PI MU EPSILON JOURNAL

THE OFFICIAL PUBLICATION OF

THE HONORARY MATHEMATICAL FRATERNITY

VOLUME 2 NUMBER 1

CONTENTS

NOVEMBER 1954

All editorial correspondence, including manuscripts, chapter reports, books for review, news items, etc., should be addressed to THE EDITOR OF THE PI MU EPSILON JOURNAL, 15 Smith College, Syracuse University, Syracuse 10, New York

PI MU EPSILON JOURNAL is published semi-annually at Syracuse University.

SUBSCRIPTION PRICE: To Individual Members, $1.50 for 2 years; to Non-Members and Libraries, $2.00 for 2 years. Subscriptions, orders for back numbers and correspondence concerning subscriptions and advertising should be addressed to the PI MU EPSILON JOURNAL, 15 Smith College, Syracuse University, Syracuse 10, New York.

PI MU EPSILON JOURNAL

THE OFFICIAL PUBLICATION OF
THE HONORARY MATHEMATICAL FRATERNITY

VOLUME 2 NUMBER 1

CONTENTS

NOVEMBER 1954

STEWART SCOTT CAIRNS

DIRECTOR GENERAL of PI MU EPSILON FRATERNITY

GREETINGS FROM THE DIRECTOR GENERAL

Having recently assumed office as Director General of Pi Mu Epsilon, I extend my greetings to the officers, the several chapters and the individual members of the Fraternity. I do this with the mixed feelings of one who says, "Hail and farewell!" for I commence my incumbency by turning my duties over for a year to Professor J. Sutherland Frame of Michigan State University, while I enjoy a sabbatical leave with a Fulbright Grant to the University of Strasbourg in France.

During approximately twenty years, since being initiated into the chapter at Lehigh University, I have participated in various Pi Mu Epsilon activities. During the same period, I have seen a great change in the status of mathematics in our society and in the nation. For several years before the war mathematicians shared in the general depressed conditions, and some of the most competent of them were unable to find suitable employment. The war and the postwar period have been accompanied by an increasing awareness of the need for mathematicians and also of the growing shortage thereof. This means that the members of Pi Mu Epsilon can look forward to a period of heavy demand for their talents and also that they may render both individual and public service by gathering and spreading information as to the prospects of those whose tastes and abilities lead them into mathematical or related careers.

It is a pleasure and an honor to become your Director General. After an easy first year, when my services will, in the mathematical sense, be vacuous, I will perform my duties for the Fraternity to the best of my ability. Meanwhile, my good wishes to all of you!

Stewart S. Cairns

DIOPHANTINE EQUATIONS[1]

W. R. Utz*
University of Missouri

Many puzzle problems are like the following question which is similar to questions found in almost any algebra of a few generations ago.

(I) A man buys some goats at \$6 each, pigs at \$4 each and rabbits at \$1 each. If he buys at least one of each animal, a total of 20 animals, and pays \$40 for them, how many of each kind of animal does he buy?

Algebraically we seek a solution x, y, z in ***positive integers*** of the system

$$x + y + z = 20,$$
$$6x + 4y + z = 40,$$

where x, y, z denote the number of goats, pigs and rabbits, respectively, purchased.

If there is a solution, upon eliminating y from the system, we obtain

$$2x - 3z = -40,$$

or $x = z - 20 + z/2$. Now, if x is to be an integer, $z/2$ must be an integer, hence $z = 2u$ for some integer u. From this we see that x must be of the form $3u - 20$. Hence

$$x = 3u - 20,$$
$$y = 40 - 5u,$$
$$z = 2u,$$

if there is a solution. Actually, for ***any*** value of u the

*This paper was presented before the Gamma of Missouri Chapter, St. Louis University, at their initiation banquet, April 27, 1954, when Dr. Utz was the guest speaker for the occasion.

[1] James E. Case Memorial Lecture, St. Louis University, April, 1954.

resulting numbers x, y, z satisfy the given system of equations. However, we need the integral solutions such that x, y, and z are each between 0 and 20. Since $x = 3u - 20 > 0$, we must have $u \geq 7$, and since $y = 40 - 5u > 0$, we must have $u \leq 7$. From these inequalities it follows that $u = 7$, $x = 1$, $y = 5$, $z = 14$.

We would have seen that only a finite number of answers were possible if we had considered the equation $5x + 3y = 20$, which follows from the given system upon eliminating z. Any values of x, y, z satisfying the initial problem give a pair of numbers x, y that satisfy this equation. Geometrically $5x + 3y = 20$ is a line, and we seek the "lattice points" (that is, the points having coordinates that are integers) on this line in the first quadrant. Since the x,y-intercepts are positive, the lattice points on the line and in the first quadrant are finite in number. Incidently, this is not the case for $2x - 3y = -40$.

A similar problem[2] is the following one.

(II) A band of 13 pirates obtained a certain number of gold coins. They tried to distribute them equitably, but found that they had 8 left. Two pirates died of small pox; then upon trying to distribute the coins equitably among the 11 remaining pirates they found that there were 3 left. Thereupon they shot 3 pirates, but still there was a remainder of 5 coins when they attempted an equal division among the 8 remaining pirates. How many coins were there?

Let N be the total number of coins. Let x, y, z be the share of each pirate in the three cases of division. Then

$$N = 13x + 8 = 11y + 3 = 8z + 5,$$

or

$$13x - 11y = -5, \qquad (1)$$
$$11y - 8z = 2. \qquad (2)$$

To solve the problem we must find positive integral solutions x, y, z of (1) and (2). In solving this problem we

[2] This problem is usually solved as a system of congruences: $N \equiv 8, \bmod 13$; $N \equiv 3, \bmod 11$: $N \equiv 5, \bmod 8$.

use nothing more than the elementary observation that when u/v is an integer, then $u = nv$ for some integer n.

Upon solving (1) for y we secure

$$y = x + \frac{2x + 5}{11}.$$

If we let $2x + 5 = 11a$, then

$$x = 5a - 2 + \frac{a - 1}{2}$$

from which we see that $a = 2b + 1$ for some integer b. Thus,

$$x = 11b + 3, \quad y = 13b + 4.$$

In equation (2)

$$8z = 11y - 2 = 143b + 42,$$

or

$$z = 5 + 17b + \frac{2 + 7b}{8}.$$

Thus $2 + 7b = 8c$, for some integer c, hence

$$b = c + \frac{c - 2}{7}.$$

If we let $c - 2 = 7d$, d an integer, we secure $b = 8d + 2$ and $z = 143d + 41$. If x, y, and z are needed,

$$x = 88d + 25, \quad y = 104d + 30, \quad z = 143d + 41.$$

For any value of d these values of x, y, z satisfy (1) and (2). However, we must select integral values of d and such that x, y, z are positive. Clearly $d < 0$ is impossible but any $d = 0, 1, 2, \ldots$ gives an answer to the problem. That is, $N = (13)(88)d + 333$ and we see that the smallest N is 333.

These problems are examples of what have come to be called Diophantine (after Diophantos, 250 A.D.) or **indeterminate** problems. Precisely, a Diophantine problem is one in which a system of equations (or possibly a single equation) is to be solved simultaneously in integers.[3] The examples we have given have involved linear equations only. When only linear equations are involved there are theorems that provide an answer to the question, "Is there a solution?" For example: the single linear equation

[3] Sometimes rational values of the variables are admitted in solutions.

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = c$$

(wherein the a_i and c are given integers, and integral x_i's are sought) has a solution if, and only if, c is divisible by the greatest common divisor of all the given coefficients a_i.

Thus $$2x + 6y = 9$$

has no solution. Similarly the system

$$\begin{aligned} 19x + 10y + 7z + 3 &= 0, \\ 5x + 2y + 2z + 1 &= 0, \end{aligned}$$

has no solution for, although each individual equation has a solution, these equations imply the equation

$$3x + 6y - 1 = 0$$

which does not have a solution.

The following linear problem is appropriate for a reader who now wants to try his hand at solving a Diophantine problem.

(111) Three men and a monkey gather N coconuts and then go to sleep for the night. During the night one man awakens, divides the coconuts into three equal piles, finds there is one coconut left which he gives to the monkey, hides his share and goes back to sleep. Another man awakens, divides what remains of the coconuts into three equal piles, discovers that there is an extra coconut which he gives to the monkey, hides his third and goes to sleep. The third man awakens and does the same thing the first two men did including giving the one coconut he secures as a remainder to the monkey. When morning comes the three men divide the coconuts that are left into three equal piles and give the monkey the one they secure as a remainder (the monkey now has four coconuts). What is the smallest possible value of N?

Systems of higher degree than one are harder to solve, and the solutions are often very individual. As an example, consider the following problem.

(IV) Find the lengths of the sides of all right triangles of integral sides and with area equal to perimeter. That is, if T is such a triangle and its area is M square units, then its perimeter is M linear units.

There are such triangles, for consider a 5, 12, 13 triangle or a 6, 8, 10 triangle. That all right triangles do not satisfy the conditions is seen by considering the 3, 4, 5 triangle.

Algebraically, we wish to solve the system

$$\frac{ab}{2} = a + b + c,$$

$$c^2 = a^2 + b^2.$$

According to the first equation

$$c^2 = \frac{a^2b^2}{4} + a^2 + b^2 - a^2b - ab^2 + 2ab$$

hence it is necessary that $ab - 4a - 4b + 8 = 0$. Then

$$a = \frac{4(b-2)}{b-4} = 4 + \frac{8}{b-4}.$$

Clearly, $b - 4$ must be a factor of 8. Then $b - 4 = \pm 1, \pm 2, \pm 4, \pm 8$. However, $b - 4 = -4, -8$ are impossible since $b > 0$; $b - 4 = -1, -2$ are impossible since $a > 0$, hence $b - 4 = 1, 2, 4, 8$ or $b = 3, 6, 8, 12$ and $a = 12, 8, 6, 5$, respectively. Thus there are only two such triangles.[4]

The indispensible reference for those people working in Diophantine problems is L. E. Dickson's 800 page volume 2 of the ***History of the Theory of Numbers***.[5] This book is a catalogue of practically all results in this subject up to 1920. Although some theorems treating a general class of problems are possible, such as the one we gave above for one linear equation, most of the work in this field in on individual special problems similar to the three problems already solved.

---

[4] If we do not insist that our triangles be right triangles, (6, 25, 29), (7, 15, 20), and (9, 10, 17) are also such triangles.

[5] Carnegie Institution of Washington, Washington, 1920.

Before examining the book it may be difficult to imagine the labor necessary to compile a comprehensive survey of this field. Dickson said that he spent 15 months reading the proof-sheets! If this gives you an idea of the magnitude of the job (There are over 4000 footnotes.) you will appreciate Dickson's remark in the preface, "Since there already exist too many papers on Diophantine analysis which give only special solutions, it is hoped that all devotees to this subject will in the future refrain from publication until they obtain general theorems on the problem attached if not a complete solution of it."

Diophantine problems come in all degrees of difficulty. Euler, Fermat, Cauchy, Legendre, Lagrange, Gauss and hundreds of lesser mathematicians of the past and present times have contributed to this subject. The tools used in solving Diophantine problems come from many fields. Infinite series, elliptic functions, gamma functions, differential equations, etc., have been used at some time. The theory of continued fractions is an example of one of the more popular elementary tools. For example, continued fractions may be used to solve the so-called "Pell Equation,"

$$x^2 - Ny^2 = 1.$$

This equation is of interest because an integral solution (x,y) gives a rational number approximation of $\sqrt{N}$, since

$$\left(\frac{x}{y}\right)^2 - N = \frac{1}{y^2}.$$

Naturally, the larger the y the better the approximation. For example, as was known before Diophantos was born, (577,408) is a solution of $x^2 - 2y^2 = 1$, hence

$$\frac{(577)^2}{(408)^2} - 2 = \frac{1}{(408)^2} < .000007.$$

We shall not discuss Pell Equations any further since some readers may be unfamiliar with continued fractions. However, we can easily treat the special case $N = 1$, and, more generally, the equation $x^2 - y^2 = q$ by elementary methods.

(V) Suppose that q is an integer. Let us seek integral points on the family of hyperbolas

$$x^2 - y^2 = q. \tag{3}$$

Let $x + y = u$ and $x - y = v$, then

$$x = \frac{u+v}{2}, \quad y = \frac{u-v}{2}, \quad uv = q.$$

If $q = 1$, $(u = 1, v = 1)$ or $(u = -1, v = -1)$ since u and v are integers and $uv = q$. Hence $(x = 1, y = 0)$ or $(x = -1, y = 0)$.

If $q = 2$, then $(u = \pm 2, v = \pm 1)$ or $(u = \pm 1, v = \pm 2)$. However, for these values of u and v, x and y are not integral, hence $x^2 - y^2 = 2$ has no solution in integers.

If q is odd, let $u = q$, $v = 1$ to secure the solution

$$x = \frac{q+1}{2}, \quad y = \frac{q-1}{2}.$$

If q is divisible by 4, so that $q = 4s$, let $u = 2s$, $v = 2$ to secure the integral solutions $x = s + 1$, $y = s - 1$. Then if q is odd or if q is divisible by 4 the equation (3) has at least one solution. However, if q is even but not divisible by 4 (i.e., q is of the form $2(2n + 1)$), (3) has no solution because either u or v is even and the other odd so x and y are not integers.

Let us summarize our observations in a theorem.

Theorem. The equation $x^2 - y^2 = q$, where q is an integer, has integral solutions when, and only when, q is odd or divisible by 4.

In case q is an odd prime, then $uv = q$ implies $(u = \pm q, v = \pm 1)$, $(u = \pm 1, v = \pm q)$, thus

$$x = \pm\frac{q+1}{2}, \quad y = \pm\frac{q-1}{2}; \quad \text{or} \quad x = \pm\frac{q+1}{2}, \quad y = \pm\frac{1-q}{2}$$

and (3) has exactly four solutions.

If q is composite, the number of solutions of (3) will depend upon the factors of q. These solutions are easily determined and are left as an exercise to those who may be interested in them. It should be clear from the foregoing discussion that for a given value of q the number of solutions of (3) is finite.

An equation related to equation (3) is the equation

$$x^2 + y^2 = z^2.$$

This equation's solutions, because of the Pythagorean Theorem, are sides of a right triangle. For example, (3, 4, 5) satisfy the equation and (3n, 4n, 5n), n an integer, do also. Moreover, there are an infinite number of independent[8] solutions of the equation; (5, 12, 13) being independent of (3, 4, 5). However, $x^3 + y^3 = z^3$ has no solution at all except for trivial solutions involving a zero. Euler is credited with solving this problem first but his proof is not clear at one place. Since Euler's time convincing proofs have been provided. Euler was prompted to treat this problem by one of the most famous conjectures of all time and certainly the most famous of all Diophantine problems. The French mathematician Fermat (1608–1665) indicated in the margin of his copy of *Diophantos' Arithmetics* that he could prove that for no three integers x, y, z (none zero) is

$$x^n + y^n = z^n,$$

if n is an integer greater than 2. Fermat gave no proof, saying the margin was not large enough for the proof. Whether he actually had a proof no one knows. He sometimes made mistakes in his notes but often stated results in the margin of his books without proof. This conjecture, known as "Fermat's Last Theorem," is still unproved.

This is an important problem, not in itself but because of the mathematics that has resulted from attempts to prove the theorem. A host of mathematicians of every quality have tried to prove the theorem and some results are known. For example, the theorem can be established for $n \leqq 600$. The problem became very popular about 50 years ago when 100,000 German marks were offered as a prize for a complete solution of the problem.

The theory of Diophantine equations is generally con-

[8]The solutions (a_1, a_2, a_3) and (b_1, b_2, b_3) are called independent provided they are not proportional. That is, they are independent if no real number k satisfies the equations $a_i = kb_i$, $i = 1, 2, 3$, simultaneously.

sidered a part of the theory of numbers and anyone wishing to make a systematic study of these equations would probably commence with the chapter usually devoted to this subject in a number theory book. Beginning textbooks on number theory are easily obtained and generally good in quality. We may take this opportunity to recommend as a readable introduction to number theory H. Davenport's recent book ***The Higher Arithmetic***.[7] The serious student of mathematics should at some time examine one of the more complete works such as Hardy and Wright's ***Theory of Numbers***.[8]

---

[7] **Hutchinson's University Library, London, 1952.**
[8] **Oxford Press, 1938.**

SOME METHODS OF SOLUTION FOR ORDINARY DIFFERENTIAL EQUATIONS

John W. Cell
North Carolina State College

In this article we relate various methods of solution for ordinary differential equations with constant coefficients but with an arbitrary forcing function. Numerical coefficients are employed for convenience and for clarity of exposition, but the same methods could be used if literal coefficients were used. Some of the methods are less well known than others, but all are to be found in various textbooks.

The problem for consideration is to solve

$$\ddot{y} + 4\dot{y} + 3y = f(t) \qquad (1)$$

where $\dot{y} = dy/dt$, and $f(t)$ is an arbitrary function with the requisite mathematical properties. As initial conditions we take at $t = 0$, $y = y_0$ and $\dot{y} = \dot{y}_0$, so that the initial conditions are arbitrary.

I. *Elementary methods*

We consider first the methods usually described in elementary texts on differential equations. We solve the homogeneous equation

$$\ddot{y} + 4\dot{y} + 3y = 0 \qquad (2)$$

for the complementary solution (we assume $y = ae^{mt}$ as a trial solution) and obtain

$$y_c = ae^{-t} + be^{-3t}.$$

The D-operator method gives the particular solution

$$y_p = (D + 3)^{-1}\left[(D + 1)^{-1} f(t)\right]$$

$$= e^{-3t}\int_0^t e^{2s}\int_0^s e^{r} f(r)\,dr\,ds.$$

Since $y_p(0) = 0$, $\dot{y}_p(0) = 0$, the solution satisfying the given initial conditions is easily found to be

$$y = \tfrac{1}{2}(3y_0 + \dot{y}_0)e^{-t} - \tfrac{1}{2}(y_0 + \dot{y}_0)e^{-3t}$$

$$+ e^{-3t}\int_0^t e^{2s}\int_0^s e^r f(r)\,dr\,ds. \qquad (3)$$

We may also use the method of ***variation of parameters*** as follows.

We assume

$$y = a(t)e^{-t} + b(t)e^{-3t} \qquad (4)$$

to be a solution of the original equation. Then

$$\dot{y} = -ae^{-t} - 3be^{-3t} \qquad (5)$$

where we arbitrarily set

$$\dot{a}e^{-t} + \dot{b}e^{-3t} = 0. \qquad (6)$$

Then

$$\ddot{y} = ae^{-t} + 9be^{-3t} - \dot{a}e^{-t} - 3\dot{b}e^{-3t}. \qquad (7)$$

We substitute from equations (4), (5), and (7) in equation (1) and obtain

$$\dot{a}e^{-t} + 3\dot{b}e^{-3t} = -f(t). \qquad (8)$$

We solve (6) and (8) simultaneously and find

$$\dot{a} = \tfrac{1}{2}f(t)e^{t}, \qquad \dot{b} = -\tfrac{1}{2}f(t)e^{3t}. \qquad (9)$$

We integrate and determine

$$a(t) = A + \tfrac{1}{2}\int_0^t f(u)e^u du,$$

$$b(t) = B - \tfrac{1}{2}\int_0^t f(u)e^{3u}du.$$

We substitute these in (4) and obtain

$$y = Ae^{-t} + Be^{-3t} + \tfrac{1}{2}\int_0^t f(u)\,[e^{u-t} - e^{3u-3t}]\,du. \qquad (10)$$

Our next determines A and B to satisfy the initial conditions. Differentiation of (10) with respect to **t** yields

$$\dot{y} = -Ae^{-t} - 3Be^{-3t} + \tfrac{1}{2}\int_0^t f(u)\,[-e^{u-t} + 3e^{3u-3t}]\,dt.$$

On placing $y = y_0$ and $\dot{y} = \dot{y}_0$ when $t = 0$, solving the resulting equations for A and B, and substituting these in (10) we obtain

$$y = \tfrac{1}{2}(3y_0+\dot{y}_0)e^{-t} - \tfrac{1}{2}(y_0+\dot{y}_0)e^{-3t} + \tfrac{1}{2}\int_0^t f(u)\,[e^{u-t} - e^{3u-3t}]\,du. \qquad (11)$$

It is interesting to note that (3) reduces to (11) by using integration by parts on the iterated integral, taking

$$u = \int_0^s e^r f(r)dr, \qquad dv = e^{2s}ds.$$

We observe that here we first obtain a general solution (equation 10) and from this the particular solution for the initial conditions (equation 11).

II. *Laplace Transform Methods*

The method of solution by **Laplace** transforms is explained in many textbooks such as, for example, ***Advanced Engineering Mathematics*** by C. **R. Wylie,** Jr. **(McGraw-Hill** Book Company, 1951) and ***Modem Operational Mathematics in Engineering*** by R. **V.** Churchill **(McGraw-Hill** Book Company, 1944). For an excellent brief discussion of **Laplace** and other transforms the reader is referred to ***Integral Transforms*** in ***Mathematical Physics*** by C. J. Tranter (John Wiley and Sons, 1951). The basic idea of this method is to utilize **Y(s)** defined by

$$Y(s) = \int_0^\infty e^{-st}\,y(t)dt. \qquad (12)$$

This relates **Y(s)** to **y(t),** where **Y(s)** is called the **Laplace** transform of **y(t).** Important theorems are obtained for this transform and tables of transforms derived so that the **ac-**

tual solution of a problem, such as the one of this article, is simple. Thus, the solution of the problem involves as the first step the transformation of the given differential equation (1) with the use of the initial conditions to obtain

$$s^2Y - sy_0 - \dot{y}_0 + 4sY - 4y_0 + 3Y = F(s), \tag{13}$$

where

$$F(s) = \int_0^\infty e^{-st} f(t)dt.$$

If the individual who is unacquainted with this method will multiply equation (1) by $e^{-st}dt$, integrate from t = 0 to t = w, utilize integration by parts, and assume that (for example) $\lim_{t\to\infty} \int_0^t e^{-st} y(t)dt = 0$, he will obtain equation **(13)**.

We solve equation (13) for Y and obtain

$$Y = \frac{(s+4)y_0 + \dot{y}_0}{(s+3)(s+1)} + \frac{1}{(s+3)(s+1)} F(s)$$

$$= \tfrac{1}{2}\left\{\frac{3y_0 + \dot{y}_0}{s+1} - \frac{y_0 + \dot{y}_0}{s+3}\right\} + \tfrac{1}{2}\left\{\frac{1}{s+1} - \frac{1}{s+3}\right\} F(s).$$

One next obtains the inverse transform by use of tables of **Laplace** transforms and obtains

$$y = \tfrac{1}{2}(3y_0+\dot{y}_0)e^{-t} - \tfrac{1}{2}(y_0+\dot{y}_0)e^{-3t} + \tfrac{1}{2}\int_0^t f(u)\,[e^{u-t} - e^{3u-3t}]\,du \tag{14}$$

which is the same result as in equation (11).

We may, if we wish, modify the above procedure and arrive at the same end result. We first solve

$$\ddot{y}\ t\ 49\ t\ 3y = \frac{1}{\delta}[S(t-\alpha) - S(t-\alpha-i)] \tag{15}$$

where

$$S(t) = 0, \quad t<O$$
$$= \tfrac{1}{2}, \quad t = 0$$
$$= 1, \quad t > 0,$$

and use as initial conditions y = 0 and $\dot{y}$ = 0 when t = 0.

The right-hand member of equation (15) has a graph as shown in Figure 1.

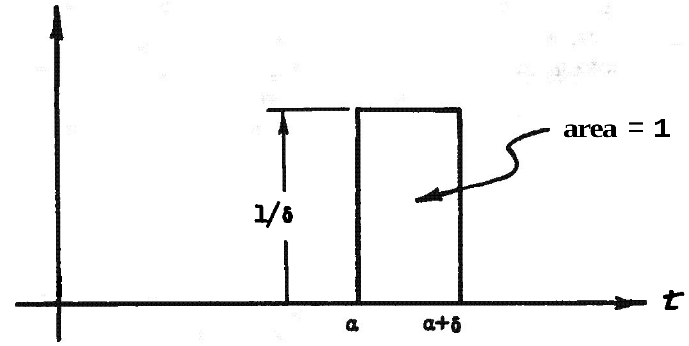


Figure 1

This function is commonly described as an impulse function, and it may be assumed that 6 is small as compared to unity.

Then on taking the **Laplace** transform we obtain

$$s^2Y + 4sY + 3Y = \frac{1}{\delta s}[e^{-\alpha s} - e^{-(\alpha+\delta)s}],$$

whence

$$Y = \frac{e^{-\alpha s}}{(s+1)(s+3)} \cdot \frac{1-e^{-\delta s}}{\delta s}.$$

One can show that it is permissible, instead of finding the inverse transform and then letting 6 approach zero, to reverse that order of operation. Letting 6 approach zero and separating the function into partial fractions we obtain

$$\text{new } Y = \tfrac{1}{2}\left[\frac{1}{s+1} - \frac{1}{s+3}\right]e^{-\alpha s}, \tag{16}$$

whence the inverse transform is

$$y = \tfrac{1}{2}[e^{-(t-\alpha)} - e^{-3(t-\alpha)}]\,S(t-\alpha). \tag{17}$$

This solution (17) may be considered as an approximation to the correct solution for (15) together with the stated initial conditions. The smaller the value of δ the more accurate is this solution. (The approximation is much like that of stating that since $\lim_{x \to 0} \frac{\sin x}{x} = 1$, $\sin x \approx x$ for small values of x.)

We next suppose that the area under **f(t)** is made up of a series of impulses or, in mathematical language, we consider the area under the curve for **f(t)** to be replaced by a sequence of rectangles as shown in Figure 2 (with $\Delta\alpha$ chosen so that $n\Delta\alpha = t$ where n is the number of rectangles.)

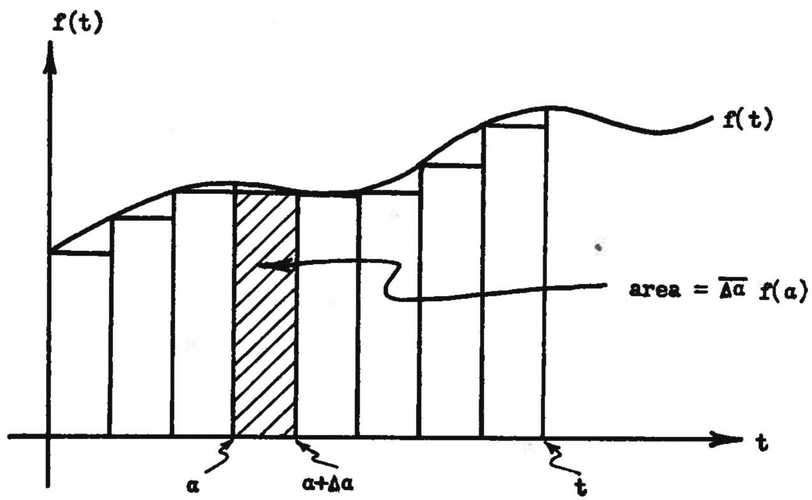


Figure 2

For each rectangle the solution of (1) with zero initial conditions is given by (17) multiplied by the area of the rectangle. Thus the solution for the rectangle between $t = \alpha$ and $t = \alpha + \Delta\alpha$ is

$$\tfrac{1}{2}\left[e^{-(t-\alpha)} - e^{-3(t-\alpha)}\right] S(t-\alpha) f(\alpha)\, \overline{\Delta\alpha}. \qquad (18)$$

The limit of the sum of these as $\Delta\alpha$ approaches zero is given by

$$\tfrac{1}{2}\int_0^t \left[e^{-(t-\alpha)} - e^{-3(t-\alpha)}\right] f(\alpha)\, d\alpha \qquad (19)$$

where we may omit the factor $S(t-\alpha)$ since it is superfluous in the integral. To complete the problem by this method we would combine the result (19) with terms to account for the original initial conditions.

The procedure in going from (18) to (19) utilizes the fundamental theorem of integral calculus. But, more important, it involves what is known as ***superposition,*** a property of linear differential equations. As applied to our example this principle states that if y is a solution of

$$\ddot{y} + 4\dot{y} + 3y = f_1(t)$$

and y_2 a solution of

$$\ddot{y} + 4\dot{y} + 3y = f_2(t)$$

then $y_1 + y_2$ is a solution of

$$\ddot{y} + 4\dot{y} + 3y = f_1(t) + f_2(t).$$

This language of impulse functions is or should be used by physicists and electrical engineers as a shorthand. Actually, the final result (19) has ***no*** approximation involved since the steps of taking limits to obtain (16) and thence **(17),** and again (18) to (19) may be combined into a single rigorous operation.

III. *Green's Function*

We solve first equation (2) with the conditions: $t = 0$; $y = 0$, $\dot{y} = 0$; $t = \alpha$; y is continuous and $\dot{y}(\alpha+) - \dot{y}(\alpha-) = 1$. This last is a shorthand for the statement that $\dot{y}(t)$ is to be continuous except at $t = \alpha$ where it is to have a unit upward jump.

The general solution of the homogeneous equation is

$$y = ae^{-t} + be^{-3t}.$$

From $t = 0$ to $t = \alpha$, one can see that $a = b = 0$. Assuming then that

$$t < \alpha, \quad y = 0$$
$$t > \alpha, \quad y = ae^{-t} + be^{-3t}.$$

We obtain from the continuity requirements on $y(t)$ at $t = \alpha$:

$$ae^{-\alpha} + be^{-3\alpha} = 0,$$

and from the "jump requirement" on $\dot{y}(t)$ at $t = \alpha$:

$$(-ae^{-\alpha} - 3be^{-3\alpha}) - (0) = 1.$$

We solve these and obtain

$$a = \tfrac{1}{2}e^{\alpha}, \qquad b = -\tfrac{1}{2}e^{3\alpha}.$$

The required solution may be written as

$$y = \tfrac{1}{2}\left[e^{\alpha-t} - e^{3-3t}\right] S(t-\alpha),$$

where $S(t-\alpha)$ is appended as a multiplier to show that $y = 0$ when $t < \alpha$ and that y is the first portion of the right-hand term when $t > \alpha$. This final result is called the ***Green's function*** and is likewise, by comparison to equation **(17)**, the solution due to the unit impulse at $t = a$.

By aid of superposition as before and with the addition of the terms due to the initial conditions we are led to the same final result as given by equation (11).

IV. Characteristic Functions

We denote by $x(t)$ the solution of the homogeneous differential equation (2) and the initial conditions $t = 0$: $y = 1$ and $\dot{y} = 0$. This solution is easily found to be

$$x(t) = \tfrac{1}{2}(3e^{-t} - e^{-3t}). \tag{20}$$

We next denote by $v(t)$ the solution of the same equation subject to the conditions $t = 0$: $y = 0$, $\dot{y} = 1$. Then

$$v(t) = \tfrac{1}{2}(e^{-t} - e^{-3t}). \tag{21}$$

The final result (11) for the given problem may now be written in the form

$$y = y_0[x(t)] + \dot{y}_0[v(t)] + \int_0^t v(t-u)\, f(u)\, du. \tag{22}$$

These two functions $x(t)$ and $v(t)$ are ***characteristic functions.*** They are the solutions of the given problem for unit initial values respectively for y_0 and $\dot{y}_0$ and with $f(t) = 0$.

The given differential equation could have arisen as a solution of an electrical circuit problem or as a vibration problem. Thus, the differential equation for the **spring-**

mass-damping system in Figure 3 could be written as

$$m\ddot{y} + c\dot{y} + ky = f(t)$$

and equation **(1)** is a special case.

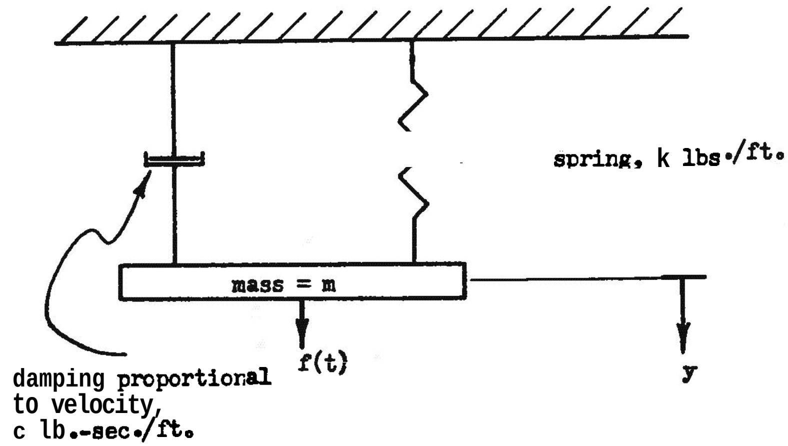


Figure 3

V. Summary and Discussion

The reader will have observed that both elementary methods, both **Laplace** transform methods, and the Green's function all lead to a particular integral for the solution of equation (1). For one who is familiar with the five methods the shortest solution of the given problem in terms of characteristic functions is by the first **Laplace** transform method. But the other three methods are important in other situations.

From the result in equation (22) we may derive an elementary but interesting theorem. For convenience we take $y_0 = 0$ and $\dot{y}_0 = 0$. Suppose that $f(u)$ is an impulse function as shown in Figure 1. Then equation (22) may be written as

$$y = \int_{\alpha}^{\alpha+\delta} v(t-u)\left[\frac{1}{\delta}\right] du$$

and

$$\lim_{\delta \to 0} y = \begin{cases} 0, & t < \alpha \\ v(t-\alpha), & t > \alpha \end{cases} .$$

Thus in the vibration-problem language the displacement of a mass initially at rest and acted upon by an impulsive function of unit area at time $t = \alpha$ is the same as the displacement of the mass after $t = \alpha$ that would result from a unit "initial" velocity at $t = \alpha$ and no impulsive force function. Stated differently, applying a unit impulse at any instant has the same effect on the subsequent motion of the mass as would result if the mass at that same instant were given an added unit velocity.

A NOTE ON QUALIFICATIONS FOR A CHAPTER OF PI MU EPSILON FRATERNITY

Chapters of Pi Mu Epsilon, national honorary mathematical fraternity, may be chartered only in colleges and universities requiring at least 8 semester hours ***in mathematics*** beyond two semesters of calculus for a mathematics major and having an average of at least 5 majors per year. The institution of the petitioning group must have at least one **Ph.D.** ***in mathematics*** on its staff and must have an active honorary mathematics club in existence and operating successfully at the institution for at least one year prior to petitioning for a charter. For further information, faculty members of institutions meeting these requirements may write to Richard V. Andree, Department of Mathematics, The University of Oklahoma, Norman, Oklahoma.

A MATRIX DERIVATION OF SOME TRIGONOMETRIC IDENTITIES

Gary H. **Meisters**
Iowa State College
Ames, Iowa

This note concerns a matrix derivation of the trigonometric identities:

$$\sin(x+y) \equiv \cos x \sin y + \sin x \cos y$$
$$\cos(x+y) \equiv \cos x \cos y - \sin x \sin y$$

In the derivation, the following four items will be assumed given.

(1) Matrix multiplication and addition.

(2) $e^M \equiv I + M + \frac{1}{2!}M^2 + \dots$, where e is the base of the natural logarithm and M is any square matrix.

(3) $e^A \cdot e^B = e^{A+B}$ if A-B = B-A, where A and B are matrices.

(4) $\sin x \equiv x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

$\cos x \equiv 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

We may write

$$e^{x\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}} \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + x\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \frac{x^2}{2!}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \frac{x^3}{3!}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + \frac{x^4}{4!}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$+ \frac{x^5}{5!}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \cdots$$

$$\equiv \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}, \text{ where x is a real variable.}$$

But $e^{x\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}} \cdot e^{y\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}} \equiv e^{(x+y)\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}$

or

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \cdot \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$$

Hence,

$$\cos x \cos y - \sin x \sin y = \cos(x+y)$$

and

$$\cos x \sin y + \sin x \cos y = \sin(x+y).$$

PROBLEM DEPARTMENT

Edited by

Leo Moser, University of Alberta

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within five months after publication. Address all communications concerning problems to Leo Moser, Mathematics Department, University of Alberta, Edmonton, Alberta, Canada.

PROBLEMS FOR SOLUTION

68. Proposed by Lean Bankoff, Los Angeles, California

An ellipse of maximum area is inscribed in a given triangle. Show that the area of the smallest quadrilateral circumscribing this ellipse is less than the geometric mean and greater than the harmonic mean of the areas of the ellipse and the triangle.

69. Proposed by C. W. Trigg, Los Angeles City College

Two frictionless planes, inclined at angles of 10° and 30° to the horizontal, are joined at their tops, where an ideal (frictionless, massless) pulley is placed. An ideal string parallel to the planes passes over the pulley. To the ends of the string are attached masses of 31-Kg. and 47-Kg., one mass resting on each plane. When the masses are released, the string will be under tension. To what angle must the inclination of the 10°-plane be changed in order that the tension in the string may be doubled?

70. ***Proposed by Pedro Piza, San Juan, Puerto Rico***

Find eight distinct numbers $a_1, a_2, \ldots, a_8$ which satisfy

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 = a_5^2 + a_6^2 + a_7^2 + a_8^2$$

and also, for arbitrary k, satisfy

$$(a_1 + k)^2 + (a_2 + k)^2 + (a_3 + k)^2 + (a_4 + k)^2 =$$
$$(a_5 + k)^2 + (a_6 + k)^2 + (a_7 + k)^2 + (a_8 + k)^2.$$

71. ***Proposed by A. J. Goldman, Princeton University***

Let S be the set of $m \times n$ matrices with all mn entries distinct, and let the matrix A be chosen at random from S. If entry a_{ij} of A is the least entry in the i-th row and the greatest entry in the j-th column, then the pair (i,j) is called a ***saddlepoint*** of A. (This concept is important in game theory.) Prove that A has at most one saddlepoint, and find the probability that A has a saddlepoint.

72. ***Proposed by Ken U. Summit, Adder College***

Evaluate the following

$$\sum_{i=0}^{\infty} \frac{\sin i}{i!}$$

SOLUTIONS

40. ***Proposed by J. H. Butchart, Arizona State College***

Three infinitely long parallel wires carry electrostatic charges e_1, e_2, e_3 per unit length, where $2e_1 = 2e_2 = -e_3$. Show that if the cross section of the wires is an equilateral triangle then the circumscribed circle to this triangle is a line of force.

Solution by the proposer

The effect of the long parallel charges is to produce lines of force in parallel planes perpendicular to the wires. In one of these planes, the charges can be replaced by point charges where the force at any point is the vector sum of the forces due to all the charges, and each of these forces varies as the strength of the charge and inversely as the distance from the arbitrary point to the charge. To prove that the circle through the equilateral triangle of charges is a line of force, invert the figure with respect to an arbitrary point of this circle. The inverses of the lines joining the arbitrary point to the charges all end on a line, and when these are multiplied by the charges, remembering algebraic signs, and summed vectorially, the resultant force is along the tangent to the circle.

Clearly the same proof applies to any number of charges lying on a circle if their algebraic sum is zero.

57. ***Proposed by F. L. Miksa, Aurora, Illinois***

(a) Prove that the number representing the area of a Pythagorean triangle must end in 0, 4, or 6 when written in base ten.

(b) Prove that the number representing the product of the lengths of the sides of a Pythagorean triangle is divisible by 60.

Solution by Mel Stover, Winnipeg, Manitoba

(a) The sides of a Pythagorean triangle have the form

$$a = 2kmn \qquad b = k(m^2 - n^2) \qquad c = k(m^2 + n^2).$$

Hence the area A has the form $A = k^2 \cdot m \cdot n(m^2 - n^2)$. Clearly A is even so it remains to find the residue of A (mod 5). Testing the various residues (mod 5) we find that $k^2 \equiv 0, \pm 1 \pmod 5$ while $m \cdot n \cdot (m^2 - n^2) \equiv 0, \pm 1 \pmod 5$ and hence $A \equiv 0, \pm 1 \pmod 5$. Combining this with the evenness of A yields the required result.

(b) The product of the sides of a Pythagorean triangle is of form $P = k^3 \cdot 2 \cdot (m^5 n - mn^5)$. Now it is easily verified (using Euler's theorem or otherwise) that $x^5 \equiv x \pmod{30}$. Hence

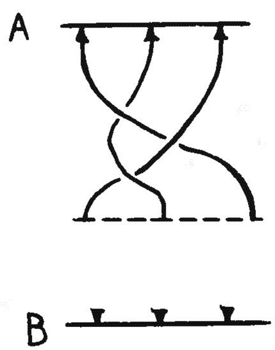
$$m^5 n - mn^5 \equiv mn - mn \equiv 0 \pmod{30}$$

so that $p \equiv 0 \pmod{60}$ as required.

Also solved by K. A. Brons and C. W. Trigg.

60. ***Proposed by J. Lambek, McGill University***

Three strings are tied to three pegs on a board A. They are tangled and interwoven as shown in the figure.



It is required to tie three other strings to the three free ends and attach the free ends of the new strings, which may be tangled, to the three pegs of board B, in such a way that the resultant entanglement can be combed out to give three loosely parallel strings. How can this be done?

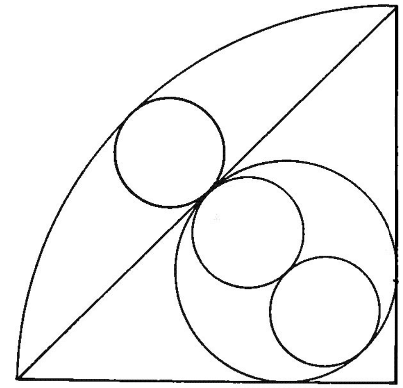
Solution by N. Grossman, Aurora, Illinois

The required pattern of strings may be obtained from the given pattern by taking its reflection in the dotted line. In the theory of braids such a pattern and its reflection are called inverse braids.

Also solved by F. Gross, N. Schecter, C. W. Trigg and the proposer.

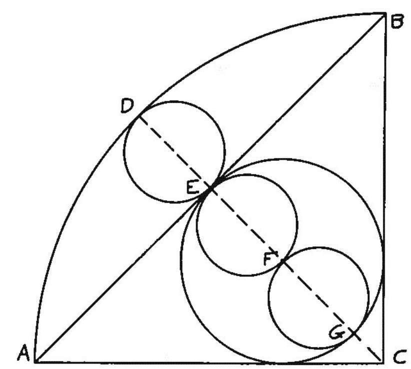
63. ***proposed by Leon Bankoff, Los Angeles, California***

State and solve the problem suggested by the following diagram.



Solution by C. W. Trigg, Los Angeles City College

Theorem: The maximum circle inscriptible in the segment of a quadrantal sector has one-half the diameter of the inscribed circle of the residual triangle of the sector.



Let the radius bisecting the right angle ACB be CD which meets AB in E. Then DE is the diameter of the maximum circle inscriptible in segment ADB.

Let CA = CD = CB = 1, then AB = $\sqrt{2}$, EC = $\frac{1}{2}\sqrt{2}$, and DE = $1 - \frac{1}{2}\sqrt{2}$. Now the diameter of the inscribed circle of the right triangle is a+b−c, so EF + FG = $1 + 1 - \sqrt{2}$. Thus DE = $\frac{1}{2}$(EG) = EF = FG = $1 - \frac{1}{2}\sqrt{2}$.

Also solved by F. Gross, T. Sumner and the proposer.

64. ***Proposed by Fred Gross, Brooklyn College***

Prove that

$$x^9 - 6x^7 + 9x^5 - 4x^3$$

is divisible by 27 for all positive integral values of x.

Solution by W. S. Stewart, University of Arkansas

$f(x) = x^9 - 6x^7 + 9x^5 - 4x^3 = [(x-2)(x-1)(x)]\ [(x-1)(x)(x+1)]\ [x(x+1)(x+2)]$.

Each square bracket is the product of 3 consecutive numbers and so divisible by 3. The product is therefore divisible by 27.

Also solved by F. Gross, L. S. Grinstein, T. Sumner, C. W. Trigg and the proposer.

Trigg further proved that for integral x, $f(x)$ is divisible by 8640.

66. ***Proposed by C. W. Trigg, Los Angeles City College***

If three circles with radii **a,b,c,** are externally tangent, there are two circles with radii **r,R,** which touch the three circles. Show that

$$1/r - 1/R = 2\ (1/a + 1/b + 1/c)$$

and that

$$1/r + 1/R = 4\sqrt{(a+b+c)/abc}.$$

Solution by the Proposer

It is shown in the solution to problem 2293, ***School Science and Mathematics, 53,*** 75, January 1953, that **r** and R are given by

$$abc/\ [2\sqrt{abc(a+b+c)} \pm (ab+bc+ca)].$$

The stated relationships follow immediately. The first of these equalities is stated without proof by Thomas Muir in the ***Proceedings of the Edinburgh Mathematical Society,*** Volume 3, page 119, (1884–5).

67. ***Proposed by Pedro Piza, San Juan, Puerto Rico***

Find four numbers in arithmetical progression and three in geometrical progression such that the sum of the squares of the four is equal to the sum of the squares of the three.

Solution by T. Sumner, University of North Carolina

Let a, a+d, **a+2d, a+3d,** be the A.P. and b, cb, c^2b the G.P. We require that

$$a^2 + (a+d)^2 + (a+2d)^2 + (a+3d)^2 = b^2 + c^2b^2 + c^4b^2.$$

This reduces to

$$4a^2 + 12ad + 14d^2 = b^2(1+c^2+c^4).$$

The left member is even but $1 + c^2 + c^4$ is odd for all c. Hence b is even and 4 divides the left hand side so that d is even. Taking minimal positive values for **a,b,d,** we set a = 1, b = 2, d = 2. This yields c = 2 and the progressions **1,3,5,7** and **2,4,8** with

$$1^2 + 3^2 + 5^2 + 7^2 = 2^2 + 4^2 + 8^2 = 84.$$

Also solved by F. Gross, J. Kaplan, C. W. Trigg and the proposer.

The proposer derives the result from the identity

$$1 + (n + 1)^2 + (s + 1)^2 + (n + s + 1)^2 = n^2 + s^2 + (n + s + 2)^2$$

by setting n = 2 and s = 4.

November 9, 1954

Dear Fellow Members of Pi Mu Epsilon:

The chapter responses received to date favor holding an annual meeting of Pi Mu Epsilon in late August 1955 at Ann Arbor, Michigan, in conjunction with the other mathematical meetings. The success of such a meeting depends, in part, upon the cooperation of ***your*** chapter. Will one of your undergraduate or first-year graduate students have a paper to present? We hope so. Now is the time to begin thinking about student speakers and delegates. We sincerely hope that ***your*** chapter will be well represented.

If you mail certificate orders to the wrong address, a delay in receiving the certificates is inevitable. Please change your records and old certificate order forms to include my address.

Several chapters apparently publish news sheets. If your chapter does, please send copies to the **Secretary-**Treasurer General and to the Editor of the Pi Mu Epsilon Journal. If your chapter sponsors other interesting activities, please mention them in your annual report. In this fashion your work may eventually help other chapters as well as your own.

Cordially and fraternally yours,

Richard V. Andree

NEW GENERAL OFFICERS OF THE FRATERNITY

DIRECTOR GENERAL*

STEWART SCOTT CAIRNS, Professor and Head of the Mathematics Department at the University of Illinois, was born in Franklin, N. H., and attended public schools in Tennessee and Massachusetts.

He graduated from Chelsea (Mass.) High School in 1922 and then studied at Harvard University, receiving his A.B. degree in mathematics in 1926. He received his M.A. in 1927 and his **Ph.D.** degree in 1931, both from Harvard and in Mathematics.

He was instructor in mathematics at Harvard University, 1927-28, and at Yale University, 1929-31. From **1931-**37 he was instructor and later assistant professor of mathematics at **Lehigh** University, moving then to Queens College (N.Y.) as assistant professor until 1946. He served as professor and chairman of the Mathematics Department at Syracuse University from 1946-48 before coming to the University of Illinois the latter year as professor and department head.

Professor Cairns is a member of the American Mathematical Society, the American Association for the Advancement of Science, and the Mathematical Association of America. His fields of special interest are topology and analysis. He is the author of numerous articles in mathematical journals.

From 1944-46 he served on various groups under the Applied Mathematics Panel of the National Defense Research Council. He was vice-chairman of the Division of Mathematics and Physical Sciences of the National Research Council 1949-50; consultant for the Research and Development Board, 1950-.

***For photograph of the new director general, please see Frontispiece of this issue of the Journal.**

In 1954, he received a Fulbright Grant for research at the University of Strasbourg, in France, for the next academic year. Professor and Mrs. Cairns are now in Europe. They have two sons, James Donald, 23, with U.S. Navy, and Charles Edward, 19, sophomore at Harvard. (The photograph, frontispiece in this number of the Journal, and biographical data kindly furnished by Bureau of Public Relations, University of Illinois)

JAMES SUTHERLAND FRAME

VICE-DIRECTOR GENERAL
(Also, Acting Director General for the year 1954-1955)

JAMES SUTHERLAND FRAME, Professor of Mathematics and Chairman of Department, Michigan State College. (For biographical data, please see Vol. 1, No. 5, p. 192, this Journal.)

WEALTHY BABCOCK

COUNCILOR GENERAL

WEALTHY BABCOCK, **Associate Professor** of Mathematics, University of Kansas. Native of Washington County Kansas. A.B., M.A., **Ph.D.,** University of Kansas. Instructor, Assistant Professor, Associate Professor, **1940—,** University of Kansas. Member of the American Mathematical Society, the Mathematical Association of **America,** ***Pi*** Mu ***Epsilon,*** Pi Lambda Theta, Phi Beta Kappa and Sigma Xi. Geometry associated with certain determinants with linear elements.

COUNCILOR **GENERAL**
and BUSINESS **MANAGER**
OF THE **JOURNAL**

HENRY WRIGHT **FARNHAM,** Associate Professor of Mathematics, Syracuse Univ. Native of Rochester, N.Y. **Ph.B.,** Yale Univ., and M.S., Syracuse Univ. Member of Delta Kappa Epsilon and Pi Mu ***Epsilon*** Fraternities. Formerly Sales Engineer for Taylor Instrument Co. of Rochester, District Manager for the Brooks Co. of Cleveland, and Actuarial! for Underwriters Ass**oc.** for N.Y. State. Joined the M**ath.** Dept. of the Coll. of Business **Ad**ministration, Syracuse Univ., **1921,** and later served as Acting Chairman of the Dept. and Counselor to Business Administration Students. Became a member of the Univ. Dept. of Math. at Syracuse Univ. when it was formed in **1943,** combining the mathematics of the three colleges: Business Administration, Engineering, and Liberal Arts.

HENRY WRIGHT **FARNHAM**

NOTE: Photographs **and** biographical data of the new secretary-treasurer general **and** the two other new councilors general will be published in the April **1955** issue.

EXCERPTS FROM LETTERS TO THE EDITOR

"I am very glad to join the $\pi\mu\epsilon$. I came from Japan 1951 and entered U. C. L. A. in 1952 fall.

"I'd like to have the journal and enclose $1.50 money order.

"I hope for progress of the fraternity."

10 June 1954 — Masako Oba

"Thank you for sending me your Journal, with its interesting article on Dr. Mina Rees.

"Please note that I have moved from Strathearn Boulevard to 67 Roxborough Drive, Toronto 5, (Canada)."

12 June 1954 — H. S. M. **Coxeter**
University of Toronto

"Of course I derived a great deal of pleasure from seeing my article in print in the Journal (Metric Extension, Vol. **1,** No. 10, **p. 400,** PME J.). I had never realized before how fine the typography and format are.. **. .**This has been a most gratifying experience, and I certainly hope to contribute again to the Journal.

"Thank you for your encouragement and interest."

21 June 1954 — Alan **J. Goldman (Graduate** Student)
Princeton, New Jersey

"I shall be away from **Cornell** for the forthcoming year, and I am enclosing a notice of my new address. I shall try to induce some of my new associates at UCLA to contribute something to the Journal.

"After September 20, 1954, my address will be:

Numerical Analysis Research
University of California
405 Hilgard Avenue
Los Angeles 24, California"

R. J. Walker
31 July 1954 Associate Editor of PME J.

"We recently acquired a third child (first son) and this as well as moving, etc., has kept me very busy. I shall have to postpone for a while sending you the ***Hexahexaflexagram*** and the article (for the Journal) I promised you."

Leo Moser
16 September 1954 Editor of the Problem Department

"Members of Pi Mu Epsilon may be interested to know that Dio L. Holl, **Ph.D.,** Head of the Mathematics Department of Iowa State College, and member of Pi Mu Epsilon, died unexpectedly May 20, **1954."**

Gary H. **Meisters**
9 September 1954 Director of Iowa Alpha Chapter

Last July, through the kindness and friendly cooperation of Professor S. S. Cairns, Head of the Mathematics Department at the University of Illinois and member of the U.S.A. Subcommittee of the International Congress of Mathematicians, for the Meeting in Amsterdam 2-9 September 1954, we were invited to send to the chairman of exhibits copies of the Journal to form a part of an exhibit of materials suited to the interests of young mathematics students of the age group from 16 to 21 years. Accordingly, we very promptly sent a copy of Vol. 1, Nos. 1-10, to M. **André Cardot,** Centre National de Documentation **Pédagogique,** Paris, France. The following letter is his **acknowledgement** of receipt of the Journals.

Ministère
de
L'EDUCATION NATIONALE

CENTRE NATIONAL
de
DOCUMENTATION PEDAGOGIQUE

Musée Pédagogique
29, rue **d'Ulm** Paris **5e**

Tél. ODEon 75-50

Réf. : MP/I/N° 2.027

Paris, **le 22 juillet** 1954

Monsieur le Directeur
Ruth W. Stokes. Editor
Howard C. Bennett, Business Mgr.
15 Smith **College**
Syracuse University
Syracuse 10, New York **(U.S.A);**

Monsieur **le Directeur,**

J'ai l'honneur de vous accuser **réception des** documents **mentionnés sur** la **liste** ci-jointe, que **vous** avez **eu l'amabilité de mettre** à **ma** disposition pour **l'exposition organisée** à **l'occasion** du **Congrès** International **des Mathématiques** (Amsterdam - 2-9 **septembre** 1954) **et qui** me sont **parvenus aujourd'hui.**

Avec **mes remerciements, je vous prie d'agréer,** Monsieur la **Directeur, l'expression de mes** sentiments **distingués.**

André Cardot

Pi Mu Epsilon Journal - the official publication of The Honorary Mathematical Fraternity.

n° : 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10

REPORTS OF THE CHAPTERS

(Send reports to Ruth W. Stokes, 15 Smith College, Syracuse University, Syracuse 10, New York.)

EDITOR'S NOTE. According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary-General and to the Director-General, an annual report of the chapter activities including programs, results of elections, etc." The Secretary-General now suggests that an additional copy of the annual report of each chapter be sent to the editor of the Pi Mu Epsilon Journal. Besides the information listed above we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships. These annual reports will be published in the chronological order in which they are received.

Beta of California, University of California

During the academic year 1953-1954, the California Beta chapter held seven regular meetings. In addition there was an initiation banquet in December and a picnic in May. Twenty-five new members were initiated during the year.

At the program meetings the following papers were presented:

"What is a Method?" by Professor Raphael Robinson

'Periodic Surfaces" by Professor Steplon Diliberto

"Games and Programs" by Philip Welfe

"Some Generalizations of Elementary Notions" by Dr. Marvin Epstein

"The Convexity of the Gamma Function" by Professor Leon Henkin

"Number-Theoretic Exploration via High Speed Computers" by Professor D. H. Lehmer

"Boolean Algebras" by Professor John L. Kelley.

Officers elected for the 1954-55 year are as follows: Director, Dana Scott; Vice-Director, Eva Kallin; Secretary, Geneva Grosz Belford; Treasurer, Isaac Namioka.

Beta of Alabama, Alabama Polytechnic Institute

The Alabama Beta Chapter of Pi Mu Epsilon held twelve meetings during the academic year 1953-54. This included two initiation meetings at which forty-four persons were initiated. During the year the following papers were presented:

"Non-Linear Differential Equations" by Dr. E. Ikenberry

"Riemann-Steinitz Theories on Rearrangement of Series" by Professor C. W. McArthur

"The Ham Sandwich Theorem" by Dr. M. K. Fort, University of Georgia.

"Determinants" by Paul Williams

"Simple Consequences of the Field Postulates" by Dr. W. A. Rutledge

"Mathematicians" by Anne Parker

"Boundary Value Problems" by John Herron

"Magic Squares" by Miriam Horton

"Non-Analytic Functions" by Dr. W. C. Royster

"The Roots of Unity" by Jewel Magee.

Officers for 1953-54 were: President, John Herron; Vice-President, Alex Taylor; Secretary-Treasurer, Anne Parker; Faculty Advisor, Dr. W. C. Royster; Permanent Secretary, Professor S. L. Thompson

Officers for 1954-55 are: President, Miriam Horton; Vice-President, Robert Lowder; Secretary-Treasurer, Jewel Magee; Faculty Advisor, Dr. N. Macon; Permanent Secretary, Professor S. L. Thompson.

Alpha of Alabama, University of Alabama

The Alabama Alpha chapter held five meetings during the academic year 1953-54 in addition to the Christmas banquet and the spring picnic.

During the year the following papers were presented:

"Strength of Systems of Differential Equations" by Dr. N. L. Balazs, Department of Physics

"High Speed Computation of Eigen Values" by Dr. Ward Sangren of the Oak Ridge Laboratories

"Corrections for Linear Interpolations" by Dr. C. L. Seebeck

"Matric Inversion" by A. W. Yonda

"A Collineation Group of Order 1536" by C. E. Robinson.

At one of the meetings Mr. C. E. Robinson discussed some of the problems from the Putnum Mathematical Contest for the current year.

Twenty-seven new members were initiated during the year.

Officers elected for 1954-1955 were: Director, Horace P. Baker; Vice-Director, William C. Bowman III; Secretary, Bernard Mulligan; Treasurer, Sherman L. Prosser; Faculty Advisor, Dr. F. A. Lewis; Scholarship Chairman, Dr. J. N. Hornback; Social Chairman, Susie Lee Ward.

Alpha of Montana, Montana State University

The Montana Alpha Chapter held six regular meetings during the 1953-1954 year. In addition, the annual banquet was held on May 4 with Director Verne Fauque serving as toastmaster. The banquet address was given by Professor H. Chatland. On this occasion ten new members were presented their certificates of membership.

During the year the following papers were presented:

"Metric Spaces" by Professor William Myers

"Algebra of Sets" by Professor J. Hashisaki

"Difference Sets in a Finite Plane Projective Geometry" by Professor T. G. Ostrom

"Borel Sets" by Mr. Sheldon Rio.

The officers for 1954-1955 are as follows: Director, James Ford; Vice-Director, Theodore Mueller; Secretary-Treasurer, Ralph Bingham.

Gamma of Kansas, University of Wichita

The Kansas Gamma Chapter held six meetings during the year including the annual dinner meeting. Mrs. Vida Grace Hildyard, Statistical Department of Boeing Aircraft Company, gave the banquet address. Her subject was: "Making Statistics Pay". Seven new members were initiated.

The officers for 1953-1954 were: Director, Ann Klein; Secretary, Gynith Griffin; Treasurer, Agnes Nibarger.

Alpha of Oregon, University of Oregon

The Oregon Alpha chapter initiated eighteen new members during the 1953-1954 year. The annual picnic was held on May 27.

Officers for 1953-1954 were: Director, Karl Stromberg; Vice-Director, Pete Mundle; Secretary-Treasurer, Forest Easton.

Officers for 1954-1955 are: Director, Barbara Jean Thomson; Vice-Director, Joy S. Heller; Secretary-Treasurer, Gerald Alexanderson; Advisor, Dr. R. L. San Soucie.

Beta of North Carolina, University of North Carolina

The following papers were presented at meetings of the North Carolina Beta Chapter during the academic year 1953-1954:

"Theory of Games" by Alex Davis

"Mathematical Examination of Spiral Compound Curves" by Dr. T. F. Hickerson

"What Is a Point?" by Dr. F. Burton Jones

"What Is a Logarithm?" by Dr. J. S. MacNerney

"Mean Value Theorem" by Dr. W. R. Mann

"What Is a Number?" by Louis F. Mc Auley

Thirty-two new members were initiated. In the spring the chapter held a picnic to which all members of the mathematics faculty were invited.

Officers for 1953-1954 were: Director, Robert W. Heath; Vice-Director, Marion B. Smith; Secretary, Marilyn L. Marshall; Treasurer, George W. Lofquist; Chairman of Awards and Scholarship Committee, Richard J. Painter.

Officers elected for 1954-1955: Director, Alex Davis; Vice-Director, Robert W. Heath; Secretary, Tiny M. Morrow; Treasurer, Bill Timlake; Chairman of Awards and Scholarship committee, Bonnie M. Baker.

Gamma of Missouri, St. Louis University

The Missouri Gamma Chapter held four meetings during the 1953-1954 year in addition to the annual banquet. The following papers were presented:

"Theory of Games" by Mr. Paul K. Ito

"Mathematical Paradoxes" by Reverend John Daly, S. J.

"Geometry of Position" by Reverend Martin Hanhauser, O. F.M.

"Diophantine Equations" by Dr. W. R. Utz, Jr., University of Missouri.

Fifty-eight new members were initiated by the chapter.

Officers for 1953-1954 were as follows: Director, Paul K. Ito; Vice-Director, Jack Lange; Recording Secretary, Barbara Riley, Corresponding Secretary, Carolina Del Mar; Faculty Advisor and Permanent Secretary-Treasurer, Dr. Francis Regan.

Officers for 1954-1955 are as follows: Director, Reverend Martin Hanhauser, O.F.M.; Faculty Advisor and Permanent Secretary-Treasurer, Dr. Francis Regan. The election of a Vice-Director, Secretary and Treasurer will be held at the first fall meeting.

Alpha of the District of Columbia, Howard University

The District of Columbia Alpha Chapter held six meetings during the 1953-1954 year. In addition there were two initiation banquets. The guest speaker at the fall banquet was Dr. **Eagleson** of the physics department. Dr. George Butcher gave the address for the spring banquet. Nine students were initiated.

The following papers were presented during the year:

"On Mathematical Puzzles" by Dr. Alan Maxwell

"Dimensional Analysis" by William H. Smith

"Frink's test for Convergence of Series" by Dr. George K. Butcher

"Precise Measurement of Engineering Materials" by Dr. Walter Daniels.

The following officers were elected for the year of 1954-1955: President, Gloria Roberson; Vice-president, Foster Walker; Secretary-Treasurer, Nelson S. **DuBois;** Director, Dr. George H. **Butcher.**

Alpha of Florida, University of Miami

The Florida Alpha Chapter held six program meetings during the year 1953-1954. The following papers were presented:

"Conditionally Convergent Series" by Dr. Charles **Capel**

"Knots" by Dr. Robert Roberts

"The Calculus" by Dr. Herman Meyer

"The Digital Calculator at Oak Ridge" by Dr. Alston Householder

"Laws of Conservation" by Dr. Clarence Rainwater

'Applications of Least Squares to a Physical Problem" by Dr. Alfred Mills.

Fifteen new members were initiated in April, 1954. The annual Christmas party was held December 19,1953, and the annual banquet was held on May 22, 1954. Dr. E. M. Miller, Dean of the College of Arts and Sciences, was the speaker at the banquet.

Officers for 1954-1955 are: Director, Dr. Robert Roberts; Vice-Director, Walter Roop; Secretary, Frank Cleaver, Treasurer, **Sam Berman;** Permanent Secretary, Mrs. Georgia Del **Franco.**

Alpha of Louisana, Louisiana State University

The Louisiana Alpha chapter held six meetings during the year of 1953-1954 including the Pi Mu Epsilon annual lecture series.

In addition there was the annual initiation banquet. On this occasion twenty-two students were initiated. Dr. Houston T. Karnes was the banquet speaker.

On March 25, 1954, the annual Pi Mu Epsilon lecturers were held. The guest speaker was Dr. W. L. Duren, **Jr.,** head of the department of mathematics at Tulane University. The titles of his addresses were:

"A Theory of Measurement and "Boundary Conditions for the Mathematics Curriculum."

Officers for the year were: Director, Jack R. Hall; **Vice-**Director, Cecil **J. Bergeron;** Secretary, Sadie Ferguson; Treasurer, John C. Jackson; Faculty Advisor and Corresponding Secretary,. Houston T. Karnes.

The officers for the year 1954-1955 will be elected at the first meeting in the fall.

MEDALS, PRIZES AND SCHOLARSHIPS

'Montana Alpha Chapter started out the academic year (1953-54) with the annual awarding of the Pi Mu Epsilon entrance prizes. These prizes are given to the three freshmen who placed highest in an examination in mathematics. The first prize of $25 was awarded to William Cogswell; second prize of $15 to James Rowland; and third prize of $10 to William Erhard and Richard Graven who tied."

- Montana Alpha Chapter

"Awards were given to an undergraduate and a graduate student presenting the best papers during the year. These respective awards were won by Miss Anne Parker and Miss Jewel Magee."

- Alabama Beta Chapter

"At the initiation held May 19, 1954, two fifty-dollar prizes in honor of the late Edgar E. DeCou, former Head of the Mathematics Department, were presented to the two outstanding mathematics students, Jack B. Goebel and Barbara Jean Thornson."

- Oregon Alpha Chapter

"The eighth Annual Prize Essay Contest was conducted by the chapter with Doctor Waldo A. Vezeau as chairman. The junior award was won by Vondell Carter of Parke College. The Garneau Award of twenty-five dollars was presented by Doctor Francis Regan to J. James Malone, the highest ranking senior in mathematics."

- Missouri Gamma Chapter

"The Annual Senior Award was won by Stephen Pool Brote. The annual Freshman Award, based on an honors examination, was won by Charles D. Russell."

- Louisiana Alpha Chapter

At the April 13, 1954 meeting, the name of Miss Patricia Conger was announced as that of the recipient of the Pi Mu Epsilon Mathematics Scholarship.

- Kansas Gamma Chapter

"Chih Han Sah and Carol L. Stewart were each awarded the chapter's prize of $25 given annually to the senior outstanding in the field of mathematics."

- Illinois Alpha Chapter

"Each year to reward excellence in Freshman Mathematics and to encourage interest in the field of mathematics ten dollars is presented to the freshman enrolled in Freshman Mathematics who attains the highest score on a test prepared by the Head of the Mathematics Department and Chapter Officers."

- Ohio Delta Chapter

DIRECTORY

of

PI MU EPSILON FRATERNITY, INC.

*

General Officers
(1954-1957)

Director General: Professor S. S. Cairns, Department of Mathematics, University of Illinois, Urbana, Illinois

Vice-Director General: Professor J. S. Frame,* 207 **Physics-**Mathematics Bldg., Michigan State College, East Lansing, Michigan

Secretary-Treasurer General: Professor **R.** V. Andree, Department of . Mathematics, University of Oklahoma, Norman, Oklahoma

Councilors General:

Professor Wealthy Babcock, Department of Mathematics, University of Kansas, Lawrence, Kansas

Professor Henry W. **Farnham,** Department of Mathematics, Syracuse University, Syracuse 10, New York

Professor R. F. Graesser, Department of Mathematics, University of Arizona, Tucson, Arizona

Professor Sophia L. McDonald, Department of Mathematics, University of California, Berkeley 4, California

Professor Ruth W. Stokes, Department of Mathematics, Syracuse University, Syracuse 10, New York

Professor **H.** S. Thurston, Department of Mathematics, University of Alabama, University, Alabama

***Also,** Acting Director General for the year **1954-1955.**

FALL 1954 ROSTER OF THE FIFTY-FOUR
ACTIVE CHAPTERS WITH CHARTER **DATE**[1]

Corresponding secretaries[2]

(5) Alabama Alpha, 1922, University of Alabama, University, Alabama;
Professor H. S. Thurston, Dept. of Mathematics

(54) Alabama Beta, 1953, Alabama Polytechnic Institute, Auburn, Alabama;
Professor S. L. Thompson, Dept. of Mathematics

(40) Arizona Alpha, 1941, University of Arizona, Tucson, Arizona;
Professor **R.** F. Graesser, Department of Mathematics

(22) Arkansas Alpha, 1931, University of Arkansas, Fayetteville, Arkansas;
Professor William **R. Orton,** Jr., Department of Mathematics

(12) California Alpha, 1925, University of California, Los Angeles 24, California;
Professor W. T. Puckett, Dept. of Mathematics

(19) California Beta, 1930, University of California, Berkeley 4, California;
Professor Sophia L. McDonald (Mrs. J. H.), Department of Mathematics

(33) **Colorado**[3] Alpha, 1936, University of Colorado, Boulder, Colorado;
Professor Jack **R.** Britton, Dept. of Mathematics

[1]**Number** appearing in parentheses before chapter designation indicates chronological order in which charter was granted.

W e give here the name and business address of the permanent corresponding secretary (to whom official correspondence for the chapter should be sent) who is a member of the mathematics staff with the rank of instructor, or higher, at the institution where the chapter is located.

[3]**Reported** "inactive" on 11 October 1954.

(50) Colorado Beta, 1950, University of Denver, Denver 10, Colo.;
Professor O. M. Rasmussen, Dept. of Mathematics

(41) Delaware Alpha, 1941, University of Delaware, Newark, Del.;
Professor E. Vernon Lewis, Department of Mathematics

(52) District of Columbia Alpha, 1951, Howard University, Washington 1, D. C.;
Professor George H. Butcher, Department of Mathematics

(51) Florida Alpha, 1951, University of Miami, Coral Gables 46, Florida;
Mrs. Georgia Del **Franco,** Department of Mathematics

(29) Georgia Alpha, 1934, University of Georgia, Athens, Georgia;
Professor W. S. Beckwith, Department of Mathematics

(**7**) Illinois Alpha, 1924, University of Illinois, Urbana, Illinois;
Professor M. Evans Munroe, 360 Mathematics Building

(42) Illinois Beta, 1944, Northwestern University, Evanston, Ill.;
Dr. J. C. E. Dekker, Department of Mathematics

(6) Iowa Alpha, 1923, Iowa State College, Ames, Iowa:
Dr. Fred M. Wright, Department of Mathematics

(16) Kansas Alpha, 1928, University of Kansas, Lawrence, **Kansas;**
Professor Wealthy Babcock, 209 Strong Hall, Dept. of Mathematics

(31) Kansas Beta, 1935, Kansas State College, Manhattan, Kansas;
Professor T. A. **Mossman,** Department of Mathematics

(49) Kansas Gamma, 1950, University of Wichita, Wichita 14, Kan.;
Professor C. B. Read, Department of Mathematics

(13) Kentucky Alpha, 1927, University of Kentucky, Lexington, Ky.;
Professor H. S. Downing, Department of Mathematics

(38) Louisiana Alpha, 1939, Louisiana State University, Baton Rouge 3, Louisiana;
Professor H. T. **Karnes,** Department of Mathematics

(39) Michigan Alpha, 1940, Michigan State College, East Lansing, Michigan;
Dr. John G. Hocking, Department of Mathematics

(4) Missouri Alpha, 1922, University of Missouri, Columbia, Mo.;
Professor Mary Cummings, 212 Engineering Building

(11) Missouri Beta, 1925, Washington University, St. Louis 5, Mo.;
Professor Jessica Young Stephens, Dept. of Mathematics

(43) Missouri Gamma, 1945, St. Louis University, St. Louis 3, Missouri;
Professor Francis **Regan,** Dept. of Mathematics

(9) Montana Alpha, 1925, Montana State University, Missoula, Montana;
Professor George Marsaglia, Dept. of Mathematics

(15) Nebraska Alpha, 1928, University of Nebraska, Lincoln 8, Nebraska;
Professor Edwin **Halfar,** Department of Mathematics, 213 **Burnett** Hall

(45) New Hampshire Alpha, 1948, University of New Hampshire, Durham, New Hampshire;
Professor F. Cunningham, Jr., Dept. of Mathematics

(56) New Jersey Alpha, 1954, Rutgers University, New **Brunswick,** New Jersey;
Professor Harold S. Grant, Dept. of Mathematics

(1) New York Alpha, 1914, Syracuse University, Syracuse 10, New York;
Professor Nancy Cole, 15 Smith College

(10) New York Beta, 1925, Hunter College, 695 Park Ave., New York 21, N. Y.;
Professor Jewel Bushy, Department of Mathematics

(26) New York Gamma, 1933, Brooklyn College, **Bedford** Ave. and Ave. H., Brooklyn 10, N. Y.;
Professor Frank E. Smith, Dept. of Mathematics

(28) New York Delta, 1933. N. Y. University, 100 Washington Sq. East, New York 3, N. Y.;
Professor John Van Heijenoort, Department of Mathematics

(30) New York Epsilon, 1935, St. Lawrence University, Canton, New York;
Professor Ruth M. Peters, Dept. of Mathematics

(53) New York Eta, 1951, University of Buffalo, Buffalo 14, N. Y.;
Dr. David D. **Strebe,** Department of Mathematics

(55) New York Theta, 1953, **Cornell** University, Ithaca, New York;
Dr. **Mervin** Muller, Department of Mathematics

(24) North Carolina Alpha, 1932, Duke University, Durham, N. C.;
Professor F. G. **Dressel,** 309 Frances Street

(46) North Carolina Beta, 1948, University of North Carolina, Chapel Hill, North Carolina;
Dr. John W. Lasley, Jr., 523 E. Rosemary Street

(2) Ohio Alpha, 1919, Ohio State University, Columbus, Ohio;
Professor Earl J. Mickle, Dept. of Mathematics

(13) Ohio Beta, 1927, Ohio **Wesleyan** University, Delaware, Ohio;
Professor Philip C. Stranger, Dept. of Mathematics

(32) Ohio Gamma, 1936, University of Toledo, Toledo, Ohio;
Professor Wayne Dancer, Dept. of Mathematics

(48) Ohio Delta, 1949, Miami University, Oxford, Ohio;
Professor H. S. **Pollard, Upham** Hall

(18) Oklahoma Alpha, 1929, University of Okla., Norman, Okla.;
Professor Dora **MacFarland,** Dept. of Mathematics

(35) Oklahoma Beta, 1938, Oklahoma A. & M. College, Stillwater, Oklahoma;
Professor James **H.** Zant, Dept. of Mathematics

(21) Oregon Alpha, 1931, University of Oregon, Eugene, Oregon;
Dr. Robert L. San Soucie, Department of Mathematics

(36) Oregon Beta, 1938, Oregon State College, Corvallis, Oregon;
Professor G. A. Williams, Department of Mathematics

(3) Pennsylvania Alpha, 1921, University of Pennsylvania, Philadelphia 4, Pennsylvania;
Dr. Pincus Schub, 4C 21 Math-Physics Bldg., University of Pennsylvania

(8) Pennsylvania Beta, 1925, **Bucknell** University, Lewisberg, Pennsylvania;
Professor J. S. Gold, Department of Mathematics

(17) Pennsylvania Gamma, 1929, Lehigh University, Bethlehem, Pennsylvania;
Professor R. R. Stoll, Department of Mathematics

(20) Pennsylvania Delta, 1930, Pennsylvania State College, State College, Pennsylvania;
Instructor David Dickinson, Dept. of Mathematics

(44) Pennsylvania Epsilon, 1947, Carnegie Institute of Technology, Pittsburgh 12, Pennsylvania;
Professor **Marlow** Sholander, Dept. of Mathematics

(47) Virginia Alpha, 1948, University of Richmond, Richmond, Va.;
Professor E. S. Grable, Box 45, University of Richmond

(25) Washington Beta, 1932, University of Washington, Seattle 5, Washington;
Professor Lee **H. McFarlan,** Dept. of Mathematics

(27) Wisconsin Alpha, 1933, Marquette University, Milwaukee 3, Wisconsin;
Professor H. P. **Pettit,** Department of Mathematics

(37) Wisconsin Beta, 1939, University of Wisconsin, Madison 6, Wisconsin;
Catherine S. **Standerfer** (Faculty), Department of Mathematics

Note. We have made a serious effort to prepare a roster free of mistakes; however, there may be many. Any errors called to the attention of the editor before February 1955, will be gratefully received and corrections will be printed in the next issue of the Journal.

INITIATES, ACADEMIC YEAR 1953 - 1954

ALABAMA ALPHA, University of Alabama
(November 11, 1953)

Ralph Bice, Jr.
Charles M. Cason
Holland C. Filgo, Jr.
Frank M. Hardy
Raymond C. Hayes
Paul F. Haughton
John Jewett
Kermit G. Johnson
Donald K. Jordan
John Richard Kemp
Lucy Messick
Patricia Oliver
Sherman Lee Prosser
Bettye Busbee Weekley

(January 15, 1954)

Horace Powell Baker, Jr.

(April 20, 1954)

Isabel Ann Barker
William C. Bowman III
Emil Eichhorn
John Wynne Evans
William P. Harrison, Jr.
Eric C. Jernigan
Joan Kochy Kulp
Arnold Lawrence Milner
Bernard Mulligan
David M. Mullins
Thy Ming Quo
Floyd E. Turner, Jr.

ALABAMA BETA, Alabama Polytechnic Institute
(May 12, 1954)

J. W. Amoss
Benny B. Barnes
Robert Austin Beckerle
Ben Campbell Blake, Jr.
James Ramsey Bonner
Shirley Ann Brown
James W. Carroll
Jerry Conrad Eskew
Terry Marlin Eskew
James Frierson
Paul Daniel Hill
William Hinds
Edward M. Holliday
Brady A. Hughes, Jr.
David Carson Leaird
Lawrence A. Lucchesi
Jewel Magee
Thomas N. Marshall, Jr.
Virginia Ann Middleton
Eldred O. Mixon, Jr.
Alice Kerr Neighbors
Robert W. O'Neill
William H. Panter
Mike A. Piazza
Charles F. Pinson
Arnold L. Rodriguez
John Rudolph Seibold
Floyd Baty Stroup
Wayne Summerford
William W. Walker
Robert E. Wallace
Vernon H. White

ARKANSAS ALPHA. University of Arkansas
(March 25, 1954)

Stacy Stevens
William S. Stewart
Buster F. Womack

(May 6, 1954)

Dorothy M. Hoover
Glen T. Clayton

CALIFORNIA ALPHA, University of California, UCLA
(May 22, 1954)

Yoichiro Fukuda
Antranig V. Gafarian
Leonard Kleinman
Eugene Malek
Gordan Matthews
William G. Melbourne
Edwin H. Mookini
Isaac Richman
Arnold Rosenbloom
Donald S. Sarris
Joseph F. Seewerker
Edsel Ford Stiel
Jack Wong
Lem Wong

CALIFORNIA BETA, University of California, Berkeley
(December 16, 1953)

Andrew Astromoff
Eugene Breitenstein
Philip Deuel
Vida Greenberg
William Hanf
Luc Huang
Stephen Jauregui
Robert Kellman
Ernest Malamud
Nina Martianoff
Dr. Wilhelm H. Meyer
Otto Plaat
Jan Tarski
Velta Vasilevskis

(May 15, 1954)

Jacqueline Chalmers
Paul Fong
Eldon Hansen
John B. Jackson
Karolyn Kaeser
Melvin Katz
William Keese
Neophytos Kockinos
Melville McClelland
John Morrissey
Edwin Towster

DELAWARE ALPHA, University of Delaware
(May 21, 1954)

Kenneth E. Curewitz
Norman N. Hochgraf
Donald A. Jardine
Raymond Kushida
Donald W. Marquardt
Lois M. McKay
Irven H. Rinard
James Francis Scott
Evelyn A. Strawbridge
L. Edward Scriven II
Harry M. Truax

DISTRICT OF COLUMBIA ALPHA, Howard University
(March 29, 1954)

Cornelius B. Baytop
Lens Celeste Brown
Francis C. Chigbo
Nelson S. DuBois, Jr.
Gloria Frances Roberson
Foster Walker, Jr.

FLORIDA ALPHA, University of Miami
(April 1954)

Sam M. Berman
Charles E. Capel (Ph.D.)
Frank Lee Cleaver
Ali Sh. Dadras
Jack E. Diamond
Arthur L. Finlelstein
Elliott Goldberg
Mark J. Goodkind
David Simon Katzln
Clarence S. Rainwater (Ph.D.)
Robert A. Roberts (Ph.D.)
Howard L. Stern
Ronald Neil Stock
Robert Nelson Watts

GEORGIA ALPHA, The University of Georgia
(May 7, 1954)

Charles Benson
Donald E. Cadwallader
John Trawick Echols
Theodore C. Getten III
Lewis S. Hall
Jane Elizabeth Heng
Dan Edwin Pratt
Mrs. Mary Mann Prescott
William Lanier Rivenbark
Ted Leon Simons

ILLINOIS ALPHA, University of Illinois
(December 10, 1953)

James Blayney Rice

(April 1954)

Kenneth W. Anderson
Evangelos D. Argoudelis
William O. Burns
Chang Sun Chang
John Doubla- Dyson
Rabindra Nath Ghose
Maryelsie Hawkins
Richard Alan Hubach
Edward Allen Huber
Dale Eugene Kaiser
Eugene E. Kohlbecker
Gerardine M. Meerman
George Elmer Milum
George Jack Mobille
Thomas M. Oneson
James G. Rudolph
Krishan Lal Sethi
Emir Hamvasy Shuford
Hing Cheong So
Sanford Stein
Ralph N. Townsend
Phillip G. Wahlbeck
Russell Arthur Welker
Leroy Whitaker
Edwin Frederick Wilde
Robert James Zaruba

(May 13, 1954)

Bruno Valentine Dal Corobbo

IOWA ALPHA, Iowa State College
(April 13, 1954)

Norman Barson
Stephen D. Beck
John Boardman
Janet Bossenberger
Marjorie D. Brown
James Cogley
Laura C. Dean
Ronald Decker
David T. Friest
Orville Goering
Bruce M. Graybill
Charles B. Grosch
Richard A. Hendrickson
Carol Hollingworth
Jack A. Horrigan
Robert E. Keim
Harry L. Knapp
Sheldon Knight
Gerda E. Kock
Karl R. Kopechy
Paul Landis
Wendell D. Lindstrom
Richard K. McMullan
Mary Alice Mershon
Alberto D. Ng
Richard J. Oedy
Richard H. Paulsen
George W. Peglar
Conrad L. Peterson
George O. Price
Paul Redin
Robert A. Sharpe
Lyndon L. Sheldon
James Sidles
David O. Stuart
Clark W. Trafton
Onnolee Anne Underwood
Martha West
Donald F. Young

KANSAS ALPHA, The University of Kansas
(March 24, 1954)

Billy Neil Carr
Ralph B. Crouch
Donna G. Davis
Mrs. Marta de Valle
Jack H. Hafferkamp
W. E. Hartnett
Calvin V. Holmes
Eugene K. McLachlan
Thomas J. McNamara
Kathryn Mueller
Mr. Mehdi Shirazi
Herbert D. Lechner

KANSAS BETA, Kansas State College
(Spring 1954)

Donald C. Anderson
Richard A. Anderson
Edward Lee Dubowsky
John D. Ferrucci
Joseph V. Guida
Jerome Luther Hartke
John M. Marr
Joanne Marie Mundell
Nancy A. Schneckloth
Danny C. Schuyler
John A. Weese

KANSAS GAMMA, University of Wichita
(April 2, 1954)

John G. Albright
Eslie L. Darter
Jack H. Hetherington
Lawrence W. Rennick
Richard D. Sinkhorn
Howard W. Smith

LOUISIANA ALPHA, Louisiana State University
(May 14, 1954)

Joanne Aycock
Sam W. Bergeron, Jr.
Dalton E. Cantey, Jr.
Cecilia Cimerman
James R. DiStefano
Kenneth Ray Efferson
Patricia J. Harrison
Melba LeRoy Harvey, Jr.
Ernest Heilberg
Lester D. Hulett, Jr.
Charles H. Kammer
Karl E. Lewis, Jr.
James E. L'heureux
John Scott Linder
Ramez Masoud Malhis
Ludger Miller
Genaro F. Pena
Billy Ray Pesnell
Easton J. Pitre
John G. Swider
Gerald Lee Tracey
Paul E. Weaver

MICHIGAN ALPHA, Michigan State College
(May 12, 1954)

Arthur W. Baker
Terry J. Bergstrom
Maurice W. Brandt
Donald G. Daus
Richard N. Devereaux
Phillip J. Douglas
Donald C. Cause
William L. Harkness
John C. Hocking
Gloria Jean Jeffery
Nawoyoshi Kikuchi
Lee Mah
Buford H. Mundale
Charles G. Nelson
Mary M. Quinn
Richard L. Reaser
Gilbert de B. Robinson
John P. Staples
Walter W. Turner

A Gift Today for a Bright Tomorrow