PI MU EPSILON JOURNAL

THE OFFICIAL PUBLICATION OF

THE HONORARY MATHEMATICAL FRATERNITY

VOLUME 2 NUMBER 4

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SPRING 1956

PI MU EPSILON JOURNAL
THE OFFICIAL PUBLICATION
OF THE HONORARY MATHEMATICAL FRATERNITY

All editorial correspondence, including manuscripts, chapter reports, books for review, news items, etc., should be addressed to THE EDITOR OF THE PI MU EPSILON JOURNAL, 350 Math. Building, University of Illinois, Urbana, Illinois.

PI MU EPSILON JOURNAL is published semi-annually at the University of Illinois.

SUBSCRIPTION PRICE: To Individual Members, $1.50 for 2 years; to Non-Members and Libraries, $2.00 for 2 years. Subscriptions, orders for back numbers and correspondence concerning subscriptions and advertising should be addressed to the PI MU EPSILON JOURNAL, 350 Math. Building, University of Illinois, Urbana, Illinois.

CREATIVENESS IN MATHEMATICS

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A traditional and somewhat tired description of the state of creativeness in mathematics would proceed approximately as follows: It is well known that there are three unsolved problems in mathematics: the trisection of an angle, the squaring of the circle, and the duplication of the cube. These problems were first studied by the Greeks and although they have received unending attention since, they have never been solved. Thus all of mathematics with the exception of these three elusive fish is already in mothballs and will be taken out by the loving curator-teacher whenever an eager student presents himself.

This is a rather easy way to dispose of a situation. Also a false way. Completely false. It is a fact that the state of our art is not so readily apparent to the college student and certainly even less so to the educated layman as is, for instance, the state of physics, chemistry, or biology. We have all heard of the intensive work done in physics, indeed it is reported to us on the front pages of our newspapers, at least those papers worthy of the name. As to chemistry, our style in shirts changes yearly because of some new plastic. Presumably these chemists do nothing but worry about ladies' stockings and men's shirts. And as to biology, or more precisely immunology, we have seen displays in recent weeks, ceremonies in the super Hollywood style, which would make even the most dispassionate of physicists green with envy. Evidently these fields are in ferment; the men and women in them are active. In their laboratories is being carried on that most prized of mid-twentieth-century American activities, namely, *research*.

What of mathematics? In the midst of this scientific fervor, surrounded by colleagues in other fields whose laboratories are doing revolutionary work, what can the mathematician say for himself? What has he accomplished? What has he created? Wherein has he led? What are his contributions? Where does he propose to take us?

These are some of the questions to which I expect to give a few answers in this paper. But later on in the course of this discussion I hope to touch on another question which is closer to my heart and whose answering will require much more finesse and artistic delicacy. What I should like to do very briefly – later on – is to describe the process of creation in mathematics.

But let us return to our immediate task, that of describing the state of mathematics as a creative subject. In the first place, let us dispose briefly of the famous three "unsolved" problems inherited from the Greeks. These problems have all been solved, and at that between 70 and 120 years ago. Angles cannot be trisected by by ruler and compass, the cube cannot be duplicated, and the circle cannot be squared by these same means. And that is that! All of us have known that and we have known moreover of the extraordinarily penetrating work done in the 17th, 18th, and 19th centuries by Descartes, Fermat, Newton, Leibnitz, Euler, the Bernoullis, and Gauss to mention but a few names. Unfortunately, the undergraduate curriculum seldom brings us beyond the mathematics of 1820. This is a grave misfortune for us. The field of physics is so much more lucky! Every physics major worth his salt will certainly have taken a course in nuclear physics, a subject which did not exist prior to 1932. But it is inconceivable to give an undergraduate a course on the mathematics of the last generation.

To get to our point, let it be said that the field of mathematics is most active. And every generation sees an increase in activity. The amount of mathematical research being carried out these days is enormous. The American Mathematical Society issues each month a publication, the *Mathematical Reviews*, which carries short reviews of every original work in mathematics done in the entire world. I have estimated that in the course of the past year, some 7300 articles were reviewed. These articles were devoted to the most varied fields – abstract algebra, harmonic analysis, theory of complex variables, functional analysis, topology, probability, differential geometry, mathematical statistics, applied mathematics, and mathematical physics. The titles of some of these might surprise you. Here are a few samples:

Cyclotomy and the converse of Fermat's theorem.

The Plancherel formula for complex semi-simple groups.

Les éléments quasi clairsemés.

Chain transforms.

Alcune disuguaglianze relative alle funzioni armoniche.

Zur Konstruktion des Resultantensystems für homogene Gleichungen.

Odnoznacnaya opredelennost obscih vypuklyh poverhnostei (Unique determination of convex surfaces).

If we assume that the population of the earth is 2½ billion men, women, and children, then the publishing of 7,300 articles on mathematics means that for every 350,000 inhabitants on this earth, there is published one paper each year. This is a high degree of activity. I should say that the rate of publication in 1955 is at least twice as high as that in 1940. And in 1940 it was twice what it was in 1920. You will see from this that mathematics is far from dead. It is certainly as much alive as any of its neighbors. How much of its product is of value is, of course, another question. I cannot pretend to say whether mathematical research is on the whole better than it was in 1920 – my own suspicion is that it is considerably better.

Aside from the amount of research being carried on in mathematics, which is certainly an index of the growth of new material, the most interesting phenomenon in present mathematics is the daily change which is visible in the presentation of subject matter, either in advanced texts or in the graduate classroom. Every two years we seem to tear down completely our approach to a given field and replace it by an entirely new one. The senior scientists in a given field are used to seeing the younger men in their midst remodeling the structure here and there and now and then. But in mathematics, this is not a correct picture. It is the senior members of the fraternity more than anyone else who are responsible for this activity. And remodeling is not the word for it. The entire old structure is periodically torn down to the last brick and an entirely new edifice arises over the old emplacement. Unless one follows developments closely, one may be caught in a state of complete ignorance of a new attitude. To illustrate this concretely: Last week I had a conference with a graduate student of some distinction who in 1945 had terminated all the course requirements for our Ph.D. degree. He had been away for 10 years and wished to go back to his studies. I had to tell him frankly that he was almost in the position of a new first-year graduate student and in addition had probably developed very bad learning reflexes in his earlier education. Thus it was almost a duty to tell this person not to proceed with further studies leading to a Ph.D. since at his age the path was too arduous and long.

You will see from this very rapid description that mathematics is in a state of ebullition. Activity on all sides is most intense. And it is a joy to be able to state that this activity has greatly been increased by grants made to mathematicians under the Fulbright Act and by the National Science Foundation. Where is this taking us? I don't know. It is difficult to foresee the course of our muse. We are not fortunate like the physicists who know (and so do all of us too) exactly what is the outstanding problem of modern physics – to determine the structure of the nucleus. Not only do we not know where we are going but I doubt if we know where we *are*. No one knows nearly enough mathematics to have a clear idea. It is frequently said that a good physicist can know generally speaking 60% of physics; on the other hand, a good mathematician can know less than 10% of mathematics. Oh, it is easy enough to come on the scene 20 to 40 years later and assert: That period was known for a great development in algebraic topology or perhaps in topological algebra. But to state that right now the big things are being done in such and such a field is mild talk. The subject is enormous, and the big things will take place where lady fortune chances to lead an unwitting genius. It is impossible now in our field to know immediately or even more remotely which are the key discoveries of our time. There is even some doubt that there are key discoveries. This is not the case in the sciences. For example, in physics, the advent of the theory of relativity was instantly recognized as earth-shaking. In biology, the promulgation of the theory of evolution was equally accorded priority in all discussions of its time. But the mathematical atmosphere of Newton and Leibnitz is definitely at an end. We are in an enormous subject, and even the most important contributions will require some time before their impact is appreciated.

And now that we have surveyed the question of mathematical creativeness from the outside, let us look at it from the inside. I have given you an idea of the brawn of our subject. Let us examine its nervous system. This is an infinitely more difficult task. It requires subtlety and introspection in quantity and quality far beyond what I possess. What I should like to do is to give some description

of the manner in which mathematics is created. This is difficult to determine. For the mental processes concerned are very delicate and do not brook examination. The very act of spying on ourselves would destroy the naturalness of our mental processes. And in any case one would have to spy for a very, very long time before there could be any chance of catching a glimpse of the mind at work. Our inner self cares no more for spies than our moral selves in our relations to other people.

As a similar problem and one which probably many of us have tried to solve, consider the problem of determining exactly what goes on when one goes to sleep at night. I have tried it. Things go along under control fairly well as I doze off. I myself am relaxed and my mind's eye is like a lazy but watchful dog lying before the fire-place next to his master. At the moment I am about to fall definitely asleep, the vigilant dog jumps up with a loud bark to get me into action so that I may see what goes on. Well, a great many things go on, but falling asleep is certainly not one of them. This is definitely the type of experiment in which making the measurement spoils the result which is being examined. This type of experiment is well known in physics.

Thus we shall be constrained in our study of mathematical creativeness to examine second-hand evidence. This second-hand evidence concerns the externals of the creative act. What do mathematicians do when they work and how do they do it? Do they sit down at their desks from 9 to 12 and from 1 to 5? Or do they lie in bed at home until twelve noon every day that they do not teach? Do they get ideas in the way which the scientific editors of our better newspapers seem to suggest, by locking themselves up with several fellow scientists as a team, working for several months on a given problem until they come out with a solution – the way a jury comes out with a verdict – arriving at final truth by sheer intellectual effort? Or do they work alone principally, and probably in a queer disconnected way, puttering about, waiting for something to happen, not unhappy at being interrupted?

Probably each of these ways is the way of some scientists and each way has its good points. Certainly, if you are writing a book, then the more time you spend within 10 feet of your desk, the more likely are you to send off a manuscript to the printer. On the other hand, a puritanic interpretation of work leads principally to work only and not to ideas. Thus I doubt that the best mathematical thinking is done by teams or by men who grind out regular hours in their offices. I definitely do not think that the future of science lies with the 9 to 5 professors. If anything they will beat it to death.

The mathematical muse must be wooed and wooed constantly. The mind must be ready and receptive under all circumstances, whether one is discussing a technical point with an assistant or whether one is changing a diaper on his youngest child. Indeed, it is my experience that truly constructive and genial thoughts come in moments of greatest relaxation and are thus less likely to be had in conversation with colleagues than in some more homely environment. Aside from mathematics, it is my personal conviction that the decisive phrase that shaped my own life at many critical points came not from a bishop or a dean but rather from a simple citizen, a bus driver, or newsvendor whose homespun remark illuminated completely my own thinking.

In this I make a plea for the natural and homely in human contacts in order to achieve penetration in thought. We are being railroaded into thinking that the way to achieve distinction is to closet ourselves with distinguished minds. I grant that it is a good way to secure a professorship but I doubt very much that it is the way either to learn how to create or to develop character.

The principal ingredient in mathematical creativeness is day dreaming, free association, a relaxed play and interplay with ideas. It is to be understood that the dreaming is concerned with one's genuinely inner life and in this case, this means mathematics. Unless the searcher is heart and soul dedicated to his subject he will not achieve much. But take a man who breathes mathematics, who fills the margin of the newspaper he reads with symbols, whose daughter cannot attract his attention because he is so deeply buried in thought, take this man and give him time, give him leisure, free him from the enervating distractions of everyday life, and he will create.

We all remember the story of Newton and the apple. According to it, Newton was sitting under an apple tree when an apple fell and struck him. Irritated by this intrusion, Newton set about trying to determine what could have been the cause of this annoyance and thus was led to the formulation of the law of gravitation. Thus goes the story.

But the story is false to the spirit of the situation as it actually unrolled itself. In the first place, why was Newton under the apple tree and what was he doing? He was there because he had withdrawn from noise and company in order to think and dream. He wished to think of the universal laws which must govern in this world in order that planets move about the sun in ellipses. And in the midst of his dreaming and toying about for the hundredth time with various possibilities present to his mind, he was struck by the falling apple. And instantly Newton conceived the hypothesis that the seemingly abstract and certainly very distant forces which keep the planets on their courses were exactly the same as that which caused bodies to fall on this earth. And thus were united two rivers of thought and all the learning which belonged to each branch of physics was put at the disposal of the other as is the case in any ideal marriage. Before we put aside Newton's story, let us underline once more the homeliness of it. Newton's experience was derived in the humblest of ways from the most primitive ingredients. This is characteristic of great mathematical creativeness.

There is another aspect of the apple story on which I should like to dwell because it reveals one of the most characteristic phenomena of mathematical creativeness. It is the pinpointing feature of discovery or if you wish the "coup de foudre." It is the extraordinarily sharp and deep experience which allows the searcher to say "Precisely at that moment, I understood." Note that Newton saw what the situation was precisely as the apple hit him. The most famous instance of that nature goes back of course to Archimedes who was in the middle of his bath when he understood the laws governing the weight of bodies in water. He was so excited that he made the moment historic by running about the city, probably in incomplete garment, crying out "Eureka!" ("I have found it!")

You will notice several things in this experience of Archimedes. His discovery was made in a moment of great relaxation, for surely man is never more relaxed than when he is in his bath. The discovery was made in the most unpretentious surroundings. He was not in the city library nor was he heading a research committee. He was in his bath, arranging and rearranging all his thoughts on the many natural phenomena which vexed him. Finally, the moment of inspiration was crystal clear to him, he saw it and seized it. We should be grateful to him not so much for the law of physics he had just found as for the law of discovery which he so brilliantly sculptured onto the annals of our culture with his cry "Eureka!"

Parenthetically, we professors should ask of the institution which appoints us, not so much a higher salary or library with still more books. More important would be a bath for every office and a small grove with many apple trees.

Pinpointing a discovery is one of the experiences shared by many men of science and is certainly well-known in the field of mathematics. One of the most famous of recent experiences of this type was that of Henri Poincare. This genius was on a geological field trip, interested – at least on the surface of things – in phenomena geological and far from his own predilection when his mind had one of its most famous strokes of insight. He recounts himself that he was just setting foot on the step of the bus which was to return him to his home when he saw with complete clarity the relation between the theory of automorphic functions and the group of motions of hyperbolic geometry. He goes on to say that his geological conversation was interrupted for only a few seconds by this event, for by the time he reached his seat in the bus the entire picture was clear and was already on file for future setting down on paper.

I do not know how many experiences of this kind a mathematician can have during a lifetime. Since the experience is a deep and precise one, I presume the number of these is somewhat limited. I believe also that the experience is less frequent in later years of one's life. In my own scientific life, I can remember four or five. Let me tell you where some of them took place so that you may have a better idea of the precision of this matter.

My first such experience was on a downtown local train of the Broadway subway. It was a Saturday evening and I was standing on the front platform of the car. I suddenly had the idea which crystallized into my dissertation. It was a question of considering the possibility that the functions with which I was dealing were connected by an algebraic relation. From this consideration the entire development followed.

The second experience was when I was a fellow at Harvard University. It was the month of June and I believe it took place on a Thursday although that part I am not very sure about. I was in my student's apartment at 111 Norway Street in Boston and I was tying my tie in front of the mirror in my bedroom. This was quite a task because I had just broken my left hand a few days before. It was just at this instant that I perceived a new method for proving that two commuting self-adjoint transformations in Hilbert space were each a measurable function of a third.

As a final experience let us go back to 1940. It was Sunday afternoon of a rather warm day. I was at my father's house and we had just had tea. I was wandering about unhappily between the kitchen and the dining room when I suddenly realized that the sums which I was considering in the theory of linear operators were the precise analogue of the Cauchy calculus in the theory of functions of a complex variable. This discovery thus made available to the study of operators the entire classic theory of functions.

This enumeration will suffice to establish the existence of a "scientific experience" which is certainly the equivalent of the esthetic or religious experiences some of us have felt in our lives. In the way of conclusion, let us investigate briefly the behavior of mathematicians instantly after they have been touched by grace. We note that Archimedes was enormously excited and probably did no serious work for some hours. Of Newton, the fable says nothing as to what happened. Poincare put the idea aside with supreme confidence and calm for elaboration and writing down in the near future. Undoubtedly, some mathematicians instantly set to work to check the correctness of their ideas.

As far as I am concerned I would definitely not have such a reaction. The excitement of the moment is too great and the fear of finding a possible error is too oppressive. One waits a long time for the spark of discovery and the supreme elation of this moment must not be cut short by a return to highly critical and humorless work. All operations for the day are suspended and one either goes for a walk in a radiant countryside or seeks the company of a faithful piano.

A CONTINUED FRACTION FOR PERIODIC RENT, LOGARITHMS, AND ROOTS

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1. *Introduction.* Several seemingly unrelated computational problems can be solved quickly and accurately by using special cases of the following continued fraction expansion

$$(1.1) \qquad f_1(x,y) = 1 + \cfrac{x^2 - y^2}{3 + \cfrac{x^2 - 4y^2}{5 + (x^2 - 9y^2)/f_4}}$$

where in general f_k is expressed in terms of f_{k+1} by the formula

$$(1.2) \qquad f_k(x,y) = 2k\text{-}1 + (x^2 - k^2y^2)/f_{k+1}$$

The value of this continued fraction is related to the periodic rent of an annuity, commonly denoted by $1/a_{\overline{n}|r}$, by the formula, to be derived presently,

$$(1.3) \qquad \frac{n}{a_{\overline{n}|r}} = \frac{nr}{1 - (1+r)^{-n}} = \frac{nr}{2} + (1 + \frac{r}{2})f_1(ny,y),$$

where $y = r/(2+r)$

This expression represents the total repayment per dollar borrowed, when a loan of $\$P$ is repaid in n equally spaced periodic rental payments of $R = P/a_{\overline{n}|r}$ at the end of each period, assuming that the payments include interest on the unpaid balance at the rate r per period. Not infrequently the given values of n and r are such that the periodic rent is either not found in readily available tables, or is not found to the required accuracy. A computation by continued fractions is then recommended, as illustrated in paragraph 3.

Continued fractions for $\log_e(1+r)$, e^x, $x \cot x$ and $(a^m + b)^{1/m}$ will be derived and used for computation in paragraphs 4, 5, and 6, but first we shall show in § 2 how we are led to the expression (1.3) for periodic rent in terms of the continued fraction (1.1).

2. *Derivation of the continued fraction.* To speed up convergence of the final result, it is advantageous to replace the periodic rent function $n/a_{\overline{n}|r}$ appearing in (1.3) by a related function that is an even function of both variables, and thus involves only the even powers of the variables in its expansion. The difference

$$(2.1) \qquad \frac{nr}{1 - (1+r)^{-n}} - \frac{nr}{2} = \frac{nr}{2}\,\frac{1 + (1+r)^{-n}}{1 - (1+r)^{-n}} = \frac{nr}{2}\,\frac{(1+r)^n + 1}{(1+r)^n - 1}$$

is an even function of n, since it is unaltered by changing the sign of n. It is not an even function of r, but can be made into an even function of a new variable y by setting

$$(2.2) \qquad 1 + r = \frac{1+y}{1-y}, \text{ or } y = \frac{r}{2+r}$$

and dividing the expression (2.1) by $1 + r/2$. The doubly even function that we get is the function f_1 defined by (1.3) and has the value

$$(2.3) \qquad f_1(ny,y) = ny\,\frac{(1+y)^n + (1-y)^n}{(1+y)^n - (1-y)^n} = \frac{g_0}{g_1}$$

This function f_1 is actually the ratio g_0/g_1 of two so-called "contiguous hypergeometric functions" g_0 and g_1 that are defined by series involving binomial coefficients:

$$(2.4) \qquad g_0 = [(1+y)^n + (1-y)^n]/2 = \sum_{s=0}^{\infty} \binom{n}{2s} y^{2s}$$

$$g_1 = [(1+y)^n - (1-y)^n]/2ny = \sum_{s=0}^{\infty} \binom{n-1}{2s}\frac{1}{1+2s}\, y^{2s}$$

The difference $g_0 - g_1$ contains the factor $(n^2-1)y^2$ since it vanishes for $n = \pm 1$ and $y = 0$. After simplifying coefficients and replacing s by $s+1$ we have

$$(2.5)\quad g_0 - g_1 = \frac{(n^2-1)y^2}{2^2-1} \sum_{s=0}^{\infty} \binom{n-2}{2s} \frac{1}{1+2s}\,\frac{3}{3+2s}\, y^{2s}$$

The right hand sum may be called g_2 if we define a general g_k by the series

$$(2.6)\quad g_k = \sum_{s=0}^{\infty} \binom{n-k}{2s} \frac{1}{1+2s}\,\frac{3}{3+2s} \cdots \frac{2k-1}{2k-1+2s}\, y^{2s}$$

A direct comparison of coefficients of y^{2s} then shows that any three successive g's satisfy the recurrence relation

$$(2.7)\quad g_{k-1} = g_k + \frac{n^2-k^2}{4k^2-1}\, y^2\, g_{k+1}, \quad k = 1,2,3,\ldots$$

When we generalize the definition $f_1 = g_0/g_1$ in (2.3) by defining

$$(2.8)\quad f_k(ny,y) = (2k-1)\,g_{k-1}/g_k,$$

it follows immediately that the relation (2.7) assumes the form

$$(2.9)\quad f_k(ny,y) = (2k-1) + (n^2-k^2)y^2/f_{k+1}.$$

Writing x for ny we get formula (1.2), and by successive applications of (1.2) for $k = 1,2,3\cdots$, we get our original continued fraction (1.1). This is known to converge for all x and for $y^2 = r^2/(2+r)^2 < 1$. Having established the formulas in § 1 we now turn to several applications.

3. *Periodic rent of an annuity.* Suppose that a multimillion dollar loan is to be repaid in equal monthly installments spread over 30 years with interest at the nominal annual rate of 3¼%, convertible monthly, and the periodic rent per dollar of loan $(1/a_{\overline{n}|r})$ is to be found to ten significant figures. Suppose further that this quantity is not found for $n = 360$, $r = 13/4800$ in available tables. How can it be computed with the aid of a ten place desk calculator?

Direct calculation of $(1+r)^{360}$ by successive squaring and multiplication would leave only 7 accurate figures of the original 10 and cannot be used. The binomial theorem is possible, but 13 or 14 terms would be needed, and there would still be divisions to perform to ten places for the final answer in (1.3). The fifth convergent of the continued fraction in (1.1) obtained by replacing f_6 by 11 gives the required accuracy.

We compute $x = nr/(2+r)$, $y = r/(2+r)$, x^2, and y^2 to 10 decimals. Then we write down $x^2 - k^2y^2$ for $k = 5,4,3,2,1$ and compute f_k successively for $k = 5,4,3,2,1$ by adding the integer $2k-1$ to $(x^2-k^2y^2)/f_{k+1}$.

$$(3.1)\quad n = 360,\ r = \frac{13}{4800},\ x = \frac{nr}{2+r} = \frac{4680}{9613},\ y = \frac{x}{n} = \frac{13}{9613}$$

$x = 0.48684\ 07365$, $\quad y = 0.00135\ 233538$

$x^2 = 0.23701\ 39027$ $\quad y^2 = 0.00000\ 18288$

$$(3.2)\quad \begin{aligned} x^2 - 25y^2 &= 0.237 & f_5 &= 9.022 = 9 + 0.237/11\\ x^2 - 16y^2 &= 0.23698 & f_4 &= 7.02627\\ x^2 - 9y^2 &= 0.23699\ 74 & f_3 &= 5.03373\ 019\\ x^2 - 4y^2 &= 0.23700\ 6588 & f_2 &= 3.04708\ 3689\\ x^2 - y^2 &= 0.23701\ 20739 & f_1 &= 1.07778\ 32505\\ & & x + f_1 &= 1.56462\ 39870\\ & & (r/2)(x+f_1) &= 0.00211\ 87616\\ & & n/a_{\overline{n}|r} &= 1.56674\ 27486\\ & & 1/a_{\overline{n}|r} &= 0.00435\ 20631\ 91 \end{aligned}$$

4. *A continued fraction for log* $(1+r)$. If in (1.3) we let n approach 0, the limit of the left member is $r/\log_e(1+r)$. By taking reciprocals we have

$$(4.1)\quad \log_e(1+r) = \frac{2y}{f_1(0,y)} \quad \text{where } y = r/(2+r).$$

Hence

$$\log_e(1+r) = \cfrac{2}{1/y - \cfrac{1}{3/y - \cfrac{4}{5/y - 9y/f_4(0,y)}}}$$

Thus to compute $\log_e 2$ we have $r = 1$, $1/y = 3$,

$$(4.2)\quad \log_e 2 = \cfrac{2}{3 - \cfrac{1}{9 - \cfrac{4}{15 - \cfrac{9}{21 - \cfrac{16}{27 - \cdots}}}}}$$

for which successive convergents are

$$
\begin{aligned}
(4.3)\qquad 2/3 &= .67\\
9/13 &= .6923\\
131/189 &= .69312\\
445/642 &= .69314\ 64\\
34997/50490 &= .69314\ 7158\\
\cdots\cdots &\\
\log_e 2 &= .69314\ 7180
\end{aligned}
$$

It can be shown that the error of the kth convergent to log 2 is less than $(.03)^k$.

5. *Continued fractions for* e^x *and cot* x. If we replace n by x/y in (2.3) and let y approach 0 we obtain as a limit

$$(5.1)\qquad f_1(x,0) = x\,\frac{e^x + e^{-x}}{e^x - e^{-x}} = x \coth x$$

Multiplying above and below by e^x and replacing x by $x/2$ we have

$$(5.2)\qquad \frac{e^x + 1}{e^x - 1} = \frac{f_1(x/2,0)}{x/2}$$

We subtract 1, take the reciprocal and solve for e^x, getting

$$(5.3)\qquad e^x = 1 + \frac{2x}{-x + 2f_1(x/2,0)}$$

$$e^x = 1 + \cfrac{2x}{2 - x + \cfrac{x^2}{6 + \cfrac{x^2}{10 + \cdots}}}$$

Special cases of (5.3) are

$$(5.4)\qquad e = 1 + \cfrac{2}{1 + \cfrac{1}{6 + \cfrac{1}{10 + \cdots}}} \qquad\qquad e^2 = 7 + \cfrac{2}{5 + \cfrac{1}{7 + \cfrac{1}{9 + \cdots}}}$$

We can change from x *coth* x to x *cot* x in (5.1) by replacing x by ix, and thus obtain

$$(5.5)\quad x \cot x = f_1(ix, 0) = 1 - \cfrac{x^2}{3 - \cfrac{x^2}{5 - \cfrac{x^2}{7 - \dots}}}$$

6. *Computation of roots by continued fractions.* If we solve equation (1.3) for $(1 + r)^n$ we obtain

$$(6.1)\quad (1 + r)^n = 1 + \frac{2y}{-x + f_1(x,y)}, \quad x = \frac{nr}{2 + r} = ny$$

By means of the substitutions

$$(6.2)\quad r = b/a^m, \quad m = 1/n, \quad u = 1/x = (2a^m + b)\, m/b$$

formula (6.1) becomes

$$(6.3)\quad (a^m + b)^{1/m} = a + \cfrac{2a}{u - 1 - \cfrac{(m^2 - 1)}{3u \quad \cfrac{(4m^2 - 1)}{5u - \cfrac{(9m^2 - 1)}{7u - \quad \dots}}}}$$

For example the cube root of 24 could be computed by taking $a = 3$, $b = -3$, $m = 3$, $u = 51$.

$$(6.4)\quad (24)^{1/3} = 3 + \cfrac{6}{50 - \cfrac{8}{153 - \cfrac{35}{255 - \dots}}} = 3.1201257345-$$

Methods for solving arbitrary equations by continued fractions are described in papers cited below as references. For the particular case of the cubic

$$(6.5)\quad y^3 - hy + h = 0, \quad h > 27/4$$

the smaller positive root can be computed from the expansion

$$(6.6)\quad y = 1 + \cfrac{1}{h - 3 - \cfrac{3}{h - \frac{10}{3} - \cfrac{26/9}{h - \dots}}}$$

References

J. S. Frame, The solution of equations by continued fractions, *Am. Math. Monthly*, vol. 60, no. 5 (May 1953), pp. 293-305.

H. S. Wall, *Continued Fractions*, D. van Nostrand, 1948.

NOTE ON THE REVOLUTION OF INTERRED MATHEMATICIANS ABOUT POST-MORTAL AXES

Oh, what can one do with the student obtuse
Whom obvious facts always strike as abstruse?
When sitting in class he's not sure any more
Whether $2 \times \frac{1}{2}$ is 1, 2, or 4.
And is $7 \div 0$ still naught? —
But, teacher, rejoice when your spirit's distraught
By the denseness of **one or two** struggling nitwits
That you aren't the corpse of Pascal or Leibnitz!

— Saddur N. Weiser

PROBLEM DEPARTMENT

Edited by
Leo Moser, University of Alberta

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within five months after publication. Address all communications concerning problems to Leo Moser, Mathematics Department, University of Alberta, Edmonton, Alberta, Canada.

PROBLEMS FOR SOLUTION

83. *Proposed by G. K. Horton, University of Alberta*

Evaluate

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\left(\sqrt{(x-1)^2+y^2}\;+\;\sqrt{x^2+(y-1)^2}\right)}dx\,dy \quad .$$

84. *Proposed by C. A. Nicol, University of Texas*

If n is a positive integer does there exist a positive integer k such that the sequence $k+1, 2k+1, 3k+1, \ldots, nk+1$ consists only of composite integers?

85. *Proposed by R. Isaacs, Van Nuys, California*

Let $x_1, x_2, \ldots, x_n$ $(n \geq 2)$ be distinct points in space. Define further x_{n+1} as the midpoint of x_1 and x_2; x_{n+2} as the midpoint of x_2 and x_3;... x_k as the midpoint of x_{k-n} and x_{k-n+1}. Show that the sequence $\{x_k\}$ converges and find its limit.

86. *Proposed by C. A. Grimm, South Dakota School of Mines and Technology*

For a, b, and x integers $(b > a)$ show that

$$x^3 + 3(a-b)x^2 + 3(a^2-b^2)x + a^3-b^3 \neq 0 \ .$$

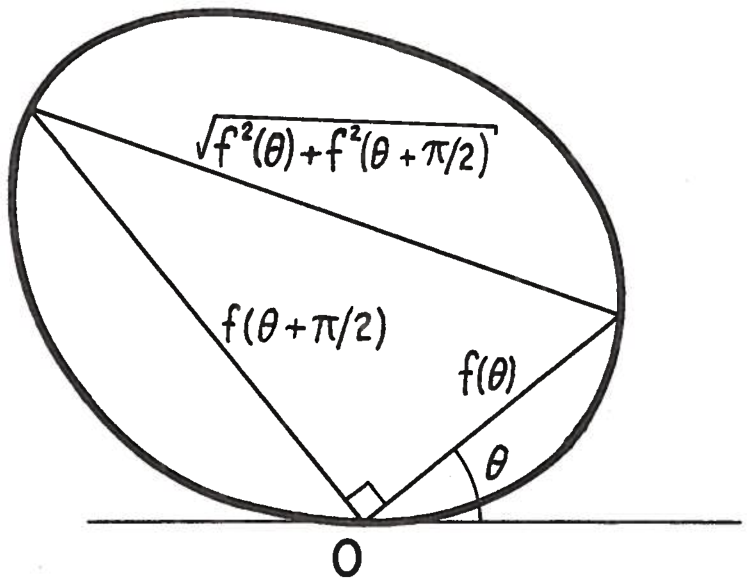
87. *Proposed by E. P. Starke, Rutgers University*

The centroid G of a triangle ABC is actually the center of area of ABC. Determine K, the centroid of the triangle considered as being composed of three linear segments. Show how to construct K and find some interesting geometric properties of this point.

SOLUTIONS

74. *Proposed by H. Helfenstein, University of Alberta*

Prove that every convex planar region of area π contains two points two units apart.



Solution by the proposer

Let the given convex region be placed so that it lies above the x axis and touches the axis at the origin. Suppose the boundary of the region in this position has polar equation $\rho = f(\theta)$ Then, since its area is π we have

$$\pi = \frac{1}{2}\int_0^{\pi} f^2(\theta)\,d\theta = \frac{1}{2}\int_0^{\pi/2} f^2(\theta)\,d\theta + \frac{1}{2}\int_{\pi/2}^{\pi} f^2(\theta)\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} (f^2(\theta) + f^2(\theta + \pi/2))\,d\theta \ .$$

Now $f^2(\theta) + f^2(\theta + \pi/2)$ represents the square of the distance between two points on the boundary of the region. Hence the assumption that no two points in the region are less than 2 units apart implies $f^2(\theta) + f^2(\theta + \pi/2) < 4$ and

$$\pi < \frac{1}{2}\int_0^{\pi/2} 4\,d\theta = \pi, \text{ a contradiction.}$$

76. *Proposed by J. J. Dodd, Aurora, Illinois*

Prove that $18! + 1$ is divisible by 437.

Solution by J. R. Trollope, University of Alberta

Since $437 = 19 \times 23$ it suffices to show that $18! + 1$ is divisible by 19 and 23. The divisibility by 19 follows from Wilson's theorem. Calculating with congruence (mod 23) we have

$$0 \equiv 22! + 1 \equiv 18!\,19\cdot20\cdot21\cdot22 + 1 \equiv 18!\,(-4)(-3)(-2)(-1) + 1 \equiv 18! + 1$$

which completes the proof.

Also solved by Leon Bankoff, J. Kiefer, and the proposer.

77. *Proposed by P. L. Chessin, Baltimore, Maryland*

If λ is the kth positive root of $\tan x = x$, prove that

$$\sum_{k=1}^{\infty} \frac{1}{\lambda_k^2} = \frac{1}{10}, \quad \sum_{k=1}^{\infty} \frac{1}{\lambda_k^4} = \frac{1}{350}\ .$$

Solution by Louisa S. Grinstein, Buffalo, New York

The roots of $\tan x = x$ are the same as those of $\sin x = x \cos x$. Using the well known series for $\sin x$ and $\cos x$ we see that

$$0 = \sin x - x\cos x = 2x^3 \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n-2}}{(2n+1)!}$$

The factor $2x^3$ yields the information that $x = 0$ is a triple root. Make the change of variable $x^2 = y$. Then the bracketed factor becomes

$$\sum_{n=1}^{\infty} (-1)^{n-1} y^{n-1}/(2n+1)!$$

Considering this as the left hand side of a "polynomial" equation of infinite degree, we proceed to make use of the theorems relating roots and coefficients and obtain,

$$\sum_{i=1}^{\infty} \frac{1}{y_i} = \sum_{k=1}^{\infty} \frac{1}{\lambda_k^2} = \frac{2}{5!}\ \frac{1}{3!} = \frac{1}{10}\ .$$

$$\sum_{i=1}^{\infty} \frac{1}{y_i^2} = \sum_{k=1}^{\infty} \frac{1}{\lambda_k^4} = \left(\frac{1}{10}\right)^2 - 2\,\frac{3}{7!}\ \frac{1}{3!} = \frac{1}{350}\ .$$

Also solved by the proposer who showed in a similar manner that

$\sum_{k=1}^{\infty} \frac{1}{\lambda_k^6} = \frac{1}{7875}$. He also showed that if $\gamma_k^{(n)}$ is the kth positive root of $J_n(x)$, (n integral), then

$$\sum_{k=1}^{\infty} \left(\gamma_k^{(n)}\right)^{-2} = \frac{1}{4(n+1)}$$

HELPFUL INFORMATION

A mathematics professor witnessed a hit and run accident. The police asked whether he recalled the license number of the fleeing car. The professor said, "No, but I did observe that the last four digits constituted the cube of the first two digits and that the sum of all six digits was odd."

WHAT'S THE DIFFERENCE BETWEEN RIGHT AND WRONG?

$$\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}, \quad \frac{19}{95} = \frac{1\cancel{9}}{\cancel{9}5} = \frac{1}{5}, \quad \frac{49}{98} = \frac{4\cancel{9}}{\cancel{9}8} = \frac{1}{2}$$

but

$$\frac{10a + b}{10b + c} \neq \frac{a}{c}$$

DEPARTMENT DEVOTED TO CHAPTER ACTIVITIES

Edited by
Houston T. Karnes, Louisiana State University

EDITOR'S NOTE. According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an annual report of the chapter activities including programs, results of elections, etc." The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the *Pi Mu Epsilon Journal*. Besides the information listed above we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Associate Editor Houston T. Karnes, Department of Mathematics, Louisiana State University, Baton Rouge 3, La. These reports will be published in the chronological order in which they are received.

REPORTS OF THE CHAPTERS

Delta of Ohio, Miami University

The Ohio Delta chapter held four meetings during the 1954-55 year. These meetings combined business and lectures by members of the department of mathematics. The papers presented were as follows:

"Highway Curves" by *Mr. Albert*

"History of Mathematics" by *Miss Wolfe*

"Measurement in Sociology" by *Mr. Dave Barr*

"Eclipsing Binaries" by *Mr. Miltenberger*

The annual banquet was held on May 2, 1955. The faculty and wives of the departments of chemistry, mathematics, and physics were guests. The guest speaker was Dr. E. W. Martin of the IBM Corporation. His subject was: "Electronic Digital Computers and their Applications".

Officers for the first semester were: President, Ted Schurman; Secretary, James Henkelman; Treasurer, James Hagias.

Officers for the second semester were: President, Gary Samuels; Vice-President, James Jones; Secretary, Nancy Lou Kirker; Treasurer, Gary Blue.

Alpha of Oregon, University of Oregon

The Oregon Alpha chapter held three business meetings and four program meetings during the 1954-55 year. The following papers were presented:

"Geometric Fallacies" by *Joy Heller*

"A Set Theoretic Approach to Nonsense" by *Barbara Thomson*

"A Problem in the Partition of Numbers" by *Leroy Warren*

"Euler and the Inductive Experimental Approach to Mathematical Research" by *Dr. George Polya*, Professor Emeritus of Stanford University

The annual initiation was held on May 23. Eighteen new members were initiated. In addition to the initiation ceremony the DeCou prizes of $50 each were presented to Barbara Thomson and Gerald Alexanderson, seniors in mathematics. The DeCou prize is annually awarded to one or two persons in memory of Edgar E. DeCou, for many years head of the department of mathematics, and his son, Edgar J. DeCou.

The Pi Mu Epsilon – Mathematics Department picnic was held May 24 at Armitage State Park.

Officers for the year were: Director, Barbara Thomson; Vice-Director, Joy Heller; Secretary-Treasurer, Gerald Alexanderson.

Officers for 1955-56 are: Director, Jack Goebel; Vice-Director, Sylvia Sommerer; Secretary-Treasurer, Loretta Humphreys.

Beta of Kansas, Kansas State College

The Kansas Beta chapter held five program meetings during the 1954-55 year. The following papers were presented:

"Some Physical Applications of the Theory of Finite Groups" by *B. Curnutte*

"Graeffe's Root-Squaring Method" by *M. P. Chrysler*

"Quaternions and Their Generalizations" by *J. D. Ferrucci*

"Legendre Functions" by *D. E. Myers*

"Mathematical Thinking and its Influence on Society" by *Y. L. Luke*

Eleven new members were initiated. Officers for 1955-56 are: Director, A. M. Feyerherm; Vice-Director, E. R. Lippincott; Secretary-Treasurer, S. T. Parker.

New Jersey Alpha Prize Awards

Alpha Chapter of Pi Mu Epsilon, Rutgers University, New Brunswick, New Jersey, offers a prize of books to the first three high school students in the prize examination given on State Mathematics Day in April. Last year the books chosen were Courant and Robbins, "What is Mathematics?", Kasner and Newman, "Mathematics and the Imagination", and Ball, "Mathematical Recreations."

Kansas Gamma Scholarship Fund

At a regular meeting of Kansas Gamma chapter of Pi Mu Epsilon, held January 31, 1956, the chapter voted to purchase five shares of Kansas Gas and Electric Common Stock to give to the University of Wichita, to increase the principal of the Pi Mu Epsilon Mathematics Scholarship fund. This scholarship is awarded annually to a student in the University, majoring in the field of mathematics.

Montana Alpha Prize Awards

Each year, the Montana Alpha chapter of Pi Mu Epsilon at Montana State University awards three cash prizes on the basis of an examination given to all freshmen who intend to register for mathematics at the University. The purpose of these awards is to stimulate interest in mathematics in the high schools of this state.

This year's winners are as follows:

First: Robert E. Sullivan, Missoula, Montana.

tied: Second: Thomas A. Branch, Cut Bank, Montana.
Third: William G. Bork, Butte, Montana.

OVERHEARD ON THE BUS

"He looks like a typical college mathematics professor, but actually he's a very pleasant fellow."

STORIES OF FAMOUS MATHEMATICIANS

A GAUSSIAN DILEMMA

The famous millionaire mathematician, Professor G. A. Miller of the University of Illinois, was an ardent and critical student of the history of mathematics in his later years. He took particular delight in finding "errors" in other people's writings on the subject, and would go to great lengths to set the matter straight. The records of these frequently bitter battles are scattered throughout a wide variety of journals.

On one occasion he discovered a remark attributed to Gauss in the writings of Professor E. T. Bell, and wrote Bell something to this effect: "I have looked through all of Gauss's works, and I could not find your so-called 'quotation' anywhere in them. I don't believe Gauss ever said that."

Bell's typically clever reply: "I challenge you to prove that he didn't say it!"

— F. E. Hohn

THE MISSING BELT

One spring day the late Professor E. H. Moore of Chicago fame walked into his class on General Analysis, in which the emphasis was on postulates and logical processes. Handsomely dressed – from spats to hat – in a new spring outfit, he greeted the three men and one woman who attended the course. Carefully, he laid aside on a chair his cane, gloves, hat, and topcoat. Muttering "It is warm!", he removed his suit coat and put that on the chair. Next he looked at his vest, removed it, and finally, looking at his suspenders, he carefully removed them also and laid them on the chair. Then he walked to the board, took up a piece of chalk, paused, turned to the class and said, "For Miss Pepper's peace of mind, we will postulate a belt."

— Echo Pepper

The Real Projective Plane. H.S.M. Coxeter. The Cambridge University Press, 1955.

This book is described very accurately by the publishers as "an introductory university textbook on projective geometry, including a thorough treatment of conics and a rigorous presentation of the synthetic approach to co-ordinates."

Following a brief introductory chapter, the author presents in Chapters 2 and 3 his axioms of incidence, order (based on separation as an undefined concept), and continuity. Projectivities and correlations are treated in the following two chapters. Chapters 6 and 7 are devoted to the conic. The author identifies the points of view of Steiner and von Staudt, and carries the development through some introductory properties of pencils of conics. Especially here does the importance of von Staudt's definition stand out sharply. Chapters 8 and 9 deal with affine and euclidean geometries – in the latter the author presents an elegant definition of congruent angles. In Chapter 10 the author returns to fundamentals; he replaces the intuitive continuity axiom of Chapter 3 (based on fixed points of projectivities in one dimension) by the more sophisticated axiom that every monotonic sequence of points has a limit, and then derives some of the fundamental properties of the real number system, expressed in geometrical language. This leads naturally to the last two chapters where coordinates are introduced and employed.

All in all, this is a most admirable book. Professor Coxeter has done a remarkable job of organization and selection of material. Our best undergraduate mathematics majors will be delighted with it; they will begin to sense the beauty of mathematical theories, for the author does not distract them with a mass of detail. The book should be a *must* for all our prospective teachers of high school geometry, but unfortunately many of these, unprepared for the discipline of a mathematical system by the inadequacies of their own high school geometry courses, may find the going rough. The remedy is the reinstatement, in the high school curriculum, of the training that a student at one time received from his course in demonstrative geometry.

Harry Levy
University of Illinois

Elementary Topology. By Dick Wick Hall and Guilford L. Spencer II. John Wiley & Sons, New York, 1955.

This book is intended as an exposition for advanced undergraduates of the basic ideas of point set topology. As far as the reviewer is aware, it is the only book on the subject directed to such an audience. It is our opinion that it will be widely used and enjoyed.

The authors indicate that they have used this book as a text with students who have not had advanced calculus. While such students might find the treatment rather abstract and difficult, students with more mathematical maturity should be able to read at least the first half of the book with satisfactory speed and comprehension. The authors pay rather careful attention, especially in the earlier chapters, to various methods of proof and a variety of problems are offered.

The book falls into two rather distinct parts. The first half (Chapters I-IV, 149 pages) is concerned with introductory set theory, the real line, and a study of topological and metric spaces. This part is elementary in character and would probably be suitable for a one-semester senior course. The basic operations with sets are introduced in Chapter I, relations and mappings are discussed, and an introduction to infinite and uncountable sets is given. Chapter II does not develop the real numbers, but treats them as decimals and introduces the basic topological ideas for the real line. In Chapter III, general topological and metric spaces are introduced and such topics as compactness, separation and continuous functions are discussed. Metric spaces are pursued further in Chapter IV, with discussions of local connectivity, countability, metrizability and completion being included.

The second part (Chapters V, VI, 124 pages) is less elementary in character. The long Chapter V is concerned with giving topological characterizations of arcs, simple closed curves, and simple closed surfaces. Peano spaces are discussed and the Jordan curve theorem and Jordan-Schoenflies theorem are proved. Chapter VI is concerned with R. H. Bing's work on partitionable spaces.

Finally, Chapter VII discusses the axiom of choice, Zorn's lemma (in the form commonly called the Hausdorff maximality principle) and the Tychonoff product theorem. Hence, this chapter is closer to the first portion of the book than the second.

The authors have tried, successfully we believe, to indicate the motivation for the abstract notions, although some additional examples would be useful. Proofs seem to be well arranged and clearly expressed and, as is appropriate, many of the easier proofs are left to the student as exercises. (However, some readers may object to the practice of having some of the proofs depend upon problems at the *end* of the section being read.) One interesting feature of the problem sets is that several term papers, dealing with Moore-Smith convergence, the Hilbert cube, category in complete metric spaces, and Peano spaces, are suggested in outline form.

The exposition is quite readable and clear. The style is easygoing and informal – although sometimes to the extent of employing classroom jargon. (For example, the hypothesis "let A and B be sets, $f:A \to B$ a mapping onto" actually occurs.) The terminology and notation are usually well-chosen (the outstanding exception being the term "totally partially ordered set") and generally adhere to standard practices. Also the printer and the proofreaders have done their jobs well, for the format is clean and pleasing and errors appear to be essentially absent.

In summary, we feel that the authors have written a good book – one which students and teachers will appreciate.

Robert G. Bartle,
University of Illinois

Elementary Differential Equations. By William T. Martin and Eric Reissner. Cambridge Mass., Addison Wesley Publishing Company Inc., 1956.

This book is organized around four central ideas, motivation, theory, application, and repetition. The entire Chapter 1 is largely motivating material. As to the theory, the material is developed with adequate regard for rigor and is presented in a readable, flowing style with sufficient discussion to bring the points considered within easy grasp of the student. The student in turn is expected to know only elementary calculus and the basic properties of power series. In one section of Chapter 2 however, a property of singularities of analytic functions is employed in the discussion of convergence of series solutions. The authors state in the preface that this section can be omitted without disturbing the continuity of the course.

Numerous examples are worked out in detail to illustrate the methods presented and an adequate number of problems are supplied for the student to solve. The answers to all problems are supplied in the back of the book.

As to the theme of repetition, most of the methods of solving differential equations introduced in Chapter 2 reappear in Chapters 3, 4, and 5, in different contexts. Obviously the idea is that by repetition the student gains a familarity with the method which enhances his understanding and insight.

Throughout, the selection of material is motivated by its usefulness in applications. This view is apparent from the emphasis on series solutions, a section devoted to the Laplace transform, and a chapter on difference equations. These are however presented from a mathematical point of view.

The book consists of eight chapters. Chapter 1, as its title suggests, is concerned with the "nature and origin of differential equations." Such topics as one dimensional steady heat flow, motion along a straight line, linear motion with variable mass, and electrical networks are discussed. Chapter 2, entitled "the differential equation of first order," introduces the idea of a singular as well as a general and a particular solution for a linear differential equation. One section is devoted to isoclines. The topics of separation of variables, integrating factors, variation of parameters, Clairaut's equation, power series solutions, undetermined coefficients, convergence of series solution, and other series methods are included. Each topic is discussed with respect to first order equations only. Chapter 3 is concerned with second order differential equations. Most of the methods of Chapter 2 are applied again. In addition two sections are devoted to series solutions in the neighborhood of a regular singular point. The chapter concludes with a study of Bessel functions. In Chapter 4 on higher order differential equations, in addition to the previously discussed methods, there is presented the Laplace transform. Chapter 5 is concerned with systems of first order differential equations. The content of Chapter 6 is described in its title, "Approximate solution of first order differential equations and Picard's Theorem." Chapter 7 considers finite difference equations, concluding with a section on approximating differential equations by difference equations. The last chapter takes up partial differential equations and their applications.

In the reviewer's opinion the book is well organized and well written, with the result that it should be of value as a text for both mathematics majors and non-majors. One misprint was noted; page one, equation (1), should read dy/dx instead of du/dx.

Wilson M. Zaring
University of Illinois

Back Issues for Sale

We have a large supply of back issues of the *Journal* on hand. We will be glad to sell these to members desiring a complete file at $3.25 for 13 issues, Vol. 1, Number 1, through Volume 2, Number 3. Short sets pro rata.

NEWS AND NOTICES

Arthur L. Sagle is Awarded Fellowship

We extend our congratulations to Pi Mu Epsilon member Arthur L. Sagle (Washington Beta, 1955) who has received a National Science Foundation Pre-Doctoral Fellowship for the year 1956-1957.

Dr. Charles H. Helliwell

Dr. Charles H. Helliwell, associate professor of mathematics at New York University's School of Commerce, Accounts, and Finance, died November 29, 1955 in New York City while on the way to his home at Dover, N. J. He was 58 years old.

Dr. Helliwell joined the NYU faculty in 1929 as an instructor in mathematics at the Washington Square College of Arts and Science. He taught at high schools in Leonardo, N. J., and Meriden, Conn., and at Stevens Institute of Technology, Hoboken, N. J., and Drew University, Madison, N. J., before coming to NYU.

He was a member of *Pi Mu Epsilon*, Psi Upsilon social fraternity, Phi Delta Kappa honor fraternity, the American Mathematical Society, and the New York University Faculty Club.

Dr. Helliwell is survived by his widow, Nanette, a son, Charles, and a daughter, Sally.

APOLOGY

We're a little thin this time, but we wanted to get back on schedule rather than wait for more material. If you readers will send in articles, news, problems, etc., **soon**, we can have a huskier fall issue and in addition get it out on time.

HELP! HELP!

We **need more stories** of famous mathematicians like the two which appear in this issue. Can you help us? Also, we need more of the little "space fillers" we use to spice up these pages. Please send in your favorites!

We are very proud of the article by Professor Lorch which appears in this issue. It was originally a speech. If any reader hears a mathematical speech of similar quality please **let us know at once** so that we can write the author airmail!

INITIATES

ACADEMIC YEARS 1954-5 (continued), 1955-6 (continued)

ALABAMA ALPHA, University of Alabama

(December 15, 1954)

George Albert Bailey
Walter Virgil Bouldin
Harold Carlton Fitz, Jr.
Bertram Fleischer
John Anthony Gioiello
Joseph Howe Hadley, Jr.
Hope Hodnette
Jo Anne Johnston
Flemy Dale Kennemur
N. Clarice Sanford
Mary Lois Singley
Theodor D. Sterling
Herman R. Wesson
James E. Winchester

(May 7, 1955)

Carol Sue Bostick
John Culver Chaty
William Oliver Criminale, Jr.
Joseph D. DeLorenzo
Alice Kathleen DeRamus
Guthrie E. Farrar
Geddie Harris Fredy, Jr.
Joe Harvey
Daniel L. Hollis, Jr.
William H. Jenkins
Myunghwan Kim
Alan H. Marshall
Patricia Lee Moore
Robert Jordan Naumann
Thomas G. Provenzano
Thomas E. Rogers
Harold L. Sellars
Gilbert O. Spencer, Jr.
Milton H. Spiegal
Joseph E. Stiles
Alton H. Wallace
John B. Winch

(December 13, 1955)

James D. Bercaw
Jo Ann Carr
Turner E. Hasty
Joan Kassner
Clarence N. Lee, Jr.
William W. Lynch
Michael K. McAbee
Emma J. J. Pitts
Joseph L. Randall
Percy H. Rhodes, Jr.
Tunstall Searcy, Jr.
Mary J. Sexton
Chester E. Thomas

ALABAMA BETA, Alabama Polytechnic Institute

(March 1, 1956)

Barbara Adams
George S. Birchfield
J. Mervin Bridges
Edward L. Daniel
John Cecil Dendy
Martin C. Dorman
W. Thomas Edwards
Albert N. Ellis
Gerald Ray Guinn
Mildred Jeannette Hurst
Harry Oscar Lindstrom
Edward H. McAdam
Joe Thomas McMillan
Robert W. McMillan
Fred W. Mace
Charles O. Ming
R. B. Morrow
Clairborne E. Myers
James F. O'Brien
George Papaiconomou
Charles H. Peterson
Franklin M. Propst
James Donley Sherman
Richard E. Slye
Sam M. Strickland
Grady Reubin Vines

ARKANSAS ALPHA, University of Arkansas

(March 31, 1955)

Harry Howlett
William Richard Mixon
Frank D. Neighbors
Thomas E. Taylor
Edward E. Williams

(April 19, 1955)

John R. Stallings
Mack R. Wells

(November 10, 1955)

Harold Lee Cornish
Robert A. Cross
Michael Dertouzos
James E. Erskine
Virgil O. Floyd
Bobby R. Frey
Jesse C. Holloway
Don Lewis
Tom Lee Powers
Jewel G. Rainwater
Benjamin F. Smith
Alfred W. Taylor

CALIFORNIA ALPHA, University of California

(January 7, 1956)

Earl R. Berkson
Judith Brostoff
Kenneth M. Ferrin
Herbert A. Gindler
Alan J. Gross
Richard Spencer Grote
Raymond B. Killgrove
Martin Posner
Edward W. Pullen
Florolyn E. Sterns

CALIFORNIA BETA, University of California (Berkeley)

(May 21, 1955)

Gary A. Aldrich
Norman Hardy
Jeremy Kilpatrick
Jerome Klotz
Myron R. Porter
Richard E. Taylor
San Fu Tuan
Jonathan D. Young
Charles E. Watts

GEORGIA ALPHA, University of Georgia

(February 1, 1956)

Joseph Jerome Day, Jr.
Robert S. Edwards
Billy Broach Sorrow

IOWA ALPHA, Iowa State College

(December 1, 1955)

Albert Ginsberg

MISSOURI ALPHA, University of Missouri

(December 1, 1955)

M. Susan Brady
Joe Edward Butler
Charles E. Campbell
Donald Payson Clark
Jack Roberts Farmer, Jr.
Elbert A. Grimsley
Ronald Clay Haynes
Kundry S. Herrmann
Kenneth Max Jobe
Harold Albert Kiehl
Charles Ralph Kost
Paul Edward Long
Carl Louis Mariz
Robert Leroy Rush
Judy Frances Wasco
Thompson F. Willis
Wallace Hunter Wilson

MISSOURI GAMMA, St. Louis University

(March 22, 1955)

Laima E. Abramikas
William M. Adams
Barbara J. Amidei
Stan W. Augustyniewicz
James W. Bailey
Ray G. Baker
Alfred J. Bilik
Edward W. Brande, S. J.
Gene Braught
William G. Burke
Sr. Mary Constance Busch, C.S.A.
Joseph H. Campana, S.J.
John C. Cantwell
John P. Cappellupo
Theodore G. Carr
Vondell Carter
Jesse T. Cherry, Jr.
D. L. Cooper
Valentine Dahlem
Richard Figueras
Joseph F. Froechtenigt
V. Gallardo
John George
Florence Haas
Edward V. Hamilton, Jr.
Eugene F. Held
Donald R. Hesser
Franz O. Hug
Galen Johnston
Joseph G. Kappel
George M. Kern
Donald L. Kettelkamp
Martin F. King
Doris M. Kirchhoff
Sr. Mary Kenneth Kolmer, AdPPS
Eugene Krupa
George R. Kuhn
Ronald P. Ladd
Janet Ann Lamm
William J. Landre
Jaclyn A. Leiendecker
Juanita M. McCarthy
Jean M. McGraw
Thomas L. Morris
Clifford J. Murino
Harry P. Mutter
Joseph Petrick
David K. Phillips
Donald L. Ringkamp
William J. Roach
Frank W. Rumreich
William F. Schallert
Gerald H. Schoemehl
William G. Schroeder
Joseph F. Shaughnessy, Jr.
Robert H. Slojkowski
Marjorie M. Soucy
Richard Spreckelmeyer
Sr. Patricia Anne Stark, SND
Thomas P. Sullivan
Patricia J. Tulley
Sarah Anne Walsh
Paul H. Wirsching
William W. Wolcott, S.J.
Carol Wonn

NEBRASKA ALPHA, University of Nebraska

(January 11, 1956)

John Allen Ball
Richard Ryan Foll
Alan Jay Heeger
Eudell Gene Jacobsen
William Mervyn Kimberly
Douglas Jason Mansfield
Russell LeRoy Nielsen
Melvin Chandler Thornton

NEW JERSEY ALPHA, Rutgers University

(March 30, 1955)

David L. Anderson
Frederick O. Bartholomew
William F. Beach
Richard P. Bernicker
Donald S. Brightly
Richard N. Brown
Ronald Caruso
Harold Christoffersen
John W. Conway
Rudolph Downs
Frederick L. Durand
Elmer W. Galbiati
William F. Goodyear
Richard J. Gowen
Michael J. Gyetvay
Richard W. Hill
Edward E. Hoffman
James B. Howell III
Alfred H. Kalantar
Paul L. Kelley
Mitri G. Khamis
Floyd M. Kregenow
Orville LaMaire
Arthur P. Luthy
Norman Madison
James Mertz
Charles E. Molnar
Eric W. Reinhardt
John J. Reiser, Jr.
David Schweizer
Frank Sparacio
William E. Steinke
M. P. Ribme Thomsen
Donald E. Troxel
Carl R. Turner
Richard W. Tweedie
Stanley Wasielewski, Jr.
George R. Weiss

NEW YORK BETA, Hunter College

(March 20, 1955)

Sybil Adler
Lilly Chin
Sandra Daniels
Marilyn Dunayer
Paul Freund
Leslie Gross
Harriet Hoof
Bernice Jaffee
Marilyn Roane
Mary Louise Shrier
Frances Testa

NEW YORK ETA, University of Buffalo

(February 1, 1956)

John H. Hodges
Joy B. Russek

OHIO GAMMA, University of Toledo

(February 6, 1956)

Marvin A. Davis
William Frederick
John L. Ginther
William Goldberg
Richard Marleau

PENNSYLVANIA BETA, Bucknell University

(December, 1955)

Alison Almy
William Beck
Charles R. Boss
David O. Fairley
Thomas J. Goblick, Jr.
Evelyn P. Grimm
William Halprin
George E. Letchworth
Lawrence E. Light
Robert L. Long
Donald E. Macaw
Georgie Ann MacKay
Clifton Martin
James J. Paugh
Robert Schiffenhaus
George A. Silfen
Doris J. Toft
Suzanne Tucker
Patricia L. Wenk
Mahlon W. Wagner

WISCONSIN BETA, University of Wisconsin

(November, 1955)

Herbert Fishman
Raynard A. Hedberg
Vilas D. Henderson
Lowell Leake, Jr.
Robert Fones Williams