

PI MU EPSILON JOURNAL

THE OFFICIAL PUBLICATION OF

THE HONORARY MATHEMATICAL FRATERNITY



VOLUME 1

NUMBER 8

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APRIL

1953

PI MU EPSILON JOURNAL
THE OFFICIAL PUBLICATION
OF THE HONORARY MATHEMATICAL FRATERNITY

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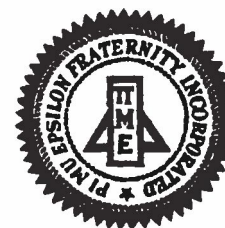
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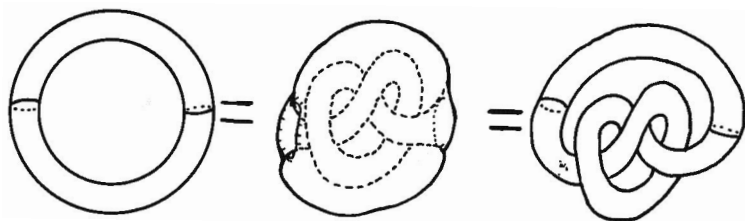
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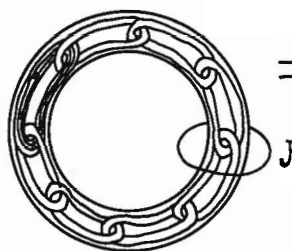
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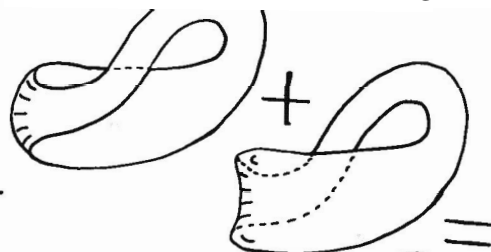


ANTOINE'S NECKLACE

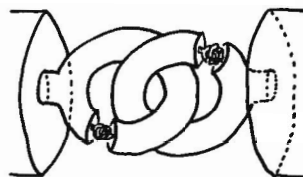
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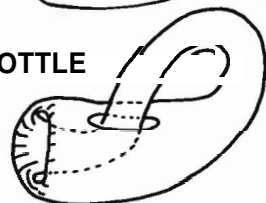
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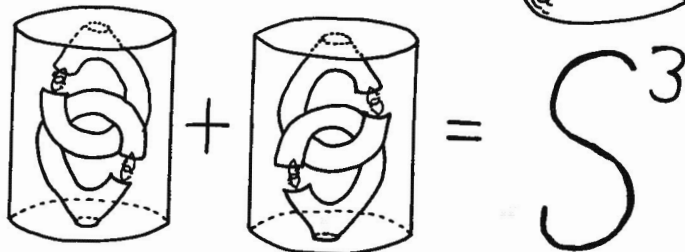
ALEXANDER'S
HORNED SPHERE



KLEIN BOTTLE



SUM OF 2 HORNED SPHERES



EXAMPLES AND COUNTEREXAMPLES

R. H. Bing, University of Wisconsin

It is not intended that each reader assimilate all the topology in the following expository article. However, it is hoped that he will finish with the impression that mathematics is a living and growing subject and that although we have learned many interesting facts about the structure of things, there is much yet to learn.

Topological Equivalence. Two figures are topologically equivalent if there is a 1-1 correspondence between the points of the two figures such that points "close together" in one figure correspond to points "close together" in the other. For example, a square, a triangle, and a circle are topologically equivalent.

One might suppose that if two sets are topologically equivalent, it is possible to deform one into the other by "pulling and stretching but without breaking or tearing". However, in E^3 (Euclidean 3-space) consider two sets each consisting of two tangent spheres; in the first set the spheres are tangent externally and in the second set one sphere lies on the interior (except for the point of tangency) of the other. The two sets are topologically equivalent because there is a 1-1 correspondence of the proper sort between the two sets. However, it is not possible to deform one set into the other in E^3 by "pulling and stretching but without breaking or tearing".

Let us consider another counterexample to the "pulling and stretching" definition. One can form a cylinder or tube by sewing two opposite sides of a rectangular rubber sheet together. If the ends of the cylinder are sewn together, the resulting figure may resemble an inner tube. However, if a knot is tied in the cylinder before its ends are united, the exterior view of the inner tube will be changed but its

interior structure will not be. If after the knot is tied and before the **final** sewing is done, one end of the cylinder is stretched and pulled over the knot before the ends are connected, another figure results. The three surfaces shown at the top of the frontispiece are topologically equivalent but not one of them **can** be stretched to make it fit on any other one of them. The interior of the unknotted surface is topologically equivalent to the outside of the one knotted inside, while the interior of the unknotted surface is like the interior of the one knotted outside.

The complement of a given figure is the set of points not in the figure. For example, on the surface of the earth, the surface of the oceans, seas, lakes, etc., is the complement of the surface of the land. In the plane, the complement of a circle is the sum of the interior and the exterior of the circle. The complement of a circle in E^3 would be connected. We shall give further examples of sets which are topologically equivalent but whose complements are not. They emphasize the contention that two sets may be topologically equivalent even though neither may be pulled to make it fit on the other.

Antoine's Necklace. Suppose M_1 is a solid torus. We will carve from it a necklace as shown on the frontispiece,

Replace M_1 by a closed chain of solid tori as shown. A person who has whittled a chain from a stick will understand how a part of M_1 may be removed so as to leave a closed chain. We call this chain M_2 .

Replace each **link** of M_2 by a closed chain to get M_3 . To avoid complications in the figure, we only show this done in one **link**. Actually each link of M_3 is replaced by a closed chain.

The process is continued a countable number of steps. The points remaining after infinitely many steps is called Antoine's necklace and may be shown (we do not do it) to have the following properties:

- (1) **Antoine's** necklace has **uncountably** many points,
- (2) The biggest connected piece of Antoine's necklace is a point.

- (3) If J is the circle illustrated, the circle cannot be shrunk to a point without touching Antoine's necklace.
- (4) Each simple surface (set topologically equivalent to a sphere) that has a point of Antoine's necklace on its interior and a point of Antoine's necklace on its exterior also contains a point of Antoine's necklace.

Antoine's necklace gets its name because it was first described by Louis Antoine and, like a set of beads, it is made up of small pieces. From properties (3) and (4) we may see that if it were hung around **one's** neck, not a point of it would break off and fall to the floor. Although the set is a thing of beauty, what is it good for except possibly to give to one's wife? However, in the last section of this **article** we discuss some of the uses to which topologists put queer examples,

Cantor Set. A Cantor set as shown on the frontispiece is obtained by starting with a straight line interval, removing its open middle third, removing the open middle thirds of each of the remaining pieces, removing the open middle thirds of the remaining pieces, etc.

A Cantor set can be shown to be topologically equivalent to Antoine's necklace (we do not show it). It has properties (1) and (2) but not property (4). Hence, E^3 cannot be pulled and stretched until Antoine's necklace fits onto a Cantor set.

We call any set topologically equivalent to a Cantor set a topological Cantor set, Antoine's necklace is a topological Cantor set.

It is possible to construct many other interesting topological Cantor sets. There is one in a square plus its interior such that each line intersecting the square intersects the topological Cantor set. There is one in E^3 that would cast a dense shade. There are others in E^3 with positive "volumes". These sets are of interest in studying surface area. Since each topological Cantor set in E^3 lies on a surface with "small area" (in a certain sense), it is possible to obtain surfaces whose "areas" are less than their "volumes",

Sum of Two Mobius Bands. A **Möbius** band is sometimes called a one-sided band. If a belt connecting pulleys is twisted like a **Möbius** band, it will wear on both sides. There is the story of the workman who **was** assigned to the task of painting one side of such a belt and removing all the paint from the other side.

The boundary of a Mobius band is a simple closed curve (set topologically equivalent to a circle). If the boundaries of two Mobius bands are sewn together as shown on the frontispiece, there results a Klein bottle. It has been called a bottle with no inside and no outside.

If one tries to construct a Klein bottle physically by sewing together Mobius bands as shown, a **kink** develops just before the sewing is finished, making it impossible to complete the construction in E^3 . There does not exist a Klein bottle in E^3 . However, a topologist does not limit his study to only those figures occurring in E^3 . The study of abstract spaces is very interesting.

In traveling around a circle **in one** direction in the plane, one either goes in a clockwise or a counterclockwise direction. Also, it is possible to assign an orientation to small simple closed curves on a sphere. However, if a cyclone follows a closed path on a Klein bottle and comes back to its starting point, its direction of spinning about the point may have changed, even though it never changed its **direction** of spinning as it moved.

Another example of a nonorientable surface is the projective plane. It may be obtained by sewing a circle plus its interior to a Mobius band along the boundary of each. Physically, this sewing could not be done in E^3 but it could be done in E^4 and some other spaces.

Horned Sphere. J. W. Alexander described the horned sphere illustrated on the frontispiece. It is topologically equivalent to the surface of a sphere but its exterior is not topologically equivalent to the exterior of a sphere. We might describe the horned sphere as follows. A long cylinder closed at both ends is folded until the ends are near each other and parallel. (In the figure we show only the ends of the closed cylinder.) Then tubes are pushed out of

each end until they almost hook as shown. The process is continued by pushing out additional tubes, pushing out additional tubes, etc. The resulting set has the property that although it is topologically equivalent to the surface of a sphere, there is a circle in the exterior of the horned sphere that cannot be shrunk in the exterior of the horned sphere to a point.

A set is simply connected if each simple closed curve (set topologically equivalent to a circle) in it can be shrunk to a point in it. It is uniformly locally simply connected if each "**small**" simple closed curve in it can be shrunk to a point on a "**small**" piece of it. The complement of a straight line interval in E^3 is simply connected but not uniformly locally simply connected. A torus is uniformly locally simply connected but not simply connected. Neither the complement in E^3 of **Antoine's** necklace nor of the horned sphere is either simply connected or uniformly locally so. However, both the complement in E^3 of a Cantor set and a sphere is simply connected and also uniformly locally so.

A simple surface is tame if there is a homeomorphism (1-1 correspondence of the sort previously mentioned) of E^3 onto itself that takes the simple surface onto the surface of a sphere. The surface of an ellipsoid is tame but a horned sphere is not. However, at certain points the horned sphere is smooth and fits into E^3 in a nice fashion. We call it locally tame here. It has been recently discovered that a simple surface is tame if it is locally tame at each point.

Here is an unanswered question. Suppose S is a simple surface in E^3 such that the complement of S is both simply connected and uniformly locally simply connected. Is S tame? Considering the rate at which we are learning new things about E^3 , I would conjecture that we shall find the answer before many years.

Sum of Two Solid Horned Spheres. In the last figure on the frontispiece we show two horned spheres. The tubes, instead of being pushed out, were pushed in. The interiors of the horned spheres are not simply connected. Each of the horned spheres plus its interior is called a solid horned sphere.

One of the horned spheres is the reflection of the other through a plane halfway between them. Suppose the two solid horned spheres are sewn together along the points on their **boundaries** made to correspond under this reflection. To what is the resulting set topologically equivalent?

If a sphere plus its interior is sewn to another sphere plus its interior along the spheres, there results a set topologically equivalent to E^3 with a point added "at infinity". This set ~~may~~ be shown to be topologically equivalent to S^3 (the surface of a sphere in Euclidean 4-space).

Is the set obtained by sewing the two horned spheres together in the above prescribed way topologically equivalent to S^3 ? The question went unanswered from 1930 to 1951. Some mathematicians conjectured that it was but others thought not. When it was discovered in 1951 that the sum was topologically equivalent to S^3 , this discovery exploded a conjecture concerning periodic transformation. (See False Statement H in next section.)

We do not yet know the answer to the following question, If the boundaries of the horned spheres were sewn together in some other fashion (we do not prescribe that a point be joined to its reflection), is the sum topologically equivalent to S^3 ?

Usefulness of Examples. If we have examples of the things we are studying, these examples may give us a better understanding of our subject. A wealth of examples helps our intuition in picking out certain theorems that are likely to be true and in labeling others as false.

A counterexample is a convincing way of showing that a theorem is false. One of the chief roles of examples is to show that certain statements are false and that it is futile to try to prove them. We illustrate this by mentioning some false statements. Consider the following.

False Statement A. If two sets in E^3 are topologically equivalent and one has the property that a simple surface in its complement separates two points of it from each other, the other set has this property also.

False Statement B. Each surface is orientable.

False Statement C. If two sets in E^3 are topologically equivalent, their complements are also.

False Statement D. The complement of a simple surface is simply connected.

Although the statements may sound very pleasing, a topologist acquainted with Antoine's necklace and a Cantor set would not make (A), one acquainted with a Klein bottle would not make (B), one acquainted with a horned sphere would not make (D), and one acquainted with either **Antoine's** necklace or a horned sphere would not make (C).

Perhaps it has been many years since any competent topologist considered as true any of the preceding false statements. However, we now mention some, each of which was conjectured as true for many years and each of which was exploded during the past five years. We use the terminology of topology and do not expect the reader to grasp the meaning of each statement.

False Statement E. The arc is the only nondegenerate plane continuum topologically equivalent to each of its nondegenerate subcontinua.

False Statement F. Each nondegenerate indecomposable continuum is 1-dimensional.

False Statement G. No set in the plane is a minimal dispersion set.

False Statement H. Each mapping of period two of E^3 onto itself is topologically orthogonal.

We could mention many other conjectures that have been exploded by counterexamples but instead mention some which still persist.

Conjecture A. Each normal Moore space is metrizable.

Conjecture B. A normal Hausdorff space is countably paracompact.

Conjecture C. A tree-like plane continuum has the fixed point property.

Conjecture D. A connected linear Hausdorff space is separable if each uncountable subset of it contains a limit point of it.

The reader may not have understood each of the preceding statements. Nevertheless he should have gotten the point that examples may be very useful in solving difficult problems.

CURVES AND POINTS
C. Stanley Ogilvy, Syracuse University

Felix Klein, in his entertaining book "Elementary Mathematics from an Advanced Standpoint," gives examples of a class of continuous algebraic curves which thread their way through the everywhere dense set \underline{R} of points both of whose coordinates are rational, without **going** through any of them except $(0,1)$ and $(1,0)$. V. E. Dietrich has shown (p. 407, Vol. 56, American Mathematical Monthly) that a very extensive class of circles also has this property: any circle whose center has at least one of its coordinates irrational passes through at most two points of \underline{R} .

Klein, later in the same book, **waxes** enthusiastic about the exponential curve $y = e^x$. Let \underline{A} be the set of all points both of whose coordinates are **algebraic** numbers. Of course $\underline{A} \supset \underline{R}$, and so in some sense the points of \underline{A} are denser than those of \underline{R} . **Yet** the point $(0,1)$ is the **only point** of \underline{A} which lies on the curve $y = e^x$. Klein exclaims, "What **would** Pythagoras have sacrificed after such a discovery if the irrational seemed to him to merit a hecatomb!" - a remark which, interestingly, has found its way into the Encyclopedia Britannica (p. 302, Vol. 14, 1950 edition).

We can point to even simpler curves with the above properties. The straight line $x = \sqrt{2}$ contains no points of \underline{R} , and $x = \pi$ contains no points of \underline{A} . But one feels somehow let down by these examples. Straight lines exhibit no delicacy in picking their way through the dense fields; they simply find a hole and blunder straight through. Consideration of this phenomenon leads to the realization that the coordinate system plays a major role in the whole question. There is nothing intrinsically rational, irrational or transcendental about a point. The point $\underline{P} (1,1)$ is in \underline{R} : but change the scale - merely magnify by π , for example; and \underline{P} now has both coordinates transcendental and thus is not

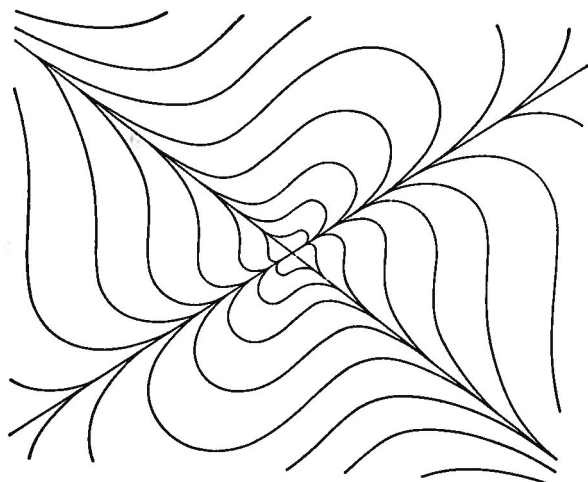
even in \underline{A} . In polar coordinates, any circle $\rho = \underline{k}, \underline{k}$ transcendental, has no points in \underline{A} . Such a circle **marches** through the (polar) points of \underline{A} very much as $x = \pi$ avoids them in the Cartesian plane.

All these curves are really far less talented than one is at first disposed to **think**. The trouble lies in our inability to appreciate "relative denseness." That the points of a denumerable set are everywhere dense is deceptive. The density of the non-denumerable continuum is so **much greater** (of a higher order of infinity, of **course**) **that** the remarkable fact is rather that a curve should manage to hit any points of \underline{A} or \underline{R} at all. Although any attempt at analogy is dangerous **guesswork**, one suspects that if we could "**see**" the way the algebraic points are distributed among the far more numerous transcendentals, the picture would somewhat resemble the way the rationals are distributed among, say, the points both of whose coordinates are integers.

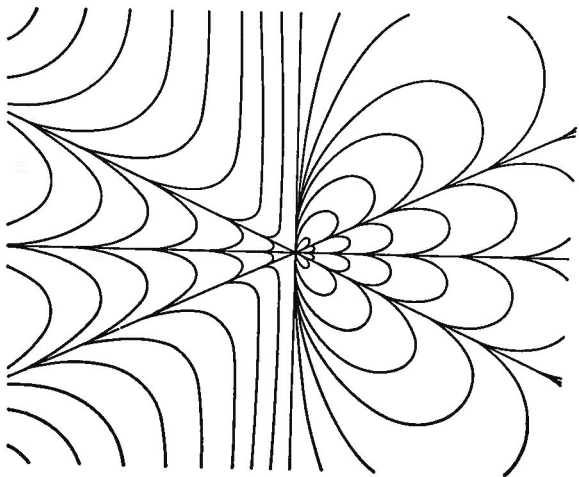
A curve truly worthy of note would be one which contained **only** points of \underline{A} ; but such a curve is an impossibility since it must be also a continuum.

Perhaps a more suitable way of thinking about these properties (since in spite of themselves most mathematicians admit that they like to get a firm grip on an intuitive handle) is not to think of curves cleverly weaving their way through points with predetermined coordinates. Rather we might do better to think of all the points of the plane as being completely nameless until a curve comes along. The points will thus **become** points of \underline{A} or of \underline{R} in certain coordinate systems if special curves happen to hit them; otherwise they will join the vast stockpile of transcendental points.

BEAUTIFUL CURVES



$$p \frac{d\theta}{dp} = - \frac{1}{\lg 5 \operatorname{arc} \lg \sin^2 \theta}$$

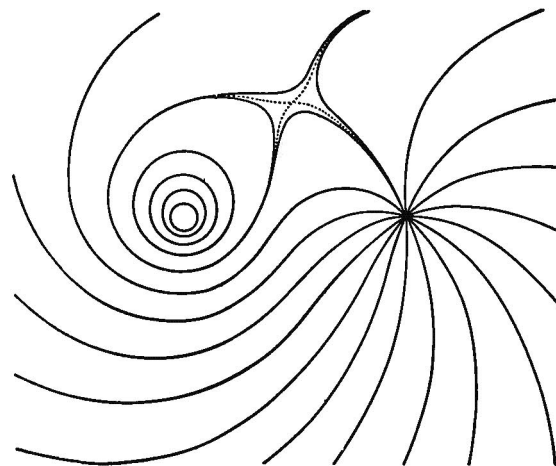


$$p \frac{d\theta}{dp} = \lg 4 \operatorname{arc} \lg \sin \theta$$

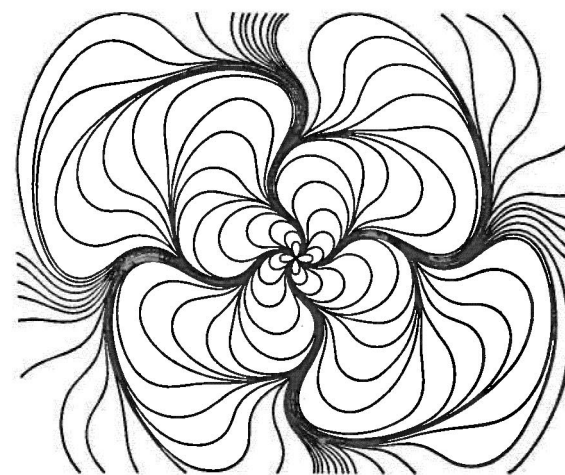
M. G. Gyllstrom

*Courtesy of SCRIPTA MATHEMATICA

BEAUTIFUL CURVES



$$\frac{dy}{dx} = \frac{x[(x-a)^2 y^2] - y(x^2 - y^2)}{(x-a)(x^2 + y^2) - y[(x-a)^2 + y^2]}$$



$$\frac{du}{dt} = \lg 2 u; u = \sin p - \theta; t = \int \frac{du}{p^2 \cos p} + \theta$$

M. G. Gyllstrom

Courtesy of SCRIPTA MATHEMATICA

PROBLEM DEPARTMENT
 Edited by
 Leo Moser, University of Alberta

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within five months after publication. Address all communications concerning problems to Leo Moser, Mathematics Department, University of Alberta, Edmonton, Alberta, Canada.

PROBLEMS FOR SOLUTION

49.* Proposed by C. S. Venkataraman, Trichur, India

If $s = (a + b + c + d)/2$ and $S = a \cdot b \cdot c \cdot d$, prove that

$$s^4 = (s - b - c)^4 + (s - c - d)^4 + (s - d - b)^4 - (s - a)^4 \\ - (s - b)^4 - (s - c)^4 - (s - d)^4 = 12S.$$

52. Proposed by R. T. Sharp, University of Alberta

A set of n smooth dominoes $1'' \times 2'' \times 1/4''$ is piled on a table, one horizontally placed domino in each layer. Find the largest distance that the top domino can be made to overhang the bottom one.

*This is the corrected form of Problem 49 as printed in Number 7 of this Journal.

53. Proposed by Leon Bankoff, Los Angeles, California

A rectangular slab of width w is moved horizontally from one corridor of width a into another at right angles to it, of width b . What is the maximum value of the length ℓ , of the slab, that will permit passage?

54. Proposed by F. L. Miksa, Aurora, Illinois

Given a right triangle ABC, with right angle at C, find a point P on AC so that the inscribed circles of the triangles BPC and BAP will be equal.

55. Proposed by Pedro Piza, San Juan, Puerto Rico

Let

$$a = 1^3 + 2^3 + 3^3 + \dots + n^3, \\ b = 1^5 + 2^5 + 3^5 + \dots + n^5, \\ c = 1^7 + 2^7 + 3^7 + \dots + n^7.$$

Prove that

$$(a + 6b + 3c)^2 + (7a + 13b + 4c)^2 = (7a + 14b + 5c)^2.$$

56. Proposed by the problem editor

Prove that

$$\left[\frac{1}{\frac{\pi^2}{6} - \sum_{i=1}^n 1/i^2} \right] = n$$

where, as usual, $[x]$ denotes the largest integer not exceeding x .

SOLUTIONS

11. Proposed by Frank Hawthorne, Hofstra College

A projectile in **vacuo** passes through two given points.

Determine the locus of foci and of vertices of the parabolic trajectories.

Solution by C. W. Trigg, Los Angeles City College

Let the equation of the parabola passing through the points (x_1, y_1) and (x_2, y_2) be $(x - X)^2 = -2p(y - Y)$, where the vertex is at (X, Y) and the focus at $(X, Y - p/2)$. Then $(x_1 - X)^2 = -2p(y_1 - Y)$ and $(x_2 - X)^2 = -2p(y_2 - Y)$.

Eliminating the parameter p between these two equations, we have the locus of the vertices of the trajectories,

$$(y_2 - y_1)X^2 + 2(x_1 - x_2)XY + 2(x_2y_1 - x_1y_2)X + (x_2^2 - x_1^2)Y + x_1^2y_2 - x_2^2y_1 = 0.$$

This locus is a hyperbola, since the discriminant

$$4(x_1 - x_2)^2 > 0, \text{ for } x_1 \neq x_2.$$

To obtain the locus of the foci of the trajectories, we eliminate the parameter p between the equations

$$(x_1 - X)^2 = 2p[y_1 - (Y + p/2)]$$

and

$$(x_2 - X)^2 = -2p[y_2 - (Y + p/2)]$$

and obtain

$$4X^2 [(x_1 - x_2)^2 - (y_1 - y_2)^2] + 8XY(x_1 - x_2)(y_1 - y_2) + 4X [2(y_1 - y_2) \cdot (x_2y_1 - x_1y_2) - (x_1 - x_2)^2(x_1 + x_2)] - 4Y(y_1 - y_2)(x_1^2 - x_2^2) + (x_1^2 - x_2^2)^2 + 4(y_1 - y_2)(x_1^2y_2 - x_2^2y_1) = 0.$$

Again we have a hyperbola, since the discriminant

$$64(x_1 - x_2)^2(y_1 - y_2)^2 > 0, \text{ for } x_1 \neq x_2, y_1 \neq y_2.$$

In the special case $x_1 = x_2$, the equations of both loci reduce to $X = x_1$, which is also the equation of all the trajectories. In the special case $y_1 = y_2$, both loci become $X = (x_1 + x_2)/2$.

14. Proposed by C. W. Trigg, Los Angeles City College

1. How may a sealed envelope be folded into a rectangular parallelepiped if overlapping is permitted? 2. What is the maximum volume so obtained in terms of the edges a and b of the envelope? 3. What must be the relative **dimensions** of the envelope in order to yield a cube? 4. What will be the volume of the cube?

Solution by the Proposer

(1) Assume that the edges of the envelope $AB = CD = b > a = AC = BD$. Now on the face of the envelope at a distance $x < a/2$ from the edges draw EF and GH parallel to AB , and IJ and KL parallel to AC . In each corner of the envelope there will now be a square. In these squares draw the diagonals AM, BN, CO and DP . Consider that the corresponding lines are on the back of the envelope with the interior vertices of the squares being $M', N', O',$ and P' .

Fold out along all lines except $OGO', PHP', NFN',$ and MEM' . Along these fold in until each of them forms a straight line. Then fold down along these lines until the triangles with vertices at the corners of the envelope lie along the faces of the resulting rectangular parallelepiped. This will then have edges $a - 2x, b - 2x$ and $2x$.

(2) $V = 2abx - 4x^2(a + b) + 8x^3$. Differentiating, setting the derivative equal to zero, and solving, we have $x = (a + b \pm \sqrt{a^2 - ab + b^2})/6$. Using the positive sign and assuming $a/2 > x$ leads to the contradictory inequality, $a > b$, hence only the minus sign holds and

$$V_{\max.} = \frac{[(a+b)(2a-b)(2b-a) + 2(a^2 - ab + b^2)^{3/2}]}{27}$$

(3) Each face of a cube is a square, and the folding process reduces each edge of the envelope by the same amount. Hence, in order to secure a square face by folding, the edges of the envelope must be equal, say to a .

(4) Then each edge of the envelope must be **divided** by folding into $a/4, a/2, a/4$, so the edge of the cube will be $a/2$ and its volume will be $a^3/8$. Due to the over-lapping

triangles, this is not the maximum parallelepiped possible from the square envelope. The maximum one will have edges of $2a/3$, $2a/3$, $a/3$, and a volume of $4a^3/27$.

31. Proposed by Victor **Thébault, Tennie**, Sarthe, France

For integers written in base B, find a number N of n digits which divides the number of digits obtained in writing the integers 1, 2, 3, ..., N.

Solution by the Proposer

The integers from 1 to N consist of

$B - 1$ numbers containing 1 digit,
 $B^2 - B$ numbers containing 2 digits,

 $B^{n-1} - B^{n-2}$ numbers containing n-1 digits,
 $N + 1 - B^{n-1}$ numbers containing n digits.

Hence the total number of digits is

$$1(B-1) + 2(B^2-B) + \dots + (n-1)(B^{n-1} - B^{n-2}) + n(N+1 - B^{n-1})$$

$$= n(N+1) - (1 + B + B^2 + \dots + B^{n-1}) = nN + n - \frac{B^n - 1}{B - 1}.$$

In order for this to be divisible by N it is clearly necessary and sufficient that N divide $(B^n - 1)/(B - 1) - n$. But this number is $11\dots 1$ (containing n digits) less n, and this has no divisors of n digits except itself. Thus the only possible value of N is $11\dots 1 - n$.

Editorial Note. For example, with $B = 10$, $n = 3$, $111 - 3 = 108$ and the numbers 1, 2, ..., 108, contain altogether 216 digits where 216 is a multiple of 108. However, for $n = 2$, $11 - 2 = 9$ contains only 1 digit. Thus there is no number N of 2 digits having the required property. It appears that the only exceptional n are $n = 2$ for any base and $n = 3$ for base 2.

32. Proposed by Francis L. **Miksa**, Aurora, Illinois

In a class in Number Theory the professor gave four

students the assignment of finding a fairly large primitive Pythagorean triangle using the well known formula for the legs:

$$A = 2mn, B = m^2 - n^2, C = m^2 + n^2,$$

where m and n are co-prime integers, not both odd. The four students produced four entirely different primitive triangles, but on comparing them it was found that two of them had the same perimeter, while the other two, also, had the same perimeter, this perimeter differing from the first one by 2. This interested the class greatly, and much time was spent in an effort to find other such sets, only to discover that there were only four such sets with perimeter less than 500,000. Can you find at least one such set?

Solution by the Proposer

m	n	A	B	C	A+B+C
106	195	41340	26789	49261	117390
215	58	24940	42681	49589	117390
184	135	49680	15631	52081	117392
232	21	9744	53383	54265	117392

Editorial Note. The triangles given above were extracted from very extensive tables recently completed by F. L. **Miksa** and A. S. Anema. These tables list primitive Pythagorean **traingles** in order of ascending perimeter and ascending area.

36. Proposed by Joan Sherley, Syracuse University

A man wished to plant an orchard with n trees in ten straight rows, five in a row. What is the smallest value n can have?

Solution by R. T. Sharp, University of Alberta

Consider the 19 points:

$$(0,0) (\pm 1, \pm 1), (\pm 1, \pm 2), (\pm 2, \pm 1), (\pm 2, \pm 2), (0,\infty), (\infty,0).$$

These yield 10 lines with 5 points in each line. It seems unlikely that this can be obtained with fewer points. If the man has some objections to planting trees at infinity he may project the configuration given above so as to bring all points within easy reach.

Also solved by C. W. **Trigg** and the proposer.

Editorial Note. It can be proved that 19 is actually the smallest value of n . The proof, however, is somewhat involved and will not be given here.

41. Proposed by Chester **McMaster**, New York City

There are more chess masters in New York City than in the rest of the U. S. combined. A chess tournament is planned in which all American masters are expected to attend. In determining the site of the tournament it is agreed that the primary consideration would be the minimization of the total inter-city distance covered by all participants. The New York masters claim that by this criterion the site chosen should be their city. The west coast players and some others claim that a city at or near the center of gravity or centroid of the players would be better. Prove that the New Yorkers are right.

Solution by the Proposer

Let the New Yorkers be labeled A_1, A_2, \dots, A_n , and let the other players be B_1, B_2, \dots, B_m , ($n > m$). Let the distance from A_i to B_j be denoted by $A_i B_j$. If the meeting is held in New York, the total distance traveled will be $T = A_1 B_1 + A_2 B_2 + \dots + A_m B_m$. Suppose now that the meeting is held elsewhere. The total distance traveled by A_i and B_i ($i = 1, 2, \dots, m$) will be at least $A_i B_i$. Hence the total distance traveled by all participants will be at least T plus the sum of the distances traveled by $A_{m+1}, A_{m+2}, \dots, A_n$.

45. Proposed by **Mel Stover**, Winnipeg, Manitoba

At a faculty meeting attended by six professors, each one left with someone else's hat. The hat taken by **Aitkins** belonged to the man who took **Baily's** hat. The man whose hat was taken by **Caldwell**, took the hat of the man who took **Dunlop's** hat. Finally, the man who took **Easton's** hat was not the one whose hat was taken by **Fort**. Who took **Aitkin's** hat?

Solution by Leon **Bankoff**, Los Angeles, California

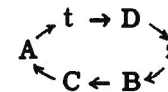
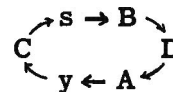
Using capital initials and the symbolism $x \rightarrow y$ to denote that x 's hat was taken by y , we have $B \rightarrow x \rightarrow A$, where x is the man who took **Baily's** hat. Also, $D \rightarrow z \rightarrow y \rightarrow C$, where z is the man who took **Dunlop's** hat and y is the man whose hat was taken by **Caldwell**.

We note at once a surplus of symbols, signifying that at least one of the unknowns duplicates a capital initial already allocated in one of the two unbroken sequences. By a double substitution the sequences can be joined in two different ways:

$$(1) B \rightarrow D \rightarrow A \rightarrow y \rightarrow C \quad (\text{where } x = D \text{ and } z = A)$$

$$(2) D \rightarrow z \rightarrow B \rightarrow C \rightarrow A \quad (\text{where } x = C \text{ and } y = B)$$

Now each sequence lacks one symbol. To supply the missing link which, at the same time, closes each unbroken sequence cyclically, use s in (1) and t in (2). We then have



All conditions are satisfied if in (1), $y = F$, $s = E$, and in (2), $t = F$, $z = E$. In either case, **Fort** took **Aitkins'** hat.

Also solved by R. M. Frisch, J. J. Greever, F. L. **Miksa**, C. Stone, **C. Strong**, C. W. Trigg, E. Zahar, and the proposer.

46. Proposed by J. Lambek, McGill University

A partial amnesty having been declared, the jailor unlocked every cell in the prison row. Next he locked every second cell. Then he turned the key in every third cell, locking those cells which were open and opening those which were locked. He continued in this way, on the n th trip turning the key of every n th cell. Those prisoners whose cells eventually remained open were allowed to go free. Who were the lucky ones?

Solution by C. W. Trigg, Los Angeles City College

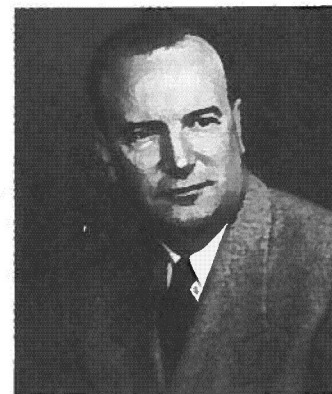
The number of times, t , that the key was turned in the q th cell is equal to the number of divisors of q . Thus, if $q = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \dots p_k^{\alpha_k}$, then $t = (\alpha_1 + 1)(\alpha_2 + 1) \dots (\alpha_k + 1)$. Now if any α_i is odd, t is even and the corresponding cell eventually remained locked. If all the α_i are even, q is a square number, t is odd, and the lucky occupants of the "square" cells found that their cells eventually remained open.

Also solved by L. Bankoff, R. M. Frisch, N. Grossman, F. L. Miksa, W. Moser, C. Stone, C. Strong, E. Zahar, and the proposer.

Men do not fail. They give up trying.

— Elihu Root.

ADDITION TO PI MU EPSILON JOURNAL STAFF*



ASSOCIATE EDITOR

HOUSTON THURMAN KARNES, Associate Professor of Mathematics, Louisiana State University, Baton Rouge, Louisiana. Native of Mt. Juliet, Tennessee. A.B. and A.M., Vanderbilt University; Ph.D., (math), Peabody College; summers, U. of Wisc. and U. of Mich. Prof. of math. and biol., Northwestern Jr. Coll.; prof. of math. and dean of men, Harding Coll.; tchr. of math. and dept. head, H.S. Nashville, Tenn.; visiting prof., Peabody Coll.; dept of math, Louisiana State University, instr., asst. prof (1938-1945), asso. prof (1945--); dean of men during the war. Hon. positions: Bd. of trustees, Harding Coll.; chmn. of natl. comm. of the Natl. Coun. of Tchrs. Math; director of the La. State Univ. Math. Inst; pres. La. State University Faculty Club. Mem: AMS; MAA; Natl. Coun. Tchrs. of Math; NEA; Omicron Delta Kappa; Pi Mu Epsilon; Phi Delta Kappa; Kappa Mu Epsilon; AAUP. Contr. to Amer. Math. Monthly; Natl. Math. Magazine; Math. Tchr. Ph.D. dissertation: Professional training of teachers of mathematics.

*Photographs and biographical sketches of the General Officers of the Fraternity and of the Editorial Staff of the Journal appeared in Vol. 1, Nos. 5 and 6, of this Journal.

REPORTS OF THE CHAPTERS

(Send reports to Ruth W. Stokes, 15 Smith College, Syracuse University, Syracuse 10, New York.)

EDITOR'S NOTE. According to Article VI, Section 3 of the Constitution: **"The** Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary-General and to the **Director-General**, an annual report of the chapter activities including programs of meetings, results of elections, **etc."** The **Secretary-General** now suggests that an additional copy of the annual report of each chapter be sent to the editor of the Pi Mu Epsilon Journal. Besides the information listed above we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships. These annual reports will be published in the chronological order in which they are received.

Epsilon of New York, St. Lawrence University

The New York Epsilon chapter held six meetings during the academic year of 1951-1952. This included the fall initiation meeting at which eight persons were initiated. During the year the following papers were presented:

"Elementary Theory of **Constructibility**" by John Taylor and Stuart Collins

"Theory of Numbers and Basesⁿ" by Bernard **Silkowitz** and George Van Wyck

"Preliminary Inspection of Algebraic Equations" by Hugh **O'Neil** and Tony **Luca**

"The Differential Analyzer and Some of Its Applications" by Donald **Hastings** and Robert Lambert.

Officers for 1951-1952 were: President, Robert Lambert; Secretary and Treasurer, John Taylor; Faculty Adviser, Dr. O. Kenneth Bates.

Officers for 1952-1953 are: President, Heidi Genhart; Secretary, Hilda Budelman; Treasurer, George Van Wyck; Director, Dr. O. Kenneth Bates; Permanent Secretary, Dr. Ruth Peters.

Alpha of Oklahoma, University of Oklahoma

The first meeting of the Oklahoma Alpha chapter for **the 1951-1952** session was held October 2, 1951. The following officers were elected for the year: Director, John E. Hoffman; **Vice-Director**, Leroy A. Guest; Secretary-Treasurer, Peter W. M. John; Faculty Adviser, Professor Arthur H. Bernhart; Corresponding Secretary, Professor Dora **McFarland**.

During the year the following papers were presented at regular meetings:

"Uses of Mathematics in Games" by Dr. Arthur Bernhart

"Some Uses of Mathematics in **Meteorology**" by Mr. Harold V. Huneke

"Consequences of the Logarithm Curve" by Mr. Roy B. Deal, **Jr.**

"Mathematics in the British School System" by Mr. Peter W. M. John

"Elementary Theory of Games" by Mr. L. A. Guest

"Polynomial Sets" by Dr. W. N. Huff

"Application of Simple Mathematics in Astronomy" by Mr. **Balfour Whitney**

"Radar" by Mr. John E. Hoffman.

The annual spring banquet, honoring thirty-six new members, was held on April 18. Professor L. Wayne Johnson, head of the department of mathematics at Oklahoma A and M College, gave the address. His subject was: "Plea for the Middle Man."

Mr. John E. Hoffman was selected as student speaker to present a paper on **"Almost Periodic Functions"** at the national meeting in September (1952).

The following were elected to serve as officers during **1952-1953**: Director, George R. **Vick**; Vice-Director, William R. Davis; Secretary-Treasurer, Mary Helen Miller; Faculty Adviser, Professor Arthur H. Bernhart; Corresponding Secretary, Professor Dora **McFarland**.

Alpha of New Hampshire, University of New Hampshire

During the 1951-1952 session the following papers were presented at meetings of the New Hampshire Alpha chapter.

"Opportunities in the Actuarial Profession" by Mr. Richard A. Leggett, Assistant Actuary of the Travelers Insurance Company, Hartford, Connecticut

"Mathematical Logicⁿ" by Dr. **Crabtree**

"Infinite Dimensional Spaces" by Mr. Cunningham

"Mathematical Logic" by Mr. Peterson

"Statistical Quality Control" by Mr. Kichline

"Calculus of Variations" by Donald Childs

"Mathematical Proof on the Impossibility of Trisecting an Angle" by Robert Hux

"Rudimentary Computing Machines" by Professor Gordon Rice

"The Four Color Mapping Problem" by Stewart Hobbs.

There were two initiation periods held during the year at which a total of twenty-two new members were received.

The by-laws were amended as follows: "All graduate students shall be extended invitations to join Pi Mu Epsilon by virtue of their positions as graduate students in mathematics."

The chapter held its annual outing on **Mendon's Pond** in **Bar-
rington**, New Hampshire, on May 21, 1952,

The following officers were elected for the year 1952-1953: Director, Donald Childs; Vice-Director, John **Oberti**; Secretary, Cornelia Cahill; Treasurer, William Peterson.

Alpha of Montana, Montana State University

During the **1951-1952** session the following papers were presented before meetings of the Montana Alpha chapter:

"**Boolean** Algebra" by Mr. G. A. Craft

"Finite Induction" by Mr. A. L. **Duquette**

"Foundations of Mathematics" by Dr. T. G. **Ostrom**.

The annual banquet was held on March 7, 1952, with Director Richard Wood as toastmaster. Thirty-five members, faculty and guests attended. Certificates of membership were presented to eleven new members.

The following were elected as officers for the year 1952-1953: Director, Larry Hunter; Vice-Director, Maynard Stevenson; Secretary, Robert Pozega; Treasurer, Dr. G. **Marsaglia**.

Alpha of New York, Syracuse University

The New York Alpha chapter held four regular meetings during the **1951-1952** session. The following papers were presented:

"**The Four Color Problem**" by Dr. William Pierce

"**Fermat's Theorem**" by Dr. **Kathryn** Morgan

"Cooperative Phenomenon" by Dr. Melvin Lax of the Physics Department

"Product **Integration**" by Dr. George **Mostow**.

The initiation banquet was held on December 14, 1951. **Thirty-**six persons were initiated. The guest speaker was Dr. W. H. **Fuchs**, Department of Mathematics, **Cornell University**. His subject was: "Unusual Applications of **Mathematics**".

On April 23, 1952, a joint meeting was held with the Mathematics Club. The speaker on this occasion was Dr. William **Hotchkiss**, of the History Department, Syracuse University. His subject was: "**Leibniz** and **Newton**".

The officers for 1951-1952 were: Director, William Rouse; Vice-Director, Murray **Falkowitz**; Recording Secretary, Joseph L. Sullivan; Treasurer, John Parley; Faculty Adviser, Dr. Paul W. Gilbert; Permanent Secretary, Dr. Nancy Cole.

The following were elected to serve as officers for 1952-1953: Director, Alan S. **Meltzer**; Vice-Director, Joseph A. Ontko; **Cor-**-responding Secretary, John E. **Klein**; Recording Secretary, Ruth M. **King**; Treasurer, Robert E. Fishlock; Faculty Adviser, Dr. Paul W. Gilbert; Permanent Secretary, Dr. Nancy Cole.

Alpha of Oregon, University of Oregon

During the 1951-1952 session Oregon Alpha initiated **twenty-**seven new members. The following paper was presented:

"Space Filling Curves" by Hugh Christensen.

Officers for 1952-1953 are: Director, Leroy Warren, Vice-Director, Calvin T. Long; Secretary-Treasurer, Pearl A. Van **Natta**.

Beta of Oregon, Oregon State College

The following papers were presented before meetings of Oregon Beta during 1951-1952:

"Errors in Linear Systems" by Dr. A. T. Lonseth

"Mathematical Paradoxes" by Mr. A. Wenner, Mr. H. Johnson, and Mr. R. Bredemeier

"Opportunities in Mathematics" by Dr. C. L. Clark

"Computer Physiology" by Mr. John **Blankenbaker**.

Thirty-four new members were initiated at the annual banquet held May 13.

The officers for the academic year 1952-1953 are: Director, Thomas **Glahn**; Vice-Director (office yet to be filled); Secretary, Richard Bredemeier; Treasurer, Professor G. A. Williams.

Alpha of Pennsylvania, University of Pennsylvania

Pennsylvania Alpha held seven meetings during the 1951-1952 session. The following papers were presented at these meetings:

"The Role of the Mathematician in Computing Machinery" by Mr. **Holmes W. Taylor**, Associate Research Engineer of **Burroughs Adding Machine Company**

"Geometric Construction of Counter-Examples" by Dr. **Richard Anderson**

"Some Remarks Concerning the Invariance Principle in Algebra and Geometry" by Dr. **George Schweigert**.

Other speakers during the year were Dr. **Nathan Fine**, Dr. **Harry Rauch** and Dr. **Pincus Schub**.

The annual banquet was held on May 13, 1952. The guest speaker for this occasion was Dr. **John R. Kline**, Chairman of the Department of Mathematics,

Beta of Washington, University of Washington

Washington Beta held ten regular meetings during 1951-1952, exclusive of two initiations and the annual picnic. Sixty-nine new members were initiated during the year. The following papers were presented at the regular meetings:

"Gamma and Beta Functions" by **Judson Smith**

"Derivation of Computing Formulae for the Quartic Equation" by **Lee Tower**

"What is Topology?" by Professor **Carl B. Allendoerfer**

"History of Geometry" by Professor **Roy M. Winger**

"Vacations for Pay" by Professor **Roy B. Leipnik**

"Theory of Braids" by Professor **J. Maurice Kingston**

"Algebra of Points" by Robert **J. Wisner**

"Magic Squares" by Professor **Ross A. Beaumont**.

The annual picnic was held on May 24, 1952, at Lake Wilderness. Ninety-one adults and thirty-two children were in attendance.

Officers for the year were: Director, **Ted R. Jenkins**; Vice-Director, **J. Richard Byrne**; Secretary-Treasurer, **Judson Smith**.

MEDALS, PRIZES AND SCHOLARSHIPS

EDITOR'S NOTE. Each chapter undoubtedly will be interested in learning what other chapters are doing along the line of prize competitions. So the editor makes the request that chapters offering prizes, scholarships, or other awards, write up their plans for such contests and submit them for publication in this journal.

The Oklahoma Alpha chapter holds an annual mathematics contest on problem solving. The contest for 1951-1952 resulted in a three-way tie. The winners were **James T. Day**, **Jack Kline** and **John D. Thomas**. The reward was five dollars worth of mathematics books to each of the winners.

Harrison E. Radford won the prize of \$10.00 given by the New Hampshire Alpha chapter for 1951-1952. This award was made to the freshman receiving the highest average in algebra, trigonometry, analytic geometry and introductory calculus.

The Montana Alpha chapter began the 1951-1952 academic year with the awarding of the Pi Mu entrance prizes. These prizes are given to the three freshmen who placed highest in an examination in mathematics.

The **DeCou** prize of \$50.00 is awarded annually by the Oregon Alpha chapter. The award is presented to a senior in mathematics due to recognition of an outstanding record and achievement in mathematics. The prize was presented to **Sam Saunders** for the 1951-1952 session.

The Oregon Beta chapter gives an annual award based on a mathematical contest. Three men were tied for first place in **1951-1952**. They were: **John Blankenbaker**, **Charles Luehr** and **John Pendleton**.

During the winter quarter of 1951-1952 the Washington Beta chapter sponsored a weekly contest in the local campus newspaper. A mathematical puzzler was published in the newspaper and solutions were turned in to the chapter. Winners were awarded cash prizes of \$2.50. The final contest involved a more sophisticated problem and the prize was \$5.00.

MISSOURI GAMMA ESSAY CONTEST

The Missouri **Gamma** chapter each year holds the Pi Mu Epsilon Essay Contest. The contestant submits his essay with a digest to the judges. Two prizes are awarded annually in this contest, one to a student in the Senior Division and the other to a student in the Junior Division. For the session 1951-1952, the topic was "The Bernoullis: Their Lives and Works." Mr. Ying Nien Yu was the winner of the Senior Division, and Mr. Roland Nokes won the Junior Division Contest. Both of these students were enrolled in Park College of Saint Louis University. A digest of the senior contest prize-winning essay is given below,

DANIEL BERNOULLI'S **HYDRODYNAMICAL** EQUATION

by Ying-Nien Yu

Daniel Bernoulli (1700-1782) was the founder of mathematical physics. He introduced innumerable mathematical **theorems** to the solutions and descriptions of problems in physics. Perhaps the most creditable contribution that Daniel bequeathed us was his work in Hydrodynamics. The term, hydrodynamics, was actually introduced by Daniel Bernoulli to comprise the two sciences of hydrostatics and hydraulics. Hence, we may justly consider him the father of Hydrodynamics.

The science of hydrodynamics is concerned with the behavior of fluids in motion. During the 18th century and the beginning of the 19th century, those masters of mathematics such as Daniel Bernoulli, Isaac Newton, Leonard Euler, and others derived many differential equations and introduced the potential theory to describe the fluid motion. The fluid they dealt with is **known** as the perfect or ideal fluid, **i.e.**, frictionless or non-viscous fluid, and the type of flow they restricted to is irrotational. In most cases, streamlines were used to describe the fluid flow. This is the so-called "Classical Hydrodynamics". Since the potential theory is a branch of mathematics, we may consider classical hydrodynamics a branch of mathematics.

The first substantial result of the theory of fluid motion was established by Daniel Bernoulli in his "**Hydraulicostatica**", written in 1738. The result can be described by Bernoulli's **hydro**-namical equation, which states that, for a steady motion, at any point along a definite streamline, the **sum** of the kinetic energy, the integral of dp/p , and the potential energy for a unit of mass is constant. This equation is of utmost importance in the hydrodynamics of non-viscous fluid.

Later, Leonard Euler, a contemporary of Daniel Bernoulli, derived Bernoulli's equation more systematically by applying Newton's second law.

Bernoulli's equation is concerned only with steady motion along a streamline. It is logical to consider the flow as one dimensional because the vector of the fluid travels in the same path as the streamline. By this idea of the one dimension flow, engineers nowadays apply Bernoulli's equation to pipes, tubes, or channels of finite cross-sections, and obtain fairly accurate results. However, mathematicians may feel apprehensive about this.

Most classical theories show discrepancies with the 20th century's experimental results due to the restrictive assumptions of the classical theories. However, in working with the theoretically perfect fluid and in studies of theories both old and new, Bernoulli's Hydronamical Equation is still a basic tool for engineers and mathematicians, and plays a greater role in modern hydraulics and aerodynamics.

 ACKNOWLEDGMENTS

For complimentary copies of the following publications received at the office of the Pi Mu Epsilon Journal, the editor is very grateful.

"A Mathematics Oklahoma University Newsletter," Vol. 1, No. 2, published in March, and No. 3, in May, 1952, by the Oklahoma Alpha Chapter of Pi Mu Epsilon.

"The Missouri Gamma News," Vol. II, No. 2, published in July, 1952, by the Missouri Gamma Chapter of Pi Mu Epsilon.

"Math Mirror," Spring 1952, published annually by the Mathematics Society of Brooklyn College.

NEWS AND NOTICES

A new name has been added to the list of associate editors of the Pi Mu Epsilon Journal, that of Professor Houston T. **Karnes** of Louisiana State University, **His** work will be closely connected with that of the chapters, and he will edit the sections "Reports of the Chapters" and "Medals, Prizes and Scholarships". For many years he has been a most cooperative and efficient corresponding secretary for the Louisiana Alpha chapter, always responding promptly to our communications and sending in annual reports of the chapter on time and in excellent form. Because he has such a genuine interest in the work of the Fraternity and its members, we are fortunate in having him join the staff of editors.

* * *

Two new chapters of Pi Mu Epsilon are soon to be added, which will bring the number of active chapters to fifty-six. Secretary-Treasurer General **J. S. Frame** reports that the petitions of Alabama Polytechnic Institute and **Cornell** University have been approved. These chapters will be installed sometime this spring, probably in April.

* * *

Two letters dated November 3, and December 4, 1952, from Corresponding Secretary Professor C. B. Read, of the Kansas Gamma chapter, are concerned with matters not only of great interest to his chapter at the University of Wichita but, also, to chapters on other campuses where there might be motivated similar action. With this object in mind, we quote, in part, Professor Read's letters (addressed to the editor):

(1)

"Kansas Gamma chapter of Pi Mu Epsilon has received two substantial cash donations to its Pi Mu Epsilon Mathematics Scholarship Fund. One contribution was made by Mrs. E. B. **Wedel** in memory of her husband (former professor of mathematics at the University of Wichita) who died last July. The other was made

by Mr. **H. K. Sears**, lecturer in mathematics and a member of the Kansas Gamma chapter, in memory of Mrs. Sears, who died about a year ago."

"Income from this scholarship fund goes to a senior, majoring in mathematics. The present holder of the scholarship is Miss **Ann Klein**."

(2)

"Kansas Gamma chapter reported earlier in the year a gift from Mr. Henry K. Sears to our Pi Mu Epsilon Scholarship Fund in memory of his wife who died last year. **Mr. Sears** has just made an additional contribution of slightly under \$100 to this fund. The purpose of the second gift is to cover the cost of exchanging securities, given earlier in the year, for other securities having a higher market value and carrying a higher dividend rate. Mr. Sears volunteered to stand all expense involved in exercising conversion rights of the original securities."

"I thought this item might be of interest for the Pi Mu Epsilon Journal."*

News Items Gleaned from Chapter Reports

The secretary of the New York Epsilon chapter reported: "Our annual, high school mathematics contest, promoted by the members of the Northern New York Interscholastic League, was held on April 26 (1952), and fourteen schools were represented."

"A mathematics 'Oklahoma University Newsletter' was produced for the first time during the **Session 1951-1952** by the Oklahoma Alpha chapter. Problems and articles of interest to high school students were included and copies sent to high schools throughout the **country**."

Gleanings from the report of the New Hampshire Alpha chapter indicate a lively interest in the Fraternity. Note the following three activities:

- (1) "Aid classes were given by the members of the chapter in courses ranging from algebra to differential equations every other Wednesday evening throughout the year for those students who had difficulty in these courses. . . ."

***Editor's comment:** It certainly is of interest to us. Thank you very much, Professor Read.

- (2) "Two volumes, a German-English and **English-German Dictionary** and 'Mathematics-Queen and Servant of Science' by Bell, were added to the Pi Mu Epsilon Shelf in the mathematics library.. .."
- (3) "Donald Childs was elected to membership in the Mathematical Association of America."

* * *

Reference is made to page 283, Vol. 1, No. 7, of this Journal. The date, in parentheses, following each name in the list of Pi Mu Epsilon members who received the Doctor of Philosophy Degree in 1951, refers to the year in which the person was initiated into the Fraternity and not to the year in which the degree was conferred.

Repeating what we said there, it is our intention to publish annually the list of names of our members who take their doctorates in that year, whenever such information is made available. It would be appreciated if when the corresponding secretaries are making their annual chapter reports they would include news items about their members earning advanced degrees and receiving notable appointments and other honors. Many of our members, after being initiated into the Fraternity, transfer to other universities to work on higher degrees, and thus identification with their home chapters is lost. It would be a matter of great pride to have a complete list of our members earning the doctorate each year, in the files of the Secretary General; then the editor of the Journal could refer to those files for such information.

* * *

We understand that many of our readers solve the problems that are published in the PROBLEM DEPARTMENT of the Journal but just do not take the trouble to send in their solutions to the problem editor. We should like to make the special plea that you do take time to send in your solutions. The editor of that department is doing a fine job. His supply of interesting problems seems to be a nondenumerable set, but his only means of knowing what type of problem has the most popular appeal to our readers is based on the response he gets. Problem solving is an important phase of mathematical study. In a recent letter from Dr. Moser he mentioned the fact that his solution to a problem which had appeared in the AMERICAN MATHEMATICAL MONTHLY

served as a starting point for a paper "On Sequences of Integers" published in the latest issue of the MATHEMATICAL GAZETTE. Won't you write him. He would be interested in any comments, favorable or otherwise, about the problems he is publishing.

Notice Concerning Changes of Addresses and Subscriptions to Journal

When communicating with Business Manager Howard C. Bennett, concerning change of address to which the Journal is to be sent, it is important to state the name of the chapter into which the member communicating was initiated.

INITIATES, ACADEMIC YEAR 1951-1952 (Continued from Vol. 1, No. 7)

ILLINOIS ALPHA, University of Illinois (May 19, 1952)

James **H. Abbott**
J. E. Ams
Dale **Ashcroft**
Jyoti Bhattacharjee
J. D. Busch
Glenn Cate
Stanley **Changnon**
Charles R. Chubb
C. L. Coates
Jeanne **Crawford**
Arno **Cronheim**
Ray Eckman
M. A. El-Hifni
R. L. Fisher
Aubyn Freed
James Henderson

Raymond Honey
R. F. Hyneman
Richard **Janer**
Q. R. **Jeffries**
Richard King
Ken **Kolence**
Philip Larsen
A. D. **Liehr**
W. L. **Masterton**
John Muerle
Marshall Nolan
R. H. F. Pao
Karl **Pister**
Ray **Lolivka**
A. L. **Promislow**
R. C. **Quinlan**
J. **M. Quinn**

P. **H. Rosenfield**
George Russell
C. T. **Sah**
Harry Schey
Norman **Shapiro**
Ronald **M. Shelton**
Sidney Singer
Edward **Stejskal**
Michio Suzuki
Kenneth Tabler
L. N. Tao
W. E. West
R. S. Wiseman
J. B. Wong
Theodore Wright
H. N. Yu

NEW YORK ETA, The University of Buffalo (April 28, 1952)

Mrs. **Allan** Brown
Walter Fleming

Louisa **Grinstein**
Donald O. **McKay**
Rita A. **Mochan**

Sidney R. Nichols
George Walker

NORTH CAROLINA ALPHA, Duke University (May 14, 1952)

Alexander Troy Cole
Edwin Richard Gabler
George Earl Gerber
Thomas **H. Harmount**

William **H. Jennings, Jr.**
Alfred **Evison Kerby**
Still-ley June Markee
George **Wilmot Marsden**
Elaine Popp

John Edward Roberts
Ronald B. Stauffer
Charles **H. Warlick**
Walter Q. **Wilson**

NORTH CAROLINA BETA, University of North Carolina (May 1, 1952)

Joseph A. Arnold
George Birkel, Jr.
Jacob F. Blackburn

Donald J. Morrison
Richard J. Painter
Steve Pugh

Herbert Spease
Ronald Telley
C. V. Williams, Jr.

OHIO ALPHA, The Ohio State University (May 16, 1952)

Charles Thomas Anderson
William Paul **Bagley**
Robert Alien **Bain**
Carlisle Brown Barnes
Robert **Poston Caren**
Wendell Arnold Cook
Thyrsa Anne Frazier
Alan Josef **Gutman**

Billy O. **Hoyle**
Jui Sheng Hsieh
William Ray **Irion**
Lawrence D. Jones, Jr.
Richard Theodore Kuechle
John Kenneth Lerohl
Robert Stanley **Marcum**

Jack Herbert Richmond
Dorothy Lee Roberts
John Everett Sandefw
Dorothy K. **Shaner**
John Andrew Stamper
Constantine Vontsolos
Sakae Yamamura
John Jacob Young

PENNSYLVANIA GAMMA, Lehigh University (March 3, 1952)

Richard A. Ash
Stanley E. **Aungst**
John F. **Barteau, Jr.**
Richard T. **Begley**
Malcolm A. **Bingaman**
James P. Bond
Donald W. **Clapp**
Nathan L. Cohen
J. David Conrad
David A. **DeGraaf**
John C. **Diercks**
Daniel D. **Dubosky**
Neil A. Fisher

James G. **Gottling**
Kenneth A. Heller
Robert S. **Knox**
William C. **Ladew**
Robert A. Lane
Thomas **H. May**
Richard A. **Moyer**
Robert W. Moore
Donald W. **Oplinger**
Donald L. Ort
Frederick A. Otter, Jr.
A. Graham Patterson
Samuel I. **Plotnick**

Bruce L. **Reinhart**
Frederick A. **Saal**
Robert G. Schilling
Lloyd R. **Schissler**
Andrew E. **Seman**
Robert C. Smith
William T. Spencer
A. Walter Stubner
Philip J. **Sturiale**
Joseph Teno
Franklin M. **Townsend**
Robert J. **Vekony**
Leroy J. Yeager

WISCONSIN BETA, University of Wisconsin (May 21, 1952)

Morton Brown
Phyllis Drews
George **Gloumousis**

Richard **Gitter**
Beverly **Iverson**

Raymond Rishel
Marilyn **Ulrich**
Donald White

INITIATES, ACADEMIC YEAR 1952-1953

CALIFORNIA ALPHA, University of California, Los Angeles (Fall, 1952)

Earl A. **Coddington**
Raymond **M. Hill**
Robert E. Jackson
Wendell S. Miller

Roger A. Moore
Mary J. Schulte
Yutaka Shiraiishi
Isadore M. Singer

Seymore Singer
James V. Whittaker
Jacob **Wolfowitz**
Dean **Zes**

COLORADO BETA, University of Denver (December 4, 1952)

Claude Peter **Coppel**
Raymond Henry **DeMoulin**

Randon Eugene Holben
John Ray **Hunsberger**

Martin **Nesenbergs**
Donald **Edwin Rugg**

Aloysia Mary German
Clifford A. **Hauenstein**

John Thomas **Irwin**
William Joseph **Kunzman**
James Robert **Mondt**

John G. **Tomkinson**
Marvin C. Warner

DISTRICT OF COLUMBIA ALPHA, Howard University
(November 18, 1952)

Mr. William F. Howard

KANSAS **GAMMA**, University of Wichita
(November 21, 1952)

John **M.** Dale
Richard P. Dodge

Thomas C. Farrar
Wayne **Holmes**

John W. Johnson, Jr.
L. K. Walker

MICHIGAN ALPHA, Michigan State College
(November 25, 1952)

James L. Bailey
Edward H. Carlson
Warren J. **Eding**
James **S.** Grimes
Mary Lu **Hamill**
William **J. Hardell**

Ching-U Ip
Delia **Koo**
Anthony **Koo**
Hubert W. **Lilliefors**
John T. **McCall**

Richard M. Meyer
Shirley Ann **Overley**
Malik M. Quraishie
Orrin E. **Taulbee**
Phillip **R. Thornton**
Donald H. Webb

MISSOURI ALPHA, University of Missouri
(December 12, 1952)

Richard **Burnap Beale**
F. Joe Crosswhite
James Eugene **Fithian**
Anna Lee Freshman
Edward Dale **Fryslie**

Dan Barton Hoagland
Robert Marshall **McKee**
Raymond Lester Owen
Joe Wiley Painter
Dorothy Lee Powell
Bob Ray Scott

Katheryne **Shoop**
Roland D. Taylor
Kenneth Oliver **Weiser**
Delmar B. Van Meter
Henry L. **S. Yee**

NEBRASKA ALPHA, University of Nebraska
(November 18, 1952)

Rolland W. Ahrens
Richard D. Ayers

Edward A. **Brong**
Jean Davis

Robert **J. Tockey**
William E. **Wageman**

NEW YORK ALPHA, Syracuse University
(December 17, 1952)

Walter Baum
Norbet Bischof
John Chase
Hellen Cooper
Robert Downing
Virginia **Feldmann**
George **Finkbeiner**

Sally Keller
Paul **Kenline**
Warren Lombard
Robert Mack
Alfred **MacRae**
Thomas Manwarren
George **Mulfinger**

Joseph E. **Rizzo**, Jr.
Charles **Serby**
Herbert **Shoen**
Joyce **Shorrin**
Harold **Siegal**
Emmanuel Stern
William Terrell

Robert I. Gray
Patricia **Hansell**
Fritz Hemmer
Charles Johnson

Ira Nemeroff
Victor Pietrafesa
Frank Raymond

Rolf Thorkildsen
Clarence **Vanselow**
John H. Van De Walker
Ralph Wiegand

NEW YORK BETA, Hunter College
(November 8, 1952)

Carmen Bell
Millicent **Karmin**
Helga **Muhlstock**

Grace Sacks
Pauline **Sakles**
Nancy Scribano

Sandra **Tausig**
Marilyn **Tuckman**
June **Zimany**

NEW YORK EPSILON, St. Lawrence University
(Fall, 1952)

Hilda **Budelman**

Heidi **Genhart**

OHIO DELTA, Miami University
(Fall, 1952)

Harvey F. Blanck
Richard Blancenbecker
Traian Cindea
Edgar B. Dally
Carol Jean Fell
Pat Flanagan

Robert E. **Gaynor**
James Henkelman
Nancy L. **Kiehborth**
Ernie A. Kuehls
John L. Madden
Joseph **Martino**

Amelia **Mattson**
Richard **S. Neddenriep**
Charles C. Robinson
Richard C. Roth
Elmer W. Schirmer
Ronald L. Siereveld

OKLAHOMA ALPHA, University of Oklahoma
(Fall, 1952)

F. W. Ashley
James C. Bradford
E. N. Brandt
Wayne T. Ford

Doyle E. **Goins**
Charles C. Grimes
Thomas J. Head
Ernest L. **Lippert**

T. Richard **McCalla**
Frank L. Miller
Franklin A. **Phillips**
J. B. Willis

OKLAHOMA BETA, Oklahoma A. and M. College
(Fall, 1952)

Charles E. Durrett
John Leroy Folks
Burt Gambill

Edward Gastineau
Dan C. Hanan
Edward D. Holstein
Therman I. **Lassley**

Frank G. Martin
William E. **Pruitt**
Lorene O. Young

OREGON BETA, Oregon State College
(November 25, 1952)

Ruth **M.** Blair
James F. Carpenter

Donald H. Cone
Charles H. **Gutzler**

Patric W. Paddock
Walter C. **Riddell**

PENNSYLVANIA ALPHA, University of Pennsylvania
(November 21, 1952)

John J. **Crawford**
Arlene **Goldman**
Gerson **Goldstick**
Edwin Kellerman
Fred **Ketterer**
Marvin **Kornblau**

Edwin LePar
Sheldon **Lisker**
Leroy **R. Loewenstern**
Clarence Reed
Donald **Rosen**

Rhoda **Rosen**
Ivan Rudolph
Alex Stogryn
Albert Whetstone
Lowell **Zeid**
Doris **Zoblan**

(December 12, 1952)

Yonathan Bard
Henry J. Greenwood

Howard Grey
Edwin Langberg
Martin Levy

Paul Venuto
Aaron Wasserman

PENNSYLVANIA BETA, **Bucknell** University
(November 19, 1952)

Stuart Evans **Athey**
Ed. **Axelrod**
Harry **Bostian**
Donald Burns
Robert Catherman
Jeanne Cooper
Lois **Cullon**
Barbara Davis
Harold Debbi

Francis Derby
Eleanor **Gilliams**
Ronald Goodman
John E. **Gorman**
Gates **Hallino**
Robert **Keller**
Al George **Koslin**
Mr. **Krzominski**

Mrs. Lieberhorr
John **McKee**
Mrs. V. Richardson
Janet **Sandford**
Wu Wen Shao
Bill **Swartz**
Tonny Wang
Donald Watson
Richard Wilson

PENNSYLVANIA GAMMA, **Lehigh** University
(November 17, 1952)

Robert J. Adler
Peter **M.** Barba
George E. **Clauser**
James Cutler
Donald J. **Glick**
Samuel **Goldberg**

Frederick A. D. **Granados**
Louis W. **Hauschild**
Charles E. **Klabunde**
James P. **Klima**
Stanley P. **Lundstrom**
Ronald B. Madison

A. **R. Middlekauff**, Jr.
Donald G. Smith
Kenneth W. Todd
Raymond P. Vogel
Lawrence J. **Wallen**
William P. **Whyland**

WISCONSIN ALPHA, **Marquette** University
(January 14, 1953)

Robert T. **Dolezal**
Rev. Lawrence W. **McCall**

William G. **Schutz**

Jerome W. **Riese**
James P. **Scanlon**

WISCONSIN BETA, University of Wisconsin
(Fall, 1952)

Archie Roy **Burks**
Robert L. Glass
Peggy Jean **Kosson**
Helen S. **MacDuffee**

Clark T. Miller
Norbert J. Nitka
Harold **Ottoson**
Eva **Perlman**

James **Pomeroy**
David **Rothman**
Catherine **Standerfer**
John A. **Standerfer**

EDITOR'S NOTE

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- (1) Professors Albert Edrei, Abe Gelbart, **Erik Hemming**-sen, Kathryn Morgan, and **Otway** Pardee;
- (2) Professor Joseph Kowalski of the Art and Industrial Design Departments;
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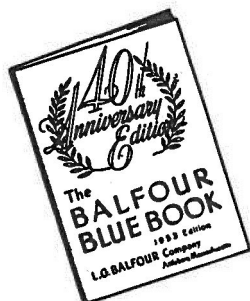


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