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Envelopes of Certain Families of Conics[1]

KATHERINE SUE LIPPS[2]
Missouri Gamma, '57

1. Introduction. It is proposed to find and discuss the envelope of a family of conics. This family has certain conditions placed upon it, one being that a fixed point on the conic has a fixed Frégier point. In explanation for those who may not be familiar with this point, it is thought proper to state the following.

Today a student of geometry usually encounters the theorem of Frégier as an exercise when studying projective geometry. Allied theorems and exercises are amendable to the methods of projective geometry. However, Frégier presented the following theorem, with many more, using the method of Cartesian geometry in Gergonne's Annales de Mathematiques (t. VI, 1816, pp. 229-241, pp. 321-326, and t. VII, 1816, pp. 95-99).

Theorem. *If a variable chord PQ of a conic subtends a right angle at any fixed point* V *on the conic, it passes through a fixed point* F*, which lies on the normal at* V.

The problem to be discussed is as follows:

If a variable conic

$$Ax^2 + 2Bxy + Cy^2 + Dx + Ey + G = 0 \tag{1.1}$$

with $B^2 - AC$ fixed is tangent to a fixed line at a fixed point V and the Frégier point F of V is fixed, then the envelope of the family of conics is determinable.

2. The Case of the Ellipse. First, let us assume $B^2 - AC < 0$; thus, neither A nor C can be zero. Let us further assume that V has coordinates (0,0) and that the conic is tangent to the y-axis. The Frégier point F lies on the normal to the tangent at V or the x-axis. Let the coordinates of F be (k,0) where $k > 0$.

Since V is at the origin and the slope of the tangent at that point becomes unbounded, the coefficents G and E are zero. Also, with $C \neq 0$, we may divide the equation (1.1) by C. Thus, the equation of the ellipse may be expressed as

$$ax^2 + 2bxy + y^2 + dx = 0. \tag{2.1}$$

Draw MN perpendicular to the x-axis at F and join $M(k,y_1)$ and $N(k,y_2)$ to V. Since by the property of the Frégier point the angle

[1]presented to the national meeting at Penn State University, August 26, 1957. Received by the editors October 15, 1957.

[2]Editor's note: Miss Lipps, whose major is mathematics, was a Sophomore in the College of Arts and Sciences at St. Louis University, when this paper was written.

MVN is a right angle, MV and NV are perpendicular lines. By solving (2.1) with $x = k$, we obtain

$$y_1 = -bk + \sqrt{[k^2(b^2-a) - dk]}$$

$$y_2 = -bk - \sqrt{[k^2(b^2-a) - dk]}$$

From the hypothesis, $B^2 - AC < 0$, $b^2 - a$ also is negative; we shall let $b^2 - a = -g$ $(g > 0)$. Hence, from above it is seen that

$$(2.2) \qquad y_1 = -bk + \sqrt{[-gk^2 - dk]}$$

$$(2.3) \qquad y_2 = -bk - \sqrt{[-gk^2 - dk]}$$

Since the slopes of MV and NV are the negative reciprocals of one another,

$\frac{y_1}{k} = -\frac{y_2}{k}$. Using (2.2) and (2.3) we obtain

$$d = -k(b^2 + g + 1).$$

Substituting values for (a) and (d) in (2.1), we obtain

$$(2.4) \qquad b^2x^2 + gx^2 + 2bxy + y^2 - b^2kx - gkx - kx = 0.$$

(2.4) is the second degree equation of a one-parameter family of ellipses with the given restrictions.

By solving simultaneously (2.4) and the partial derivative of (2.4) with respect to the parameter, b, we obtain the envelope of this family of curves,

$$(2.5) \qquad y\ k = x(x-k)[gx - k(g+1)].$$

Investigating $x < 0$ for (2.5), we find that no negative value of x will give real values of y. When considering $x > 0$, we find values $0 \leq x \leq k$ and $x \geq \frac{k(g+1)}{g}$ will give real values of y. Further investigation reveals a maximum and minimum when $x = k[\frac{(2g+1) - \sqrt{(g^2+g+1)}}{3g}]$ and a point of inflection when

$$2x = \frac{2k(2g+1)}{3g} + \sqrt{A - B}$$

where

$$A = \frac{4k^2(5g^2+5g+2)}{9g^2} + \frac{4k^2(2g+1)}{9g^2}\sqrt{2(g^2+g+1) + 3g(g+1)\sqrt[3]{2/g}}$$

$$B = \frac{2k^2(g+1)}{3g}\left[\sqrt[3]{2/g} + 2 - 2\sqrt{\sqrt[3]{4/g^2} + 2\sqrt[3]{2/g} + 4}\right].$$

Thus, using the property of symmetry, the curve for the envelope for the family of ellipses (2.5) appears in Fig. 1.

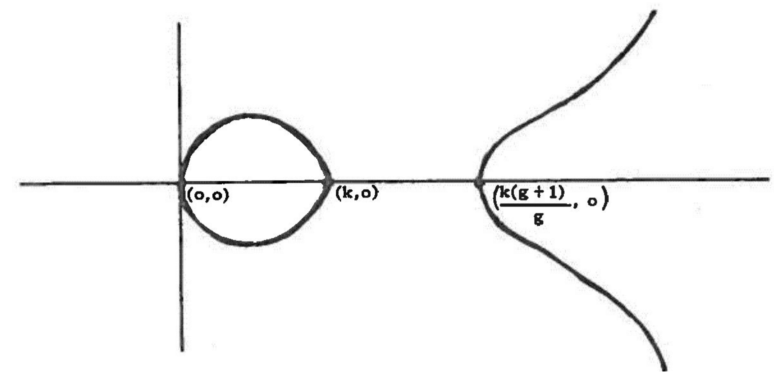


Figure 1

3. ***The Case of the Hyperbola.*** Let us assume $B^2 - AC > 0$. (The remaining assumptions for the case of the ellipse also pertain to this case.) It is not difficult to show that envelopes of the family of hyperbolas can only occur when $A \nleq 0$, $B \nless 0$, and $C \nless 0$.

Using these conditions, we notice that the equation for the envelope of the family of hyperbolas will differ from that for the family of ellipses only in the sign of g. Thus, following the method presented in §2, we find that the equation for the envelope for the family of hyperbolas upon which the given restrictions have been placed is

$$(3.1) \qquad y^2 k = x(x-k)[k(g-1) - gx].$$

It is noticed that the sign of (3.1) depends upon the range of g. Let us first consider $g > 1$.

Investigating $x < 0$ for (3.1), we find that all $x \leq 0$ give real values of y. When considering $x > 0$, the values of x such that $\underline{k(g-1)} \leq x \leq k$ are found to give real values of y. Further investigation reveals a maximum and minimum when $x = k[\frac{(2g-1) + \sqrt{(g^2-g+1)}}{3g}]$ and a point of inflection when $-g$ is substituted for g in the expression given for the point of inflection for the case of the ellipse. Using again the property of symmetry, the graph of (3.1) is in Fig. 2.

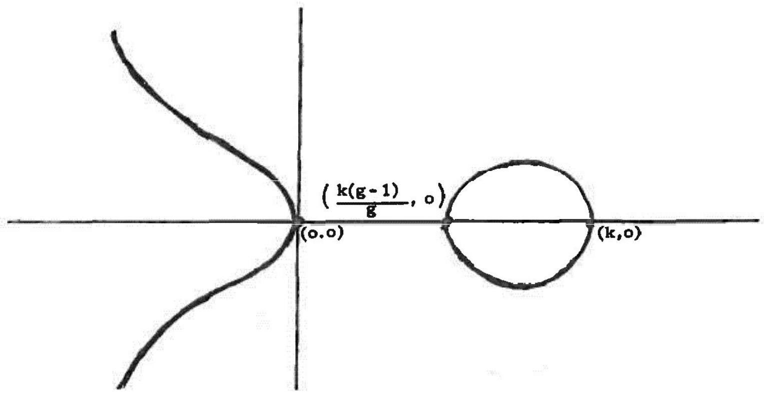


Figure 2

Let us now consider $g = 1$. (3.1) becomes

(3.2) $$y^2 k = -x^2 (x-k).$$

Investigating $x < O$ for (3.2), we find that all $x \leq O$ will give real values of y. For $x > O$, the values of x are $x \leq k$. The maximum and minimum occur when $x = \frac{2k}{3}$, and there is a double point at (O,O).

In Fig. 3, we have the sketch when $g = 1$.

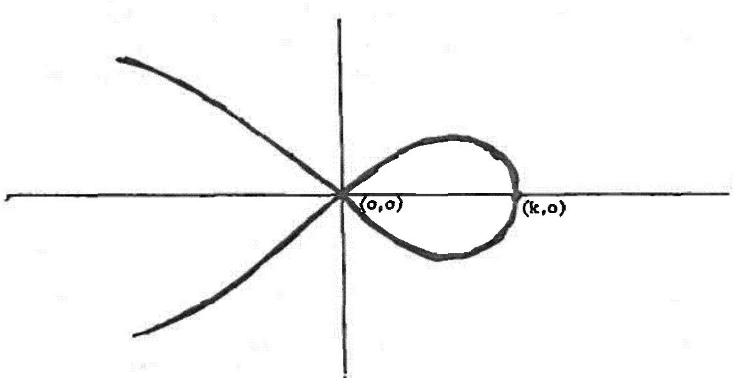


Figure 3

Let us consider $O < g < 1$. We find the values of y are real in equation (3.1) with $O < g < 1$ when $x \leq \frac{k(g-1)}{g}$. For $x > O$, the range of values for x is $O \leq x \leq k$ when y is real. The maximum and minimum occur again at the point given for $g > 1$. The points of inflection occur when

$$2x = \frac{-2k(1-2g)}{3g} - \sqrt{C + D}$$

where

$$C = \frac{4k^2(5g^2-5g+2)}{9g^2} - \frac{4k^2(1-2g)}{9g^2}\sqrt{2(g^2-g+1)-3g(1-g)\sqrt[3]{2/-g}}$$

$$D = \frac{2k^2(1-g)}{3g}\left[\sqrt[3]{2/-g} + 2 + 2\sqrt{\sqrt[3]{4/g^2} + 2\sqrt[3]{2/-g} + 4}\right]$$

Thus, Fig. 4 is the graph of (3.1) with $O < g < 1$.

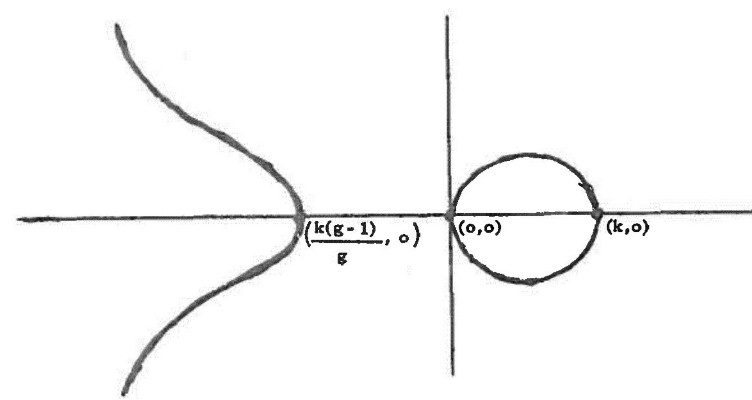


Figure 4

4. *The* **Case of** the Parabola. Let us now assume $B^2 - AC = O$ or that $b^2 a = -g = O$. We now have for the equation for the envelope of the family of parabolas upon which the above-mentioned restrictions have been placed

(4.1) $$y^2 = x(k-x).$$

Equation (4.1) is seen to be a circle with center at $(k/2,0)$ and radius of $k/2$.

St. Louis University

An Elementary Method for Finding All Numbers Both Triangular and Square[1]

RALPH M. WARTEN[2]
New York Gamma, '55

A triangular number is a positive integer of the form $\frac{x(x+1)}{2}$, where x is an integer. The problem of finding triangular numbers that are also squares was solved many years ago. In particular Euler[1] proved that $\frac{x(x+1)}{2}$ is a square, y^2, only when $x = \frac{a+b-2}{4}$, $y = \frac{a-b}{4\sqrt{2}}$, where $a = (3+2\sqrt{2})^n$, and $b = (3-2\sqrt{2})^n$. We also have the recursion formulas

$$x_n = 6x_{n-1} - x_{n-2} + 2, \text{ and } y_n = 6y_{n-1} - y_{n-2}.$$

S. Roberts[2], basing his work on Euler's, proved, moreover, that the triangular numbers which are squares are given by

$$\left(\frac{(1+\sqrt{2})^{2n} - (1-\sqrt{2})^{2n}}{4\sqrt{2}}\right)^2.$$

However, I found no place in the literature where the following elementary method for producing all positive integers that are both triangular and square is metioned.

It may also be observed in passing that the sequence $\{K_n\}$ defined below gives the solutions in integers of the equation $2a^2-b^2=+1$.

To my regret I found that in this respect my work had been accomplished some 2500 years ago by the Pythagorians[3] who had used precisely the algorithm described in this paper for solving $2a^2-b^2=\pm 1$. Although the Pythagoreans were interested in polygonal numbers (including triangular numbers), they apparently did not realize the significance of their method with respect to the problem of this paper, for, strangely enough, there is no mention of the fact that the integral solutions of $2a^2-b^2=\pm 1$ have anything to do with triangular numbers.

Problem: Find all positive integers that are both triangular and square.

Lemma 1. K is triangular and square if and cnly if $K = a^2b^2$, where $2a^2 - b^2 = \pm 1$. Proof: If $K = a^2b^2$ with $2a^2-b^2 = 1$, set $b^2 = n$. Then $2a^2 = n+1$ and $K = \frac{n(n+1)}{2}$. Thus K is both square and triangular.

A similar argument applies when $2a^2 - b^2 = -1$.

[1]Received by editors October 9, 1957.

[2]Editor's note: Mr. Warten graduated from Brooklyn College in June, 1957 and is now a graduate assistant in the Department of Mathematics at Purdue University. Mr. Warten was a student at Brooklyn College when this paper was written.

Conversely, if $K = \frac{n(n+1)}{2} = y^2$, suppose first that n is even. Then $\frac{n}{2}$ and $n+1$ are relatively prime. Moreover, since $K = (p_1 p_2 \cdots p_r)^2$, where $p_1, p_2, \ldots p_r$ are powers of distinct primes, we have

$$\frac{n}{2} = (p_1 p_2 \cdots p_m)^2 = a^2, \text{ and } n+1 = (p_{m+1} \cdots p_r)^2 = b^2.$$

Thus $K = a^2b^2$ with $2a^2 - b^2 = -1$.

If n is odd, a similar argument shows that $K = a^2b^2$ with $2a^2-b^2 = 1$.

Lemma 2. If $2A^2-B^2 = \pm 1$, where A and B are integers and $A > 1$, $B > 1$, then $2A > B > A$.

Proof: Assume $A \geq B$. Then $A^2 \geq AB \geq B^2 > 1$. Therefore, $2A^2 \geq A^2+B^2 > B^2+1$, and $2A^2 - B^2 > 1$. Accordingly $B > A$.

Now assume $B \geq 2A$. But $B > A$. Therefore, $B \geq A+1$. Hence, $B^2 \geq 2A(A+1) = 2A^2+2A > 2A^2+1$, and thus $2A^2 - B^2 < -1$. Consequently, $2A > B$.

Theorem: Define the sequence $\{K_n\}$ of positive integers as follows:

(1.) $K_1 = a_1^2 b_1^2$ with $a_1 = 1$, $b_1 = 1$.

(2.) $K_n = a_n^2 b_n^2$ for $n = 2, 3, 4, \ldots$, where $a_n = a_{n-1} + b_{n-1}$, $b_n = 2a_{n-1} + b_{n-1}$.

Then $\{K_n\}$ contains all and only those positive integers which are both triangular and square.

Proof: All terms of $\{K_n\}$ are both triangular and square, for: K_1 is triangular and square. Assume K_n is triangular and square. Then, by lemma 1,

$$K_n = a_n^2 b_n^2, \text{ where } 2a_n^2 - b_n^2 = \pm 1. \; K_{n+1} = a_{n+1}^2 b_{n+1}^2 = (a_n+b_n)^2(2a_n+b_n)^2. \text{ But } 2a_{n+1}^2 - b_{n+1}^2 = b_n^2 - 2A_n^2 = \mp 1.$$

Hence, K_{n+1} is triangular and square.

Any positive integer that is both triangular and square is a term of $\{K_n\}$, for: Let $P_1 > 0$ be triangular and square but not included in $\{K_n\}$.

Then $P_1 = A_1^2 B_1^2$ with $2A_1^2 - B_1^2 = \pm 1$, $A_1 > 1$, $B_1 > 1$ and $2A_1 > B_1 > A_1$.

Form the number $P_2 = A_2^2B_2^2$, where $A_2 = B_1 - A_1$, $B_2 = 2A_1 - B_1$. Using lemma 2 we know $2A_1 > B_1 > A_1$. But this inequality implies $A_1 > B_1 - A_1 > 0$. Therefore, $A_1 > A_2 > 0$. The same inequality also implies that $2B_1 > 2A_1$, so that $B_1 > 2A_1 - B_1$. Finally from $2A_1 > B_1$ we have $2A_1 - B_1 > 0$. Hence, $B_1 > B_2 > 0$. Moreover

$$2A_2^2 - B_2^2 = 2(B_1 - A_1)^2 - (2A_1 - B_1)^2$$
$$= B_1^2 - 2A_1^2 = \mp 1.$$

Hence, P_2 is triangular and square, and $P_1 > P_2 > 0$. Induction therefore shows that $\{P_n\}$ is a sequence of integers that are both triangular and square, and $P_1 > P_2 > \dots > P_n > \dots > 0$.

By hypothesis P_1 does not belong to $\{K_n\}$. Now suppose that P_2 belongs to $\{K_n\}$. But $P_1 = A_1^2B_1^2$ and from $A_2 = B_1 - A_1$, $B_2 = 2A_1 + B_1$, we obtain $A_1 = A_2 + B_2$, $B_1 = 2A_2 + B_2$, and so P_1 belongs to $\{K_n\}$.

Accordingly we must agree that P_2 does not belong to $\{K_n\}$, and that none of the other integers P_n belong to $\{K_n\}$. However, $\{P_n\}$ must have a least positive term, $P_k \neq 1$. But if this term P_k is greater than 1, then by lemma 1, $P_k = A_k^2B_k^2$, where $2A_k^2 - B_k^2 = \pm 1$. Hence, both A_k and B_k are greater than 1, for, if not, suppose $A_k = 1$. Then $2 - B_k^2 = \pm 1$ and $B_k^2 = 2 \mp 1$, so that either $B_k = \sqrt{3}$ or $B_k = 1$. Therefore, B_k is not an integer or $P_k = 1$, both of which we must exclude.

Next suppose $B_k = 1$. Then $2A_k^2 = 1 \pm 1$, so that $A_k = 1$ or 0, both of which are again not allowable. Accordingly both A_k and B_k are greater than 1, and P_{k+1} can be formed, where $P_k > P_{k+1} > 0$. Thus P_k is not the least positive integer in $\{P_n\}$. We conclude that $P_k = 1$ and that P_1 belongs to $\{K_n\}$.

References.
1. L. E. Dickson, History of the Theory of Numbers, Vol. II, p. 16.
2. Ibid., p. 27.
3. Heath, Diophantus of Alexandria, Cambridge Press, 1910.

Brooklyn College and
Purdue University

The Gravity Meter and Its Use in Oil Prospecting[1]

WILLIAM B. HERR, JR.[2]
Missouri Gamma, '56

The object of Exploration Geophysics as applied to the search for petroleum is the location of geologic structures which may represent conditions favorable to the accumulation of oil deposits. Thus the geophysicist does not look for oil directly, but rather attempts to ascertain the location of features associated with petroleum.

Essentially all types of exploration geophysics consist in the application of the laws of the physical sciences to the earth in an attempt to obtain information regarding some of the earth's properties. Foremost among these properties are the structure and composition of the upper portion of the earth's crust, or that region which may be economically valuable to man. Geophysical methods are subdivided on the basis of the field of force with which they deal. Thus the gravitational method of geophysical exploration utilizes the properties of the earth's gravitational field as a basis for determining various factors concerning the earth. The fundamental physical law governing the gravitational method is Newton's Law of Universal Gravitation:

$$F = \gamma \frac{m_1 m_2}{r^2}.$$

F = force of mass attraction
γ = gravitational constant
m_1 = mass of attracted body
m_2 = mass of attracting body
r = distance between m_1 and m_2.

If the earth were a homogeneous spherical body of uniform composition the force of gravity would be constant over the entire surface of the earth. However, gravity measurements made at the surface are seen to vary. Now it may be demonstrated from potential theory that a solid homogeneous sphere attracts as if it were a point mass located at the center of the sphere, that is, as if its mass were concentrated at the sphere's center. Since the earth is actually not spherical and is also rotating the field intensity (or force of gravity) at a point depends both on the elevation and latitude of the point.

[1]This paper was presented to the Missouri Gamma Chapter at its December, 1957 meeting. Received by the Editors Jan. 31, 1958.

[2]Editor's note: Mr. Herr worked last summer for the California Oil Company in the field, using a gravity meter. Mr. Hen, who is majoring in Geophysical Engineering in the Institute of Technology of St. Louis University will be associated with California Oil after graduation in June, 1958.

The elevation is a factor since it represents the distance from the center of the attracting mass. The latitude is a factor since it too represents the distance from the earth's center (because of polar shortening) and also since the field intensity at a point (except at the poles) is opposed by the centrifugal force of rotation. The centrifugal force varies with latitude, therefore the force of gravity does also. Taking these two factors into account the field intensity for a specific point may be calculated. However, this value, in general, will not agree with the observed value for gravity at this point. This variation represents an anomaly and is due to a non-homogeneous condition of the earth. This property is the lateral variation of the earth's density which is generally an accompanying feature of geologic structures and is termed a density contrast. (The anomaly, or difference between observed and calculated gravity values for a point, is determined in this manner in practice: The contribution of the elevation and latitude to the deviation take the form of correction factors which are applied to the observed value. There are two other correction factors as well. The **Bouguer** correction which compensates for the attraction of surface material and the terrain correction which compensates for surface irregularities.) The purpose of gravity exploration is therefore to determine subsurface mass distribution as characterized by density contrasts from the surface observations of gravity.

These surface determinations of gravity are now generally made with an instrument called the gravity meter. Because the gravity variations are very minute the gravity meter is an extremely sensitive device.

From Newton's Law of Universal Gravitation and his second law of motion:

$$F = \gamma \frac{m_1 m_2}{r^2}$$

$$F = m_1 g$$

$$g = \gamma \frac{m_2}{r^2}$$

m_1 = unit mass

g = acceleration of gravity.

Gravity may therefore be considered as a force per unit mass or an acceleration. For geophysical purposes the unit 1 gal = $1\,\mathrm{cm/sec^2}$ is defined. It is named after Galileo. Normal acceleration of gravity is about 980cm/sec^2 and the minimum sensitivity for a gravity meter may be considered to be .01 milligals or very roughly one part in one hundred million of the normal acceleration of gravity.

Essentially a gravity meter may be considered as a mass suspended from a spring. Changes in gravity produce corresponding changes in the spring's length. Because these changes are so small some form of

magnification (usually optical) is used. Practical problems from the point of view of construction arise from the fact that the meter, although extremely sensitive, must be rugged enough to endure field work and also be portable. Other problems result because a device of the required accuracy is inevitably influenced by factors unrelated to gravity. These factors include seismic disturbances and even tidal effects. These may be compensated for by an appropriate field technique.

The actual field procedure consists in taking gravity readings with the meter at surveyed points (usually at one quarter mile intervals). After applying the corrections the results represent the anomalous conditions in the region. These data are plotted and an isogam (map with points of equal gravity contoured) made.

This isogam then represents the gravity effects of some subsurface mass distribution. The problem of the geophysicist is to interpret these results in terms of some feasible geologic feature. The difficulty lies in the fact that the shape, size, depth and subsurface densities are unknown to the interpreter. A shallow low density body or feature might produce the same gravity effect as a deep higher density structure. The overall picture is generally masked by some regional or large scale gravity trend as well. Therefore, no one interpretation of a gravity survey may be considered unique, and all available geologic information for the area is utilized in the interpretation. This ambiguity of the gravity exploration result may be demonstrated thus:

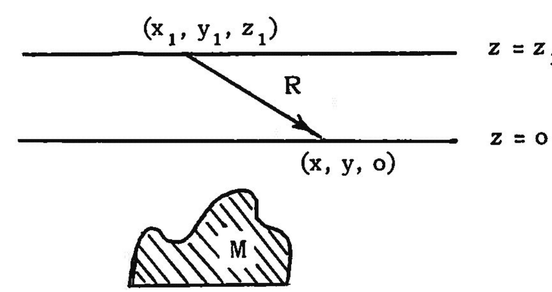


Figure 1.

Definition:

The gravational potential at a point (P) may be defined as the work done by the gravity field of a mass M in bringing a unit mass test particle to the point (P) from infinity.

From Potential Theory it may be proven that the gradient of the potential is equal to the field intensity,

or $\frac{\partial U}{\partial z}$ = vertical gradient.

From Figure 1:

U = potential at (x_1, y_1, z_1)

From Potential Theory, a corollary of Green's Theorem is

$$(1)\quad U = \frac{1}{2\pi}\int_S \frac{1}{R}\left(\frac{\partial U}{\partial z}\right)_{x,y,o} ds$$

where

$\int_S$ is the integral over the surface

and

$\left(\frac{\partial U}{\partial z}\right)_{x,y,o}$ is the vertical gradient at (x,y,o).

Let

$$(2)\quad \left(\frac{\partial U}{\partial z}\right)_{(x,y,o)} = g\,(o)\,.$$

Then $g\,(o)$ represents the vertical component of attraction due to the mass M at any point on the $z = o$ plane.

Therefore

$$U(x_1, y_1, z_1) = \frac{1}{2\pi}\int_S \frac{1}{R}\, g\,(o)\, ds$$

and

$$\left(\frac{\partial U}{\partial z}\right)_{x_1, y_1, z_1} = g(x_1, y_1, z_1) = \frac{\partial}{\partial z_1}\left[\frac{1}{2\pi}\int_S \frac{1}{R}\, g\,(o)\, ds\right].$$

Since $g\,(o)$ is independent of z_1 we have

$$(3)\quad g(x_1, y_1, z_1) = \frac{1}{2\pi}\int_S \frac{\partial}{\partial z_1}\left(\frac{1}{R}\right) g\,(o)\, ds$$

which represents the gravitation at a point (x_1, y_1, z_1) due to the mass M.

Now removing the mass M and replacing it by a variable density layer at the $z = O$ plane, we have (Fig. 2)

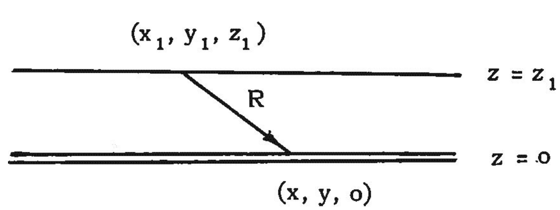


Figure 2.

For Fig 2, let σ be the variable density, where

$\sigma = f(x, y)$,

and Δt is the thickness of the variable density layer. The mass of ds is Δm, where

$\Delta m = \sigma\, ds\, \Delta t$.

If Δt is very small, then the gravitational attraction at (x_1, y_1, z_1) is

$$(4)\quad \Delta g(x_1, y_1, z_1) = \gamma\, \frac{\partial}{\partial z_1}\left(\frac{1}{R}\right) \sigma \Delta t\, ds.$$

The attraction of the entire sheet is

$$(5)\quad \int_S \Delta g(x_1, y_1, z_1) = \gamma \int_S \frac{\partial}{\partial z_1}\left(\frac{1}{R}\right) \sigma \Delta t\, ds$$

or

$$(5a)\quad g(x_1, y_1, z_1) = \gamma \int_S \frac{\partial}{\partial z_1}\left(\frac{1}{R}\right) \sigma \Delta t\, ds.$$

Now this $g(x_1, y_1, z_1)$ represents the gravitation at (x_1, y_1, z_1) due to the attraction of the variable density layer. Now equations (3) and (5a) are of the same form and become equal if

$$\sigma = \frac{g\,(o)}{2\pi\gamma\Delta t}$$

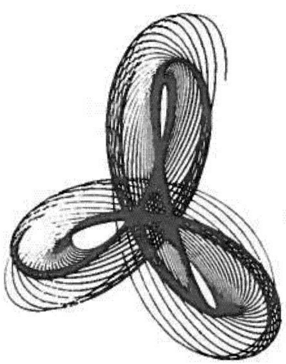
Therefore this demonstrates that the same gravitational effect may be produced at the earth's surface ($z = 0$) by two radically different mass distributions at different depths.

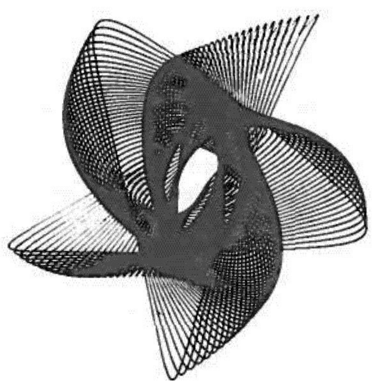
Because the isogam represents a composite of the gravity effects of all the material from the earth's surface to its center, various methods have been developed in an attempt to separate these effects and thus possibly gain more information as to the particular geologic feature giving rise to each effect. These methods are for most part graphical. The obscuring regional gravity may usually be determined and its relatively uninteresting effect (uninteresting from the petroleum prospector's point of view) removed. The remaining gravity is due to shallow and usually small-scale features which are economically interesting and deeper, usually large-scale features which underlie the petroleum bearing sediments. An interesting method which has been developed to separate these two effects is called the Second Vertical Derivative Method. The isogam represents the vertical rate of change of gravity with depth. Thus the possibility presents itself that a second vertical derivative plot would magnify the effect of the shallow feature at the expense of the deeper feature. If the original isogam is sufficiently accurate this may be done by a graphical process and has produced rather good results in practice.

One method of interpertation of a gravity effect of a single body consists in the comparison of this effect with that produced by various known shapes of bodies. The effect of a regularly shaped body may be calculated and various combinations of shapes may be used in an attempt to gain some insight into the shape of the actual body. To determine the gravity effect of irregularly shaped bodies the graphical computer is used which may be described as a "three-dimensional planimeter".

Thus it is that the interoretation techniques of gravity work determine the quality of the resulting geologic picture. The instrumentation problems have, for the greater part, been solved and the challenge to geophysics lies in the development of more accurate and reliable methods of interpretation. Geophysicists are at work in an attempt to solve this major difficulty and produce a unique solution to the gravity problem thus increasing the utility of the gravitational method for exploration purposes.

St. Louis University





ON "A PARADOX"

Murray S. Klamkin*
Avco Research and Advanced Development Division

I am in total disagreement with the answer reached by George R. Sell in his recent note (this journal) "A Paradox".

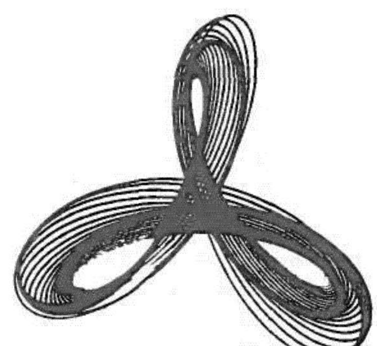
Given a sequence (a , a , a , . a_n); the question of deciding which member can be "tossed out" is meaningless. For one can arbitrarily toss out any one of the n members of the set and justify it by providing, not one, but an infinite number of formula representations for the remaining set, each of which not containing the tossed out number as a member. One simple way of doing this would be to use generalized Lagrangian interpolation formulae.

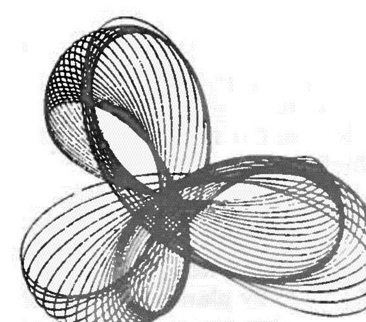
I sometimes wonder why our IQ tests do not include the following question which is just as meaningful (or meaningless) as the previous one:

If it rains n—days in succession, what does it do on the $(n+1)^{st}$ day?

One answer which is just as good as any other but perhaps more pertinent (or perhaps impertinent) is that on the $(n+1)^{st}$ day our modern IQ test makers are still making up the same type of nonsensical questions as the one being answered.

* On leave of absence from Polytechnic Institute of Brooklyn.





PROBLEM DEPARTMENT

Edited By
C. W. Trigg, Los Angeles City College
and
Leon Bankoff, Los Angeles, California

This department welcomes problems believed to be new and demanding no greater ability in problem solving than that possessed by the average member of the Fraternity. Solutions of these problems should be submitted on separate, signed sheets within three months after publication. Address all communications concerning problems to C. W. Trigg, Los Angeles City College, 855 N. Vermont Avenue, Los Angeles 29, California.

Interest in this department can be evaluated by one criterion only --- contributions which are submitted. This evidence of your interest is earnestly solicited.

PROBLEMS FOR SOLUTION

97. ***Proposed by Alan J. Goldman, Princeton University***

Prove that a triangle of area $1/\pi$ has a perimeter greater than 2.

98. ***Proposed by C. W. Trigg, Los Angeles City College***

Each of the letters in (AH) (ME) = EEE represents a distinct digit. Decode the equation.

99. ***Proposed by D.C.B. Marsh, Texas Technological College***

A student, asked to write the equation of $79x^2-2xy+79y^2=1$ after the xy-term has been removed by rotating the axes, uses the facts that (B^2-4AC) and $(A+C)$ are invariants, whence $A'+C'=158$ and $-4A'C'=2^2-4(79)^2$, or $A'C'=(78)(80)$. The transformed equation must be $78x'^2+80y'^2=1$ or $80x'^2+78y'^2=1$, but which is it? How can he tell?

100. ***Proposed by Leon Bankoff, Los Angeles, California***

A right triangle ABC (AC > CB) is inscribed in a semicircle (0) whose diameter is AB. The radius OS, perpendicular to AB, cuts AC in R, and CD is the altitude upon AB. Find the ratio SO/RO for which triangles ODC and CDB are both Pythagorean.

101. ***Proposed by Norman Anning, Alhambra, California***

In Dantzig's "Bequest of the Greeks" (Scribner, 1955) p.38, the statement is made, "Eudoxus...discovered the sections of this surface (a Torus) by planes parallel to the axis of revolution, quartic curves which today are called Cassinian ovals." Show that these sections are, or are not, Cassinian ovals.

SOLUTIONS

48 (November, 1953). ***Proposed by Victor Thébault, Tennie, Sarthe, France.*** Find bases B and B' such that the number 11, 111, 111, 111 consisting of eleven digits in base B is equal to the number 111 consisting of three digits in base B'.

Comment by C. W. Trigg, Los Angeles City College

If $B^{n-1}+B^{n-2}+\dots+1=A^2+A+1$, then $B(B^{n-2}+B^{n-3}+\dots+1)=A(A+1)$. That is, $B(B^{n-1}-1)/(B-1)=A(A+1)$. Now since R, the right-hand member, is the product of two consecutive integers, R must end in 0, 2, or 6, and $R\equiv 0, 2, 3$ or $6 \pmod 9$.

(1) Now if B = 2, we have $2^n-2=A(A+1)$. Since $A^2<A^2+A+2<A^2+2A+1$ for all $A>1$, the equation has no solution for n even and > 2. We now compare the residues of 2^n-2 and $A(A+1)$ modulo 11, 13, 17, 19 and 23, and find that no solutions exist if n is of the forms $10k+7$, $10k+9$; $12k+7$, $12k+11$; $8k+7$; $18k+9$, $18k+11$, $18k+15$, $18k+17$; $22k+7$, $22k+9$, $22k+11$, $22k+17$, or $22k+19$. Thus all values of $n<113$ are eliminated except 3, 5, 13, 21, 25, and 93. For these values we extract the square roots of 2^n-2 and find that the only values for which the remainder equals the root correspond to (n,A) = (3,2), (5,5), and (13,90).

Hence the solution given on page 366 of the November, 1953 issue is in error. The values B = 2, B' = 90 apply to a number consisting of thirteen digits (all ones) in the base 2.

(2) If n = 11, then we have $B(B^9+B^8+\dots+1)=A(A+1)$ or $B(B^{10}-1)/(B-1)=A(A+1)$. Now there are ten terms in the parentheses in the left-hand member, so B may be odd or even. We now compare the residues of $B(B^{10}-1)/(B-1)$ and $A(A+1)$ modulo 3, 7, 11, 13, 17, 19 23 and 29, and find that for values of $B<1101$ no solutions exist except possibly for 258, 350, 576, 597, 713, and 714. For these values of B we extract the square root of $B(B^{10}-1)/(B-1)$ and find that the problem has no solution for $B<1101$.

75 (April, 1955). ***Proposed by Leon Bankoff, Los Angeles***

A line parallel to hypotenuse AB of a right triangle ABC passes through the incenter I. The segments included between I and the sides AC and BC are designated by m and n. Show that the area of the triangle is given by

$$\frac{mn(m+\sqrt{m^2+n^2})(n+\sqrt{m^2+n^2})}{m^2+n^2}.$$

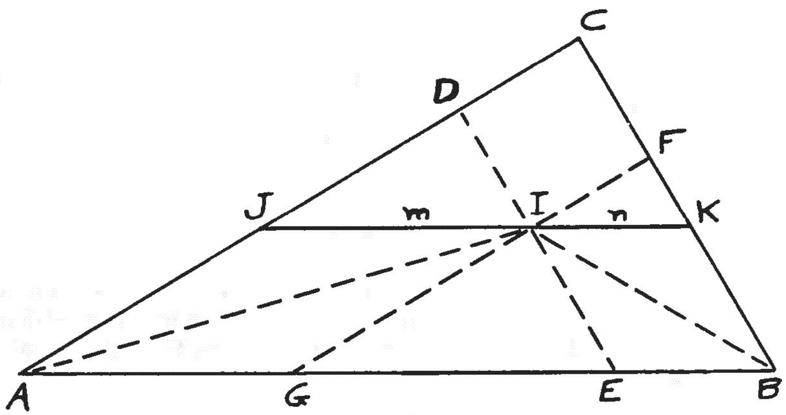


Figure 75

Solution by the proposer. The line through I parallel to AB cuts AC in J and CB in K. Through I draw a line parallel to CB cutting AC in D and AB in E; also a line parallel to AC, cutting CB in F and AB in G. AGIJ and IEBK are parallelograms. BI bisects angle CAB and AI bisects angle CBA. It follows that angles JAI and JIA are equal, as are angles KBI and KIB. Hence m = JI = AJ = GI = AG and n = IK = KB = IE = EB, so $(GE)^2 = (GI)^2 + (IE)^2 = m^2 + n^2$. From similar triangles,

$$BC = (AB)(IE)/(GE) = (m + n + \sqrt{m^2+n^2})\,n/\sqrt{m^2+n^2}.$$

$$AC = (AB)(IG)/(GE) = (m + n + \sqrt{m^2+n^2})\,m/\sqrt{m^2+n^2}.$$

Hence the area of the triangle is (BC) (AC) / 2 or

$$\frac{2mn[m^2+n^2+mn+(m+n)\sqrt{m^2+n^2}]}{2(m^2+n^2)} \text{ or } \frac{mn(m+\sqrt{m^2+n^2})(n+\sqrt{m^2+n^2})}{m^2+n^2}$$

Also solved by ***C. W. Trigg.***

88 (Fall, 1956). ***Proposed by A. R. Aumalis and R. B. Wright, University of Nebraska***

A mathematics professor witnessed a hit-and-run accident. The police asked whether he recalled the license number of the fleeing car. The professor said, "No, but I did observe that the last four digits constituted the cube of the first two digits and that the sum of all six digits was odd".

At this point a student with the professor piped up, "But, sir, did you not also observe that the greatest prime divisor was less than one hundred!"

Would this information enable you to obtain the license number?

Solution by P. G. Can, University of New Mexico. Of the twelve 4-digit cubes, N^3, only five are such that the sum of the digits of N and N^3 is odd. Then since

$$121728 = 2^7\cdot 3\cdot 317$$
$$132197 = 13\cdot 10169$$
$$174913 = 17\cdot 10289$$
$$185832 = 2^3\cdot 3^2\cdot 29\cdot 89$$
$$219261 = 3\cdot 7\cdot 53\cdot 197$$

it follows that the license number was 185832.

Also solved by ***The proposers and by Leon Bankoff,*** who noted that the sum of the digits of 185832 is also a cube; and by ***C. W. Trigg.*** who observed that the student has a potentially great future as a computer. Even if the confused professor were uncertain about the end of the number at which the "first" two digits lay, the solution would still be unique, for each of 172812, 219713, 491317, 583218, and 926121 has a prime factor > 100.

90 (Fall 1956). ***Proposed by Vern Hoggatt, San Jose College.***

Show that

$$\lim_{n\to\infty} \sum_{j=1}^{n-1} [j(n-j)]^{-1/2} = \pi.$$

Solution by the proposer.

$$\lim_{n\to\infty} \sum_{j=1}^{n-1} [j(n-j)]^{-1/2} = \lim_{n\to\infty} \sum_{j=1}^{n-1} \left(\frac{1}{n}\right)\left[\left(\frac{j}{n}\right)\left(1-\frac{j}{n}\right)\right]^{-1/2}$$

$$= \lim_{\substack{\epsilon_1\to 0\\ \epsilon_2\to 1}} \int_{\epsilon_1}^{\epsilon_2} [x(1-x)]^{-1/2}\,dx = \int_0^1 [x(1-x)]^{-1/2}\,dx = \pi.$$

Also solved by ***M. A. Krasny.***

92 (Fall 1956). Proposed by Leon *Bankoff,* Los Angeles.

It has been said that algebra is but written geometry and geometry is but diagrammatic algebra (Sophie Germain, *Memoire sur* les Surfaces *Elastiques).* In the spirit of this quotation, show geometrically that

$$\sum_{n=2}^{\infty} \binom{n}{2}^{-1} = 2.$$

First solution by Thomas *Porsching,* Carnegie Institute of Technology.

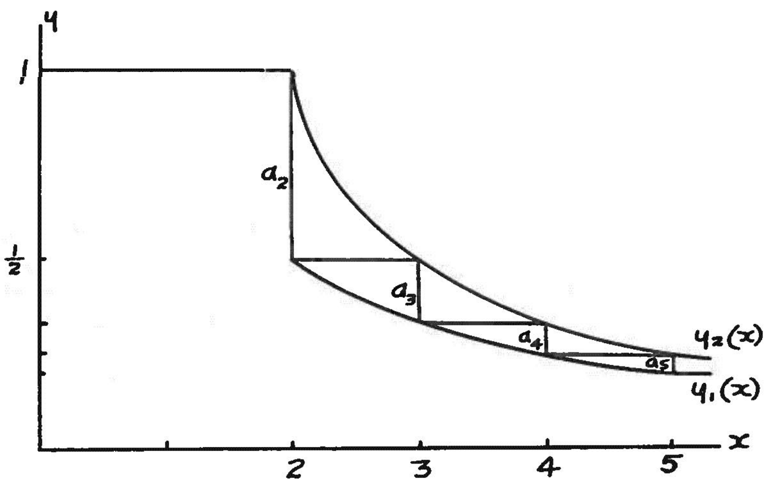


Figure 92 Solution No. 1

Clearly,

$$\sum_{n=2}^{\infty} \binom{n}{2}^{-1} = 2 \sum_{n=2}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n}\right) = 2S.$$

Now consider the hyperbolas, $y_1(x) = 1/x$ and $y_2(x) = 1/(x-1)$, $x \geq 2$, both of which are asymptotic to $y = 0$. If we let $a_n = y_2(n) - y_1(n)$, as in the figure, then S is the total projection of a a,, ... onto the y-axis, for the terminal point of the projection of a, is the initial point of the projection of a $_{,+1}$. Hence, S = 1.

Second solution by the proposer.

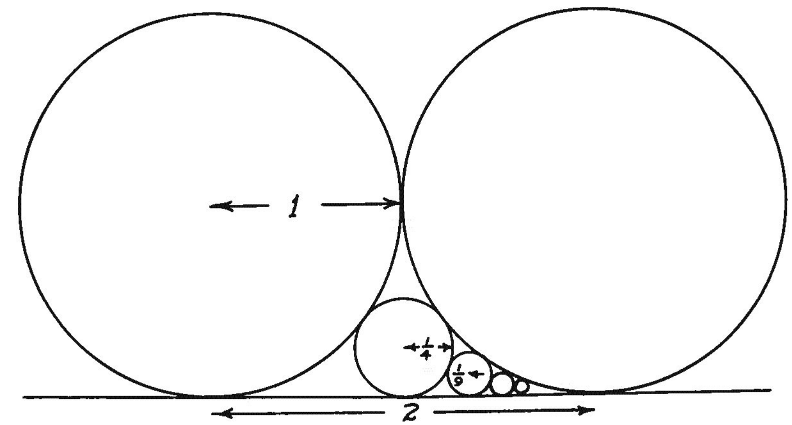


Figure 92 Solution No. 2

Inscribe a circle in the space bounded by two externally tangent unit circles and one of their common external tangents. Then construct a chain of consecutively tangent circles touching the common tangent and either of the unit circles externally.

The radius of any circle of the chain is $1/n^2$, where n is the order number in the chain, starting with the unit circle. (This follows from E 432, American Mathematical Monthly, April 1941). The segment of the common tangent included between its contacts with two consecutive circles, c_{n-1} and c_n, is $[(1/(n-1)^2+1/n^2)^2-(1/(n-1)^2-1/n^2)^2]^{1/2}$ or $2/n(n-1)$ or $\binom{n}{2}^{-1}$. It is evident from the figure that the sum of the segments to infinity is 2.

93 (Spring 1957). Proposed by Michael *J.* Pascual, *Siena* College

Derive a formula for the solutions of the equation

$$(a_1 + a_2 i)z^2 + (b_1 + b_2 i)z + (c_1 + c_2 i) = 0$$

which gives the roots in the form A + Bi.

Solution *by* the proposer. Using the standard formula for the quadratic we get

$$z = \frac{-(b_1 + b_2 i) \pm \sqrt{(b_1 + b_2 i)^2 - 4(a_1 + a_2 i)(c_1 + c_2 i)}}{2(a_1 + a_2 i)}$$

So we must find the square root of $(b_1 + b_2 i)^2 - 4(a_1 + a_2 i)(c_1 + c_2 i)$

$$= (b_1^2 - b_2^2) + 2b_1 b_2 i - 4(a_1 c_1 - a_2 c_2) - 4(a_1 c_2 + a_2 c_1)i$$
$$= (b_1^2 - b_2^2 - 4a_1 c_1 + 4a_2 c_2) + (2b_1 b_2 - 4a_1 c_2 - 4a_2 c_1)i$$
$$= x_1 + y_1 i.$$

Now

$$\sqrt{x_1+y_1i} = \sqrt{(\sqrt{x_1^2+y_1^2}+x_1)/2} + \sqrt{(\sqrt{x_1^2+y_1^2}-x_1)/2} = x_2+y_2i.$$

Hence

$$z=\frac{-b_1-b_2i\pm(x_2+y_2i)}{2(a_1 t a_2i)} = \frac{(-b_1\pm x_2)+(-b_2\pm y_2)i}{2(a_1 t a_2i)}$$

$$= \frac{(-b_1\pm x_2)a_1+(-b_2\pm y_2)a_2}{2(a_1^2+a_2^2)} + \frac{[(-b_2\pm y_2)a_1-(-b_1\pm x_2)a_2]i}{2(a_1^2+a_2^2)}$$

and since x_2, y_2 can be expressed in terms of a_1, a,, b_1, b_2, c_1, c_2, we have corresponding to the t and - , respectively, the two roots

$z_1 = A_1 + B_1i, \qquad z_2 = A_2 + B_2i.$

94 (Spring 1957). *Proposed by Pedro A. Piza, San Juan, Puerto Rico*

Let $S_k = S_k(n) = 1^k$ t 2^k t ... t n^k.

Prove that

$$225(S_1^2 \text{ t } S_2^2 + S_3^2 \text{ t } S_4^2 \text{ t } S_5^2) =$$

$$400\, S_1^6 + 88\, S_1^5 + 190\, S_1^4 + 196\, S_1^3 + 251\, S_1^2.$$

Solution by C. W. Trigg, Los Angeles City College.

Since $S_1 = n(n+1)/2$, $S_2 = n(n+1)(2n+1)/6$, $S_3 = n^2(n+1)^2/4 = S_1^2$,

$S_4 = n(n \text{ t } 1)(2n \text{ t } 1)(3n^2 \text{ t } 3n - 1)/30$,

and $S_5 = n^2(n \text{ t } 1)^2(2n^2+2n-1)/12$, it will be sufficient to show that

$$225(S_2^2+S_4^2+S_5^2) = 400S_1^6 + 88S_1^5 - 35S_1^4 \text{ t } 196S_1^3 \text{ t } 26S_1^2.$$

Upon substituting the values of the S_i and simplifying, each side of the last equation reduces to

$$n^2(n \text{ t } 1)^2(100n^8 \text{ t } 400n^7 \text{ t } 644n^6 + 532n^5 \text{ t } 197n^4 - 26n^3 \text{ t } 357n^2 \text{ t } 392n \text{ t } 104)/16.$$

95 (Spring 1957). *Proposed by Hüseyin Demir, Zonguldak, Turkey.*

Find the probability thay any given four points on a plane be the vertices of a convex polygon.

Solution by the proposer. The probability that three points form a triangle is 1. Let ABC be the triangle formed by three of the given points. For the points A, B, C, D to form a convex polygon it is necessary that D be exterior to ABC in the regions where there are excircles. These regions of the plane correspond to the angles*, β, γ of triangle ABC. Hence p = $(\alpha \text{ t } \beta+\gamma)/$(whole plane) $= \pi/2\pi = 1/2$.

96 (Spring 1957). *Proposed by Leon Bankoff, Los Angeles.*

A circle (P) touches the diameter AB of a semicircle (0) in D, and arc AB of the semicircle in R, (AD<DB). The perpendicular to OR at P cuts the arc RB in S. If $(RS)^2 = (DB)^2 - (AD)^2$, find the ratio AD/DB.

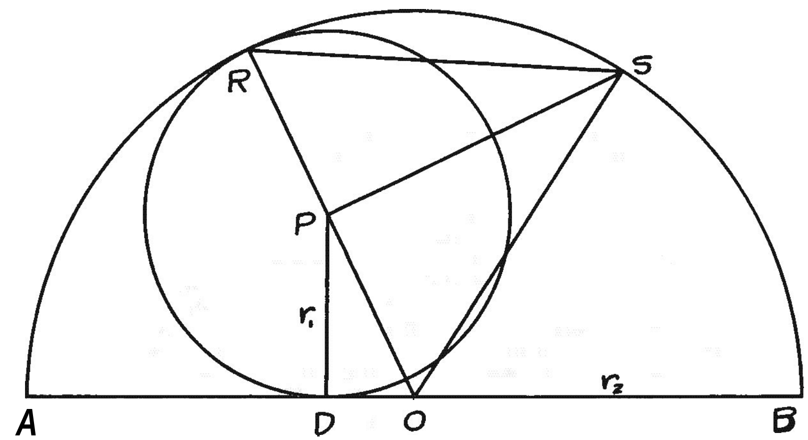


Figure 96

Solution by Albert F. Oilman, 111, Montana State University, Missoula, Montana. In the following expressions, the substitutions made become evident upon reference to the figure.

$$(RS)^2 = (RP)^2 + (PS)^2 = (PD)^2 + (SO)^2 - (PO)^2$$

$$= (PD)^2 + (AO)^2 - (PD)^2 - (DO)^2 = (AO+DO)(AO-DO)$$

$$= (OB+DO)(AD) = (DB)(AD).$$

Now if $(RS)^2 = (DB)^2 - (AD)^2$, we have

$$(AD)^2 + (DB)(AD) - (DB)^2 = 0.$$

Then $\quad AD = DB(-1\pm\sqrt{5})/2.$

So $\quad AD/DB = (\sqrt{5}-1)/2.$

Also solved by *the proposer*, and by *C. W. Trigg*, who further observed that $(AD+DB)/(DB) = (\sqrt{5}+1)/2$, so the radius, r_2, of the semicircle is AB/2 or $(DB)(\sqrt{5}+1)/4$. Let $RP = r_1 = PD$; then from the triangle PDO, $(r_2-r_1)^2 = (r_2-AD)^2 + r_1^2$. Upon simplifying and making the obvious substitutions, $r_1 = (DB)(3-\sqrt{5})/2$, whereupon $r_1/r_2 = 2(\sqrt{5}-2)$.

BOOK REVIEWS

Edited *by*
Franz E. Hohn, University of Illinois

An Introduction to Probability Theory and Its Application. By William Feller. John Wiley and Sons, Inc., New York, **1957,** Vol. **1,** 2nd **Ed. xv + 461** pp., **$10.75**

Since this book is a second edition to a well known and accepted first edition, one might wisely ask wherein the second edition differs from the first. After noting these differences then one can ask which is the better. No doubt, the author and publisher felt the need of improving an already good book or this second edition would never have appeared.

All chapters in the book have been rewritten with some sections deleted and others placed in a different location; many examples have been added to give clarity to the theory. Many additional exercises have been added.

It appears that much more revision has taken placed within the first thirteen chapters than in the last four.

Chapter 111 dealing with fluctuations in coin tossing and random walks is an entirely new chapter. It may be a little misleading to say the material found in Chapter 111 is entirely new, because one might claim that at least the results in Section **5,** Chapter 111 may substantially be found in the first edition in Chapter XII. The proofs of theorems **in** this section are new.

The material in Chapters **II** and III of the first edition has been streamlined and is now found in Chapter 11 of the second.

Chapter **XII** dealing with compound distributions and branching processes has much new material added. However, some of the material of this chapter did appear in Chapter XI of the first edition.

Much of the work found in Chapters XII and **XIII** of the first edition is contained in Chapter XIII entitled "The Renewal Equation".

In the first edition definitions and theorems are given but not named as such, whereas in the new edition these are clearly indicated. For example, Definition 2 (Chapter V, p. **119)** has a caption, whereas it appeared in the first **edition** without a caption (p. **90).**

Emphasis has been placed upon certain topics in this edition, for example waiting times. This topic appears in various places throughout the book. It first makes its appearance in Chapter II. Some of the other places one may encounter it are Chapters VI, IX, XI, XIII, and XVII.

The plan used in the first edition has not been altered here, namely the probability theory has been developed rigorously, avoiding non-mathematical concepts. It does describe the empirical background and gives many practical applications of the theory developed. To aid those reading, stars are used to indicate sections and chapters which may be omitted without interrupting the continuity of the discourse.

The publisher has done an excellent job of printing and of using larger print to indicate new sections within chapters.

It is felt that this new edition, even though the price is much higher, will receive an even more favorable reception than the first edition did.

St. Louis University — Francis **Regan**

Modern Mathematics for the Engineer. Edwin F. Beckenbach, Editor. New York, **McGraw-Hill, 1956,** xx **+ 514** pp., **$7.50**

This book is the second publication in the University of California Engineering Extension Series. The book consists of nineteen lectures on mathematical theory which are useful to the modern **engineer.** The lectures were given by fourteen mathematicians, three engineers, and one chemist. Their purpose was to "generate **in** the minds of engineers and applied scientists engaged in research, design, and administration an awareness of the recent rapid advancement in applied mathematical thought".

If the book is judged by this standard it is quite successful. On the whole the articles are well written and practically every mathematical topic of interest to the modern engineer has been included. Each author has also provided a short bibliography where the interested reader can obtain a more detailed and mathematically complete discussion of the topic under consideration.

All of the papers are expository and most, if not all, do not **contain** any new results. Occasionally one may find a more modern approach to an old problem, but otherwise the material will be of little help to the individual engaged in mathematical research. The book will prove to be of greatest value to its intended audience, to the engineering student who is not interested in a formal mathematical developement, and to the mathematics student who is interested in the use of advanced mathematics in engineering.

The book is divided into three parts:

(1) Mathematical Models. This part is devoted to the treatment of engineering problems which **involve** ordinary differential equations, partial differential equations, the calculus of variations, and many types of boundary value problems.
Contributors: Solomon **Lefschetz,** Richard **Bellman, Magnus R.** Hestenes, Richard Courant, **Mehahen** M Schiffer, and Ivan S. **Sokolnikoff.**

(2) Probabilistic Problems. This part considers problems which have or can have stochastic features. The main topics covered are prediction theory, game theory, linear programming, dynamic programming and monte **carlo** methods.
Contributors: Norbert Wiener, H. Frederic Bohnenblust, Gilbert W. King Richard **Bellman,** and George W. Brown

(3) Computational Considerations. In this section several topics of applied mathematics and numerical analysis (matrix theory, functional transforms, conformal mapping, high speed computing devices, etc.) are shown to be quite useful in engineering.
Contributors: Louis A. Pipes, John L. Barnes, Edwin F. Beckenbach, Charles B. Morrey, **Jr.,** George E. Forsythe, Charles B. Tompkins, and Derrick H. Lehmer.

The addition of a very complete name and subject index by Professor Beckenbach enhances the book's value for reference purposes. The book also includes an introduction by Royal Weller which discusses the fact "that engineering of today is becoming more of a science and less of an art and that better mathematical foundation is required to participate in this transition".

Carnegie Institute of Technology, and
University of Pittsburgh — Kenneth S. Kretschmer

Calculus with Analytic Geometry. By Richard E. Johnson and Fred L. Kiokemeister. Bostor, Allyn and Bacon, Inc., **1957.** xi **+ 650** pp. **$7.95**

As the title suggests, this is primarily a calculus book with just enough analytic geometry included so that it can be used as the textbook of an integrated analytic geometry-calculus course. It can also serve as a calculus textbook by omitting the three analytic geometry chapters, which do not affect the continuity of the rest of the book.

Consequently, as might be expected, the portion of analytic geometry which is served up is somewhat meager, while the calculus portion is generous and meaty and is even supplemented by a chapter on differential **equations** which features **a rather complete** discussion of second-order linear differential equations with constant coefficients. The book reflects quite clearly the modern teaching tendency to slight analytic geometry and emphasize calculus.

The authors stress in the preface that this book was "...designed to be used by students who have had a good training in algebra, plane geometry, and trigonometry", and suggest further on in the preface that "...the book might be used for a second calculus course following a variety of different first course."

Some of the features of the book are:

(1) In Chapter **9** sequences are introduced for the purpose of showing that the integral equals the limit of a sequence of **Riemann** sums. Sigma and delta notations are used.

(2) Chapter **12** gives a general substitution formula which validates trigonometric and related substitutions for evaluating integrals.

(3) In Chapter **14** the boundedness properties of a continuous function are proved and the mean value theorem is used in the discussion of indeterminate forms, improper integrals, and finite Taylor's theory.

(4) In Chapter **16,** least upper bounds are used in defining the radius of convergence of a power series, and proofs are given that a power series **may** be differentiated and integrated **termwise** within its interval of convergence.

In most cases an intuitive discussion precedes the rigorous development of a new concept. A number of example problems and their solutions are given in the section preceding most of the problem sets. The answers to the odd numbered problems are given at the end of the book. The format of the book is good. It is printed and bound attractively and arranged so that it is easy to refer to and easy to read. It is a carefully written, well organized book in which the rigorous theoretical development of many of the topics is emphasized. It is a book which ought to be very stimulating to the students of above-average ability, but parts of it may be rather difficult for some students.

Ohio University R. F. Spring

Linear Algebra for Undergraduates. **By D. C.** Murdoch. John Wiley and Sons Inc., New York, **1957.** xi **+ 239** pp. **$6.95**

The problems of bridging the gap between College Algebra as it is traditionally presented and the abstractions of Modern Algebra is a difficult one. More generally, the problem is that of the gap between the mathematics usually taught to the Freshman and Sophomore in college and the type of mathematics encountered, for example, in a first course of Modem Algebra or Topology. The answer to this problem varies with different people. Some would introduce the modern concepts in weakened form to the Freshman. Others would propose a sink-or-swim answer of introducing no modem mathematics until the first course of that type is encountered by the student. Professor Murdoch would introduce a transition course to bridge the gap and he presents Linear Algebra for Undergraduates as a text for such a course.

The subject matter of the book furnishes necessary information for any young mathematician. Much of the material usually covered in a course in Determinants and Matrices is presented. The algebra of matrices is well covered and the general theory of matrices is developed to include such topics as similarity, diagonalization and reduction of quadratic forms. Properly so, certain standard topics, such as matrix polynomials, are not included in this text.

Professor Murdoch treats matrices as linear transformations of vector spaces. His first chapter deals with the fundamental concepts involved in vector spaces. In order to keep the material within reach of the beginning student, he makes no attempt to treat the topic generally but considers only ordered n-tuples of real or complex numbers.

It is in the method of presentation and in restricting the generality of the concepts that the author attempts to bridge the gap mentioned above. The proofs are well written and the form of the presentation insures that the student be aware of the fact that a proof is based on definitions and results proven earlier. The author hopes that after such experience a student would feel at home in a course where such methods were used even though the concepts were abstract and unfamiliar. Again, it is important to note that the subject matter treated furnishes desirable background for more advanced courses.

It is the opinion of this reviewer that this book can be used successfully as the author intends it to be used. Yet the instructor can easily modify certain definitions and proofs to attain greater generality if he so so desires. The book includes an excellent selection of exercises to aid the student in understanding the subject matter. Many of the exercises are designed to keep the course down to earth. They are numerical and tend to make the concepts more "concrete".

The form of the book is very good. The definitions and theorems are labeled and numbered adequately and the notation throughout is clear and consistent. This reviewer, then, recommends this book as one whose subject matter is highly desirable and whose presentation might well be a solution to the problem of easing the first contact with the concepts of modern mathematics.

St. Louis University John W. **Riner**

Vector Spaces and Matrices. By Robert M Thrall and Leonard Tornheim. John Wiley & Sons, Inc., New York, **1957. 318** pages.

An alternate but lengthy title for this book could very well have been "**Finite** Dimensional Vector Spaces, Transformations of Finite Dimensional Vector Spaces, and Matrices." The content is described in the following excerpts from the preface.

"The first two chapters introduce the concepts of vector space, linear transformation, and matrix. The third chapter applies these concepts to the problem of solving systems of linear equations." "Chapter **4** gives a self-contained developement of the theory of determinants." "Chapter **5** is intended to introduce the student to the general concept of invariant and serves as a summary of what has preceded. It also includes a brief introduction to the theory of similarity. Chapters 6 and 7 present the usual material on bilinear and quadratic forms and on orthogonal equivalence." "**The** first part of Chapter **8** gives the standard theory of polynomials in one indeterminate. The last six sections develope the general theory of simple algebraic extensions of a field." "In Chapter **9** we present the standard canonical forms for matrices with integral or with polynomial elements. These results are applied to differential equations with constant coefficients, and also to finitely generated abelian groups to obtain the fundamental theorem. Chapter **10** presents the similarity theory with the usual canonical forms. It includes applications to geometry and to differential equations." "The final chapter on linear inequalities presents a brief introduction to this important topic."

The book is self contained in that a knowlegde of elementary mathematics through analytic geometry will suffice as background knowledge for the majority of the book. This does not mean that a student at the sophomore level can digest this book. The book requires a degree of mathematical maturity of the student if he is to feel at ease with the **definition-lemma-**

theorem-corollary-approach used by the authors. There is also the problem of motivating the student. A knowledge of transformations obtained from analytic geometry and the desire to solve systems of simultaneous linear equations might suffice for the material in the first five chapters. Beyond the fifth chapter a considerably broader background would be required of the student for proper motivation.

The first half of the book gives a relatively detailed exposition of the material at hand. The authors state that "the last four chapters are more advanced in nature, and the level of exposition is somewhat higher." The reviewer feels that at certain places in the last five chapters the level of exposition approaches a degree of undesirable terseness found in found in many research publications.

The authors are to be applauded for the commendable manner in which they have made extensive use of linear transformations. This use is manifested by considering matrices as concrete representations of such transformations and thus developing the theory of matrices by observing the linear transformations which they represent. It is to be regretted that many of their discussions were not extended to infinite dimensional vector spaces. It is pleasing to note that determinants have been reduced to their proper position in the theory of matrices.

The reviewer believes that the first five chapters of the book along with selected topics from the remainder of the book would make a good one-semester course for the advanced undergraduate and the beginning graduate student. The book is well supplied with exercises which would be indispensable in such a course.

The few typographical errors that were observed were easily corrected when viewed in context.

University of Maryland J. W. Brace

BOOKS RECEIVED FOR REVIEW

C. F. Adler: Modern Geometry, an Integrated Course, New York, McGraw-Hill, 1958, xiv + 216 pp., $6.00

R. V. Andree: Selections from Modern Abstract Algebra, New York, Holt, 1958, xii + 212 pp., $6.50

R. H. Atkin: Mathematics and Wave Mechanics, New York, John Wiley, 1957, xv + 348 pp., $6.00

H. Eves and C. V. Newsom: An Introduction to the Foundations and Fundamental Concepts of Mathematics, New York, Rinehart, 1958, xv + 363 pp., $6.75

H. Lass: Elements of Pure and Applied Mathematics, New York, McGraw-Hill, 1957, xi + 491 pp., $7.50

P. J. McCarthy: Introduction to Statistical Reasoning, New York, McGraw-Hill, 1957, xiii + 402 pp., $5.75

P. M. Morse: Queues, Inventories, and Maintenance, Publications in Operations Research, No. 1, New York, John Wiley, 1958, ix + 202 pp., $6.50

C. C. Richtmeyer and J. W. Foust: Business Mathematics, Fourth Edition, New York, McGraw-Hill, 1958, xii + 412 pp., $5.75

NOTE: All correspondence regarding books and reviews should be addressed to FRANZ. E HOHN, 374 MATHEMATICS BUILDING, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

INSTALLATION OF NEW CHAPTERS

The New Jersey Beta chapter of Pi Mu Epsilon was installed at Douglass College, New Brunswick, on Friday, February 28, 1958. Professor Frame conducted the installation and initiated six charter members and six initiates at a banquet held in a private dining room at Rutgers University. Miss Carol E. Crowder was elected president of the chapter. After the initiation, Professo Frame spoke on "Elementary Notions in Relativity Theory."

George J. Marks, Director of the Evendale Affiliate Chapter of Pi Mu Epsilon, receives the fraternity charter from Professor J. S. Frame, Director-General, at the chapter's installation ceremonies in Cincinnati, Ohio, on November 29, 1957.

The first industrial affiliate chapter in the history of Pi Mu Epsilon was installed at the Evendale plant of the General Electric Company in Cincinnati, Ohio, on November 29, 1957.

The installation of the Evendale chapter was conducted by Professor J. S. Frame of Michigan State University, Director General of the fraternity. For the installation, which Professor Frame called an "historic occasion," the fraternity's constitution had been amended and broadened to include the mathematically trained scientist in industry. In his remarks to the charter members and initiates at the installation banquet, Professor Frame urged the new members to establish a chapter which could be a guide for other industrial chapters which might be admitted to the fraternity if this chapter is successful.

After a few remarks on the history of Pi Mu Epsilon, Professor Frame presented the chapter charter to George Marks, of the General Electric Company, who had organized the chapter and who was elected temporary chapter director. At the conclusion of the installation and initiation ceremonies, Professor Frame gave a short talk on "Continued Fractions" with examples of applications in engineering calculations.

St. Louis University Edwin E. Eigel

OPERATIONS UNLIMITED

Science needs you

You need science

This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essent**ial as today.**

Your election as members of **Pi Mu Epsilon Fraternity is an** indication of scientific p**otential. Can you pursue advanced study** in your field of specializ**ation?**

To point out the need of **advanced study, the self-satisfaction** of scientific achievement, **the rewards for advanced preparation,** the assistance available f**or qualified students, etc., it is planned** to publish editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

We are pleased to have in this issue editorials from International Buisness Machines Corporation and Bendix Aviati**on Corporation.** We are grateful to these scientific **institutions for their editorials** and their material support of this publi**cation.**

The Fall issue will contain an editorial **"Mathematics and** the Acturial Profession" prepared by the **General American Life** Insurance Company, **encourag**ing advanced study in the Acturial Sciences.

Through these and **future e**ditorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

The following is a list of contributing corporations with the issue in-which their individual editorials appeared or will appear.

Bendix Aviation Corporation	Vol. 2, No. 8
Emerson Electric Company	Vol. 2, No. 7
General American Life Insurance Company	Vol. 2, No. 9
International Business Machines Corporation	Vol. 2, No. 8
McDonnell Aircraft Corporation	**Vol. 2,** No. 7
Monsanto Chemical Company	Vol. 2, No. 7
Olin Mathieson Corporation	Vol. 2, No. 7

The future holds a scientific career for those who have the fortitude to seek it. Don't undersell "perspiration" in your "preparation".

INTERNATIONAL BUSINESS MACHINES CORPORATION

THE CHALLENGE OF INDUSTRIAL MATHEMATICS

Dr. Shader

by
DR. M. A. SHADER, Manager,
International Business Machines University
and
Research Institute Program

A growing technology which is such an integral part of our society has created unprecedented opportunities for the mathematician. Improved products, new products and entire new industries appear on the scene almost daily — all, at least in part, the results of basic and applied research in which mathematics plays a prominent role. A modern research facility without its share of mathematicians is inconceivable; a research facility without an automatic computer would be equally inconceivable. Most research industry, government and the university is a complex activity drawing on many diciplines with mathematics and computers underlying the whole effort.

For the mathematician the most exciting development of our era undoubtedly has been the advent of the high speed automatic digital computer. It is estimated that by 1963 there will have to be 70,000 professional mathematicians engaged in the analysis, planning and programming of problems for machine solution. Except for the assurance of many job opportunities this statistic, however overwhelming, does not really give us much insight into the impact of these developments on the career opportunities for the individual mathematician. What branches of mathematics will be used? What are the prerequisites for the field? what will the individual be doing?

Traditionally, mathematics has had dual motivation. The pure mathematician studied mathematics as an end in itself while the applied mathematician was usually motivated by some physical problem. But the line of demarcation has never been clear; the "mathematics for mathematics sake" may quickly become the commonplace tool of the engineer and scientist. In today's research laboratories there is need for pure and applied mathematicians alike; it is impossible to predict which discipline will be required next. It is evident then that all branches of mathematics probably will be useful. With the knowledge that all areas of mathematics can play a role, how does the student select his course of study to prepare for a career as an industrial mathematician? The very nature of the question points to the answer. One should strive for the broadest and most thorough background possible including classical analysis, statistics and probability, algebra topology, logic and numerical methods. Both the subject matter and discipline will be significant. And, of course, since research today is so highly interdisciplinary, a good grounding in physics, chemistry and economics will be invaluable.

Most frequently the industrial mathematician is studting some physical phenomonon together with the engineer and the scientist, attempting to bridge the gap between theory and application. This calls for a basic understanding of the phenomonon itself, the mathematics of the theory and the solutions of the equations involved. The mathematician has the responsibility for producing solutions to the equations which predict the behavior. of the phenomonon being investigated — the computer is frequently his indispensable tool. The mathematician's task may include proper formulation of the problem (without which all else is useless), analysis of the resultant problem, and its formulation for machine solution. Each phase of this can present a real challenge and each can depend heavily on formal mathematics.

Out of these efforts to apply computers as a tool have arisen some new problem areas which may someday be regarded as formal disciplines in themselves but which, for the present at least, seem particularly suited to the mathematician because of his background and training. One is the problem of communication between man and machine. This is a complex question involving logic, linguistics and computer design. Perhaps the outcome will be a good symbolic language easily understood by both human and machine.

Another promising area is the use of the computer to simulate any system for which a precise mathematical model is not known or for which a mathematical model is known but not the method of solution. One may construct a model which is composed of a complex set of relations including equations, inequalities and probabalistic statements. By use of trial and error techniques (some quite sophisticated) it is sometimes possible to arrive at answers which give real insight into the system being studied or which enable one to control the system. Model building or simulation techniques have been used to study production job shops, flow of traffic through street intersections, and flow of information through a business complex, to name just a few.

Industrial mathematicians are tackling yet another class of problems. This is in the area of management science where the objective is to better understand the operations of business and business systems and thereby to control these operations in a more profitable way. Associated with most of these problems are very fundamental mathematical questions drawing heavily on probability, statistics, analysis and economics. The areas under study include marketing, scheduling, transportation and manufacturing. Here, again the mathematician and the computer play essential roles.

The mass of data which must be handled in today's business and in today's research has created yet another problem area in which the mathematician and the computer may come up with answers. The amount of technical material being published today makes it virtually impossible for anyone to keep up with the work in anything but a small field of specialization. It is essential to develop procedures for automatic processing and retention of technical data in files with automatic means for recall of the pertinent information on a given topic. This area, called information retrieval, is one which mathematicians are giving a great deal of attention. Here again one may have to draw on many branches of mathematics including logic and statistics.

The final area to be mentioned is that of using machines to perform elementary routine tasks heretofore necessitating human effort. Any thought or idea may be decomposed into elementary logical elements. Computers, of course, are capable of manipulating elementary logical notions with great facility. There are a tremendous number of potential applications for machines once this decomposition can be mastered. Again this appears to be a natural area for the mathematician to make significant contributions.

It is clear from the foregoing that all branches of mathematics are involved in the work being done by industrial mathematicians. The more background and mathematical maturity the individual possesses, the better able he will be to face the diverse and challenging problems of our era. The problems exist and the jobs exist. The rest is up to the mathematician himself — truly there are Operations Unlimited!

BENDIX AVIATION CORPORATION

WHY GRADUATE STUDY

By
C. B. CRUMB, JR.
Project Engineer, Research Laboratories Div.

Industrial employers today are vigorously seeking graduate degree holders in mathematics, science, and engineering. Why is this so?

It is because the projects which we tackle now are more and more fraught with highly technical problems. We must solve those problems; and to do that we must have more employees trained in advanced problem solving techniques. Let's look at this situation in some detail.

First, let us consider some examples of employers' need for graduate degree personnel. Assume that you have a contract for development of instruments to be carried in an earth satellite for determination of the characteristics of ambient atmosphere. You would need men who could determine the instrumentation requirements for making measurements upon an atmosphere where its pressure is only **1/000,000,000** as great as at the earth's surface, and must be observed while passing through it at 18,000 miles per hour; select instruments for design and development; evaluate designs; and estimate the effects of ambient conditions, such as practically zero ambient pressure, extremes of temperature, and a high level of cosmic radiation.

You may protest that that is a remote example, that only a very few people are concerned with satellite instrumentation. Perhaps a more down-to-earth example would be a project for the development of air traffic control procedures and equipment. You would find that analysts of air traffic control problems are plagued by the mathematical difficulties of a dynamic system of many bodies and many largely unrelated variables. You would almost certainly find that only probabilistic mathematical techniques of a rather sophisticated type would be effective in analysis of such systems. What kind of mathematicians would you want for such analytical tasks?

Similarly, if you were proposing an anti-aircraft defense system composed of manned interceptors, air-launched guided missles, surface-launched guided missiles, and radars both airborne and surface, how would you combine all these elements? Material and effort which is assigned to one element is denied to another. How much should go for each? This would be a worthy problem in operations research if all of the elements were existing and well known. Where some are still not only undeveloped but unspecified, the problem of allocation of effort is challenging, to put it mildly!

There are still more frequently encountered examples of projects requiring technical background of more than bachelor degree level. At present, virtually all electronic apparatus that faces severe restriction of power consumption or size is now designed to incorporate solid state components. Who is capable of analyzing such circuits? Analysis of the dynamics of solid state components and circuits often requires application of partial differential equations; the analyst must be skilled in their use.

What all of this means is, in short, that our technical employees must be more highly educated. The frontiers of technical knowledge are moving outward with ever increasing rapidity. The content of the normal undergraduate technical curriculum cannot possibly keep up with this accumulation of technical knowledge and the opening up of new technical fields. Professor Draper of M.I.T. described this problem very well in a talk on engineering education: "Subject matter revisions are generally in the direction of adding new material, usually without any solid justification for greatly reducing the coverage of old areas." Thus we are faced with an already crowded undergraduate curriculum, which every day falls farther short of preparing students for an active place in the modern technical world. Furthermore, undergraduate training tends to be presented as blocks of substantially unrelated techniques and information. Before he can see these tools as parts of an integrated whole, the student must have an opportunity to apply them and perceive their interrelation.

There are two important ways in which graduate study by qualified students helps us out of this dilemma. First, it provides industry with a much more valuable employee. He is one who has taken the time to acquire a greater part of knowledge and skill needed for technical work, — he's pushed humself farther outward toward those advancing frontiers we mentioned before. He has used the material of his undergraduate work and begun to acquire the appreciation of their interrelation which is a sign of technical maturity. Often the graduate student assumes some teaching load while he is still working on his advanced degree. Thus he begins to acquire the all-important capabilities for effective human relations, — communication with people is the key. At the same time he consolidates his grounding in the subjects he teaches; what teacher will deny the old saying "You don't really know a subject until you have to teach it." As a result of these important gains and the things which qualified him for admission to graduate school, the advanced degree holder is immediately a condidate for positions of leadership in advanced technical work.

The second great contribution of the people with graduate training is of course in teaching. The same qualified which were described just above are essential to the good teacher. These people are desperately needed to make available the knowledge and skills required by the problem solvers. They operate both in our schools and in ourindustrial organizations. They are the ones who can interpret technological advancements for less highly trained individuals, and lead the way in their application.

We have seen that the challenging problems of our day are presenting the demand for a generally increased level of education of technical workers; and that graduate degree holders are making special contributions to this problem both by their own advanced knowledge and their teaching capabilities. When they have achieved an appreciation of the interrelation of technical studies, have moved farther outward toward the advancing frontiers of knowledge, and have learned to communicate with real effectiveness with their fellow men, then they are ready to assume positions of rapidly increasing importance in our world of rapidly increasing technical complexity.

Edited By
Mary L. Cummings, University of Missouri

We are glad to report that Mu Alpha Theta, the National High School and Junior College Mathematics Club, which Pi Mu Epsilon sponsored, is now a thriving one year old youngster. It will be remembered that Pi Mu Epsilon contributed **$300** for initial expenses in the organization of the club.

The first chapters received their charters in April, **1957.** There are now **157** chapters scattered all over the country. National officers are the following:

President: Henry L. Alder, University of California, Davis, California
Vice-president: Edward L. Walters, William **Penn** Senior High, York, Pennsylvania
Secretary-Treasurer: Josephine P. Andree, Box **1127,** University of Oklahoma, Norman, Oklahoma
Governor: George R. Hunt, Odessa Junior College, Odessa, Texas
Governor: Nellie M. Kitchens, **Hickman** High School, Columbia, Missouri
Governor: John R. Mayor, Science Teaching Improvement Program, A.A.A.S., **1515** Mass. Ave., N.W. Washington **5,** D.C.
Governor: Virginia Lee Pratt, Central High School, Omaha, Nebraska

The club has an organ, ***The Mathematical*** Log, containing articles of interest and summaries of chapter activities. Regional meetings are being planned.

Congratulations, Mu Alpha Theta! We believe this new organization is just what young people in the high schools and junior colleges need to ecourage their interest in mathematics.

Throughout this issue of the Journal, the readers may enjoy some complicated but beautiful drawings. These drawings were loaned to us for use in the Journal by Dr. R. F. Piper, Professor Emeritus at Syracuse University. Professor Piper received these drawings from the author, H. Martyn Cundy, Sixth Form Mathematical Master, **Sherborne** School, England

Mr. Cundy has collaborated with another mathematician, Mr. **Rollett** in the book ***Mathematical Models,* 1952,** Oxford, Clarendon Press. These same drawings are found on pp. 224, **230.**

News items should be sent to Mary L. Cummings, Mathematics Department, University of Missouri, Columbia, Missouri.

ALABAMA ALPHA, University of Alabama (December **12, 1957)**

William R. Champion	Maurice E. **Harrell**	Palmer D. Peterson
Mirt Cleon Davidson	John B. **Hendricks**	G. E. Smallwood
Samuel A. **Dickson,** Jr.	James V. **Herod**	Nora K. Truelove
James S **Dupuy,** Jr.	Robert C. Lewis	Thomas O. White
Gale Saben Fly	Fred H. Mitchell, **Jr.**	Emily E. Yow
Rayburn Hammer	David W. **Murrell**	

ALABAMA BETA, Alabama Polytechnic Institute (February **11, 1958)**

Glen Porter Brock	Fred William **Isbell,** Jr.	James Loren Perry
Donald Ray **Connell**	Kwangil Koh	Billy Russell Phillips
John **Allan Dinkel**	Owen Walker **Livingston**	Charles **Lowe** Rogers
William H. Evans, Jr.	Glen Paul Love	Charles Edward Smith
Milton Louis Flucker	Jose' **Meji'a**	Arthur **Gordan** Strum
Walter Luther Green	**Cletus** Eugene Morris	Billie Ann Walling
Walter Robert **Hanley**	James Lawaynne Nix	Don Ray Wood
		Mary Jane Wright

CALIFORNIA GAMMA, Sacramento State College (January **8, 1958)**

Ross H. Brown	John C. Green	Philip **Mishler**
Lawrence G. **Bryans**	Roy D. Hardy	Bobby **J.** Morrison
Timothy Dennis Cavanagh	Dr. Stanley P. **Hughart**	William Patterson
Paul Duncan	William Hunt	Carla Stevenson
John W. **Fothergill**	Ursula Annette Mahlendorff	Betty Thomas
Dr. Gordon R. **Glabe**	Marcella May	Brandon William Wheeler
	Howard P. **McDonough**	

FLORIDA BETA, Florida State University (Jan **15, 1958** and Feb. **12, 1958)**

Charles A. Brown	Charles W. Nabors	William W. **Upchurch**
Walter James **Koss**	Jon V. **Patton**	**Volney** H. Wells
Sherwin Levy	David C. **Rimm**	Robert P. **Whittier**
	Ellen **M. Starbuck**	

GEORGIA ALPHA, University of Georgia (October **30, 1957)**

Mario G. Barbato	Reginald G. **Joiner**	Suzanne Walk**er**
O. **Lexton Buchanan,** Jr.	Wayne Miller	Nancy E. Ward
	Eric Fontelle Thompson, Jr.	

(January **31, 1958)**

B. Jean Clarke	Anna C. Johnson Green	Mary W. **McRitchie**
George S. **Dixon,Jr.**	Frances Faye **Hagan**	Jean Wall Morgan

ILLINOIS BETA, Northwestern University (October, **1957)**

Roy Rene Douglas		**Emil** Christopher Muly

ILLINOIS GAMMA, DeP**aul University (October 14, 1957)**

Louis E. **Aquila**	**William R. Kropp**	Leo **Liolios**
Anthony F. Behof, Jr.	**Jon A. Laboda**	Charles W. Mitchell, Jr.
A. R. Chenot	Gerard P. Lietz	Anthony **Patricelli**
George D. Gaspari		John **A.** Synowiec

KANSAS GAMMA, University of Wichita (December **9, 1957)**

Glen Leon Boroughs	Carolyn Lee Hildyard	Jackson T. Huang
Carl Albert Greene		Marion **M.** Shropshire

OHIO BETA, Ohio Wesleyan University (October **14, 1957)**

Barbara **Briesmeister**	Robert P. Hunt	John H. Mi**ller**
Sarah C. Crooke	Richard **J.** Johnson	William D. **Moorhead**
Frederic E. **Fulmer**	**Jin** Young Kim	**Donley** R. Olson
Sheldon G. Gray	Judith A. **Kraver**	Lyman C. Peck
Richard H. Hagenlocker	Yong **Wooh** Lim	Anne W. **Pusterhofer**
Thomas C. **Hockman**		**Wendel** W. Walty

OHIO GAMMA, University of Toledo (November 20, 1957)

Kenneth D. Friddell
Michael L. Kelley
Edward W Kleppinger
John W. McDonald
Clifford W Thomson

OHIO DELTA, Miami University (November 7. 1957)

John F. Beerman
(Robert) Dave Brown
William W Brown, Jr.
Charles F. Dugan
Robert L. Ellis
Edward J. Fries
Jack E. Graver
Larry D. Kenworthy
Willard C. Loomis
Joseph Mayor
Richard Paul Middleton
James M. Mosser
Waldo A. Patton
ChoKyun Rha
James H. Stamper
Conrad Ray Sturch
Joseph F. Tinney

PENNSYLVANIA BETA, Bucknell University (November 20, 1957)

Doris E. Abbot
Robert Charles Appleman
William Deshel Bandes
Dorothea Louise Bell
Harvey J. Berk
John Roland Coulter
Marion Dennis Douglas
Henry Charles Farrell
Mary Emma Fetter
Nancy Ann Honker
Charlotte Taylor Jones
Joseph David Kissinger
Elizabeth Schrenk Krupka
Donald Kenworthy Miller
Gary Kenneth Munkelt
Gayle Eleanor Myers
Elva Mae Nicholson
Joan Lorraine Obert
Fredric Phillip Olsen
David Dert O'Sullivan
Arthur Leonard Reenstra
George Henry Schneer
Martin Michael Sokoloski
Ronald Glen Staley
Janet Semple Thompson
Richard James Wildenberger

PENNSYLVANIA GAMMA. Lehigh University (October 9, 1957)

Jon Armstrong
Donald E. Bachman
Richard Chichester
Raymond J. Coates
William H. Comerford
George S. Egeland
Armand F. Girard
Willard L. Kauffman
John McMurtrie
Hugo B. Schwandt
John J. Sember
Robert H. Shabaket
Mitchell E. Sisle
Andris Suna
Charles E. Tallman
Frederick Vescial

PENNSYLVANIA DELTA, Pennsylvania State University (October 10, 1957)

William S. Adams
Herman Beisterfeldt
George R. P. Bulman
Donald E. Catlin
Samuel E. Chappell
Paul H. Cutlet
Ronald L. Duty
Roger L. Fritz
Lewis J. Geiger
Ann D. Greene
William Griffin
Ralph L. Hasland
Edward E. Headington
Robert E. Henderson
Mary Hickey
Leslie E. Hoffman
Kostas Hrisoulas
Alan L. Jones
Richard H. Jones
Donald C. Klick
Joseph H Mitchell
Richard B. Noll
George T. Onega
Donald P. Rozenberg
Frank E. Rupp
George Simkovich
Lloyd V. Slocum
Rene A. Steigerwalt
John M. Tomlinson
Robert Welch
Jack D. Wolf
Carlton S. Young

MISSOURI ALPHA, University of Missouri (December 4, 1957)

Wallace Junior Austin
John Norman Baker
Frederick Dorn Bean
Ivan Leroy Berry
John Wesley Brown
Albert Donald Epperly
Charles George Ernst
Dean David Froerer
Richard Allen Hart
Robert Henry Johnson
Mary Dale Jolly
Carol Lee Kirby
Andrew James Macaulay
Patrick L. McClung
Jo Ann Nelson
Christiaan Jan Penning
Edward Clifton Scott
Sue Lynn Strait
Eldo Leroy Throckmorton, Jr.
Thomas Arthur Warring
Robert Allen Wenski

NEBRASKA ALPHA, University of Nebraska (January 14, 1958)

Robert Duane Anderson
Vernon Peter Bolleson
Gary Gordon Frenzel
Burton Eugene Greiner
Charles Duane Grimsrud
Mildred Lucile Gross
John Orlando Herzog
Ervin Henry Hietbrink
Dean Harlan Hohnstein
Clarence Glenn Houser
Marvin Gale Kesler
Donald Eugene McArthur
C. Robert Nelson
Haki Halil Ozbek
Dwaine William Rogge
Dr. Hubert H. Schneider
Larry Lee Smalley
Paul David Smith
Margaret Ann Tevis
Gordon J. Warner
James Alex Wees
Joyce Darlene Wichelt
James Austin Williams
Richard Lee Woolley

NEW YORK BETA, Hunter College (October 14, 1957)

Laura Adams
Hilda Berglas
Rose Bokser
Stephanie Farbman
Myrna Gittel
Natalie Horowitz
Helen Cohen Kaufman
Katherine Lee
Mildred Malek
Kathryn Miranda
Patricia Ortler
Marianne Sinapi
Gilda Solomon

NEW YORK GAMMA, Brooklyn College (Jan, 7, 1958 and Jan. 13, 1958)

Mendel Beer
Alfred Brandstein
Barry Bressler
Myrna Cohen
Harold Engelsohn
Ellen Hirsch
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