) + f^1(\bar{\ell}_i) = f^1(\ell_c) = 0;$$

then, adding $f^1(\bar{\ell}_i)$ to both sides (using 2.1) we get

$$f^1(\ell_i) = f^1(\bar{\ell}_i)$$

which is just what we wanted. If ℓ_i and $\bar{\ell}_i$ have other intersections, then ℓ_i and $\bar{\ell}_i$ taken together form (apart from edges they may have in common) a collection of simple closed paths and the same result clearly holds.

Now the theorem is immediate, for we assert that, with f_0 as constructed above, $f^1 = \delta f_0$. To see this, suppose e_a is any edge in S and let its vertices be v_i and v_j; since, by the argument above, we may use any simple path ℓ_j from v_1 to v_j in determining $f^0(v_j)$, let us choose for ℓ_j the same path ℓ_i as used in determining $f^0(v_i)$ followed by the edge e_a. Then it is clear that $f^0(v_i) = f^0(v_j)$ if $f^1(e_a) = 0$ whereas $f^0(v_i) \neq f^0(v_j)$ if $f^1(e_a) = 1$. From this and 2.1 we see that δf^0 will assign to e_a the same value as does f^1, and since this is true for every edge e_a in S the theorem is proved.

Remark. We have considered above the surface of a sphere subdivided into polygonal regions. Our definitions of cochain, cocycle, coboundary here obviously make sense also for any figure similarly subdivided into polygonal regions; the figure might be some other surface such as the surface of a doughnut or of a pretzel, or even just a part of such a surface. The theorem we have proved is, however, a theorem about spheres and need not hold in general for other figures.

4. A theorem on map coloring. We are going to consider here a theorem on the coloring of maps. For us a map will mean a subdivision of the surface of a sphere[2] into a number of polygonal regions (the "states*). For such a map to be "properly" colored means that no two states with a common edge are to have the same color.

Theorem II. If a map on the sphere has the property that there is an even number of edges meeting at each vertex, then the map can be properly colored using only two colors.

This theorem is well-known and there are many ways to prove it. Let us observe first, however, that it is a particular property of maps on a sphere and does not hold, for instance, for maps on the surface of a doughnut. (Let the reader construct an example to show this.)

Thus some characteristic property of the sphere must come into account in the proof. Can we perhaps single out and describe algebraically in some way this particular property and base our proof only on it? The answer is that we can and indeed theorem I contains precisely the information about the sphere which we need.

In proving theorem II then, the only way in which we shall use the fact that our map is on the sphere is to use the conclusion of theorem I. It follows that our theorem will in fact hold for any map which is a subdivision of a figure sharing the property of theorem I.

Proof of theorem II. Theorem II is a consequence of the following proposition:

(4.1) A closed curve drawn on our map which does not go through any vertices must cross state lines (edges) an even number of times.

To see first that this implies theorem II, suppose we use red and black for colors and select one state p_1 and color it red. Now for any other state p_i we draw an arbitrary curve from the center of p_1 to the center of p_i which does not go through any vertex and color p_i red or black according as this curve crosses state lines an even or odd number of times. It is clear from 4.1 that any two curves from p_1 to p_i must yield the same color for pi. Coloring each p_i this way then, we see that adjacent states p_a, pi, with a common edge e must have different colors, for one possible curve from p_1 to p_b is the curve from p_1 to p_a followed by the path from p_a to pi, crossing the one additional state line e. Thus this is a proper coloring for our map.

[2] If the reader prefers to think of a map as just a country divided into states, then his map can be made into one of ours by simply adding on an extra "state," namely the whole of the globe exterior to the country Clearly, if our map can be properly colored in any assigned number of colors, then so can the original map without this extra "state."

It remains to prove 4.1. Given the closed curve of 4.1, let us construct a 1-cochain f^1 as follows: For each edge e_i we set $f^1(e_i) = 0$ or 1 according as the curve crosses e_i an even or odd number of times (zero is an even number here.)

Inasmuch as the curve must cross the boundary of each state in toto an even number of times (since whenever it enters a state it leaves it again), it follows that this f^1 must be a 1-cocycle. But then, by theorem I, f^1 must be the coboundary of some 0-cochain f^0.

Let $v_1, v_2, \ldots, v_r$ be all the vertices to which this f^0 assigns the value 1. Now the total number of edges meeting these vertices must, according to the hypotheses of theorem 11, be even, provided we count twice any edge which contains two of these vertices; it follows that the total number of edges which meet just one (but not two) of these vertices must be even. Since (by the definition of the coboundary in § 2) it is these and only these edges to which $\delta f^0 = f^1$ assigns the value 1, we see that f^1 assigns the value 1 to an even number of edges.

But then, from the definition of f^1 above, there is an even number of state lines which the closed curve of 4.1 crosses an odd number of times and this proves 4.1 and hence the theorem.

ON THE EQUATION $\phi(n) = \pi(n)$

Leo Moser, University of Alberta

As is usual, let $\phi(n)$ denote the number of integers not exceeding n and relatively prime to n, and $\pi(n)$ the number of primes not exceeding n. The main object of this note is to show that the only solutions of the equation $\phi(n) = \pi(n)$ are n = 2, 3, 4, 8, 14, 20, 90. Thus, for example, for n = 14 the numbers 1, 3, 5, 9, 11, 13 are relatively prime to n, while the primes under n are 2, 3, 5, 7, 11, 13. Hence $\phi(14) = \pi(14) = 6$. Note that 1 is not counted as a prime. All solutions of the inequality $\phi(n) < \pi(n)$ will also be found and some related results will be discussed.

The main tool required is the following lemma:

Lemma 1. For $x > 1$, there is a prime p with $x < p < 2x$.

This is an important result in the theory of the distribution of primes. It is usually known as Bertrand's postulate, having been conjectured by J. Bertrand in 1845. It was first proved by P. Tchebychef in 1852. Tchebychef's proof was rather complicated and simpler proofs were subsequently given by E. Landau, S. Ramanujan, P. Erdös and the author ([1][2][1]). The last two proofs make use of elementary properties of integers only.

Let p_r denote the r^{th} prime. We prove the following lemma:

Lemma 2. For $r > 4$, $\pi(\sqrt{p_1 p_2 \cdots p_r}) > 2r$.

Proof: For r = 5 and r = 6, the lemma may be checked directly. For r > 6, we use induction over r; i. e., assume that

$$\pi(\sqrt{p_1 p_2 \cdots p_r}) > 2r$$

and prove that

$$\pi(\sqrt{p_1 p_2 \cdots p_r p_{r+1}}) > 2(r+1).$$

[1] Numbers in square brackets refer to the bibliography at the end of the paper.

For $r > 6$, $\sqrt{p_r + 1} > 4$. Hence

$$\pi(\sqrt{p_1p_2 \cdots p_rp_r + 1}) \geq \pi(4\sqrt{p_1p_2 \cdots p_r})$$

$$= [\pi(4\sqrt{p_1p_2 \cdots p_r}) - \pi(2\sqrt{p_1p_2 \cdots p_r})]$$

$$+ [\pi(2\sqrt{p_1p_2 \cdots p_r}) - \pi(\sqrt{p_1p_2 \cdots p_r})] + [\pi(\sqrt{p_1p_2 \cdots p_r})].$$

By lemma 1, each of the first two square brackets is at least 1, while by the induction hypothesis the last bracket is greater than 2r so that

$$\pi(\sqrt{p_1p_2 \cdots p_rp_r + 1}) > 1 + 1 + 2r = 2(r + 1).$$

Hence, the induction is complete.

Consider now the following definitions:

Let $A(n)$ be the number of prime divisors of n.

Let $B(n)$ be the number of non-primes, which do not exceed n and are relatively prime to n.

Let $C(n)$ be the number of primes, not exceeding n and relatively prime to n.

To fix these definitions in mind, consider, for example, n = 20. The prime divisors of 20 are 2 and 5 so that $A(20) = 2$. The numbers 1 and 9 are non-primes relatively prime to 20 so that $B(20) = 2$. Finally, the primes 3, 7, 11, 13, 17, 19 are relatively prime to 20 so that $C(20) = 6$.

It follows immediately from these definitions that

$$\phi(n) = B(n) + C(n)$$

and

$$\pi(n) = A(n) + C(n).$$

Hence $\phi(n) - \pi(n) = B(n) - A(n)$ and the equality $\phi(n) = \pi(n)$ is equivalent to $B(n) = A(n)$.

Lemma 3. For $n > 360$, $\pi(\sqrt{n}) \geq 2A(n)$.

Proof: Consider first the case $p_1 p_2 \cdots p_r \leq n < p_1p_2 \cdots p_r + 1$, $r > 4$. Clearly $A(n) \leq r$, while, by lemma 2, $\pi(\sqrt{n}) > 2r$, so that $\pi(\sqrt{n}) > 2A(n)$ for $n > 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 = 2310$. For $361 \leq n \leq 2310$, we have $A(n) \leq 4$, while $\pi(\sqrt{n}) \geq \pi(\sqrt{361}) = 8$, so the lemma holds here too.

Lemma 4. $B(n) > \pi(\sqrt{n}) - A(n)$.

Proof: Consider the primes under $\sqrt{n}$ and relatively prime to n. There are at least $\pi(\sqrt{n}) - A(n)$ of these. The squares of these numbers, and 1, are non-primes under n, and relatively prime to n, so the lemma is proved.

Theorem 1. The only solutions of the equation $\phi(n) = \pi(n)$ are $n = 2, 3, 4, 8, 14, 20, 90$.

Proof: Combining lemmas 4 and 3, we have $B(n) > A(n)$, for $n \geq 361$. Since $\phi(n) - \pi(n) = B(n) - A(n)$, this yields $\phi(n) > \pi(n)$ in this range. Direct examination of the numbers under 361 reveals that $\phi(n) > \pi(n)$ for $n > 90$, while the only cases of equality are the ones listed above. Such an examination also yields the following result:

Theorem 2. The only solutions of the inequality $\phi(n) < \pi(n)$ are $n = 6, 12, 18, 24, 30, 42, 60$.

As an application of these results we give a simple proof of the following theorem recently proved in this Journal [3].

Theorem 3. For every positive integer r, there exists a number $N(r)$ such that for all $x > N(r)$, $\phi(x) > r$.

Proof: Let $N(r) = \max [91, p_r]$ where as before p_r denotes the r^{th} prime. For $x > N(r)$ we have

$$\phi(x) > \pi(x) \geq \pi(N(r)) \geq \pi(p_r) = r.$$

Finally, we note that theorems 1 and 2 also enable us to prove the following property of the number 30 discussed by several authors [4].

Theorem 4. Thirty is the largest number such that all the numbers under it and prime to it are unity and primes.

Proof: If n is a number having the required property then in our notation

$$\phi(n) = \pi(n) + 1 - A(n).$$

Since $A(n) \geq 1$, we have only to examine the numbers for which $\phi(n) \leq \pi(n)$, as listed above, to obtain all numbers with the required property.

REFERENCES

1. R. C. Archibald, "Bertrand's Postulate," Scripta Mathematica, vol. 11 (1945), pp. 109-120.
2. L. Moser, "A Theorem on the Distribution of Primes," Amer. Math. Monthly, vol. LVI, pp. 624-625.
3. R. V. Andree, "Number of Solutions of $\phi(x) = B$," Pi Mu Epsilon Journal, April 1950.
4. Uspensky and Heaslet, Elementary Number Theory. New York, 1939.

AN INTERESTING THEOREM

Pedro A. Piza
San Juan, Puerto Rico

We shall demonstrate the following theorem which we have found and consider new:

One hundred ninety-two times the cube of the sum of the first x squares is equal to the sum of the cubes of the first 2x triangular numbers plus twice the sum of their biquadrates.

The identity to be proved for any integer x is the following:

$$(A)\quad 192\left[\sum_{a=1}^{x} a^2\right]^3 = \sum_{a=1}^{2x}\left[a(a+1)/2\right]^3 + 2\sum_{a=1}^{2x}\left[a(a+1)/2\right]^4.$$

Equation (A) is valid for $x = 1$, since

$$192 = 1 + 3^3 + 2(1 + 3^4) = 28 + 164.$$

We know that

$$\sum_{a=1}^{x} a^2 = \frac{x(x+1)(2x+1)}{6}.$$

Let us suppose that (A) is valid for any $x > 1$, so that

$$(B)\quad \frac{192}{216}\,x^3(x+1)^3(2x+1)^3 = \sum_{a=1}^{2x}\left[a(a+1)/2\right]^3 + 2\sum_{a=1}^{2x}\left[a(a+1)/2\right]^4 = B.$$

If we now prove that (A) is also true when we substitute $x + 1$ for x, we shall have proved the theorem by induction.

With $x + 1$ we have

$$(C)\quad \frac{192}{216}(x+1)^3(x+2)^3(2x+3)^3$$

$$= B + \frac{(2x+1)^3(2x+2)^3}{8} + \frac{(2x+2)^3(2x+3)^3}{8}$$

$$+ \frac{2(2x\ 1)^4(2x\ 2)^4}{16} + \frac{2(2x+2)^4(2x+3)^4}{16}.$$

Subtract (B) from (C). This difference must be proved to be an identity.

$$\frac{8}{9}(x+1)^3\left[(x+2)^3(2x+3)^3 - x^3(2x+1)^3\right]$$

$$= (x+1)^3(2x+1)^3 + (x+1)^3(2x+3)^3$$

$$+ 2(x+1)^4(2x+1)^4 + 2(x+1)^4(2x+3)^4.$$

Divide by $(x+1)^3$ and multiply by 9:

$$8(x+2)^3(2x+3)^3 - 8x^3(2x+1)^3$$

$$= 9(2x+1)^3 + 9(2x+3)^3$$

$$+ 18(x+1)(2x+1)^4 + 18(x+1)(2x+3)^4.$$

Whence

$$(2x+3)^3\left[8(x+2)^3 - 9 - 18(x+1)(2x+3)\right]$$

$$= (2x+1)^3\left[8x^3 + 9 + 18(x+1)(2x+1)\right],$$

$$(2x+3)^3(2x+1)^3 = (2x+1)^3(2x+3)^3.$$

Q. E. D.

PROBLEM DEPARTMENT

Edited by Leo Moser, University of Alberta

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's degree. Solutions of these problems should be submitted on separate, signed sheets within five months after publication. Address all communications concerning problems to Leo Moser, Mathematics Department, University of Alberta, Edmonton, Alberta, Canada.

PROBLEMS FOR SOLUTION

30. Proposed by J. H. Butchart, Arizona State College

A well known construction for the roots of $x^2 - px + q = 0$ is to find the x intercepts of the circle having the join of $(0,1)$ and (p,q) as diameter. Show that if the roots are complex, the real part is the abscissa of the center and the coefficient of i is the tangent from $(p+2,0)$ to this circle.

31. Proposed by Victor Thébault, Tennie, Sarthe, France

For integers written in base B, find for every n a number N which divides the number of digits obtained in writing the integers 1, 2, 3, ..., N.

32. Proposed by Francis L. Miksa, Aurora, Illinois

In a class in Number Theory the professor gave four students the assignment of finding a fairly large primitive

Pythagorean triangle using the well known formula for the legs:

$$A = 2mn, \quad B = m^2 - n^2, \quad C = m^2 + n^2,$$

where m and n are co-prime integers, not both odd. The four students produced four entirely different primitive triangles, but on comparing them it was found that two of them had the same perimeter, while the other two also had the same perimeter, this perimeter differing from the first one by 2. This interested the class greatly, and much time was spent in an effort to find other such sets, only to discover that there were only four such sets with perimeter less than 500,000. Can you find at least one such set?

33. Proposed by C. W. Trigg, Los Angeles City College

It is well known that the elements of the fourth row (or column) of the Pascal triangle are tetrahedral numbers. Establish the following properties of the fourth row.

1. The difference of two consecutive elements is a triangular number.
2. The difference of two alternate elements is a square.
3. The difference of the $(n+2)$nd and the nth elements increased by the $(n+1)$st element of the third row is a pentagonal number.
4. Six times the nth element added to the $(n+1)$st element of the second row is a cube.
5. The nth element is equal to the sum of the first n elements of the third row.

34. Proposed by J. S. Frame, Michigan State College

For what values of k are the following twelve points the vertices of a regular icosahedron? $(0, \pm k, \pm 1), (\pm 1, 0, \pm k), (\pm k, \pm 1, 0)$?

SOLUTIONS

16. Proposed by W. J. Jenkins, Livingston, Alabama

Given a circle and two exterior points not in a straight line with the center. Construct a circle passing through these two points and dividing the given circle into two equal arcs.

Solution by Mel Stover, Winnipeg, Manitoba

Denote the given circle by P and its center by O. Let the required circle be Q and let the two given points on it be A and B. Let C be the second point of intersection of the line OA and the circle Q. The circle Q is to cut P in end points of a diameter EF. The circle P will therefore have two cords intersecting at O so that AO·CO = EO·FO. Of these four lengths only CO is unknown. The length CO can therefore be determined by using a similar triangle construction. Once this is done we can locate C and use the well known construction for a circle through three points to obtain Q as the circle through A, B, and C.

Also solved by Donald A. Swenson, University of Alabama, and the proposer.

18. Proposed by Lindley J. Burton, Bryn Mawr College

Points A_1, B_1, C_1 are chosen on the sides BC, CA, AB of the triangle ABC such that $AC_1 = \frac{1}{2}C_1B$, $BA_1 = \frac{1}{2}A_1C$, $CB_1 = \frac{1}{2}B_1A$. The lines AA_1, BB_1, CC_1 determine a triangle $A_2B_2C_2$. Show that the area is one seventh the area of ABC.

Editorial note: C. W. Trigg submitted a proof of a generalization of this theorem and also an extensive list of references of earlier treatments of this problem. This list includes: Nouvelle correspondance mathématique, 1875, p. 105, 1876, p 310; National Mathematics Magazine, vol. 14 (1939), p. 109; School Science and Mathematics, vol. 39 (1939), p. 282, vol. 41 (1941), pp. 765-7; H. Steinhaus, Mathematical Snapshots, Stechert and Co., p. 7.

In Trigg's generalization the constant $\frac{1}{2}$ is replaced throughout by m/n, where $m < n$. The ratio 1/7 must then be replaced by $(n - m)^2/(m^2 + mn + n^2)$.

25. Proposed by Polly Tope, Institute for Hyper Study

A square has 4 lines of symmetry and a cube has 13. Derive a formula for the number of lines of symmetry of an n-dimensional cube.

Solution by the proposer

Consider an n-dimensional unit cube embedded in an n-dimensional cube 3 units to a side. Each line of symmetry, when extended, enters two outside unit cubes. On the other hand each outside unit cube has exactlyone line of symmetry through it. Thus the number of lines of symmetry is just one half the number of outside cubes, that is, $(3^n - 1)/2$.

27. Proposed by Arthur B. Brown, Queens College

The number 3 can be expressed as a sum of one or more positive integers in 4 ways, namely as 3, $1 + 2$, $2 + 1$, $1 + 1 + 1$. Show that any positive integer can be so expressed in $2^n - 1$ ways.

Solution by William Moser, University of Toronto

Consider n one's in a row with spaces between them. There is clearly a 1-1 correspondence betweenexpressions for n as a sum and ways of disposing of the n-1 spaces by entering plus signs or leaving the spaces blank. This gives us n-1 tasks to perform and two ways of handling each one. Thus the total number of expressions for n as a sum is 2^{n-1}.

Also solved by Ding Hwang, University of California, and the proposer.

29. Proposed by Francis L. Miksa, Aurora, Illinois

For a given positive integer k, find integers m and n such that

$$1+2+3 \ \ldots + m = (m + k) + (m + k + 1) + (m + k + 2) + \ \ldots + n.$$

Solution by E. P. Starke, Rutgers University

Using the familiar formula for the sum of an arithmetic progression, we may simplify this equation and put it in the form

$$(1) \qquad (2n + 1)^2 - 2(2m + k)^2 = (2k - 1)^2 - 2k^2,$$

for which one obvious solution is

$$n_0 = k - 1, \qquad m_0 = 0.$$

Now if, for any value of k, m and n are integers satisfying (1)then

$$(2) \qquad m' = 2n + 3m + k + 1, \quad n' = 3n + 4m + 2k + 1$$

are also integers satisfying (1), as may be verified by direct substitution. Starting with n_0 and m_0 we arrive at solutions

$$n_1 = 5k - 2 \qquad m_1 = 3k - 1$$
$$n_2 = 29k - 9 \qquad m_2 = 20k - 6$$
$$n3 = 169k - 50 \qquad m_3 = 119k - 35$$

and so on.

The above is a simple adaption of the usual treatment of the Pell equation

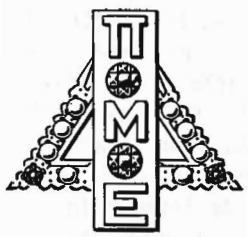
$$x^2 - 2y^2 = c \tag{1'}$$

for which, if (x,y) is a solution, then so is (x', y') where

$$x' = 3x + 4y, \qquad y' = 2x + 3y. \tag{2'}$$

In general, we may get all solutions by the above procedure unless the right member of (1) is divisible by a perfect square, t^2. In this case there may be additional solutions in which $2n + 1$ and $2m + k$ are both divisible by t. For example, $k = 6$, $n = 59$, $m = 39$.

Also solved by the proposer.



THE FOUNDER'S PIN

THE spring of 1929 marked the fifteenth anniversary of the founding of Pi Mu Epsilon. Dr. Alan D. Campbell, the president of the Syracuse Chapter at that time, suggested that at the annual banquet some special tribute be given to Dr. Edward Drake Roe, Jr., the founder of the fraternity. The chapter responded enthusiastically and the other chapters were approached and asked if they would care to contribute to the fund for that purpose. In all, about seventy-five dollars was contributed.

After consulting with Mrs. Roe, the committee for the gift selected a very handsome scarf pin at Stetson and Crouse, Syracuse jewelers and agents of the official jewelers of the fraternity. This pin, set with a large aquamarine and two small diamonds, was presented to Dr. Roe at the banquet as an expression of appreciation of his devotion and services to the fraternity. He was deeply touched and made a charming speech of acceptance.

Several weeks later, Dr. Roe decided that since the pin was a gift from all the chapters it would mean more to him to have a large jeweled badge in the form of the fraternity pin. Accordingly, the scarf pin was exchanged for the handsome jeweled badge (pictured above), set with diamonds and

rubies, Dr. Roe's favorite jewels. He wore it often for its significance meant much to him.

Several weeks after Dr. Roe's death, in loving memory of her husband's friendship, Mrs. Roe gave the pin to Dr. Campbell. It was worn upon special occasions by Dr. Campbell until the time of his death.

Last year, while our sons and I were looking over some family treasures that had long been away in a strong box, we came across the jeweled Pi Mu Epsilon pin. We decided to return it to the fraternity in the names of Dr. Roe and Dr. Campbell, with the hope that in the future other high officers of the fraternity might take pleasure in wearing it. So it was turned over to the editor of the Pi Mu Epsilon Journal, Dr. Ruth Stokes, who graciously accepted the pin and promised to see that it was placed in the proper hands.*

Gertrude G. Campbell

*Editor's Note: We decided that no more fitting disposition could be made of the pin than that it be worn by each succeeding director-general of the fraternity while in office. Accordingly, the pin was sent in June of this year to the present incumbent, Director-General C. C. MacDuffee, Professor of Mathematics at the University of Wisconsin.

CYRUS COLTON MacDUFFEE

DIRECTOR GENERAL

CYRUS COLTON MacDUFFEE, Professor of Mathematics, University of Wisconsin. Native of Oneida, N. Y. B.S. and hon. Sc.D, Colgate; M.S. and Ph.D, Chicago. Instr, Colgate; instr. and asst. prof, Princeton; asst. prof, assoc. prof. and prof, Ohio State; prof, Wisconsin, Hunter Col; Wisconsin, 1943—. Fellow, Inst. for Advanced Study, 37-38; visiting prof, Puerto Rico, 47. Summer, visiting asst. prof, Chicago, 28. A.A; Math. Sco (v. pres, 42); Math. Asn (pres, 45). Algebra; theory of matrices; linear algebras.

WILLIAM MARVIN WHYBURN

VICE-DIRECTOR GENERAL

WILLIAM MARVIN **WHYBURN**, Professor of Mathematics and Department Head, University of N.C. Native of **Lewisville,** Texas. **B.A, M.A, Lipsitz** fellow, **Ph.D, U. of Tex; LL.D,** Tex. Tech. Col. Instr. **S.** Park **Jr.** Col, Tex; **asst.** prof, Agr. & Mech. Col, Tex; assoc. prof, Tex. Tech. Col; nat. research fellow, Harvard; **asst.** prof, assoc. prof, prof. and chairman of dept, Calif, at **L.A;** pres. Tex. Tech. Col; **Kenan** prof. and dept. head, U. of **N.C,** 1948—. Chief **operations** anal. sect, Air Force **(1944),** Fla. **A.A;** Math. **Soc.** (councilor, assoc. **sec'y);** Math. Asn. **(past** lstv. pres); **Philos. Soc.** Tex; **cor.** mem. La Acad. **Nacional** de Ciencias **Exactas, Fisicas** y Nat. de Lima. Differential equations; boundary value **problems;** critical **point** theory; theories of integration.

SECRETARY-TREASURER GENERAL

JAMES SUTHERLAND FRAME, Professor of Mathematics and Chairman of Department, Michigan State College. Native of New **York,** N.Y. **A.B,** A.M. **Ph.D,** Harvard. Instr, Harvard; **Rogers** traveling fellow, Harvard, Gottingen and Zurich; **instr.** and **asst.** prof. Brown Univ; assoc. prof. and chairman dept, Allegheny Col; prof. and chairman dept, Mich. State Col, 1943—. fast. for Advanced Study (50-51). Assoc. ed. 'Am. Math. Monthly' (42-46); assoc. ed. **'Pi** Mu Epsilon Journal' (49-). Am. Math. **Soc;** Math. Asn (Bd. of Govnrs, 50-); **Inst.** Math. Statist; AAUP (Nat. Council, **48-**50); Phi Beta Kappa, **Sigma Xi.** Theory of representations of finite groups; approximations and short-cuts in computational problems.

JAMES SUTHERLAND FRAME

STEWART SCOTT CAIRNS

COUNCILOR GENERAL

STEWART SCOTT CAIRNS, Professor of Mathematics and Head of Department, University of Illinois. Native of Franklin, N.H. **A.B, A.M, Ph.D,** Harvard; also, Harvard traveling fellow, **Gottingen** and Szeged, Hungary. Instr. Harvard, Yale, Lehigh; **asst.** prof, Lehigh, Queens Col. (N.Y.); prof. and dept. chairman, Syracuse; prof. and head of dept, Illinois, 1948—. Mem. **Inst.** for Advanced Study, 36-37; consultant and research worker in various groups under the Applied Math. Panel of the NDRC, 44-46; vice **chmn.** of Div. of Math. and Phys. Sciences of **the NRC,** 50-51; consultant for the Research and Development Board, **1950—.** Topology; analysis.

COUNCILOR GENERAL

SOPHIA LEVY **McDONALD**, Professor of Mathematics, University of California, Berkeley. **B.S, Ph.D, University** of California. Instr. math, **asst.** prof, assoc. prof, prof, California, 1949—. Am. Math. **Soc;** Math. Asn; Comm. 20, Int. Ast. Union; Astron. **Soc.** Pacific. Theoretical Astronomy— Development of Tables of **General Pertubations** of a Group of Minor Planets which Includes the Group One-Half, with Applications to Minor Planets belonging to this Group.

SOPHIA LEVY McDONALD

NOTE: Photographs and biographical sketches of the two other councilors general will be published, along with those of the Journal Staff, in the April 1952 issue.

REPORTS OF THE CHAPTERS

(Send reports to Ruth W. Stokes, 15 Smith Hall, Syracuse University, Syracuse 10, New York.

EDITOR'S NOTE. According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary-General and to the Director-General, an annual report of the chapter activities including programs of meetings, results of elections, etc." The Secretary-General now suggests that an additional copy of the annual report of each chapter be sent to the editor of the Pi Mu Epsilon Journal. Besides the information listed above we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships. These annual reports will be published in the chronological order in which they are received.

Alpha of Louisiana State University

The first meeting of the Louisiana Alpha chapter for the 1950-1951 session, held October 5, was organizational. The following officers were elected: Director, James M. Turner; Vice-Director, Horace C. Hearne; Secretary, Delilah Stokes; Treasurer, Roger W. Richardson; Corresponding Secretary, Professor Houston T. Karnes.

Papers presented during the year were:

"The Number System" by Professor F. A. Rickey

'Plane Continua" by Professor N. E. Rutt

'Development of Quantum Mechanics" by Professor V. E. Parker

"Some Applications of Mathematical Concepts to Chemical Problems" by Professor H. B. Williams

"Groups in Crystal Structure and Theory of Equations" by Professor Eugene Schenkman

"Another Way of Doing It" by Professor Paul K. Rees

"Calculus of Variations" by Professor B. B. Townsend

"The Exterior Differential Calculus of Cartan" by Professor Eugenio Calabi

"Curves in Minkowski Space" by Professor C. C. MacDuffee.

At the initiation banquet, April 17, twenty-one new members were initiated. Professor C. C. MacDuffee, Director-General, gave the banquet address. It was the chapter's very great pleasure to have Mrs. MacDuffee present on this occasion.

Beta of North Carolina, University of North Carolina

Five meetings of the North Carolina Beta chapter were held during the academic year 1950-1951. Two of these meetings were purely business. At the remaining three, the following talks were given:

"Logical Foundations of Mathematics" by Professor L. O. Kattsoff

"Lommel Functions" by Mary Nunn Morrow

"Von Staudt Property of Bernoulli Numbers" by George W. Carow.

Under the direction of the social committee, Mary Morrow and Emilie Haynesworth, two bridge parties were held.

Nine new members were initiated during the year.

The newly elected officers for the year 1951-1952 are: President, Tullio J. Pignani; Vice-president, Mary Nunn Morrow; Treasurer, Margaret Butler Seelbinder; Secretary, John Jones.

Alpha of Oklahoma, University of Oklahoma

The first meeting of the Oklahoma Alpha chapter for the academic year, 1950-1951, was held on October 24. At this meeting the following officers were elected: Director, Charles C. Williams; Vice-Director, Leigh Ortenburger; Secretary-Treasurer, Michael Famiglietti.

On January 21, Mr. Charles Williams was called to active duty with the Armed Services. Mr. Leigh Ortenburger became director and Mr. Howard Prier was elected to serve as vice-director.

The activities included business, social and regular meetings at which the following talks were given:

"Some Methods of Summation of Divergent Series" by Charles C. Williams, graduate assistant, department of mathematics

"A Postulational Development of Real Numbers" by Mr. Roy Deal

"Pythagorean Angles" by Dr. Arthur Bernhart

"Applications of Symbolic Logic" by Dr. Carlton Berenda

"A Problem of Eclipsing Binaries" by Mr. Howard White

"The Group Concept in Geometry" by Dr. C. E. Springer.

The initiation banquet was held in the Copper Kettle, May 4. The guest speaker was Dean E. D. Meacham of the College of Arts and Sciences. A total of 28 new members were inducted into the chapter.

(Note. With the annual report was also a most attractive printed banquet program giving the menu served and list of initiates; also, a copy of the examination used in the annual contest. We regret space does not permit the printing of these.)

Beta of Oregon State College

For the academic year 1950-1951, the Oregon Beta chapter of Pi Mu Epsilon reported six meetings including the annual initiation and banquet. The following papers were presented:

"Linear Diophantine Equations" by Mr. Philip Anselone

"Arithmetic of the Complex Domain" by Mr. Robert Brown

"Theory of Runs" by Mr. Gene Thompson

"Different Sizes of Infinites" by Dr. James Price.

The initiation of new members and the annual banquet were held on May 17. Forty-one new members were initiated.

Officers for 1950-1951 were: Director, James Nickel; Vice-Director, Verner Hoggatt; Secretary, Arthur Wirshup.

Officers for 1951-1952 are: Director, Patricia Pearson; Vice-Director, Dallas Banks; Secretary, Robert Brown; Treasurer, Professor George A. Williams.

Alpha of Illinois, University of Illinois

In addition to the usual business meetings, the Illinois Alpha chapter of Pi Mu Epsilon held two meetings during the academic year 1950-1951.

Approximately one hundred members and guests attended the first meeting, held December 5, at which time Professor A. T. Nordsieck of the Physics Department of the University of Illinois gave a demonstration lecture on his analog computer.

The second meeting, May 16, was for the annual, spring initiation banquet, at which ninety-nine new members were initiated. Professor R. H. Bing of the University of Wisconsin, guest speaker for the occasion, spoke to the one hundred and eighty-six initiates, old members and their guests on some theorems in topology which are valid in two-space but not in three space.

Two important actions were taken in business meetings: (1) An amendment to the local constitution was passed, setting the initiation fee at seven dollars. This was thought to be necessary because of the increased cost of the banquet and the recent change in the national constitution, setting the fee for each certificate at one dollar.

(2) Election of the following officers for 1951-1952: President, Thomas B. Elfe; Vice-president, John William Toole; Secretary, Beverly A. Marshall; Treasurer, Richard E. Priest.

Gamma of Missouri, St. Louis University

The annual report of the Missouri Gamma chapter of Pi Mu Epsilon for the academic year 1950-1951 was so very good we regret not being able to publish the entire report of the chapter's activities, but limited space does not permit us to do so. Topics of papers presented are given below; while certain news items, awarding of prizes and scholarships will appear elsewhere in this journal.

"Foundations of Mathematics" by Mr. Bernard Derwort

"Partial Differential Equations" by Mr. John Hoffs-Chwells

"Line Geometry" by Dr. Paolo Lanzano

#Chance" by Dr. Paul R. Rider, banquet speaker.

Seventy-five new members were initiated during the year.

Mr. Eugene Bold, graduate student, was elected Director for the year 1951-1951. Again Dr. Francis Regan graciously accepted the post of Faculty Adviser and Permanent Secretary-Treasurer of the chapter. The election of a Vice-Director and a Secretary-Treasurer will be held at the first meeting in the fall of 1951.

Alpha of California, University of California, L. A.

The California Alpha chapter of Pi Mu Epsilon held ten meetings during the year 1950-1951 of which two were initiation meetings, two were purely business meetings and six were lecture meetings.

The Fall initiation meeting was held at the home of Peter Severling, and about sixty members were present for the initiation of eighteen new members. The Spring initiation meeting, held at the home of Professor **W. T. Puckett,** was attended by about sixty, and sixteen new members were initiated.

The following lectures were given before the chapter during the year:

"Theory of Games" by Irving Glicksberg

"Theory of Braids and a Graphic Approach to Permutation Groups" by James R. Jackson

"Problem Types in Plasticity" by George **Zizicus**

"Some Results Related to Fixed-Point Theorems — and Stuff" by Dr. Robert Steinberg

"Automatic Computing Machinery" by Dr. Harold **Luxenberg**

"Transforms and Tautochrones" by Dr. G. Milton Wing.

Officers for 1951-1952 are: Director, James R. Jackson; Vice-Director, Sharla Rita Perrine; Secretary, Mervin Miller; Treasurer, W. T. **Puckett;** Faculty Adviser, Phil Hodge.

Alpha of Florida, University of Miami

The Florida Alpha chapter of Pi Mu Epsilon was installed at the University of Miami on March 21, 1951. A banquet was held on this occasion, and Professor Tomlinson Fort of the University of Georgia represented the national Fraternity on this occasion. There were twenty-nine new members who signed the charter and one each from the Wisconsin Beta and New York Gamma. Later seven new members were initiated.

In addition to the banquet two other meetings were held, one a business meeting and the other a program meeting at which Dr. Meyer spoke on **"An** Axiomatic Approach to Trigonometry**".**

The chapter officers for Pi Mu Epsilon are as follows: Director, Mrs. Del **Franco; Vice-Director, David Foulis;** Secretary, Mary M. Magner; Treasurer, **Tadeus Patla.**

Alpha of Missouri, University of Missouri

The Missouri Alpha chapter of **Pi Mu Epsilon,** during the academic year 1950-1951, initiated forty-three new members, nine in December and thirty-four in May. Monthly meetings were held on the campus. Talks presented included:

"Some Applications of Mathematics in Electrical Engineering" by Professor Bert Gastineau

"Some Mathematical Aspects of Human Behavior" by Dr. Herman **Betz**

"A Demonstration of Soap Film Surfaces" by Ben **Jaeger**

"Some Famous Mathematicians and What They Did" by Miss Mary Cummings

"Importance of Mathematics to the Navy" by Lt. Com. Cobb, **U. S.** N.

"Infinite Series and **Summability"** by Dr. Paul **Burcham.**

Officers for 1951-1952 are: President, Harley **Newsom;** Vice-president, Paul Sims; Secretary, Carl Spohr; Treasurer, Don **Putnam;** Faculty Sponsor, Miss Mary Cummings.

Alpha of Pennsylvania, University of Pennsylvania

The Pennsylvania Alpha chapter of Pi Mu Epsilon, during the year 1950-1951, initiated thirty-one new members into the Fraternity.

Officers for 1951-1952 are: President, H. Newton Garber; Treasurer, Fred W. Aron; Secretary, **Anita** Bredt; Director, Dr. H. E. Campbell.

MEDALS, PRIZES AND SCHOLARSHIPS

EDITOR'S NOTE. Each chapter will undoubtedly be interested in learning what other chapters are doing along the line of prize competitions. So the editor makes the request that chapters offering prizes, scholarships, or other awards, write up their plans for such contests and submit them for publication in this journal.

At the annual banquet, the Louisiana Alpha of Pi Mu Epsilon made the following awards: The Freshman Award, based on an Honors Examination, went to Herbert W. Kelley; the Senior Award, based on the amount of work taken in mathematics and the quality of work done, involved a tie. Those who tied for this award were Horace C. Hearne and Roger W. Richardson, Jr.

Secretary Michael Famiglietti of the Oklahoma Alpha chapter made the following report: "The annual, university-wide competitive mathematics examination was given on 26 April. Mark Melton was first place winner and Charles Reich was second place winner. First place winner will receive $10.00 in mathematical books." (Mr. Famiglietti kindly sent us a copy of the examination questions used. We regret that lack of space prevents our printing it, but if any of the chapters request a copy the editorial office will be glad to furnish it.)

From the Oregon Beta chapter was received the following report on contests:

"An annual mathematics contest was inaugurated(1950-1951). Prize winners were William Gribble, first prize; Richard Lee Adey, second prize; Marshall McMurran, third prize.

"Following the untimely death of the second prize winner, the chapter decided to perpetuate his memory by designating the first prize as the Richard Lee Adey Award."

The Illinois Alpha awards annually a prize called the Pi Mu Epsilon Award. This year there was a tie, so on the occasion of the annual spring initiation banquet Professor J. W. Peters, making the presentation, presented duplicate awards to G. E. Modesitt and to Lloyd R. Welch.

The Missouri Gamma chapter reported four awards as follows: "The fifth Annual Prize Essay Contest, open to undergraduate students only, was conducted by Mr. Alois Lorenz. The senior division award was won by Miss Helen Fagan, a senior of Maryville College. The title of her paper was "The Mind of Newton as Reflected in the Principia." Her award was D. E. Smith's SOURCE BOOK OF MATHEMATICS. Miss Maureen Burke, a sophomore of Fontbonne College, won the junior award for her essay "Isaac Newton: His Life and Works!'. She was awarded E. T. Bell's MEN OF MATHEMATICS.

"James Krebs received the Chemical Rubber Publishing Company's BOOK OF TABLES for being the freshman with the highest scholastic standing in mathematics. Roger Ahrens was given the Garneau Award of twenty-five dollars for being the highest ranking senior majoring in mathematics. All of these awards were made at the banquet."

From the California Alpha chapter we received the following announcement: "Our annual Calculus Prize was won by George S. Rasmussen and W. H. Root (tie), who each received a prize of ten dollars."

"In May the Missouri Alpha chapter conducted a competitive examination in the calculus and awarded prizes as follows: First prize of fifteen dollars to Donald Garnett; second prize of ten dollars to Frederick M. Cash; and third prize of five dollars to Clifford H. Brown.

"One of our graduating seniors, Charles H. Propster, Jr., received a Fulbright award for study in physics at the University of Amsterdam, and he will spend next year abroad."

From the Alabama Alpha chapter we learned: "For the third consecutive year, the chapter sponsored a campus-wide competitive examination in mathematics. Prize winners were Donald A. Swenson, Walter B. Mitchell, Billy Letson, J. J. Samuels and Robert Woltman."

The Iowa Alpha chapter awards a Pi Mu Epsilon prize each year to an outstanding mathematics student. It was won this year by Robert H. Tweedy.

NEWS AND NOTICES

The Iowa State College "Math Club" is sponsored by the Iowa Alpha chapter of Pi Mu Epsilon. The fraternity's vice-director is the student-director of the undergraduate Math Club. It is his duty to provide the initiative which is necessary to keep Math Club an active organization. Richard E. Johnson is the retiring and John Druyor is the new student-director of Math Club.

The following Iowa State College students were awarded Math Club membership cards as a result of their attendance at three meetings: Jeannine Sarchott, Stanley Petrick, Mary Ethel Buxton, Roger S. Hanson and Richard E. Johnson.

The Kentucky Alpha chapter each year devotes a portion of its treasury to the purchase of mathematical volumes for the library. Since the granting of the chapter's charter (1927), two hundred fifty-six new members have been initiated.

The Louisiana Alpha chapter maintains a Pi Mu Epsilon shelf in the mathematics library and each year several new volumes are added.

Since the charter was granted to the Missouri Gamma chapter (1945), five hundred fifty-three new members have been initiated. During the last year we have had fifty-one paid subscriptions from this very active chapter.

The Pennsylvania Alpha chapter is running the Missouri Gamma a close second in the matter of paid subscriptions, the former having sent us an order for twenty-five individual subscriptions at one time and then following that order with another large one.

At the annual banquet of the Missouri Alpha chapter of Pi Mu Epsilon, four students presented a dramatization of the article "On the Set of Legs of a Horse", which appeared recently in this journal. We believe this subject may have great possibilities for an amusing skit, and one of the members of the New York Alpha chapter has expressed a desire to see the skit mimeographed and loaned to other chapters for use at meetings.

To date, in 1951, there have been three new chapters installed in the Pi Mu Epsilon Fraternity: Florida Alpha, District of Columbia Alpha and New York Eta.

INITIATES, ACADEMIC YEAR 1950-1951

ALABAMA ALPHA, University of Alabama
(April 25, 1951)

Roy B. Applequist
Gladys Irene Blalock
Emil A. Braunlich
Betty Lou Campbell
John A. Dyer
Betty Ellis
Thomas L. Hicks
J. W. Hoover
James C. Johnson, Jr.
Maxwell McBrayer
James B. McGuffin
Robert G. Tate, Jr.
Ying Victor Wu

ARIZONA ALPHA, University of Arizona
(April 19, 1951)

Frank W. Anderson
Fernando J. Astiazaran
John Kipp Becker
Benjamin Cato
Russell Denker
Donald Duncan
George E. Eckert
Robert W. Edwards
J. W. Haake
Michael Heileman
John Earl Hickman
James E. Householder
James F. Hulet
Marvin W. Karlin
Thomas S. Kasparian
Robert Krans
Andrew D. Lauver
Daniel P. Lee
John K. Matlock
Jack P. Middlekauff
David M. Nelson
Robert L. Pirtle
Clarence R. Robinson
Waldo Rogers
Daniel J. Rundell
George A. Scholey
Douglas E. Scott
Clyde H. Skinner
Jack N. Smith
Leonard M. Snyder
Richard H. Thomas
Harold G. Watson

ARKANSAS ALPHA, University of Arkansas
(March 21, 1951)

Theodore L. Beeler
Carl Natho

Allan T. Controy
F. Robert Goldammer
Alfred B. Herrin
John W. Keesee
Thomas E. Lewis
James R. Seabock
Charles E. Stanley
Charles W. Stebbins
Dale Thompson
Dallas Vire

CALIFORNIA ALPHA, University of California, Los Angeles
(Fall, 1950)

Robert R. Brown
Allen H. Flagg
Neel W. Glass
Bertrand M. Hall
K. J. Harker
Theodore Harris
John S. Herman
Alfred E. Mann
John F. Matousek
Charles W. Perkins
Raymond Redheffer
Sharla Rita Perrine
John C. Shaw
Phil Siegel
Thoralf A. Skolem
Seymour Soll
Roland M. Suarez
Dante V. Susco
James R. Vine
Alvin M. White
Hsien Shih Yu

(May 19, 1951)

Ali Reza Amir-Moez
Sigurd L. Andersen
Patricia Childress
George W. Fairchild
Robert K. Froyd
Sheldon Green
Charles J. Halberg
Albert M. Jaqua
Thomas J. Kelley
John L. Kuhns
Leon J. Lander
Benjamin Mittman
Darrell J. Peterson
Lewis W. Rambo
Wayne E. Smith
William M. Swan

CALIFORNIA BETA, University of California, Berkeley
(December 14, 1950)

Clarence S. Badger
(Mrs) Marion Baker
Richard E. Bateman
H. S. Bear, Jr.
Victor O. Brady
Leo O. Breiman
John J. Harton, Jr.
David W. Hullinghorst
George Jeromson
Walter T. Kyner, Jr.
Richard C. MacCamy
Albert W. McKinney III

(Mrs) Regina W. Butler
Paul L. Chambré
Lensey Chao
Clyde E. Corson
Glen J. Culler
Newman H. Fisher, Jr.
Robert J. Mercer
Gabriel Raab, Jr.
Henry B. Sarrail
Schiller Joe Scroggs
Donald C. Wilfong
Joseph A. Winokur

(May 5, 1951)

Waleed A. Al-Salam
John D. Brillhart
Susan Chakmakjian
Chien-Chung Chang
Edward Chu
Margaret Fuller
Colonel D. Gardner
John William Haynes
Donald G. Hess
Eugene H. Holderbach
Teruo Ishihara
Bradley Johnston
LeRoy Krueger
John C. Long
Magoroh Maruyama
Sylvester W. Mead
Virginia E. Miles
Norman K. Nystrom
Bayard Rankin
Edward S. O'Keefe
Marvin Rosenblum
Daniel Saylar
Joseph Sider
Hamilton O. Smith
Henry P. Stapp
Thomas B. Steel, Jr.
Peggy Tang
Ina Bell Tucker

COLORADO ALPHA, University of Colorado
(February 28, 1951)

Charles Robert Class
James Arthur Cooley
David Devol
George Euclid Kersey
Thelma M. Kohl
Ruel Coe Mercure
Vernon Ronald Nelson
Gordon C. Savage
Kim Richard Schuette
Richard Louis Sharp
Joseph Stanley Zinns

DELAWARE ALPHA, University of Delaware
(April 20, 1951)

Harold A. Birkness
Earl Edgar Bomberger
Thomas Christy Clements
Robert Monroe Eissner
Francis John Lerch
Everett Vernon Lewis
Kenneth William Millett
Joseph Robert O'Donnell

Norman Paul **Harberger**
Harold Joseph Hasenfus
Harold Murray Hurlburt
Carl Elwood Kerr
Mae Kathryn Kerr
Robert **Winfield** Knox
Keith Gordon Parthemore
James Oliver Porteus
Neal Jules Rothman
Gerald Bruce Shpeen
Leon **Berton** Shore
Henry Teicher

DISTRICT OF COLUMBIA ALPHA, Howard University
(March 29, 1951)

Charter Members

David H. Blackwell
Jonelle L. Burr
(George H. Butcher, member **Penna.** Alpha)
Jeremiah Certaine
William W. **S. Claytor**
Thelma A. **Cooley**
Elbert F. Cox
(Walter T. Daniels, member Iowa Alpha)
David D. **Dinkins**
John A. Doggett
Eleanor V. Green
Theodore R. **Mikell**
Melba Chloe Roy
Ralph B. Turner
Ethel M. Tyree

Non-Charter Members

Charles **D. Batchlor**
Algernon Lorenza Brown
Calvin **Conliffe**
Eugene K. Cox
Frank W. Douglas
Benjamin **Dunmore**
Donald C. Fontaine
David A. Franks
Benjamin F. Handy
Bobbie Eskia Jones
Irving Jones
Young Lee
Bustov Lounderman
Norman E. **McAdory**
Robert **Minton**
Lincoln **J.** Oliver
Alonzo Smith, Jr.
Robert N. Smith
Thurman Spriggs
Lovell W. Sutherland
Loretta Wilson Walker
Andretta **Adkins** Yeldell

FLORIDA ALPHA, University of Miami
(March 21, 1951)

Forrest Edwin Adams
Stanley Edward **Asplund**
Lybrandt Gray **Barbee**
Helen Alice Butcher
Marilyn Hess Cross
Katherine Zorsch Cunningham
Georgia Knox Del **Franco**
Mayme Irwin Logsdon
Harris Franklin **MacNeish(N.Y.** Gamma)
John Howard Maecher
Mary Martha Magner
Herman Meyer
Carolyn Alice Palmer

Gordon **McCrea** Fisher
David James **Foulis**
Edith **Hjort** Franzen
William Gerald Franzen
William Morris **Gaylor**
Emanuel Friedrich Globisch
Harold Greenberg
William Lawrence Harkness
Albert Bruce **Hawkins**
Ernest **LeRoy** Hunt
John Ernest Kelley **(Wisc.** Beta)
Morris Joseph Liss
Tadeus **Patla**
Mabel Agnes Pauley
Agnes Young Rickey
Melanie **Rohrer** Rosborough
Ira Rosenbaum
Harry Shaw, Jr.
Marvin Stanley Shinbaum
Morris David Snyder
Robert **Cowan** Strong, **Jr.**
Paul **Mecartney** Swingle
Richard Britain Tuggle
Howard Raymond Wright, Jr.
Joseph Zucker

GEORGIA ALPHA, University of Georgia
(May 16, 1951)

Edward Carlton Allmon
Stamatios Konstantinos Asselanis
Thomas Leroy Austin
Gilda Madeline Bloom
Mildred Smith Darby
Barbara Joan **Deiters**
James Frederick Dilworth
Elmore Gordon Douglas
Mahlon Cooper Garrett
Harold Milton **Heckman, Jr.**
Welcome Ann Lancaster
George Victor Luellman
James Walter Lynch
Cecil **Nesbitt** Martin
Elizabeth **Segrest** Price
Harold Jack Sherman

ILLINOIS ALPHA, University of Illinois
(May 16, 1951)

W. O. Ackerman
Glenn C. Bailey
Joseph A. **Barkson**
Frederick G. Bauling
Jerome **S.** Becker
R. Linsey Belford, **Jr.**
S. E. Benesch
M. H. Bert
Sheldon F. Best
William H. **Birkett**
A. P. **Boresi**
Charles W. Bostick
B. M. Brown
Austen F. Lindley
J. T. Littlefield
Frank **Litz**
Chester Lob
Elwyn R. **Lovejoy**
Myron E. Lunchick
Robert **J.** Malach
Bernard **J.** Marks
Beverly A. Marshall
D. V. McKinley
Leon H. Meyer
G. E. Modesitt
Edward H. **Mottus**

Walter W. Cannon
Sai-Pak Chan
A. S. Chodakowski
Howard G. Cooper
R. B. Cuddeback
B. V. Dean
Robert B. Dillaway
Thomas B. Elfe
Earl O. Embree
James T. Ephgrave
James E. Etter
Jason H. S. Fan
Frank J. Fishman, Jr.
Meyer Garber
James E. Gindler
Gordon Goldman
William K. Green
A. E. Guia-Monasterio
David S. Heeschen
Neil Hilvety
Charles W. Hurter
Austin E. Idleman
N. S. Inoue
Shigeru Ishii
Donald H. Janney
Richard C. W. Kao
Harold W. Katz
Martin Krakowski
Ora M. Kromhout
V. B. Kurfman
George Kvitek
Joseph Landin
Boyd T. Larrowe
Vernon L. Larrowe
Robert L. Lebduska
James E. Leiss
E. A. Mueller
Robert H. Nau
J. O. Neuhaus
Thomas S. Noggle
Martin S. Osman
Tarik Ozker
J. F. Phelan
Ervin L. Piper
Joe Ploczatek
Robert G. Pohl
William M. Portnoy
Richard C. Price
Robert A. Reitz
Arthur Ross
Joseph A. Saloom
W. E. Schmidt
Leon P. Schnepper
Alan Schoen
R. H. Schwaar
E. J. Schweppe
John W. Shelton
Charles P. Slichter
Robert W. Sloan
A. Sosin
Ray F. Spring
E. C. Steiner
Ruth R. Struik
D. R. Sullivan
Kurt Toman
Philip K. Trimble
Robert M. Turner
Anestis S. Veletsos
Alfredo D. Vergara
Ira Weissman
Joyce W. Williams
Dale E. Woerner

James Y. Wong

IOWA ALPHA, Iowa State College
(November 14, 1950)

Om Prakash Aggarwal
Lalitkumar Bhagwati
William L. Hughes
Warren A. Hunt

Robert G. Brown
Eugene C. Byrne
Esther Chivers
Leonard Cohn
Charles O. Cole, Jr.
Robert L. Doty
Katherine J. Douglas
Yndalescio J. Elizonda
Richard L. Ewen
Jarrett M. Goodman
Keith W. Halvorson
Charles L. Hawley, Jr.
George Robert Karlson
Martin Glen Keeney
John M. Kohout
Herbert Loeschen
John R. Lyall
Fred McCarron
William Noble Nelson
Margaret Oehmke
Frank R. Parchen
Arthur Paskin
John Pauls
Charles K. Titus

Jan Van Schilfgaarde

KANSAS ALPHA, University of Kansas
(January 10, 1951)

Delmar L. Boyer
Dean Brown
Ruth Barbara Hurwitz
Lucy Helen McAneny
Paul Wayne Ott
Dorothe Schuepbach
Yvonne Settle
Alan B. Showalter

Robert H. Thompson

(April 23, 1951)

Dorothy Jane Boyer
Gordon Irvin Gaston
Harvey M. Grandle
Ruth Heilbrunn
Lester E. Laird
Kenneth E. Lake
Mary Elizabeth Mann
David G. Murcray

KANSAS BETA, Kansas State College
(May 2, 1951)

Phil A. Arnold
Jocelyn A. Butcher
R. Dean Dragsdorf
Louis D. Ellsworth
James Earl Faulkner
Clarence M. Fowler
Abraham Franck
William J. Griebstein
John G. McEntyre
Doris B. Meyer
Betty M. Navratil
Milton E. Raville
Robert M. St. John
Lawrence W. Van Meir
Robert J. Vidensek
Stewart E. Wagner

Wesley G. Wilson

KANSAS GAMMA, University of Wichita
(November 29, 1950)

Roger L. Huckins

(April 27, 1951)

Mary Una Hamilton
Roy Lester Horn
Ann Klein
Charles A. Reagan
Roscoe Raymond Reagan
Vernon Victor Vlcek

KENTUCKY ALPHA, University of Kentucky
(January 16, 1951)

Ralph C. Brown, Jr.
Benny R. Coleman
Raymond **Distler**
Dr. Albert C. English
Richard Graves
Elbert Harber
Annette Siler
William Swift
Wilson M. Zaring

(April 19, 1951)

John Biggerstaff
Robert Causey
Virgial Christian
Martha Sue Creal
Richard Hood
Rowland Layson
A. G. **McGlasson**

LOUISIANA ALPHA, Louisiana State University
(April 17, 1951)

Jack Bertrand
Claude **J.** Cantrell
Daniel Bo-Yen Chen
Barbara Marie **Coco**
Charles H. Cunkle
Harold Paul Dupuy
Alvaro Garcia
Henry Clyde Kerr
William Emmett Kidd
Samuel James Kniffen, Jr.
Marguritte Yvonne Leach
Van Be Luong
Gleb Momantov
Burt **Mullin**
Andre Edovard Rouillard
David Alexander **Sandberg**
Lloyd B. Smith
Robert Miller Smith
Roy Melvin Steele
Van Carl **Vives**
Albert Henry **Wehe,** Jr.

MICHIGAN ALPHA, Michigan State College
(May 8, 1951)

Richard C**. Beckwith**
Robert L. Berry
Arthur Boggs
Robert G. Brown
Adrain R. Chamberlain
Paul T. Chan
Enayat B. **Dorosti**
George T. Hazelworth
Edward R. Holland
Dwight **F.** Kampe
Thomas P. Lee
John M. Long
Lawrence I. Lowell
Richard E. May
Robert F. **McCauley**
Wm. W. Schroeder
Mamon **S.** Talib
Alavi **Yousef**

MISSOURI ALPHA, University of Missouri
(December **11,** 1950)

Edith L. Beckett
Wm. E. Brewer
John Colston
Don Edwards
John **Lauchli**
Paul Sims, **Jr.**
Carl Spohr
Jesse H. Wright

(May 2, 1951)

Robert **C.** Baker
Charles B. **Basye**
Elizabeth Becker
Clifford H. Brown
Charles C. **Burks**
Donald Calvert
Frederick M. Cash
Michael Chiarottion
Leonard C. Fuller
Leopoldo Gomez
Cecil L. Gregory
Joe D. **Hankins**
Joseph L. **Holman**
James F. Jakobsen
Darrell E**.** Kirkendall
David H. Lillard
Frank Lloyd
Donald H. **McInnis**
Robert M. Montogmery
Theral **O.** Moore
Robert M. Pendergrass
Richard K. Reider
Lois Jane Roper
Martin G. Rudroff
Robert C. Sanford
William **Schwartz**
Frederick D. Smith
Paul R. Stapp
William A. Steele
Anna Lee Taylor
Milton A. Tegethoff
Ernest W. Wagner
Charles S. **Whitmore**
Wilbert H. Woodruff

MISSOURI GAMMA, St. Louis University
(April 18, 1951)

Khalid **Amin**
Ahmet Aytekin
Robert Aubuchon
Herbert A. Baur, **Jr.**
Anne M. Blanton
Angeline A. Bolesina
Francis **J.** Brock
George A. **Buckner**
Roy Lee Clay
Bart **O.** Coleman
John D. Corbett
Charles F. Deck
Warren **J.** Deshotels
George A. Donaldson
Helen **Fagan**
Rev. Norbert Feld
Thomas Flaut
Carl F. Flipper, **Jr.**
Donald **Fogarty**
Leonard M. Gaines
Robert Galiano
Eugene S. Gall
James John Gilchrist
Andrew K. Grier, **Jr.**
Martin A. Hanhauser, O.F.M.
Patrick A. **Heelan, S.J**
Robert P. Hewitt, **Jr.**
Victor D. Hewitt
Robert W. Hippe
Robert F. Loulihan, S.J.
Walter Huebner
Bernard **J. Jansen**
Curtis Kellogg
Phillip H. Kief
Robert Leo Kisslinger
Gerald Klosterman
Sharon Lamb
Josephine Leong
James G. Lewellen
Rev. Richard **J. Lubeley**
Francis X. Mara
Jeanette Mary Maschmann
Richard D. Milford
Paul Morgenstern
James L. Munier
Gloria Nirgenau
Frank C. Nothnagle
George A. **O'Sullivan**
Margaret Mary **Padberg**
Sterling F. Patrick
Robert L. Peace, **Jr.**
Mother Felicia Plaza, MM.**B.**
Valerian A. Prevallet
Sister **Pius** Regnier
Charles Reinhardt
Louise **Renard**
William L. Reitmeyer
George Reichmann
James John Ruddick, S.**J.**
James S. Sheehan
Alvin **Simpson**
Floyd D. Songer
Peter W. Soule
William F. Sprengnether, **Jr.**
Barbara N. Sullivan
J. Miles Turner
Alfred **J.** Valcourt
Paul E. Waltman
James Weidenborner
Joseph Witko, **Jr.**
Sarah Williams
George **J. Wooley**
Albert H. Wuerz, **Jr.**
Alice Wuest
Ying-nien Yu

NEBRASKA ALPHA, University of Nebraska
(January 1951)

Nestor E. Acevedo, **Jr.**
Paul H. **Chismar**
Robert G. Crook
William E. Eagan
R. Bruce Emmons
F. Dale Flood
Donna Mae Grueber
Robert E. Haight
Richard H. Holze
Masahiko Iwahara
Hans Jeans
Peter L. Keene
Ralph W. Kilb
Arthur C. Lindberg
Naremba Loomba
Edward R. Maunder
Richard T. **Pusateri**
A. Kellam Rigler
Thomas E. Reinhardt
Andrew Sheets
Kenneth **J. Whitcomb**
Jack H. Yelken
Winfred C. **Zacharius**

(May 1951)

John Robert Anderson
Richard **Cutts**
Lt. Marvin W. **Greenstein**
Charles A. Harvey
Myron **J.** Holm
Nolan T. Jones
Norman G. Lind
Don Jerome Nelson
James A. Nelson
Lt. Victor Utgoff
Norman Dale Williams
Kellogg V. Wilson

NEW HAMPSHIRE ALPHA, University of New Hampshire
(May 15, 1951)

John Charlton
Philip Hoyt
John Kovalik
Norman Landry
Christos E. Mandravelis
Donald Montgomery
Elizabeth Stone

NEW YORK ALPHA, Syracuse University
(December 1950)

Charles **Aldridge**
Richard Carey
Kenneth Cramer
Robert Dilley
Joseph Early
Murray **Falkoivitz**
Harry Kagan
David Kelley
Joan **Kraft**
Paul Loewner
Angelo Margaris
Richard **McKinney**

John **Farley**
Irwin **Goldberg**
Lillian Golub
Robert Gattuso
Al Graham
John Hall
Robert **J.** Hart
Marvin Hass
Herbert Hellerman
David **Hiser**
Sanford Meltzer
William Penney
Louis Robinson
Donald Rogers
William Rouse
Judson Spencer
Charles Stodard
Stewart Suttenburg
Willard **Tremlett**
Robert A. Wright

(February **1951**)

Bernard **Baschkin**
George **Hallo**
Elmer Juneau
Ludwig Karl
Ross T. Nelson
Joseph Sullivan
Eugene Wells

NEW YORK BETA, Hunter College
(May 2, **1951**)

Barbara Ciliotta
Veronica Coletti
Agnes **Duffy**
Alice Gersh
Helen **Grossman**
Evelyn Horvath
Ann Jicha
Myran Knopf
Amelia Lindner
Leila Singh
Susan **Yost**

NEW YORK GAMMA, Brooklyn College
(Fall **1950**)

Sol Aisenberg
Sol Davis
Herbert Gelernter
Jerome Glick
Alan **Goldman**
Arthur Hausner
Hartley Leavitt
Ronald **Rockmore**
Melvin Schwartz
Joseph Sucher
Miriam Jacobs

(May 28, **1951**)

Julius Barnathan
Davis Bienenfeld
Martin Bondy
Ruth Beller
Allen Carlan
Abraham Karrass
Sidney **Kissen**
Ruth Last
Joel Lebowitz
Martin **Milgram**
Lucy Molnar
Muriel Paragamon
Susan Pollack
Millie Tratner

QUALITY LITHOPRINTING

including

Typewriter and Letterpress Composition

GUSHING - MALLOY, Inc.

1350 North Main Street

Ann Arbor, Michigan

The most economical process
for limited editions and out-of-print material.