PI MU EPSILON JOURNAL

THE OFFICIAL PUBLICATION OF

THE HONORARY MATHEMATICAL FRATERNITY

VOLUME 3

CONTENTS

FALL 1959

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PI MU EPSILON JOURNAL is published semi-annually at St. Louis University.

SUBSCRIPTION PRICE: To Individual Members, $1.50 for 2 years; to Non-Members and Libraries, $2.00 for 2 years. Subscriptions, orders for back numbers and correspondence concerning subscriptions and advertising should be addressed to the PI MU EPSILON JOURNAL, Department of Mathematics, St. Louis University, 221 North Grand Blvd., St Louis 3, Mo.

RELATIONS AS MODELS OF PHYSICAL SYSTEMS

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Illinois, '37

1. The Postulational Method and its Application to the Study of Nature.

It is the purpose of this paper to outline briefly the philosophy of the construction of abstract mathematical systems and of the application of these to the study of systems occurring in nature. We shall then illustrate the principles we present with some simple but significant examples.

It is important at the outset to recognize that because one cannot define every word in terms of simpler words, the formal construction of every mathematical system necessarily begins with some *undefined terms.* Similarly, because one cannot deduce every theorem from more primitive theorems, every mathematical system must also contain unproved theorems or *postulates.* These postulates relate the undefined terms and give them such mathematical meaning as they possess.

For example, in Euclidean geometry, the undefined terms might include *point, line,* and *pass through.* Then one of the possible postulates is: *Through two distinct points there passes one and only one line.*

From the undefined terms and postulates, one deduces *theorems* by means of the rules of logic. When this process has yielded all the useful conclusions it can, one introduces *definitions* of new concepts in terms of the undefined ones and then states more postulates and proves more theorems. For example, in Euclidean geometry, having defined *parallel lines* in terms of *point, line,* and *pass through,* one might state the postulate: *Through a given point not on a line there passes one and only one line parallel to the given line.* Then one could prove the theorem: *If a line passes through exactly one point of one of two parallel lines, it passes through exactly one point of the other.*

The choice of the undefined terms and of the postulates of a mathematical system is by no means simple. Those of Euclidean geometry are the outgrowth of several thousand years' experience with experimental and intuitive geometry in ancient Babylonia, Egypt, and Greece. In all other examples of postulational systems – and there are many – the postulates are likewise selected, on the basis of appropriate experience, in such a way as to yield useful results.

1 This paper was presented as the initiation lecture of the Missouri Gamma Chapter in April, 1958 and also before Sigma Xi at Southern Illinois University, Carbondale in April, 1958.

When mathematics is applied to the world of nature, it is relatively rare that any very extensive natural system being studied is well-enough understood that even a reasonably complete set of undefined terms and postulates can be stated formally. To illustrate, no such system has ever been given for the science of electricity; we simply do not know enough about the physical aspects of the subject to reduce it to a simple, formal, postulational kind ofmatliematical scheme. Hence we must, at present, be content with several independent and not always consistent systems which explain different aspects of the subject.

Often, however, it is possible to construct a formal mathematical system which is a useful description of a suitably restricted part of nature. This involves first of all abstracting from our experience with nature a set of undefined terms, postulates, and definitions for a mathematical system which describes to a suitable degree of approximation the part of nature we wish to study. Such a system is called a ***mathematical model*** of the part of nature it represents. We then manipulate this mathematical system according to known laws of logic and mathematics and draw such ***mathematically valid*** conclusions as we can.

The next step is to interpret these mathematical conclusions as conclusions about the part of nature under study. If these conclusions can be verified by experiment, then our model is a good one, at least to the limits of our ability to detect, for we can then use the model to make ***physically valid*** and useful predictions about the part of nature being observed.

On the other hand, no mathematical model has ever provided all the answers to all the problems concerning its corresponding physical system. This is because it does not - and in fact cannot - take into account ***all*** of the conditions which affect the physical system in question. Normally one ignores all but what appear to be the most vital factors when one is constructing a model. Taking these most vital factors into account, one builds a mathematical model which, if it is cleverly constructed, produces theorems which correlate closely with what is observed in nature. When this is the case, the model is a useful one. When the correlation is not good, the model is unsatisfactory and at least one additional factor must be added to the list of vital ones.

Newtonian mechanics provides the classic example of this latter situation. Adequate to explain the mechanical phenomena of ordinary experience, it is inadequate to explain all observable phenomena at either the sub-atomic or the astronomical levels. Hence the theory of relativity, a generalization of Newtonian mechanics which includes the latter as a special case, was invented by Einstein to account for the apparently irregular observations.

An interesting sidelight on the history of science is related to the fact that Einstein also derived at one time a formula for the potential of an ion in solution. He assumed that electrical forces of attraction or repulsion between the ions were not significant because the distances between the ions appeared large compared to their radii. The formula did not agree with what was observed in experiment. It remained for Nernst, who recognized that these same electrical forces are indeed significant, to derive the correct equation.

The most characteristic aspect of modern mathematics is its exploitation of the postulational method described above to create new mathematical systems and to analyze familiar ones. Properly used and understood, the method yields a level of rigor and a degree of insight not otherwise attainable. Moreover, the mathematical systems obtained by these abstract methods are with increasing frequency found to be well-adapted to the analysis of physical, social, and biological systems that have not been mathematized before.

We shall now illustrate these matters with some examples. All these examples are based on the mathematical concept of a ***relation.*** I have chosen this mathematical concept not only because it is a fundamental one, but also because it is applicable in a very simple way to a wide variety of problems. However, by employing the concept of a relation here, I do not mean to ascribe to it an undue significance. It is just one of a large number of basic mathematical tools.

2. The Concept of a Relation.

The most familiar example of a relation is that of family relationship in a group of people. If x is father or brother, mother, aunt, cousin, etc., of y, we say x and y are "related." If flipping a certain switch customarily has the consequence of turning on a certain light, we say these two events are "related". Rainfall and grain yield are also "related", though in a more complex way. In each example, however, we are concerned with certain special pairs: pairs of people, pairs of events, pairs of numbers, and also in each case ***the first member of the pair bears a certain relation to the second.***

This familiar notion of a relation can be made mathematically precise as follows. Let X and Y be arbitrary sets of objects, where Y is not necessarily different from X. We define first the ***Cartesian product*** X x Y of X and Y to be the set of all ***ordered pairs*** (x,y), that is, the set of all pairs (x,y) whose ***first*** member x belongs to X and whose ***second*** member y belongs to the set Y.

It is customary to write the symbol "ϵ" for the words "belongs to", "belong to" or "belonging to" so that "x belongs to X" is written simply "$x \epsilon X$". Then we write

$$X \times Y = \{ (x,y) \mid x \epsilon X, y \epsilon Y \}$$

to mean "X x Y is the ***set of all*** ordered pairs (x,y) ***such that*** $x \epsilon X$ and $y \epsilon Y$."

It should be noted that Y x X, which is $\{ (y,x) \mid y \epsilon Y, x \epsilon X \}$, is not ordinarily the same thing as X x Y because the orders of the elements in the pairs are opposite in the two cases. When Y is the same set as X, then X x Y and Y x X are of course the same.

As an example, let X and Y each denote the set of all real numbers so that X x Y is the set of all ordered pairs of real numbers **(x,y).** This set has as one geometrical representation the familiar system of rectangular Cartesian coordinates in the plane where x is the abscissa and y is the ordinate of the point **(x,y).** This example is of course responsible for the name "Cartesian product."

As another example, let X denote the set of all male human beings living in a certain township and let Y denote the set of all female humans living in that **township.** Then X x Y denotes the set of all possible pairs **(x,y)** of where x is a man and y is a woman from this township. This particular Cartesian product is of course a major object of masculine concern.

Our earlier examples of relations now suggest the following definition: An ***abstract relation*** from ***the set X to the set Y,*** more simply a relation in X x Y is any subset of the set of all ordered pairs **(x,y).** For given sets X x **Y,** some of the relations in X x Y may have familiar meanings; others may correspond to no familiar relation at all, thus simply having a formal mathematical meaning. To illustrate, in the example given above, of male and female humans in a certain township, we might select from X x Y those pairs **(x,y)** such that x is the husband of **y,** thereby obtaining a familiar relation. On the other hand, we could select 10 males at random and likewise 10 females, pair these in some arbitrary order, and obtain thus a perfectly valid but probably useless example of a relation in X x Y.

The set of all x's in the pairs of a relation $\mathcal{R}$ is called the ***domain*** of $\mathcal{R}$ and the set of all **y's** in the pairs of $\mathcal{R}$ is called the ***range*** of $\mathcal{R}$. To illustrate further, when X and Y are both the set of all real numbers, we could obtain a subset of X x Y by requiring that x and y simultaneously satisfy the restrictions

$$\begin{cases} 0 < x < 1 \\ x^2 < y < \sqrt{x}. \end{cases}$$

The requirement $x^2 < y$ means that the point **(x,y)** is above the parabola with equation $x^2 = y$. The requirement $y < \sqrt{x}$ means that the point **(x,y)** is below the parabolic arc represented by $y = \sqrt{x}$. The region to which the pairs **(x,y)** of the relation are restricted by these requirements is shown shaded in Figure 1.

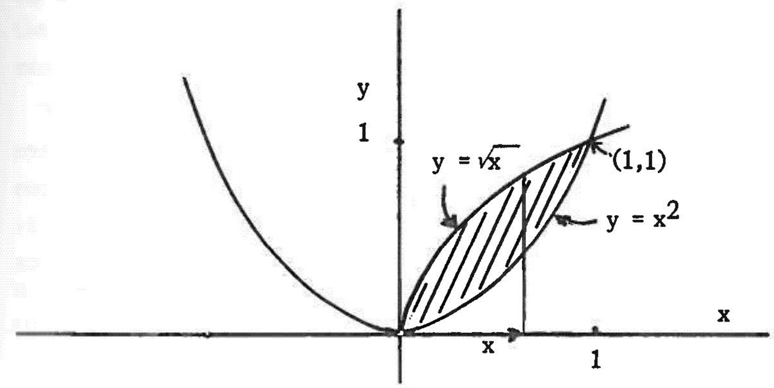

FIGURE 1.

Here for each x belonging to the domain $0 < x < 1$ of the relation, **there** are infinitely many y's of the range $0 < y < 1$ such that the pair $(x,y) \in \mathcal{R}$.

In many physical situations, to each possible value of one **variable** x **(e.g.,** rainfall) there corresponds a range of possible values of **a** second variable **y(e.g.,** grain yield) so that a graphical **representation** of the relation between the two variables is a two-dimensional **region,** often roughly similar to that shown in Figure 1.

A more restricted example of a relation is given by the following definition: Again let X and Y each be the set of real numbers. We shall say that a given pair **(x,y)** belongs to a relation $\mathcal{F}$

if and only if

$$\begin{cases} y \geq 0 \\ x^2 + y^2 = 1. \end{cases}$$

These conditions imply further that

$$\begin{cases} -1 \leq x \leq 1 \\ 0 \leq y \leq 1 \end{cases}$$

which give respectively the domain and the range of this relation. This relation is representable geometrically as the upper half of a unit semi-circle (Figure 2).

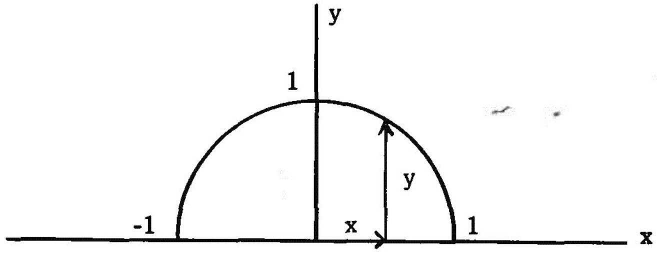

FIGURE 2

In this case, for each x belonging to the domain of $\mathcal{F}$, there exists ***exactly one*** y such that the pair (x,y) belongs to $\mathcal{F}$.

Any relation $\mathcal{R}$ in which to each x of the domain of $\mathcal{R}$ there corresponds exactly one y of the range of $\mathcal{R}$ is an example of what is called a (single-valued) ***function.*** Thus the very general concept of a relation includes the concept of a function as a special case.

The sets X and Y do not need to be sets of numbers for a relation in X x Y to be a function. The function concept extends in fact to arbitrary sets. The only requirement is that ***to each x in the domain of $\mathcal{R}$ there should correspond exactly one y of the range of $\mathcal{R}$ such that $(x,y) \in \mathcal{R}$.*** We might, for example, let Y denote the set of positive numbers but let X denote the set of all women in this country. We could then say that (x,y) belongs to a relation $\mathcal{W}$ if and only if y is the weight in pounds of x. This well-known relation is a function which has been subjected to largely irrelevant but highly profitable study by many manufacturers of "reducing aids."

If the elements of each of X and Y can be put into one-to-one correspondence with subsets of the set of all real numbers, two-dimensional graphical representation of a relation in X x Y is always possible. However, if X itself consists of ordered pairs (x_1,x_2) which can be represented on a 2-dimensional graph, then a 3-dimensional graph of a relation in X x Y is possible. In this case the graph may consist of a 3-dimensional region, a two-dimensional region, a 1-dimensional region (curve) or just some isolated points.

3. Arithmetic Representation of a Finite Relation

Now let us consider a relation $\mathcal{R}$ in X x Y where X and Y are finite sets with elements $x_1,x_2,\ldots, x_n$ and $y_1,y_2,\ldots, y_m$ respectively. We can give such a relation a uniquely defined arithmetic representation by constructing an array of n rows (one for each element x_i) and m columns (one for each element y_j). In the ith row and jth column of this array, we record a "1" if $(x_i,y_j) \in \mathcal{R}$, otherwise

we record a **"0".** The resulting array of **0's** and **1's,** stripped of the row and column headings, but enclosed in brackets, we call the ***matrix*** R ***of the relation*** $\mathcal{R}$.

To illustrate, let $X = \{x_1,x_2,x_3\}$ where x_1,x_2,x_3 are dormitory roommates. Suppose x_1 likes x_2 and x_3 and that the feeling is reciprocated. Suppose on the other hand that x_2 detests x and vice versa. Finally, suppose x_1 likes himself, but that x_2 and x_3, subconsciously regarding themselves as rascals, do ***not*** like themselves. Then we have the following array and the matrix L of a ***liking*** relation $\mathcal{L}$ in X x X:

$$\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & 1 & 1 & 1 \\ x_2 & 1 & 0 & 0 \\ x_3 & 1 & 0 & 0 \end{array} \qquad L = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Conversely, given any n x m matrix R of zeros and **1's,** we may interpret it as representing a uniquely defined abstract relation in X x Y where X and Y are arbitrary sets having n and m elements respectively and where, given any pair (x_i,y_j), a 1 in the **i,j-position** of R is taken to mean that (x_i,y_j) belongs to $\mathcal{R}$ and a **"0"** is taken to mean that it does not.

Now of what use are these matrices of **0's** and **1's?** There is a good deal of information available about the algebraic properties of such matrices. **These** properties may often be interpreted as properties of relations corresponding to the matrices in question. Thus the algebra of matrices affords computational means of deducing properties of relations.

To illustrate, a relation $\mathcal{R}$ in X x X is called ***reflexive*** if and only if every pair $(x,x) \in \mathcal{R}$ where $x \in X$. If a relation $\mathcal{R}$ on a finite set is reflexive, then its matrix R will have **1's** down the main diagonal and conversely.

A relation $\mathcal{R}$ in X x X is called ***symmetric*** if and only if whenever $(x_a,x_b) \in \mathcal{R}$, $(x_b,x_a) \in \mathcal{R}$ also. If X is finite and $\mathcal{R}$ is symmetric, the corresponding matrix R will have a 1 or 0 in the **a,b-position** whenever it has a 1 or (respectively) 0 in the **b,a-position,** and conversely.

The matrix L of the liking relation given above shows that L is, in this instance, ***not*** reflexive but that it is symmetric. The relation of liking is not always a symmetric one, however, as many a frustrated lover has discovered.

The quantitative study of various relations which appear in relatively small groups of individuals is currently of great interest to psychologists and sociologists. In these studies the **0's** and **1's** we have used above are often replaced by numbers from a scale with which it is attempted to measure the intensity of the relation in question.

Another possible property of a relation in X x X is that of ***asymmetry:*** If (x_a,x_b), $a \neq b$, belongs to $\mathfrak{R}$, then (x_b,x_a) does ***not*** belong to $\mathfrak{R}$. In this case, if X is finite, the **b,a-entry** of R is 0 whenever the **a,b-entry** is 1.

Finally, there is the property of ***transitivity:*** a relation $\mathfrak{R}$ in X x X is called transitive if and only if whenever $(x_a,x_b) \in \mathfrak{R}$ and $(x_b,x_c) \in \mathfrak{R}$, then $(x_a,x_c) \in \mathfrak{R}$ also.

A simple example of an asymmetric, transitive relation is the ancestral relation in a group of people: If x_a is an ancestor of x_b, then x_b is ***not*** an ancestor of x_a. If x_a is an ancestor of x_b and x_b is an ancestor of x_c, then x_a is an ancestor of x_c.

4. Relations and Switching Circuits.

We now turn to a totally different kind of application of the relation concept. The basic element of many telephone and computing circuits is a switch which has the property of being open or closed. Consider a switch S in a conducting wire from a point p_1 to a point p_2:

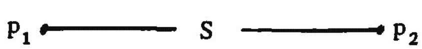

FIGURE 3

If **S** is closed the vertices p_1 and p_2 are electrically connected to each other; if S is open they are not. We represent this symbolically by means of a variable s such that $s = 1$ if S is closed but $s = 0$ if **S** is open. This is a symmetric relation if the switch is such that current can flow through it in either direction.

Now consider the circuit shown in Figure 4 which contains switches S_1,S_2,S_3,S_4. (The two switches labeled S_4 are assumed to open and close simultaneously).

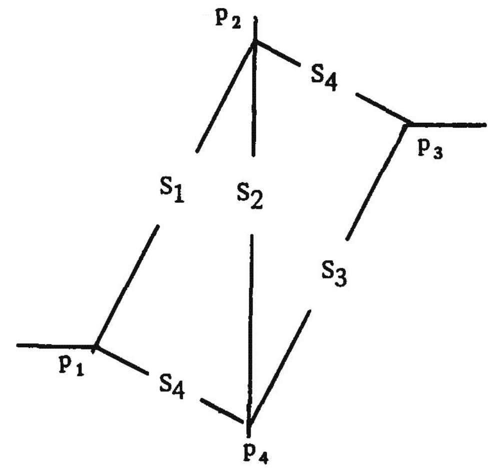

FIGURE 4

We examine the relation of electrical connectedness between the terminals p_1,p_2,p_3,p_4 as controlled by the conditions of the four switches. We shall always regard a terminal as being connected to itself electrically. However, the connection of one terminal to another, by means of a path not passing through a third terminal, is ordinarily a ***variable*** relation depending on the closed or open condition of a switch. We therefore write in the matrix for this connection relation not 0 or 1 but rather a variable s_j which takes on the value 0 or 1 according as S_j is open or closed. The result is the matrix

$$C = \begin{bmatrix} 1 & s_1 & 0 & s_4 \\ s_1 & 1 & s_4 & s_2 \\ 0 & s_4 & 1 & s_3 \\ s_4 & s_2 & s_3 & 1 \end{bmatrix}$$

Here the entry in the **1,3-position** is 0 because there is no wire from p_1 to p_3 in the circuit. For any given set of values of the circuit variables s_1,s_2,s_3,s_4, this matrix indicates which vertices are connected to which others through a closed switch.

Many switching circuits may be analyzed completely and may often be simplified with the aid of suitable computations on matrices like these. Moreover, from verbally stated requirements for the operation of a circuit, one can often develop a simple matrix similar to the one above by means of systematic techniques. From the matrix one can then draw a circuit which meets the given requirements. It should be added (hat the purely **mathematical** study of these matrices and of similar algebraic systems is highly rewarding. In the study of such circuits and matrices, it is convenient to regard the variables $s_1, s_2, \ldots s_n$ which appear, as elements of a Boolean algebra.

5. Relations and Computing Machines

As a simple example of the kind of machine we have in mind, consider a box with an input wire and two lights, one labeled **E**, one labeled θ. The box is set so that initially the light labeled **E** is on. (Figure 5.) Via the input wire we now send pulses of voltage, say high voltages of brief duration, into the box.

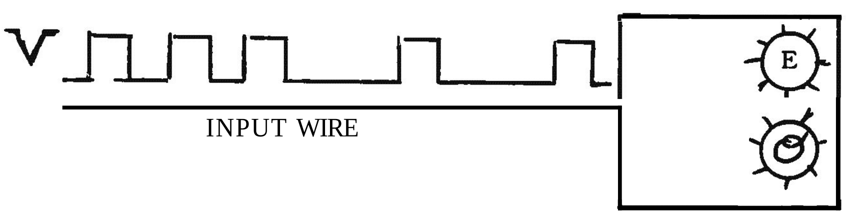

FIGURE 5

The circuits inside the box are so constructed that whenever such a pulse is applied, the light which is on goes off and the light which is off goes on. Evidently then, when an even number of pulses has come into the box, light E will be on and when an odd number has come in, light (Twill be on. Such a machine is called a binary counter and circuits which perform essentially these operations are basic components of every electronic computer.

Now this machine may be thought of as being in an **"E-state"** or an "a-state." The vital relation between the two states is one of transition: There is a transition from one state to the other when an input pulse, represented below by the symbol H, comes along. The transition is from a state back to itself, **i.e.,** really no transition at all, if no input pulse, represented below by the symbol L, comes along.

These facts suggest the following graph as an abstract picture of the machine. (Figure 6.)

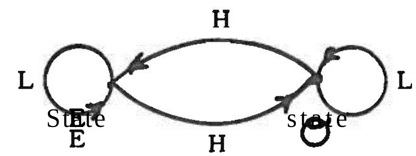

FIGURE 6

Here we have selected one point called a vertex for each possible state of the machine. Since a transition is directed from a state to a state we now draw one arrow, called a branch, from one vertex to another vertex corresponding to each possible transition of the machine. With each arrow we associate the input symbol H or L which accounts for the transition.

A matrix which summarizes all this information is the transition matrix **T** of the machine:

$$T = \begin{array}{c} E \\ \theta \end{array} \begin{bmatrix} L & H \\ H & L \end{bmatrix}$$

In this generalization of a relation matrix we have a powerful tool for the systematic study of abstract relations in computers and in other automata.

6. Conclusion

The examples indicate only sketchily the fact that the concept of a relation underlies much of modern mathematics and its applications. Moreover, this concept and the devices for computation associated with it have led to many useful models of systems in the physical, biological, and social sciences. In turn, the study of these applications has led to the study of more general, abstract mathematical systems that had not been investigated before. This situation is a revealing illustration of the perpetual interplay between mathematics and its applications.

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DEFINITION OF A TOPOLOGY BY MEANS OF A SEPARATION RELATION

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One aspect of point set topology is a **generalization** of some concepts encountered earlier in mathematical studies, such as "arbitrarily near", "**sufficiently** small", "however large", etc. The usual definition of a topology brings out this aspect, but the connection is not ordinarily apparent to the young topologist. Thus, there would seem to be a need for a definition that would be more appealing to the intuition and would reveal the important connections between topology and its academic and historical predecessors. The definition of a topology by means of the separation relation is offered herein as one answer to this need.

In order to bring this definition by separation relation to light, the ordinary definition is first recalled; then the elementary results and definitions following upon the usual definition are presented and then the definition by the separation relation is introduced. The equivalence of these two definitions is proven, showing their sameness, while they are then contrasted to show their differences.

The usual definition of a topology is as follows.

Definition O.
Let X be an arbitrary set and let T be a family of subsets of X; $T = \{A_i : A_i \subset X\}_{i \in I}$; I an index set.

Then T is a topology for X if and only if the following hold:

O-1. The union of any number of members of **T** is again a member of T, or symbolically, $\bigcup_{i \in J \subset I} A_i \in T$ for any $J \subset I$.

0-2. If A and B are members of **T**, $(A \cap B) \in T$.

The definitions and theorems listed below will be used throughout this discussion.

1) A set A is open if and only if $A \in T$.
2) A set B is closed if and only if $B' \in T$. [Here B' denotes the complement of B in X.]
3) A set A is a neighborhood of a point x if and only if there exists an open set **V** such that $x \in V \subset A$.
4) A point x is an accumulation point of a set A if and only if for every neighborhood V of x, $V \cap (A - \{x\}) \neq \emptyset$.
   Let $h(A) = \{x: x$ is an accumulation point of $A\}$ and $A = A \cup h(A)$.
5) A necessary and sufficient condition that T be a topology for a set X is that the family $F = \{A_i : A_i' \in T\}_{i \in I}$ satisfy:

C-1. The intersection of any number of members of **F** is again a member of **F**; that is, $\bigcap_{i \in J \subset I} A_i \in F$ for any $J \subset I$.

C-2. If A and B are members of F, $(A \cup B) \in F$.

This theorem may be proved by applying **DeMorgan's** theorem to the usual definition.

6) An open set is a neighborhood of each of its elements.

The definition of a topology by the separation relation may now be introduced.

Definition S.
Let X be an arbitrary set and let a binary relation s be defined on the set $P(X)$ of all subsets of X. This relation will be denoted by AsB (A is separated from B) where A and B are subsets of X. Let $A \not{s} B$ mean that A is not separated from B and let xsA mean that the singleton set $\{x\}$ is separated from A. A set function k: $P(X) \to P(X)$ is defined by : $k(A) = \{x: x \not{s} A\}$ for all $A \in P(X)$.

The following axioms are assumed to characterize the relation and the function:

1. $As0$ for every non-empty subset A of X;
2. AsB if and only if BsA;
3. If AsB and $C \subset A$, then CsB;
4. If AsB, then $A \cap B = 0$;
5. If AsC and BsC, then $(A \cup B)sC$;
6. $k(A)'sA$; [$k(A)'$ again denotes the complement of $k(A)$ in X]
7. If xsA, then $xsk(A)$.

With s and k so characterized, a topology may now be defined for **X**. Let F_s be a family of subsets of X defined by $F_s = \{A: A = k(A)\}$. Then the family **T** of subsets of **X** defined by $T = \{B: B' \in F_s\}$ is a topology for X.

In order to show the equivalence of the last definition of the topological space and the usual one, the last definition will be shown to imply C-1 and C-2 and the relation of topological separation under the usual definition will be shown equivalent to the separation relation as used in the last definition. With these shown, the definition by separation relation implies the usual definition while the usual definition implies the characterizing axioms of the last definition, so the equivalence is shown. A few lemmas will first be proved.

Lemma A. If $A \subset B$, then $k(A) \subset k(B)$.
Proof: Let $A \subset B$ and suppose that $x \notin k(B)$. If $x \notin k(B)$, xsB; but xsB and $A \subset B$ implies, by axiom 3., that xsA, that is, $x \notin k(A)$. Hence, $x \notin k(B)$ implies that $x \notin k(A)$, so $k(A) \subset k(B)$.

Lemma B. If $A \subset B$ and $A \neq \emptyset$, then $A \not{s} B$.
Proof: Suppose $A \subset B$ and AsB. Then, by axiom 3, AsA, which would mean that $A \cap A = \emptyset$, by 4. But this is impossible since $A \neq \emptyset$, so $A \subset B$ implies $A \not{s} B$.

Lemma C. For every $A \subset X$, $A \subset k(A)$.
Proof: If $x \in A$, $\{x\} \subset A$, which means, by B, that $x \not s A$, that is $x \in k(A)$. Thus $A \subset k(A)$.

Lemma D. $k(k(A)) \subset k(A)$.
Proof: Suppose $x \notin k(A)$. If $x \notin k(A)$, then xsA, which implies, by 7, that $xsk(A)$, that is $x \notin k(k(A))$. Therefore, $k(k(A)) \subseteq k(A)$.

Lemma E. $k(k(A)) = k(A)$; that is $k(A) \in F_s$.

Proof: By lemma D, $k(k(A)) \subset k(A)$. By lemma C, $A \subset k(A)$; so, by lemma A $k(A) \subset k(k(A))$. Therefore, $k(k(A)) = k(A)$.

Lemma F. $k(A \cup B) = k(A) \cup k(B)$.
Proof: Suppose $x \notin k(A \cup B)$. Then $xs(A \cup B)$. Since $A \subset A \cup B$ and $B \subset A \cup B$, by 3, xsA and xsB, that is, $x \notin k(A)$ and $x \notin k(B)$; hence $x \notin k(A) \cup k(B)$. Therefore, $k(A) \cup k(B) \subset k(A \cup B)$. Now suppose $x \notin k(A) \cup k(B)$. Then xsA and xsB so, by 5, $xs(A \cup B)$, that is, $x \notin k(A \cup B)$. Hence $k(A \cup B) \subset k(A) \cup k(B)$. Therefore, $k(A \cup B) = k(A) \cup k(B)$.

Lemma G. $k(\emptyset) = \emptyset$; that is, $\emptyset \in F_s$.

Proof: By 1, $\emptyset s x$ for any $x \in X$, which means that $x \notin k(\emptyset)$ for all $x \in X$, that is, $k(\emptyset) = \emptyset$.
With these results of the definition by separation, it may now be proved that:

Theorem: The family F_s satisfies C-1 and C-2.

Proof: C-1. Consider $A = \bigcap_{i \in J} A_i$, where $A_i \in F_s$ for each $i \in J \subset I$.

If $A = \emptyset$, C-1 is satisfied by lemma G. If $A \neq \emptyset$, $A \subset A_i$ for each $i \in J$. Thus, by A, $k(A) \subset k(A_i) = A_i$ for each $i \in J$; hence $k(A) \subset \bigcap_{i \in J} A_i = A$.
Also, by C, $A \subset k(A)$; hence $A = k(A)$, that is, $A \in F_s$ and C-1 is satisfied.

C-2. Suppose A and B are members of F_s. Then $A = k(A)$ and $B = k(B)$.
Since $k(A \cup B) = k(A) \cup k(B)$, by F, $k(A \cup B) = A \cup B$.
Thus $k(A \cup B) \in F_s$ and C-2 is satisfied.

By lemmas C, E, F, and G, k is a function from $P(X)$ into $P(X)$ such that;
1. $k(\emptyset) = \emptyset$
2. $A \subset k(A)$ for every $A \subset X$;
3. $k(k(A)) \subset k(A)$;
4. $k(A \cup B) = k(A) \cup k(B)$.

But these are exactly Kuratowski's closure axioms so $k(A) = \bar{A}$. Hence, $(\bar{A} \cap B) \cup (A \cap \bar{B}) = (k(A) \cap B) \cup (A \cap k(B))$. We will show, therefore, that:

Theorem: AsB if and only if $(k(A) \cap B) \cup (A \cap k(B)) = \emptyset$.
Proof: First, suppose that AsB and let $x \in B$. Then, since AsB and $\{x\} \subset B$, xsA, by 3. This means that $x \notin k(A)$, that is $x \in k(A)'$. Hence $B \subset k(A)'$ or $k(A) \cap B = \emptyset$. Now let $y \in A$. Since AsB and $\{y\} \subset A$, ysB, that is, $y \notin k(B)$, so $A \subset k(B)'$. Therefore, $A \cap k(B) = \emptyset$.
Thus AsB implies that $(k(A) \cap B) \cup (A \cap k(B)) = \emptyset \cup \emptyset = \emptyset$.
Now suppose that $(k(A) \cap B) \cup (A \cap k(B)) = \emptyset$. Then $k(A) \cap B = \emptyset$ and $A \cap k(B) = \emptyset$.

a) $k(A) \cap B = \emptyset$. If $x \in B$, $x \notin k(A)$, that is, $x \in k(A)'$. Thus $B \subset k(A)'$, but $k(A)'sA$, by 6, so, by 3, BsA, that is AsB.

b) $A \cap k(B) = \emptyset$. If $y \in A$, $y \in k(B)'$; hence $A \subset k(B)'$. But $k(B)'sB$, by 6 so, by 3, AsB. Therefore, AsB if and only if A and B are topologically separated. This completes the proof of the mathematical equivalence of the two definitions.

In order to see how the definition of a topology by the separation relation more clearly emphasizes the connection of topology to earlier mathematics and how it is more intuitive than the usual definition, a definition important to topology will be formulated under each definition and then proved equivalent. But first some preparation is needed.

Lemma H. Let $K = \bigcap_{i \in I} \{A_i : A_i \in F_s \text{ and } A \subset A_i\}$. Then $K = k(A)$.

Proof: Since F_s satisfies C-1, $K \in F_s$, that is $K = k(K)$. Also $A \subset A_i$ for each $i \in I$, so $A \subset K$; hence, by A, $k(A) \subset k(K) = K$. Further, $k(A) \in F_s$, by E, and $A \subset k(A)$, by C, so $k(A) = A_i$ for some $i \in I$. But $K \subset A_i$ for every $i \in I$, so $K \subset k(A)$.

Thus $k(A) = K = \bigcap_{i \in I} \{A_i : A_i \in F_s \text{ and } A \subset A_i\}$.

Applying DeMorgan's theorem to H yields $k(A)' = \bigcup_{i \in I} \{A_i' : A_i \in F_s \text{ and } A \subset A_i\}$.

That is, $k(A)' = \bigcup_{i \in I} \{A_i' : A_i' \in T_s \text{ and } A \cap A_i' = \emptyset\}$;

Hence, $k(A)' = \bigcup_{i \in I} \{B_i : B_i \in T_s \text{ and } A \cap B_i = \emptyset\}$.

The usual definition of an accumulation point x of a set A (a point which is arbitrarily close to the set) is repeated as follows: A point x is an accumulation point of a set A if and only if for every neighborhood V of x, $V \cap (A - \{x\}) \neq \emptyset$.

The definition of an accumulation point x of a set A is:
A point x is an accumulation point of a set A if and only if $x \not s (A - \{x\})$.

In order to show the equivalence of these two definitions, the following lemma is proved:

Lemma I. A necessary and sufficient condition that $x s A$ is that there be a neighborhood V of x such that $V \cap A = \emptyset$.
Proof: First suppose that $x s A$. Then $x s A$ implies that $x \notin k(A)$, that is, $x \in k(A)'$. Also $k(A) \in F_s$, so that $k(A)' \in T_s$, so $k(A)'$ is a neighborhood of x. **But,** by axiom 6, $k(A)' s A$; hence, by 4, $k(A)' \cap A = \$$. Thus $k(A)'$ satisfies the conditions.
Now suppose that there is a neighborhood V of x such that $V \cap A = \emptyset$. Then there is an open set V_o such that $x \in V_o \subset V$, and so $V_o \cap A = \emptyset$. Then, by the remark after lemma H, $V_o \subset k(A)'$; hence $x \in k(A)'$, that is, $x \notin k(A)$. So $x s A$.

If "$A - \{x\}$" is substituted for A in the last lemma and if both implications are contraposited, the result is:
$x \not{s} (A - \{x\})$ if and only if for every neighborhood V of x, $V \cap (A - \{x\}) \neq \emptyset$. This proves the equivalence of the two definitions.

Contrasting these two definitions (and implicity the respective basic definitions of a topology) from the viewpoint of the beginning student in topology, the advantage would seem to accrue to the separation method. It clearly indicates the connection between topology and the familiar notion of "arbitrarily near". It is intuitively logical and the intervention of the notion of a neighborhood is not needed, which enhances its directness. Although it may seem somewhat rambling to the professional, the more succinct fomulations may be introduced as necessary and sufficient conditions and, of course, more advanced notions may be approached more concisely.

St. Louis University and
McDonnell Aircraft, St. Louis, Mo.

PROBLEM DEPARTMENT

Edited by M. S. Klamkin,
Avco Research and Advanced Development Division

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication. Address all communications concerning problems to M. S. Klamkin, Avco Research and Advanced Development Division, T-430, Wilmington, Massachusetts.

112. Proposed by J. S. Frame, Michigan State University.

Find all real analytic functions F such that

$$F(x+y)\,F(x-y) = [F(x) + F(y)]\,[F(x) - F(y)].$$

113. Proposed by Leo Moser, University of Alberta.

Prove that it is impossible to enter the integers 1, 2, . . . 10, on the 10 intersections of 5 lines of general position in such a way that the sum of numbers on every line is the same (22).

114. Proposed by D. J. Newman, Brown University.

Solve the four simultaneous equations

$$\begin{aligned} x + y &= a, \\ ux + vy &= b, \\ u^2x + v^2y &= c, \\ u^3x + v^3y &= d, \end{aligned}$$

for x, y, u, and v.

115. Proposed by Francis L. Miksa, Aurora, Illinois.

What is the smallest integral set for which

$$(10a + b)^2 + (10b + a)^2 + (10c + d)^2 + (10d + c)^2 = R^2.$$

116. Proposed by *M.* S. Klamkin, AVCO RA DD.

Problem No. 147, due to **Auerbach-Mazur,** in the **"Scottisch"** book of problems **is** to **show** that if a billiard ball **is** hit from **one** corner of a billiard table having commensurable sides at an angle of **45°** with the table, then it will hit another corner. Consider the more general problem of a table of dimension ratio **m/n and initial** direction of ball of $\theta = \tan^{-1} a/b$ (m, n, a, and b, are integers). Show that the ball **will** first strike another corner after $\frac{an + bm}{(an, bm)} - 2$ cushions (**(x,y)** as usual denotes the greatest common divisor). Furthermore, determine which other corner the ball will strike.

Solutions

103. Proposed by Lawrence Shepp, Princeton University.

If

$$\frac{F(x) - F(y)}{x - y} = F'\left(\frac{x+y}{2}\right)$$

for all x and y in a bounded **interval,** then $F(x) = ax^2 + bx + c$.

Solution by Norman *Padnos,* University *of* Rochester.

By differentiating

$$F(x) - F(y) = (x - y) F'\left(\frac{x+y}{2}\right)$$

with respect to x and then with respect to y, we obtain

$$0 = \frac{(x-y)}{4} F'''\left(\frac{x+y}{2}\right)$$

Whence, $F'''(x) = 0$, and $F(x) = ax^2 + bx + c$. Also solved by H. Kaye, Paul Myers, M. Wagner and the proposer.

104. Proposed by *D. J. Newman, Brown* University.

If $X_{n+1} = a_n X_n + b_n X_{n-1}$

where $a_n, b_n \geqslant 0,$

$a_n + b_n = 1,$

find a necessary and sufficient condition on the a_n, b_n such that $\{X_n\}$ converges for all initial conditions,

Solution by Paul Myers, New York, N.Y.

$$X_{n+1} - X_n = -b_n (X_n - X_{n-1}). \quad \text{Thus,}$$

$$X_{n+1} - X_n = (-1)^n (X_1 - X_0) \prod_{r=1}^{n} (1 - a_r).$$

In order for $X_{n+1} - X_n \to 0$, the infinite product must diverge to zero or equivalently that $\sum_{1}^{\infty} a_r$ diverge.

Also, solved by L. Shepp, J. Thomas, M. Wagner and the proposer.

105. Proposed *by C.* D. Olds, *San* Jose State *College.*

Show that

$$\int_1^{\sqrt{2}} \frac{(x^4 - 2x^2 + 2)^n dx}{x^{2n+1}} = \int_1^{\sqrt{2}} \frac{(x^2 - 2x + 2)^n dx.}{x^{n+1}}$$

Solution by Norman **Padnos,** University of Rochester.

By letting $z = x^2$,

$$\int_1^{\sqrt{2}} \frac{(x^4 - 2x^2 + 2)^n dx}{x^{2n+1}} = 1/2 \int_1^{2} \left(z - 2 + \frac{2}{z}\right)^n \frac{dz}{z}$$

Now we need only show that

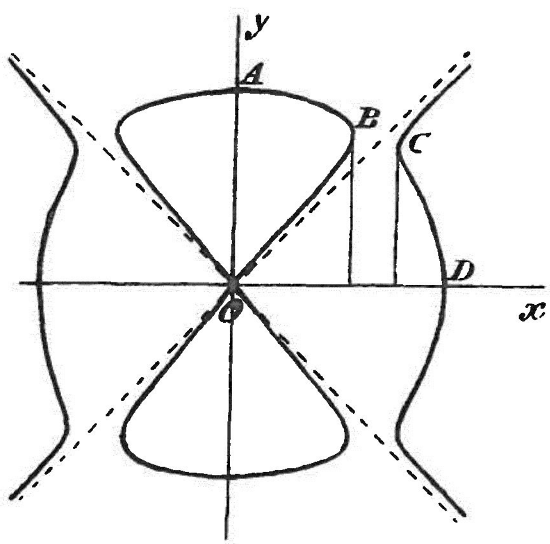
$$\int_1^{\sqrt{2}} \left(z - 2 + \frac{z}{2}\right)^n \frac{dz}{z} = \int_{\sqrt{2}}^{2} \left(z - 2 + \frac{z}{2}\right)^n \frac{dz}{z}$$

But this follows by letting $w = 2/z$.
Also solved by Paul Myers, D. J. Newman and the proposer.

Editorial note:

This problem is a special case of $\int_1^a F(x^2)\frac{dx}{x} = \int_1^a F(x)\frac{dx}{x}$,

provided that $F(a^2/x) = F(x)$.



$y^4 - 96a^2y^2 + 100a^2x^2 - x^4 = 0$

BOOK REVIEWS

Edited by
Franz E. Hohn, University of Illinois

Introduction to Statistical Reasoning. By Philip J. McCarthy. New York, McGraw-Hill, 1957. xiii t 402 pp., $5.75.

Intended for a "one-semester, nonmathematical course in statistics", this book is not strikingly different from several other such books on the market. The topics discussed are what one has come to expect in such a course, except that the χ^2 and Student's distribution are not covered Many examples are given from the social sciences. The problems seem adequate but answers for them are not given. There are quite a few errors such as in the table on page 18 where an interval must have been omitted. The author's idea of emphasis on "probability models" in the last half of the book is a good one. While the pace seems slow, it may well be the correct one for students approaching statistics for the first time.

Carleton College — Frank L. Wolf

Engineering Mathematics. By Kenneth S. Miller. New York, Rinehart, 1956. 417 pp., $6.50.

This book has been written to strengthen the mathematical training of the typical engineering student who has had only the calculus and some differential equations. The author, an Associate Professor of Mathematics at New York University, has selected six mathematical topics and devoted a chapter to each of them. The remaining chapter utilizes much of the mathematics developed and applies it to electrical network theory.

The subjects chosen for exposition are determinants and matrices, integrals, linear differential equations, Fourier series and integrals, the Laplace transform, and random functions. The book is well-printed and this reviewer found few errors.

Professor Miller's book is a book on mathematics for engineering students and is devoted primarily to presenting the mathematical development of his chosen subjects. Readers may have some difficulty with it in some places due to a lack of simple, concrete examples. It is a well written book aside from the extremely condensed three appendices. The exercises are not cases of numerical substitution but are genuinely mathematical problems whose solution adds to the theoretical developments and are really an integral part of the text.

The selection of subjects and application make this a fine mathematics text foı electrical engineering students. Other engineering students may find they will need to supplement it. It lacks such topics as vector analysis, numerical methods, and complex variable theory. However, it is the opinion of the reviewer that the reader will find that he is challenged, will learn much mathematics, and will come in contact with some recent developments in the field of communication engineering that are not usually included in an engineering mathematics book.

Monsanto Chemical Co. — Lawrence A. Weller

Engineering Mathematics. By Robert E. Gaskell. New York, **Holt-Dryden, 1958, xvi t 462** pp., **$7.25.**

In engineering curricula, the trend seems to be to follow the student's study of calculus with a course in mathematics as applied specifically to **engineering** rather than with a course, as has been the custom, in differential equations. The engineering mathematics course is intended, then, to border on two fields, being most certainly a course $mathematics but at the same time being intended for the express benefit of **thé engineering** student.

In this book, which is newer than most of the many books on the subject, the author realizes the difficulties of attempting to treat the subject of engineering mathematics adequately and to the complete satisfaction of both the engineer and the mathematician. The author chooses to make the subject as clear as possible to the student and as a result the mathematician "will find rigor **sacrificed** on many occasions. He may see repetition, tautology, triviality interspersed with strained interpretations, and questionable demonstrations based on plausibility."

The reviewer finds that the author treats the general subject of **engineer**ing mathematics quite thoroughly, being quite complete in both his presentation and range of topics covered.

The author has included in his book a section on dimensional analysis, indeed a welcome addition to the list of topics generally covered in a text of this nature. Admittedly, the discussion is not complete, being intended only to serve as a supplementary illustration of matrices. Nevertheless, teacher and student alike will find the treatment adequate for the purpose of the book.

The book includes an ample **number** of examples and exercises. The exercises for solution, particularly the "word problems", are relevant to modem engineering and should serve well to make the usefulness of mathematics apparent to the engineering student.

To further aid the student, a convenient list of references is included and the answers to exercises are given at the end of each chapter.

Robert L. Gallawa

An Introduction **to** the Foundations and Fundamental Concepts of Mathematics. By H. Eves and C. V. **Newsom.** New York, Rinehart, **1958.** xv **t 363** pp., **$6.75.**

The purpose **of** this book is "... to make available to advanced undergraduate students an introductory treatment of the foundations of mathematics and of concepts that are basic to mathematical knowledge." The authors have been highly successful in accomplishing their aims.

The excellence of the exposition, at the sophomore and **junior** level, makes this book particularly useful for prospective teachers of secondary school mathematics as well as for others **seeking** an early orientation in modem mathematics.

The treatment is strongly historical and the order of topics is "**...** in a rough way a chronological development of the basic concepts that have made mathematics what it is today."

Starting with an historical survey of ancient empirical mathematics, the authors then compare Euclid's "Elements" with Hilbert's "Grundlagen". The long search for a proof of Euclid's parallel postulate, which culminated in the non-Euclidean geometries of the nineteenth century, is shown to have motivated some of the early critical examination of the foundations of geometry.

The problem of how to base the irrational numbers on the rationals is sketched **from** Pythagoras to Dedekind and Cantor. The latter's set-theory and his transfinite numbers are introduced, and the **present** crisis in the foundations of set theory is touched upon.

Finally, symbolic logic as developed in the propositional calculus of the "Principia" is explained very clearly and simply.

Some might prefer that the first hundred pages, mostly historical, be considerably compressed so that the topics introduced in the last three chapters could be further developed. In this reviewer's opinion the book gets better with each passing chapter.

There are very many exercises at the end of each chapter, those in the later chapters contributing more to the stated purpose of the book than those in the earlier.

Altogether, this is an excellent book.

University of Arizona — Edwin J. Purcell

Mathematics of Physics and Modern Engineering. By J. S. **Sokolnikoff** and R. M. Redheffer. New York, **McGraw-Hill** Book Company, **1958. ix t 810 pp., $9.50.**

In effect this book offers in one volume nine brief texts on those branches of mathematics which, in the authors' judgment, give the minimum mathematics needed by the modem engineer or physicist. The areas covered are roughly indicated by the chapter headings: Ordinary Differential Equations; Infinite Series; Functions of Several Variables; Algebra and Geometry of Vectors; Matrices; Vector Field Theory; Partial Differential Equations; Complex Variable; Probability; Numerical Analysis. There are appendices on Determinants, the **LaPlace** Transform, Comparison of **Riemann** and **Lebesgue** Integrals. The book ends with a one page table of the probability integral, answers, and an index.

Each chapter is sectioned, with most of the sections ending with a set of exercises. These are usually formal applications, but in the more advanced topics lead to a deeper insight into the ideas involved. As typical of the scope of the text, consider the chapter on series. After treating the usual topics of a first calculus course, the authors discuss uniform convergence, series of complex terms, series solutions of differential equations, and in the last twenty-five pages of the hundred page chapter, present an introduction to Fourier series, integrals, and transforms, which includes a discussion of mean and pointwise convergence, **termwise** integration and differentiation. Some proofs are given.

The chapters are self contained, and **independent.** Thus several courses can be taught from this book, and it is also well adapted to self study. The exposition is in general clear, and the **format** attractive, Some users might wish additional references, and the purist may take exception at some places. But the reviewer feels the authors accomplish, in a thoroughly satisfactory manner, their objectives, and warmly recommends this book to the audience for whom it is written.

Saint Louis University — John D. Elder

Linear Programming and Economic Analysis. By Robert Dorfman, Paul **A.** Samuelson, and Robert M. **Solow.** New York, **McGraw-Hill, 1958. ix t 527** pp., **$10.00.**

This new book in the RAND series is a general exposition of the relationship of linear programming to standard economic analysis. The book is designed primarily for the economist who knows some mathematics but "does not pretend to be an accomplished mathematician". It should be of interest also to the mathematician who knows only a little economics and would like to see the **significance** of linear programming in economic theory. Of course some economists will find the mathematics too difficult, and mathematicians may find the economics obscure. But on the whole the authors seem to have succeeded fairly well in determining the level of presentation so as to reach those to whom the book may be most useful. Mathematicians may find it necessary to refer to books on intermediate economic theory or mathematical economics in order to appreciate the meaning of some of the discussion. Perhaps the best chapters are those on the algebra of linear programming, the linear programming analysis of the firm, elements of game theory, and interrelations between linear programming and game theory. The simplest ideas of matrix theory are given in an appendix.

The book is marred somewhat by an occupational disease of economists — the irresistible impulse to play the smart aleck. For example, in presenting the basic concepts of linear programming in Chapter II, the authors lead the reader through two pages of calculations and then remark "We have laboured hard to get the best solution. The only trouble with our solution is that it is wrong." Such manoeuvres are calculated to intimidate the reader and convince him that he is not as smart as the authors, but they are of doubtful expository value. There are several other places in the book where the authors seem to be playing a game with the reader in which their own superiority and the reader's supposed ignorance is the main source of amusement. This reviewer did not notice any mathematical errors more serious than an occasional misleading statement.

The basic difficulty in writing a book of this kind is the lack of common mathematical background among economists. Let us hope that the day will come when writers may assume that a well trained economist is familiar with the elements of analysis and linear algebra. Then books on economics could deal with their subject without having to instruct in mathematics at the same time.

Carleton College — Kenneth O. May

Introduction to Mathematical Analysis with Applications to Problems of Economics. By P. H. Daus and W. M. Whyburn. Reading, Mass., Addison-Wesley, 1958. viii + 244 pp., $6.50.

This book is designed as a text for a one-semester terminal course in mathematics for students of economics and business. It presumes one course in college mathematics as a prerequisite and would work best if used concurrently with or following a course in principles of economics.

The title contains the phrase "Mathematical Analysis," and after an introductory chapter on economic models, the book tums to a good introduction of the analysis of real variables. The pace is not maintained, however, and after Chapter Two no formal statement of theorems and proofs is given, and the discussion becomes largely one of hueristic explanations for the remainder of the book. This tends to make the level of the book somewhat uneven.

The content of the book breaks down into about 30 to 35 per cent mathematical economics, 55 to 60 per cent mathematics and 10 per cent descriptive statistics. The mathematical economics is a discussion emphasizing economic definitions and analyses which utilized topics from mathematics to achieve more power and rigor. The approximately 110 pages of mathematics cover a very abridged version of the usual topics in mathematics through the sophomore level plus two topics, Lagrange multipliers and least squares, which usually appear in advanced calculus. The emphasis is on curve tracing, conic sections, and differential calculus, including partial differentiation and maxima and minima problems, and there is a very brief treatment of integration.

On the whole the authors have succeeded admirably in their aim to write a text for a terminal course emphasizing mathematical topics which are extensively used in economic theory. The book is well written and it contains very few typographical errors. Considering the limitations outlined above, the text deserves serious consideration for a course which is in line with the book's objective. It does, however, pose problems of reentry into the usual mathematics sequence for students who change their minds after selecting a terminal course and then decide to go further in mathematics.

University of Illinois — T. A. Yancey

Ordinary Differential Equations, By Wilfred Kaplan. Addison-Wesley, Reading, Mass., 1958. xii + 529 pp., $8.50.

Books on differential equations vary in content from those which list methods to be used when encountering a specific differential equation to those which are concerned primarily with existence and uniqueness theorems. The present text lies between these extremes. While not eschewing formulas and methods, since these have their value, the author treats differential equations from the point of view of "functional analysis". That is, a differential equation is looked upon as specifying certain functions whose properties are sought from the differential equation itself. A means of acquiring a deeper insight into differential equations is achieved by considering the notions of input and output as well as stability which were suggested to the author by that branch of engineering known as systems analysis or instrumentation.

The less difficult theorems are proved in the main body of the text, whereas the more difficult ones are relegated to the last chapter. It is there that uniform convergence, the Weierstrass M-test, Lipschitz condition, Picard's method of successive approximations, complete solution, uniqueness theorems for systems of first order equations as well as order n, and dependence of solutions on initial conditions are considered.

The first two chapters deal with basic definitions, the isocline method, the step by step method of solving first order differential equations, level curves, systems of equations, separation of variables, homogeneous equations, exact equations, orthogonal trajectories, the first order linear equation and applications to physical problems. The notion of input and output is introduced in the third chapter for the first time and applied extensively to the first order linear differential equation. In the fourth chapter the author considers linear equations of arbitrary order with emphasis on those equations with constant coefficients. The notions of input, output, stability and transients are used to study the properties of solutions of linear differential equations in chapter five. Chapter six is devoted to the study of simultaneous linear equations. An appendix to chapter six applies the notion of matrices to simultaneous linear differential equations. Exact differential equations, special methods for linear equations together with applications are treated in chapter seven. Equations not of the first degree, envelopes and singular solutions are taken up in chapter eight. Chapter nine gives in some detail the method of solving differential equations in terms of power series. Numerical methods suitable to digital computers are considered in chapter ten. The analysis of non-linear equations by the phase-plane method is the subject matter of chapter eleven.

This excellent text of over five hundred pages covers a wide range of topics that will be useful to engineer and physicist alike. The format is pleasing and the drawings are extremely well done.

University of Illinois — E. J. Scott

Introduction to Difference Equations. By Samuel Goldberg. New York, 1958. xii + 260 pp., $6.75.

If one important stimulus to the currently reviving interest in difference equations is modem machine computation, surely another is the recent development of many discrete "models" in all branches of science, and particularly in the social sciences. Machine methods lead to vastly extended concepts of "solution" and thus bring back within the range of active investigation many hitherto abandoned problems. New social science models contribute to the reawakening interest differently, by posing new questions in difference equations and reinforcing our interest in other old questions.

Professor Goldberg's book is a sign of the generally reviving interest in difference equations which places special emphasis on social science applications. Fundamentally a very elementary book, it is nevertheless distinguished by several unusual features. One of these is the great care with which the author introduces each idea and explains even extraneous pieces of theory if he wants to use them. Within the main line of the book's development — difference calculus, general **properties** of difference equations, linear equations with constant coefficients, stability and equilibrium of solutions — he gives very full and clear treatment to the logical unfolding of the basic concepts. In constant interplay with the formal theory is a barrage of examples from economics, **psychology, sociology,** inventory analysis, communication theory, and even one from anthropology.

A remarkable last chapter offers fascinating glimpses of several deeper pieces of mathematics: boundary-value problems and eigenfunction of a second-order linear operator, generating functions and transform methods, matrix operators and their application to some simple problems in Markov chains. This discussion is necessarily restricted to some very special cases so that all difficulties but those essential to the underlying concepts can be stripped away; nevertheless, it should afford a fine appetizer for the more ambitious reader.

All of this has been prepared for students with no training beyond freshman mathematics. The book is said to be "primarily intended for social scientists who wish to understand the basic ideas and techniques involved in setting up difference equations". In this it should be a success. It should also make excellent supplementary reading for students just finishing calculus or beginning differential equations.

University of Virginia — Robert L. Davis

Introduction To Advanced Dynamics. By S. W. **McCuskey.** Reading, Mass., Addison-Wesley, 1959. **viii** + 263 pp., $8.50.

This book is designed for a one-semester course on the advanced undergraduate level. Its aim, according to the author, is to familiarize students of science and mathematics with some of the ideas of classical dynamics not ordinarily treated in courses in elementary mechanics, thus bridging the gap between the latter course and a **graduate-level** course in theoretical physics. Prerequisites are given as differential equations and advanced calculus including some vector analysis. A knowledge of matrices and tensors is not assumed or **used;** consequently the discussions of **rigid-**body motion and oscillatory systems are somewhat more cumbersome than necessary. The mathematical tools used have not been elaborated upon and, again according to the author, if the student is forced to seek some supplementary mathematics, so much the better. However, in such cases there are footnotes with the appropriate references.

The outline of the book is similar to that of Goldstein's *Classical Mechanics* but it is written at a lower level in keeping with its aim. In the reviewer's opinion, the author has achieved his aim in a well written text containing quite a few interesting topics **(i.e.,** motion of a spinning projectile, motion of a rocket, phase plane analysis, relativistic dynamics, the Wall continued fraction alternative to the **Routh-Hurwitz** stability criterion, etc.). However, there are some minor criticisms and these are as follows:

1. In the reviewer's opinion, more space should have been alloted to some of the mathematical preliminaries, especially since dynamics, no matter how physical one gets, is still a highly mathematical subject. In view of the importance of variational principles for the physicist, the two page (pp. 49-50) preliminary on variational techniques hardly seems sufficient. Furthermore, the author mistakenly treats the case of the fixed length hanging cable problem for the case of the free coiled hanging cable problem.

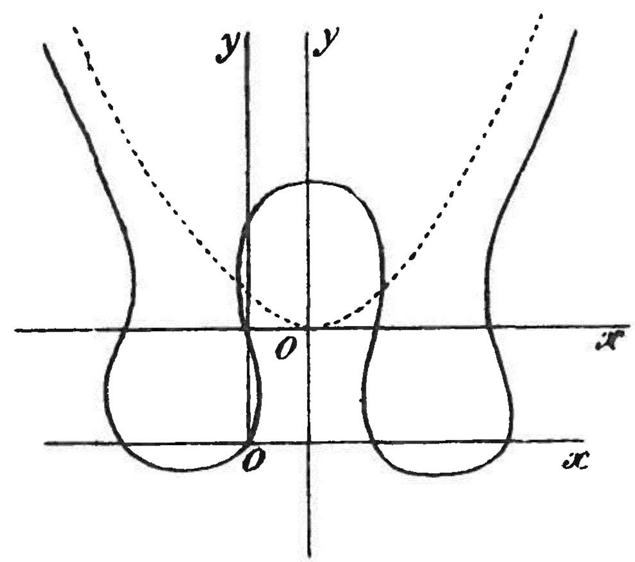
2. In the discussion of the trajectory of a particle being attracted by a central force (p. **81),** it is claimed that on physical grounds the motion is planar and is determined by the initial velocity and the initial force since "there is no force component, and hence no motion, perpendicular

to this plane". This is an argument used in many texts and is fallacious. Just consider $\ddot{z} = \sqrt{z}$, where $t = 0$, $z = \dot{z} = 0$. One solution is $z = 0$ but there is another one, $z = t^4/144$, and the motion is unstable (see note of O. D. Kellog, Amer. Math. Monthly, 37, p. 521). It should be noted, however, that previous to this argument the author did establish mathematically that the motion was planar from the equations of motion.

3. If the author feels that it is necessary to give a reference for the solution of the equation $\ddot{x} + a^2x = 0$ on **p. 93,** then he should have given it previously on p. 7.

4. The typography and many diagrams are excellent as is to be expected from Addison-Wesley, but the price of $8.50 for a 263 page undergraduate book on dynamics seems a little high.

Avco Research & Advanced Development Division — Murray S. Klamkin



$y(y - 1)(y - 2) = x(x^2 - 1)(x - 2)$

BOOKS RECEIVED FOR REVIEW

R. W. **Brink:** ***Plane Trigonometry*** (with tables), 3rd Edition. New York, Appleton-Century Crofts, **1959, $4.00.**
J. R. **Britton** and L. C. Snively: ***Intermediate College Algebra,*** Revised Edition, New York, Rinehart, **1959, $3.00.**
D. **K.** Cheng: ***Analysis of Linear Systems,*** Reading, Mass., **Addison-**Wesley, **1959, $8.50.**
H. Chernoff and L. E. Moses: ***Elementary Decision Theory.*** New York. Wiley, **1959, $7.50.**
*P. H. Daus and **W. H. Whyburn:** ***Introduction*** to ***Mathematical Analysis with Applications to Problems of Economics.*** Reading, Mass., **Addison-**Wesley, **1958, $6.50.**
C. Dennan and M **Klein:** ***Probability and Statistical Inference for Engineers.*** New York, Oxford University Press, **1959, $3.75.**
D. A. S. **Fraser:** ***Statistics, an Introduction.*** New York, Wiley, **1958, $6.75.**
G. Fuller: ***Plane Trigonometry,*** Second Edition, New York, **McGraw-Hill, 1959, $3.50.**
A. W. Goodman: ***Plane Trigonometry,*** New York, Wiley, **1959, $3.75.**
E. M. **Grabbe,** S. Ramo, and D. E. Wooldridge (Editors): ***Handbook of Automation, Computation, and Control.*** New York, Wiley, **1959, $17.00.**
S. **Kullback:** ***Information Theory and Statistics,*** New York, Wiley, **1959, $12.50.**
B. W. **Lindgren** and **G.** W. **McElrath:** ***Introduction to Probability and Statistics.*** New York, **Macmillan, 1959, $6.25.**
*S. W. **McCuskey:** ***Introduction*** to ***Advanced Dynamics,*** Reading, Mass., Addison-Wesley, **1959, $8.50.**
N. O. Niles: ***Plane Trigonometry.*** New York, Wiley, **1950, $3.95.**
W. **R.** Ransom: ***Calculus Quickly — A Part Term Text Book,*** **$1.** ***Rapid Analytics — A Part Term Textbook.*** **$1.** ***Algebra Can Be Fun.*** **$2.50.** J. **W. Walsh,** Portland, Maine, **1958.**
L. H. Rose: ***A Modern Introduction to College Mathematics.*** New York, Wiley, **1959, $6.50.**

*See review, this issue.

NOTE: ALL CORRESPONDENCE CONCERNING REVIEWS AND ALL BOOKS FOR REVIEW SHOULD BE SENT TO PROF. FRANZ E. HOHN. **374** ALTGELD HALL, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

OPERATIONS UNLIMITED

Science needs you

You need science

This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essential as today.

Your election as members of Pi Mu Epsilon Fraternity is an indication of scientific potential. Can you pursue advanced study in your field of specialization?

To point out the need of advanced study, the self-satisfaction of scientific achievement, the rewards for advanced preparation, the assistance available for qualified students, etc., it is planned to publish editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

Through these and future editorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

A CENTENNIAL SALUTE TO THE OIL INDUSTRY

One hundred years ago in August **1859** Edwin L. Drake succeeded in drilling America's first oil well at **Titusville,** Pennsylvania. This, however, was not the discovery of oil. As long ago as **3500** B.C. asphalt was used as an adhesive agent. This form of "rock oil" implemented the development of public building in the early empires. Oil drilling to depths of **3500** feet by the use of bamboo poles and crude brass bits had been achieved by the Chinese in **200** B.C. Natural gas too was used by the Chinese for illumination and heat in the pre-Christian era. It was over **2000** years later that the Drake well signaled the start of the oil industry in this country.

The next fifty years was the AGE OF KEROSENE. The expensive whale oil and the coal oil (an oil extracted from coal) that had been used for illumination was now replaced by a relatively cheap kerosene distilled from crude petroleum. Illumination was the main use of oil in this period. By-products, however, were rapidly developing.

With the automobile came the AGE OF GASOLINE, a forty-year struggle of production, marketing, distribution, and transportation. This period was spurred by technological developments. Today crude oil yields approximately 45% gasoline, 4% kerosene, 35% fuel oil, 3% jet oil, 2% **lubs,** 3% asphalt, and 8% other products including today's miracle makers, the **petrochemicals.** In the United States 318 refining plants have a capacity for processing 9,000,000 barrels of crude oil daily. The never ending flow of oil and gas provides two-thirds of the total power of the most highly industrialized nation of all times.

No review of these developments would be possible without reference to some of the men and companies who played important roles in this one-hundred-year story of oil. Here in this country the names John D. Rockefeller and Standard Oil were synonymous with "oil industry". Standard interests dominated the early development of the industry in the United States. Meanwhile on the other side of the world The Royal Dutch Company and the Shell Transportation & Trading Company were experiencing similar struggles. In 1902 they combined to form Royal Dutch-Shell. In 1912 the predecessors of Shell Oil Company began business on the Pacific Coast and in the Midwest as American Gasoline Company and **Roxana** Petroleum Company. Shell Oil Company and subsidiary companies are today among the leaders in the oil industry.

We are most pleased to publish in this issue an editorial from Shell Development Company, Emeryville, California, one of Shell's six research centers in the United States.

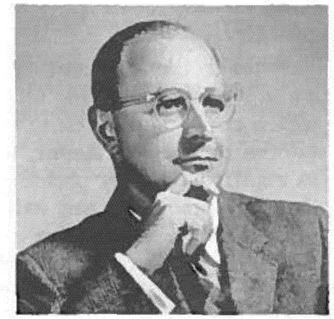
The following lists contributing corporations with the issue in which their editorials appeared.

Army Ballistic Missile Agency	Vol. 2, No. 10
AVCO, Research and Advanced Development	Vol. 2, No. 10
Bell Telephone Laboratories	Vol. 2, No. 10
Bendix Aviation Corporation	Vol. 2, No. 8
Emerson Electric Company	Vol. 2, No. 7
General American Life Insurance Company	Vol. 2, No. 9
Hughes Aircraft Corporation	Vol. 2, No. 9
International Business Machines Corporation	Vol. 2, No. 8
McDonnell Aircraft Corporation	Vol. 2, No. 7
Monsanto Chemical Company	Vol. 2, No. 7
North American Aviation, Inc.	Vol. 2, No. 9
Olin Mathieson Corporation	Vol. 2, No. 7
Shell Development Company	Vol. 3, No. 1

SWELL DEVELOPMENT COMPANY

RECENT DEVELOPMENTS IN APPLIED MATHEMATICS

By J. B. ROSEN
Shell Development Company
Emeryville, California



J. B. Rosen

The front page of your daily newspaper gives convincing evidence of the phenomenal advances in technology that have taken place in the last 15 years. This progress has been matched by, and to a considerable extent is the result of, equally great strides that have been made in the application of mathematical and statistical methods to the solution of applied problems. It is now not only possible but practical as well to develop accurate mathematical models of many technological problems and to obtain useful solutions by means of these models. When successful, such a mathematical approach can result in substantial savings in the time and money normally required for research and development of new and improved products and processes for industrial or military use. Such savings are possible because a valid mathematical model will permit a great reduction in the amount of time-consuming and expensive experimental work. The behavior of the actual system under many different conditions is studied by means of the model, with only a minimum amount of experimental data required for confirmation. On the other hand, for many important industrial situations the only possible experiment may be to actually carry out the operation itself (for example, the oil production problem described below). An incorrect decision in such a case can be very costly. A mathematical model, verified by past experience, is therefore extremely valuable, and permits the effect of alternative decisions to be investigated prior to carrying out the actual operation. The formulation and solution of a problem of this type often lies in the area of operations research, and uses techniques associated with such subjects as game theory, statistical decision theory, simulation and mathematical programming.

It seems likely that the single most important reason for this greatly increased usefulness of applied mathematics is the appearance on the scene of the high speed computer and the new mathematical techniques which have been developed specifically for its use. In this connection it is significant that the rapid development of high speed computers and related mathematical methods is due in large measure to the late John von Neumann, who is believed by many to have been the outstanding applied mathematician of this century.

Prior to these recent developments it was usually necessary to make many simplifying assumptions in order to obtain equations which could be solved. Two such assumptions or simplifications are: (1) the problem is linear, and (2) the number of variables or unknowns is small. For many problems in basic science, physics in particular, these assumptions are valid, and the solutions obtained do in fact accurately represent the physical system being investigated. These problems motivated much of the development of applied mathematics and the field of analysis as well as some other branches of pure mathematics. A considerable portion of the work in linear ordinary and partial differential equations was carried out in order to solve problems arising in mathematical physics and celestial mechanics. Series solutions of differential equations in terms of orthogonal functions, transform methods, and power series in terms of small parameters are among the tools developed in this connection. These analytic methods are most valuable when they show clearly the behavior of the solution for a range of equation parameters and a variety of initial and boundary conditions. Limiting cases and. asymptotic behavior can also be determined by these analytic methods in many cases. To be of use for comparison with experiment or for prediction of behavior, a solution must be in form suitable for the calculation of numerical results. An analytic solution in the form of a slowly convergent infinite series may be of no more than the original formulation as a differential equation for which an existence theorem is known. In either case, all that is known is the existence of a solution to the stated mathematical problem. The value of a solution to an applied problem is therefore largely determined by the ease with which numerical results can' be obtained.

The success ot these analytic methods in physics has unfortunately not been matched by their equal success in the mathematical solution of problems in many other fields of technology. Important problems arising in industry are frequently such that inherent nonlinearities cannot be neglected without destruction of an essential aspect of the problem. Furthermore, certain important problems in operations research and economics require formulation in terms of a large number of variables, several hundred in some cases. For such problems the essential nonlinearity or the large number of variables makes pre-computer methods of solution totally inadequate (these difficulties may even occur together in some particularly troublesome problems). More powerful methods are therefore required which utilize fully the capabilities of a modern high speed computer. The remainder of these remarks will be devoted to a discussion of such methods, the need for a rigorous analysis of them and to some typical industrial problems which are being solved by these modern techniques.

Most successful high speed computer methods consist essentially of the repetitive application of a basic computational procedure or algorithm. Such an iterative procedure is started with an initial set of numbers, for example, the given initial values for a differential equation. The computational algorithm is carried out with these numbers, the result being a new set of corresponding numbers. This procedure is repeated as many times as necessary to obtain answers with the desired accuracy. A large number of such iterations is often required.

The two main difficulties (the nonlinearity of the problem and the large number of variables) can often be handled by such methods, with the additional advantage that the desired numerical solutions are obtained directly. A good numerical process, developed for a particular type of problem, should not require a change in the basic procedure as the number of variables increases. The practical limitation on size is usually determined by the machine time (and cost) required to obtain a solution. Numerical methods capable of solving nonlinear problems are often extensions of those suitable for linear problems. One of the most effective such extensions is based on successive local linearizations of the nonlinear problem. A sequence of linear problems is solved and this sequence convergers to the solution of the original problem.

Important requirements for a satisfactory numerical method are suggested by the previous remarks. First, since a large number of iterations may be required, it is essential that any errors introduced are decreased in subsequent iterations. If tins is not the case small errors, due for example to truncation or round-off, may build up to the point where the computed results are meaningless. A method is called stable if errors do not increase as a result of many iterations. The second and closely related requirement is that the approximate numerical solution approaches the true solution as the number of iterations increases. Provided the approximation converges to the true solution in this way, any desired accuracy may be obtained. Furthermore, the accuracy of the approximation can usually be estimated and the iteration procedure continued only until the desired accuracy is achieved.

In order to apply a numerical method with confidence it is essential to know that it is a valid one for the type of problem to be solved. This has given impetus to important theoretical work in numerical analysis, where a particular numerical method is studied from the point of view of stability, convergence and certain other related questions. Once its validity has been established for a certain type of problem. A method can be used with confidence for any problem of that type. These investigations require the same kind of rigorous mathematical analysis typical of pure mathematics. In this sense then, the use of high speed computers has reemphasized the need for rigorous analysis in applied mathematics, in contrast to the heuristic approach of much of the earlier work in this field.

The preceding remarks will be illustrated by a brief statement of two important applied problems which arise in the petroleum industry, together with some discussion of the numerical methods by which they are being solved. The first problem, logically enough, arises in getting oil out of the ground. Oil is found in large underground reservoirs where the pressure may be as high as 10,000 psi. When one or more wells are drilled into a reservoir the gas pressure is usually sufficient to drive the oil through the porous sand and out of the wells. Important factors in determining the total amount of oil which will be obtained from a given reservoir are the number of wells drilled, their location, and the rate of production of oil at each well. After a number of years the reservoir pressure may decrease to the point where secondary recovery is advantageous. This can be carried out by pumping water into some of the wells in the reservoir, thereby forcing more of the remaining oil out of other wells. In addition to the basic decision to proceed with secondary recovery, the choice of wells and the rate of injection of water into the chosen wells are important factors in the success of the operation. It is clear that improved methods for predicting the effect of these and other factors on total recovery are of great value since incorrect decisions may either be impossible or very costly to remedy.

This general problem of two-phase flow through a porous medium has been formulated mathematically in terms of a system of two nonlinear partial differential equations subject to various initial and boundary conditions. The time-dependent solutions in one, two or three space dimensions are desired, depending on the geometry of the particular problem. Satisfactory numerical methods have been developed for solving these equations in one and two dimensions, although considerable amounts of machine time are still required for two-dimensional cases. The methods are based on a finite difference approximation to the original partial differential equations. Since the original equations are nonlinear the implicit method used to solve the finite difference equations requires the solution of a set of simultaneous nonlinear algebraic equations at each time step. This is done by the iterative solution of approximating linear equations. A much simpler explicit method of solving the finite difference equations is not practical because the method is stable only for very small time steps. The accuracy of the solution depends on thenumber of spatial grid points and time steps used. Greater accuracy can be achieved by using more grid points and smaller time steps, but only at the cost of a considerable increase in the computing time required. This illustrates clearly the need for a careful study of the accuracy required and the grid size needed to achieve this accuracy, so as to minimize the computer costs. Some of these problems may require ten or more hours of machine time at a cost of approximately $300 per hour. These methods have been examined for certain types of problems in one and two dimensions, and shown to be stable and convergent. Considerable work still remains to be done before satisfactory three-dimensional methods are available.

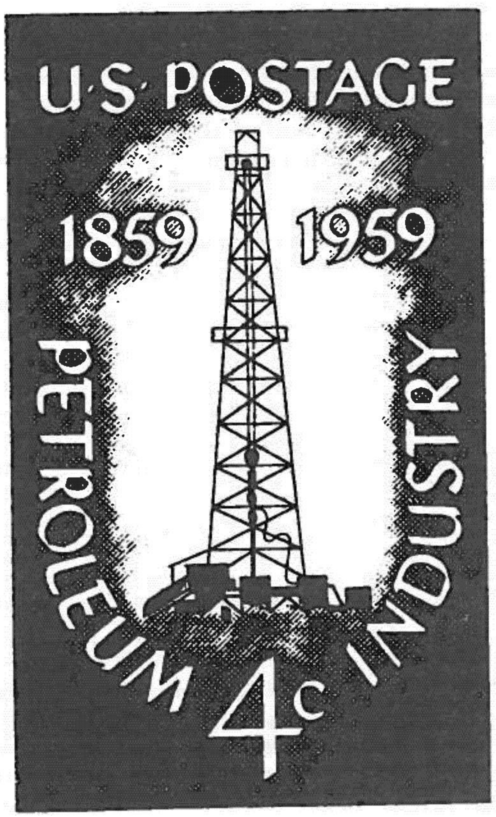
The second problem, arising farther along the road to the ultimate consumer of oil industry products, is concerned with the operation of an oil refinery. Specified amounts of many different products (premium and regular gasoline, aviation fuels, fuel oil) must be blended from various components. These blending components may either be purchased or come from a number of refinery processing units. Each product must meet a set of specifications (octane number, vapor pressure, viscosity) which can be written in terms of the blending component properties. The operating conditions of each refinery unit determine the amount and the properties of each component, as well as the cost of operation. It is desired to operate the units and blend the available or purchased components so as to make the required specification products at the lowest cost.

A typical mathematical model of such an operation may involve over a hundred unknown quantities. The quantity and quality specifications will be represented by equations or inequalities involving the unknown variables. Most of these constraints will be linear, but some nonlinear constraints may be required. The total cost of carrying out the operation depends on how the processing units are operated and how much material it is necessary to purchase. Thus, the cost is a known linear or nonlinear function of the variables. The mathematical problem therefore consists of determining the variables so as to minimize the cost, and still satisfy all of the constraints. This situation differs basically from the classical problem of minimizing a function subject to auxiliary equations. The difference is that some or all of the constraints are inequalities, so that for example, certain variables must be greater than or equal to zero.

This mathematical programming problem may be stated in geometric terms. Assume that the problem consists of N variables which form a Euclidean N-dimensional space. Those points satisfying the constraints form a convex region R in the space. It is desired to find a point in R at which the objective function attains its minimum value. If both the constraints and the objective function are linear in the variables the problem is one in linear programming, and the desired minimum point will be a vertex of R. Very efficient machine programs are now in use for the solution of linear programming problems. If either the objective function or some of the constraints are nonlinear, a more difficult nonlinear programming problem must be solved. A practical method has been developed for this situation based on following the steepest descent path subject to constraints. The path follows the gradient of the objective function, or its projection on a sequence of appropriately chosen intersections of constraint hyperplanes, until the minimum point is reached. A satisfactory computer program is being used for problems with a nonlinear objective function and linear constraints. Further work on this, as well as the nonlinear constraint problem is being carried out.

Mathematical programming techniques are now being used for many different kinds of problems. The refinery optimization discussed above is, however, one of the most important applications. This becomes clear when it is realized that the value of products from a large refinery may be as high as one million dollars per day. It is obviously worth a considerable amount of effort to achieve even a small percentage saving in such an operation.

These examples emphasize the need for new and improved methods of solution for such problems. A successful method in this important area of applied mathematics is usually developed by a combination of ingenuity and mathematical rigor. Ingenuity is required to think of a new computational procedure, and the application of rigorous methods from other branches of mathematics is required to establish the validity and limitations of the technique. Research in this field therefore offers a challenging opportunity to a mathematician with the necessary graduate training who is interested in applied problems.



NEWS AND NOTICES

Edited By
Mary L. Cummings, University of Missouri

Robert J. Myers, Chief Actuary of the Social Security Administration, Department of Health, Education, and Welfare, since 1947, has been chosen by the National Civil Service League as one of the ten top career men in the Federal Government for 1959. Mr. Myers, educated at Lehigh University and the State University of Iowa, is a member of Pi Mu Epsilon His address is 9610 Wire Avenue, Silver Spring, Maryland.

Dr. Ruth Stokes, former editor of the Journal, is retiring from the staff of Syracuse University. We, the present editors, wish her many pleasant years in retirement.

Ted J. Cullen, Illinois Gamma, '55, has recently accepted an appointment as Assistant Professor of Mathematics at Los Angeles State College.

From Missouri Gamma Chapter (St. Louis University):

Congratulations to Katherine Lipps, '57, who received the Garneau Award for being the top graduating senior for the year 1958-59. Miss Lipps also had an honorable mention from the National Science Foundation She received a Woodrow Wilson fellowship, and will study graduate mathematics at Tulane University.

Robert Rownd, '58, and Michael Sain, '57, have won National Science Foundation graduate fellowships, the former in medical sciences at Harvard University, and the latter in engineering at Stanford University.

Sister Gregory Meyer, '56, mathematics, Robert Rutledge, '56, mathematics, and J. Willard Hannon, '57, geophysics, received graduate NSF co-operative fellowships, and will study at St. Louis University.

Edwin Eigel, Jr., '56, received an NSF Summer Fellowship and will do research on his Ph.D. dissertation

Sam Lomonaco, '59, won the Senior Prize for problem solving, while John Martin for the second year won the Junior Award.

David Lee, Missouri Alpha, University of Missouri, won a Woodrow Wilson fellowship, and will study at Massachusetts Institute of Technology. Mr. Lee served as director of Alpha Chapter during the past year.

William Brinkman, Jr., newly elected director of Missouri Alpha Chapter, has a Gregory Scholarship in physics at the University of Missouri.

Winners of the Annual Prizes in Calculus at the University of Missouri are Vladi Malakhof, first, John Huber, second, and James C Dunn, third. All three winners were initiated into Pi Mu Epsilon in May.

Professor R V. Andree, secretary-treasurer general, is announcing a major meeting, with delegates and papers, to be held during the summer of 1960 at the East Lansing, Michigan, meetings. Will chapters take note of this, and be planning to send delegates?

The Journal is eager to print news from chapters and individuals. Please mail any news to Mary Cummings, Department of Mathematics, University of Missouri, Columbia, Missouri.

Edgar P. King, D.Sc., has been named head of the statistical research department at Eli Lilly and Company. A new department, it will provide statistical services to all components of the research function

Dr. King is a member of the American Statistical Association, Institute of Mathematical Statistics, Operations Research Society of America, Society for General Systems Research. American Association for the Advancement of Science, and Pi Mu Epsilon Fraternity.

INSTALLATIONS OF NEW CHAPTERS

The Montana Beta Chapter of Pi Mu Epsilon was installed at Montana State College, Bozeman on January 26, 1959, as the sixty-seventh chapter of the Fraternity. Secretary General Richard V. Andree conducted the initiation, and gave a talk on the history and meaning of the organization. Professor J. Eldon Whitesitt, Faculty Advisor of the Montana State Math Club, was responsible for the arrangements.

The Texas Alpha chapter of Pi Mu Epsilon was installed as the sixty-eighth chapter at Texas Christian University, Fort Worth on April 15, 1959. Director General J. S. Frame addressed the members at 5: 00 p.m. on the topic "Functions of a Matrix", presided at the installation of the chapter and the initiation of 14 charter members at 6:15. and made some remarks concerning the history of Pi Mu Epsilon after a 7:00 p.m. banquet. Elected as the first officers of the chapter were Fred Womack. Director; Jane Harlan, Vice Director; Joyce Hubenak, Secretary; and Professor Landon A. Colquitt, Corresponding Secretary.

The Georgia Beta chapter of Pi Mu Epsilon was installed as the 69th chapter at the Georgia Institute of Technology, Atlanta on April 16, 1959. Director General J. S. Frame addressed the members and other guests at 3:00 p.m. on the topic "Elementary Concepts in Relativity Theory", and presided at 5:00 p.m. at the installation of the chapter and the initiation of six charter members: R. M. Crownover, K. B. Dunham, D. G. Herr, C. M. Johnson Jr., C. L. McCarty, V. H. Smith. Following a 5:30 banquet, Professor Frame gave a 20 minute talk on the history and aims of Pi Mu Epsilon. Professor J. M. Osbom served as Faculty Adviser and was responsible for the arrangements.

ANNOUNCEMENT OF SCHOLARSHIPS
AWARDED PI MU EPSILON MEMBERS

Georgia Alpha: Clarence Wayne Patty - Alumnia Foundation Fellowship; Robert Everett - Woodrow Wilson Fellowship; Dr. Robert P. Hunter - Sarah Moss Fellowship.

Georgia Beta: Robert King - National Science Foundation fellowship; Robert Sacker - National Science Foundation Cooperative Graduate Fellowship.

Kansas Alpha: Jane Crow - Elizabeth M. Watkins Scholarship; Roger T. Douglas - Summerfield Scholarship; George Gastl - Summerfield Scholarship; Alfred Gray - U. G. Mitchell Honor Scholarship in Mathematics; Joanne L. Halderson - U. G. Mitchell Honor Scholarship in Mathematics; Richard Speer - U. G. Mitchell Honor Scholarship in Mathematics; Janice A. Wegner - U. G. Mitchell Honor Scholarship in Mathematics.

Kansas Gamma: Lynn Leslie Hershey - Pi Mu Epsilon Scholarship; Bana Kartasasmita - Foreign Student Scholarship.

Kentucky Alpha: Hugh Commes - National Science Foundation Cooperative Graduate Fellowship; Betty C. Detwiler - National Science Foundation Cooperative Graduate Fellowship; Jackson B. Lackey - National Science Foundation Cooperative Graduate Fellowship; Charles Sampson - National Science Foundation Cooperative Graduate Fellowship; Clay Ross - Woodrow Wilson Fellowship; Charles Sampson - Southern Fellowship Fund Award at Rice Institute.

Michigan Alpha: John Roderick Smart - National Science Foundation Cooperative Graduate Fellowship; Preston Bard Britner - National Defense Act Fellowship; Donald Leroy Fisk - National Defense Act Fellowship; Gretchen Louise Brown - L. C. Plant Scholarship Award; William Charles Cassen - L. C. Plant Scholarship Award; Philip Ralph Humbaugh - L. C. Plant Scholarship Award; Russell Frederick Peppet - L. C. Plant Scholarship Award; Andrew Peter Soms - L. C. Plant Scholarship Award.

Missouri Alpha: Edward Z. Andalafte - National Science Foundation Cooperative Graduate Fellowship; Raymond Freese - National Science Foundation Cooperative Graduate Fellowship.

New York Gamma: Alfred Brandstein - Brown University Fellowship; Harold S. Engelsohn - Woodrow Wilson Fellowship; Rochelle M. Friedlieb - Cornell University Fellowship; Robert Greenblatt - Woodrow Wilson Fellowship; Carole Hoohman - New York University Fellowship; Rosalie Steinroth - College Teaching Fellowship, New York Regents; Susan R. Balsam - Harvard University Fellowship.

Pennsylvania Epsilon: Arthur Evans - Socony Mobil Oil Fellowship; David G. Hill - National Science Foundation Cooperative Fellowship; Melvin Hinich - National Science Foundation Fellowship; John G. Moore - General Electric Fellowship; Nicholas J. Sopkovich - National Science Foundation Cooperative Fellowship; Larry Turner - National Science Foundation Fellowship.

Wisconsin Beta: Howard E. Bell - Wisconsin Alumni Research Foundation Fellowship; Lawrence O. Cannon - National Science Foundation Cooperative Graduate Fellowship; Douglas A. Clarke - University Fellowship; Simon J. Doorman - IBM Fellowship; Eugene F. Krause - National Science Foundation Cooperative Graduate Fellowship; D. Russell McMillan - National Science Foundation Fellowship; Richard Sinkhorn (Kansas Gamma) - National Science Foundation Fellowship; Maynard DeWayne Thompson - National Science Foundation Summer Fellowship; National Science Foundation Cooperative Graduate Fellowship; Beverly R. Femer - National Science Foundation Summer Fellowship; Roy H. Goetschel (Illinois Beta) - National Science Foundation Summer Fellowship; Joan H. Rohrer - National Science Foundation Summer Fellowship; Charles P. Seguin - National Science Foundation Summer Fellowship.

NOTICE TO INITIATES

On initiation into Pi Mu Epsilon Fraternity, you are entitled to two copies of the Journal. It is your responsibility to keep the business office informed of your correct address, at which delivery will be assured. When you change address, please advise the business office of the Journal.

DEPARTMENT DEVOTED TO CHAPTER ACTIVITIES

Edited by

Houston T. Karnes, Louisiana State University

EDITOR'S NOTE: According to Article VI, Section 3 of the Constitution: 'The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an annual report of the chapter activities including programs, results of elections, etc." The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the Pi Mu Epsilon Journal. Besides the information listed above, we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Chapter Activities Editor Houston T. Karnes, Department of Mathematics, Louisiana State University, Baton Rouge 3, Louisiana. These reports will be published in the chronological order in which they are received.

REPORTS OF THE CHAPTERS

ALPHA OF NEBRASKA, University of Nebraska.

The Nebraska Alpha Chapter held seven meetings during the 1957-58 year. The following papers were presented:

"Algebra Courses at the University of Nebraska", by Dr. D. W. Miller
"Pascal's Work In Mathematics", by Miss Sharon Hecker
"A Knotty Problem", by Dr. Walter Mientka
"Two Recipes for π", by Mrs. Mildred Gross
"Steiner's Network Problem", by Mr. Ervin Hietbrink
"Seven Sevens", by Miss Margaret Tevis
"Some of the Works of Nicholas Bourbaki" by Dr. Hubert Schneider

At the January 14, 1958 meeting 29 new students were initiated and at the Initiation Tea on April 20, 1958, 13 new members were initiated.

William Thomas White was awarded the Freshman Algebra Award while the award for Prize Examinations went to Jack Kent Nyquist and James Otis Jirsa in the Senior Division, and Richard Ronald Berns and John Patrick Anderson in the Junior Division.

Officers for 1958-1959 are: Director, Vernon Schoep; Vice-Director, Jerrold Bebernes; Treasurer, John Herzog; Secretary, Bill Gingles; and Faculty Advisor, Dr. Donald W. Miller.

BETA OF KANSAS, Kansas State College

The Kansas Beta Chapter held five meetings during the 1958-59 year. The following papers were presented:

"The Use of Rank Order Statistics in a Genetic Experiment", by Dr. Stanley Wearden
"Matrices Over the Ring of Integers Modulo a Power of a Prime", by Dr. Leonard E. Fuller
"Initial and Boundary Value Problems For a Partial Differential Equation of Higher Order," by Dr. Phil G. Kirmser
"Approximation and Improvement of the Time Delay Operator", by Dr. C. A. Halijak
"Mathematics Curriculum Revision in the High School", by Mrs. Marjorie French

At the annual banquet on May 4, 1959, 30 new members were intiated.

Officers for 1959-60 are: Director, Stanley Wearden; Vice-Director, William Kimel; Secretary. Helen Moore; Treasurer, S. T. Parker.

ALPHA OF CALIFORNIA, University of California at Los Angeles

The California Alpha Chapter held eleven meetings during the 1958-59 year. The following papers were presented:

"One Hundred Oxen", by Professor Redheffer
"How to Crate A Rock", by Professor Straus
'What Every Young Girl Should Know", by Dr. Thorp
"Voyage to the Center of the Earth", by Professor Green
"Remembrance of Things Past", by Professor Taylor
"Some Problems in Harem Staffing", by Professor Ferguson
"You Don't Have to be Looped to be Tangled", by Professor Tompkins
"Solomon and the Triplets", by Professor Arens
"The Four Color Problem", by Mr. Gilbreath
"Discussion of bell ringing in England and recordings of these permutation "Campanological Groups", by Mr. Mercer
"Problems Computable on the "Turing Machines' but not Computable on the Ordinary Digital Computer", by Mr. Ralston

An experimental semi-monthly Newsletter, π ME *News Notes of UCLA*, was sponsored by the California Alpha Chapter in order to keep mathematics students informed of the chapter programs and other news of interest. The experiment was considered successful and a News Note Editor was included among the new appointive officers.

In addition to the program meetings, a Fall Initiation, a Spring Initiation, and the Annual Picnic were also held. Thirty-nine new members were added to the chapter.

Officers for 1958-59 were: Director, Andrew Bruckner; Vice-Director, Herbert Gindler; Secretary, Raymond Killgrove; Treasurer, Professor T. Ferguson; Faculty Advisor, Professor R. Steinberg.

Officers for 1959-60 are: Director, John Lindsay; Vice-Director, Joseph Mount; Secretary, Edward Sallin; Treasurer, Professor R. Blattner; Faculty Advisor, Professor R. Redheffer.

ALPHA OF GEORGIA, University of Georgia

The Georgia Alpha Chapter held seven program meetings, which included business sessions and social periods, and four business meetings during 1958-59. At three of these meeting initiations were held in which a total of 18 new members were inducted.

At the program meetings the following papers were presented:

"On Semigroups", by Dr. R. P. Hunter
"The Idea Of a Group", by Dr. G. B. Huff
"Cosmologies", by Dr. M. L. Curtis
"Some New Rational Distance Sets", by Dr. G. B. Huff
"The Isoperimetric Problem", by Dr. A. W. Goodman
"Simplexes on the Twisted Cubic", by Mr. R. P. Everett
"Applications of the Intermediate Value Theorem," by Dr. M. K. Fort, Jr.

In addition to the regular business meetings, the chapter held one party during the Fall quarter, the Annual Banquet, and the Annual Spring Picnic.

Officers for 1959-60 are: Director, Curtis Bell; Vice-Director, Britain Williams; Secretary, Marvin Atha; Treasurer, Nancy Herner; Faculty Advisor, Dr. T. R. Brahana.

DELTA OF NEW YORK, New York University

The New York Delta Chapter held two program meetings during the 1957-58 year. The following papers were presented:

"The Graph of a Group", by Professor Wilhelm Magnus of the Institute of Mathematical Sciences
"Bertrand Russell and the Number Two", by Professor John Van Heijenoort

The last meeting was a joint meeting with the Philosophy Club of Washington Square College.

Thirteen new members were added to the Chapter during the year.

GAMMA OF MISSOURI, St. Louis University

The Missouri Gamma Chapter held four meetings during the 1958-59 year. The following papers were presented:

"Hyperbolic Geometry", by Mr. Joseph Moser

"Hyperbolic Trigonometry", by Mr. Edwin G Eigel

'Hyperbolic Geometry", by Mr. Jose Padro, University of Puerto Rico

"Computers and Today's Mathematics", by Professor R V. Andree, Secretary-Treasurer General of Pi Mu Epsilon

After the final meeting a reception was held in honor of professor Andree and the 57 new members who were initiated into the chapter.

At the Chapter's 22nd annual banquet Dr. Waldo Vezeau presented awards to Mr. John Martin, winner of the Pi Mu Epsilon Junior Problem Contest, and Mr. Sam Lomonaco, winner of the Senior Contest. Mr. Tom Jerrick received the Mathematical Award of the Chemical Rubber Company. Miss Katherine Lipps received the annual James W. Garneau award of $25.00 for being the highest ranking senior in mathematics.

Officers for 1959-60 are: Director, James Thomas; Vice-Director, Grattan P. Murphy; Secretary, Sister Mary Paul Buser, C S. J.; Faculty Advisor and Corresponding Secretary, Dr. Francis Regan.

GAMMA OF PENNSYLVANIA, Lehigh University

The Pennsylvania Gamma Chapter held five program meetings during the 1958-59 year, which included the annual banquet. The following papers were presented*.

"Odd Numbers and Odd Mathematicians", by Dr. Albert Wilansky

"Statistics and Numerical Methods", by Professor Latshaw

"Space Mechanics", by Professor Beer, Head of the Department of Mechanics

"Advanced Coupon Collecting", by Dr. Milton Sobel, Bell Telephone Laboratories

The March 19, 1959, meeting was held jointly with the American Institute of Physics. During the year the Chapter also witnessed a demonstration of Lehigh's new LGP 30 computer as presented by Mr. Smith of the Department of Industrial Engineering.

Officers for 1959-60 are: Director, William F. Parks; Secretary, Ralph H. Weyer; and Treasurer, Peter S. Shoenfeld.

ALPHA OF NEW HAMPSHIRE, University of New Hampshire.

The New Hampshire Alpha Chapter introduced a weekly student seminar during the 1958-59 year at which talks were given on a student level by students and faculty members. The primary object of the seminar was to stimulate activity and interest in mathematics. Some of the topics discussed were:

"The Fundamental Theorem of Algebra"

"Seven Bridges of Konigsberg"

"Transfinite Numbers"

"Solutions of the Cubic Equation"

The annual banquet was held December 11, 1958. Dr. Donald Thornsen, head of the Data Processing Division of Watson Laboratories of IBM Corporation presented a very interesting and informative talk on "Opportunities Available for Students Majoring in Mathematics". Fourteen new members were initiated on this date.

An award was presented to the student in the class of 1961 who had the highest scholastic average in mathematics during his freshman year. Galen R. Courtney was recipient of this award

Throughout the year Pi Mu Epsilon members conducted weekly classes for students in freshman and sophomore mathematics courses who wished to receive assistance in their studies.

Officers for 1958-59 were: Director, Earl Legacy; Vice-Director, George Enos; Secretary, Nancy Porter; Treasurer, Jean Macomber; Faculty Advisor, Dr. Robert H. Owens.

ALPHA OF LOUISIANA, Louisiana State University

Louisiana Alpha chapter held five meetings during the 1958-59 year. At the annual Spring initiation twenty-five new members were inducted The following papers were presented at program meetings:

"Wrong Method - Right Answer", by Dr. Frank A Rickey

"Sputniks - Their Motion in Space", by Dr. Dan R Scholz

"Geometrical Solutions and Proofs to Problems of Maximum and Minimum", by Dr. Henry G. Jacob, Jr.

Louisiana Alpha gives two annual awards which are based on honors examinations. The Senior Award was won by Paul M. Brown, and the Freshman Award was won by Charles Sparks Rees.

Officers for 1958-59 were: Director. Paul M. Brown; Vice-Director, Ronald C Folse; Secretary, Sandra Passantino; Treasurer, Patrick J. Haddican; Faculty Advisor, Dr. Haskell Cohen; Corresponding Secretary, Dr. Houston T. Karnes.

Officers for 1959-60 are: Director, James H. Carruth; Vice-Director, Richard P. Lowry; Secretary, Sandy Ann Hundley; Treasurer, John C Wiese; Faculty Advisor, Dr. Haskell Cohen; Corresponding Secretary, Dr. Houston T. Karnes.

ETA OF NEW YORK, University of Buffalo

The New York Eta Chapter held five combination business-program meetings and one business meeting during the 1958-59 year. The following papers were presented:

"Theories and Implications of Milltivalued Logic", by Professor William T. Parry, Department of Philosophy

"Mirrors and Honeycombs", by Dr. Bruce Chilton

"The Mathematics of Redundancy", by Dr. R L. San Soucie, Sylvania Laboratory, Center for Communication Research and Development

"Potpourri of Mathematical Trivia", by Dr. Paul Schillo

'Some Comments on Peano's Postulates", by Dr. Samuel Stearn

"Linear Programming", by Fred Miller, Applied Science Representative of I. B. M.

Eleven new members were initiated during the year. Bruce Chilton was presented with the Chapter's award for the senior earning the highest grade on the mathematics comprehensive examinations.

Officers for 1958-59 were: Director, Alexander Bednarek; Vice-Director, Sam Stem; Secretary-Treasurer, Dolores M. Crapsi; Faculty Advisor, Professor Paul Schillo.

Officers for 1959-60 are: Director, Sam Stern; Vice-Director, Bruce Chilton; and Secretary-Treasurer, Virginia Snow; Faculty Advisor, Professor Paul Schillo.

DELTA OF PENNSYLVANIA, Pennsylvania State University

The Pennsylvania Delta Chapter held seven meetings during the 1958-59 year which included the annual banquet. The following papers were presented:

'Turing Machines and Unsolvability", by Bruce Lercher

'A Paradox of Topology", by Walter Sillars

"Squares Within Rectangles", by William Beyer

"Mathematics in Music", by Professor E H. Umberger

"Parity in Non-Relativistic Quantum Mechanics" by Professor Blanchard

"Transcendence of Pi", by Professor Raymond Ayoub

The guest speaker at the banquet was Professor Vladimir Vand.

Officers for 1959-60 are: Director, Kenneth Magill; Vice-Director, Michael Dutko; Secretary, James Sieber.

EPSILON OF OHIO, Kent State University

The Ohio Epsilon Chapter held nine meetings during the 1958-59 year. The following papers were presented:

"The Golden Section", by Dr. Kenneth Cummins
"Groups", by Mr. Russell Line
"Leonhard Euler", by Mrs. Carole Kyser
"Opportunities in Mathematics", a panel discussion by Don Dimitry, William Kintz, Russell Line, Maureen Weber and Joann Wirbel
"Mathematics, Logic, and Digital Computers - Today and Tomorrow", by Dr. John Lawrence, International Business Machines
"Beta and Gamma Functions", by Mr. Russell Line
"Field Theory", by Mr. William Etling
"Introduction to Topology", by Miss Jacqueline Chabot
"Gems From the Mathematics Classroom", by Professor John Kaiser

Eighteen new members were inducted at the Initiation Banquet in February. At the annual University Honors Day Assembly, Mr. Dennis Gilliland was presented with a plaque and $25.00 as winner of the Pi Mu Epsilon Award in mathematics.

ALPHA OF MISSOURI, University of Missouri

The Missouri Alpha Chapter held three meetings during the 1958-59 year, which included the annual banquet. The following papers were presented:

"The Number Pi - Some Historical Comments", by Professor James Younglove
"A Film on the Fourth Dimension with Comments", by Professor Joseph Zemmer
"The Importance of Scholarship", by Dean Thomas Brady

At the annual banquet a citation for long and faithful service was presented to Professor Herman Betz.

Fifty-four new members were initiated during the year.

Winners of the annual Pi Mu Epsilon prizes in calculus were Vladi Mlalakhof, first place; John Huber, second place; and James C. Dunn, third place.

Officers for 1959-60 are: Director. William Brinkman; Vice-Director, Gerald MaGee; Secretary, Glen Edwards; and Treasurer, John MaGee.

BETA OF PENNSYLVANIA, Bucknell University

The Pennsylvania Beta Chapter held its meetings during the year jointly with the Bucknell Mathematics Club. The following papers were presented:

"Summability Methods", by Dr. Stanley Dice
"Continuity, Differentiability and Intuition", by Dr. William K. Smith
"Mathematical Problems", by Dr. Richard Johnson, Mathematical Association of America Lecturer

The first initiation meeting at which 19 new members were initiated was followed by a dinner. Dr. William L Smith of the Department of Mathematics was the speaker.

A special initiation was held on March 8, 1959, for one student who was absent from the first initiation meeting.

Officers for 1959-60 are: Director, Professor Donald Ohl; Vice-Director, Norman Edgett; Secretary, Joan Piersol; Treasurer, Sherry Rhone.

ALPHA OF ARIZONA, University of Arizona

Arizona Alpha Chapter held two student seminars during the 1958-59 year. These seminars were conducted on the level of sophomore calculus students. The following papers were presented:

"Conic Sections of Conic Sections", by Richard Sommerfield
"Elementary Set Theory", by Dr. Robert Williamson

Officers for 1959-60 are: Director, Martin Halpern; Secretary. Stanley Dea; Treasurer, David McArthur.

ALPHA OF OKLAHOMA, University of Oklahoma

The Oklahoma Alpha Chapter held eight business and program meetings during the year. The following papers were presented:

"Additive Functions", by Mr. Robert Strong
"Calculus of Variations", by Mr. Carter
"Topology", by Mr. Jack Porter
"LaPlace Transforms", by Mr. Jerry Evans

Officers for 1959-60 are: Director, Jack Porter; Vice-Director, Harry Sims; Secretary-Treasurer, Joni Sue Williams. Corresponding Secretary, Dr. Dora McFarland; Faculty Advisor, Professor Earl La Fon

ALPHA OF MICHIGAN, Michigan State University

The Michigan Alpha chapter held meetings twice monthly during the 1958-59 year with speakers from the department of mathematics and other related fields, and student speakers.

Special events for the year included the annual picnic, two initiation meetings, and the annual Winter banquet with Mr. Wallace Givens from Wayne State University as speaker. On this occasion the L. C Plant award was presented to five outstanding mathematics students.

Officers for 1958-59 were: Director, Dale Lick; Vice-Director, Dick Klinkner; Secretary, Gretchen Brown; Treasurer, Ben Smith; and Faculty Advisor, Dr. Campbells.

ALPHA OF NEVADA, University of Nevada

The Nevada Alpha Chapter held five business meetings during the 1958-59 year.

Mr. LeRoy Wentz spoke at one meeting and showed slides of a trip made to the San Francisco Bay area where he visited the many reactors in the area.

At the meeting open to the high school students in the area the following paper was presented:

"Cones and Orbits," by Dr. M. R Demers

This chapter again co-sponsored the statewide Nevada High School Mathematics Prize Examination. Approximately 600 students took the examination

Initiation ceremonies were conducted at the annual Spring banquet for 11 new members.

Officers for 1959-60 are: Director, Hans Lindblom; Vice-Director, Ed Wagner; Secretary-Treasurer, Jean Best.

ALPHA OF OHIO, Ohio State University

The Ohio Alpha Chapter held five meetings during the 1958-59 year. The following papers were presented:

"The Gambler's Ruin", by Dr. D. Ransom Whitney

"The Inequality $\sum_{r=1}^{n} \frac{x_r}{x_{r+1} + x_{r+2}} \geq \frac{n}{2}$ (Cyclic)
by Dr. L. J. Mordell, Sadlerian Professor Emeritus of Pure Mathematics, St John's College, Cambridge

At the Initiation Banquet 36 new members were inducted into the chapter. The guest speaker for the occasion was Dr. Marshall Hall, Professor in the Ohio State Mathematics Department, who spoke on "Codes and Ciphers".

Officers for 1959-60 are: Director, Walter B. Laffer, II; Secretary-Treasurer, Glenna G. Williamson.

ALPHA OF WISCONSIN, Marquette University

The Wisconsin Alpha Chapter held eight meetings during the 1958-59 year.

The following papers were presented:

"Institute for Advanced Study", by Fr. Leser Heider
"Opportunities for Graduate Study at Home and Abroad", by Dr. John Reidl
"Relation of Science and Philosophy", by Fr. Gerard Smith

'The Evolution of Naturalistic, Impressionalistic, and of Purely Abstract Art", by Dr. Rudolph Morris, Department of Political Science

The annual banquet was held on May 10, 1959, with Dr. Arnold E. Ross, Department of Mathematics, University of Notre Dame, as the speaker. His subject was "A Metric, Defined in Terms of Primes, in Which the Triangular Inequality is Expressed in Terms of the Absolute Values of Number Pairs".

Thirty-four new members were initiated during the year.

The annual Pi Mu Epsilon Frumveller Examination in Mathematics was held May 2, 1959. The contest is open to all high school seniors of Milwaukee County who have had at least six semesters of mathematics. Award for first place is a $300.00 scholarship for the following academic year at Marquette University or, if this is declined, a token prize not to exceed $25.00. The top three winners were as follows: Sue Spoden, first place; James Tylicki, second place; and Dennis Pipkorn, third place.

Officers for 1959-60 are: Director, Ken Batinovich; Vice-Director, Leo Heiting; Recording Secretary, Joanne Kolasinski; Corresponding Secretary, Sue Leslie; Treasurer, Jerry Jacobsen; and Librarian, Phil Tonne.

ALPHA OF NORTH CAROLINA, Duke University

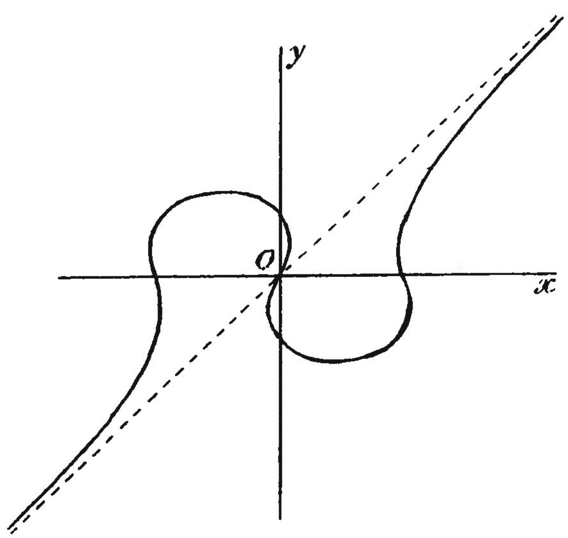
The North Carolina Alpha chapter held 3 meetings during the 1958-59 year. The following papers were presented:

"The Laplace Transform", by Mr. Charles B. Duke.

"Introduction to Topological Ideas", by Miss Priscilla Irene Edson

Forty-one new students were initiated during the year.

Officers for 1959-60 are: Director, Edward Dennis Theriot, Jr.; Vice-Director, Terry Scott Carlton; Secretary, Claudine Evelyn Fields; Treasurer, Janice Elaine Turner.



$(y^2 - 1)\, y = (x^2 - 4)\, x$

INITIATES

ALABAMA ALPHA, University of Alabama (May 5, 1959)

James E. Asquith
Robert N. Braswell
Vito An. Destito
James Howard Darden
Lyle R. Dickey
Bernardo N. Dorta
Dwight L. Eddins
William B. Gandrud
William R. Garrett
Charles E. Hall
Jack W Lehner
Arthur B. Lewis
William L. Mason
Ursula Mrazek
Peggy J. Mullins
Carl P. Rautenstrauch
John M. Reeder, Jr.
Betty Sheffield
Exton L. Spencer
Thomas C. White, Jr.
Karen H. York

ALABAMA BETA, Alabama Polytechnic Institute (May 6, 1959)

Roy B. Bogue
Sylvia G. Bowers
Joseph M. Campbell
Edmond D. Dixon
Howard S. Fogelson
Manuel A. Gonzalez
Connie J. Green
Hammond Green, Jr.
Roger L. Hamner
George M. Jones
Emil F. Kluever
Pansy Light
Nicolaia C Mitsiani
Winston P. Newton
Russell S. Pimm
Michael N. Riddle
Carolyn R. Schaefer
Donald P. Schambeau
Ronald R. Schambeau
Joe F. Sharp
Rufus W Shepherd, Jr.
Donald R. Smith
Joe B. Smith
Mary H. Smith
Margaret A. Spicer
Sally Turner
Larry Thompson
Charles T. Walters
John A. Woller
Wilfred W. Yeargen

ARIZONA ALPHA, University of Arizona (April 17, 1959)

Francis R. Ashley
Lamar Bentley
Lawrence F. Bens
John N. Conoualoff
Stanley J. Dea
Dennis A. DePasse
Wally Geniec
Thomas E. Grassman
Michael A. Gusinow
Martin B. Halpern
Peter Kertesz
Melton L. Laflen
Don E. Lovell
David A. McArthur
George C. Onodera
Robert W Rader
Robert B. Scott
Howard Sherwood
Lee J. Supowit

CALIFORNIA ALPHA, U. C. L. A. (January 15, 1959)

Monty Adler
James R. Baugh
Stanley E. Dunin
James Dyer
David E. Ferguson
Aviezri S. Fraenkel
Myron Goldstein
Daniel L. Halliday
Alexander Hurwitz
Arnold Levine
Thomas G. McLaughlin
Kathy K. Puckett
Arthur L. Satin
George E. Smith
Frederick B. Strauss
Sandra Zeitlin

(May 9, 1959)

Takeo Akasaki
Gabriele Asher
James A. Ayers
Charles B. Barker
Lawrence P. Belluce
Ernest Bramblett
Jack Brass
Kenneth M. Brown
Mohindar S. Cheema
Samuel G. Councilman
Harold Einstein
Norman L. Gilbreath
Harold Gumbel
Alice James
Keith M. Kendig
Gabriel E. Lowitz
Leon Nower
John Pendleton
Milton Rosenthal
James F. Russell
Michael Scully
Ben Wada
Robert J. Weiss

CALIFORNIA GAMMA, Sacramento State College (January 10, 1959)

August J. Bodhaine
Lee W Carrier
Yu Chang
Roy D. Draper
Virgil T. Greenfield
Harry Harold Jonas
Robert E. Marshall
Marjorie R. Martin
Nancy K. Squires
Richard M. Thomas
Robert O. Watkins
Charles H. Wiggins, Jr.
Richard E. Winkler
Fan Yee

(June 17, 1959)

Lynn J. Champomier
Diana E. Chatham
Robert L. Livermore
Robert H. Prescott
Robert A. Smith
Vivian S. Ichimura
Asa Whetstone

DISTRICT OF COLUMBIA ALPHA, Howard University (May 23, 1959)

James H. Blow
Noel Bryan
Bertha M Butler
Caroline C. Calloway
Joan A Davis
Matthew Douglas
Vernon Drew
Johnny Dunn
Gail T. Finley
Guenter Hagadarn
Carroll Harvey
Charles W Johnson
Nancy Logan
Clarence London
Kenneth Marius
Patrick H McClain
Daisy L. McKelly
Norman D Mills
Alvin Robinson
Elroy Smith
Jamie R Young
Theodore J. Wang

EVENDALE AFFILIATE, Ohio (June 1, 1959)

Marvin R. Broz
Robert G. Frank
Samoil Moise
Wesley S. Shaw

FLORIDA ALPHA, University of Miami (August 2, 1959)

Sylvan C. Bloch
Paul E. McDougle

(March 2, 1959)

Wanda S. Abel
Harold L. Beck
Daniel S. Kamis
Elliot L. Kramer
Daniel S. Levine
Michael Mahoney
Ronald D Nelson
Ellard V. Nunnally
Allen R Roth
Carl D. Sikkema
Philip Spim
David Statlander
Jack L. Tunstall

FLORIDA BETA, Florida State University (May 4, 1959)

Ann hi. Clemente
Robert B. DesJardins
Gerald W. Findley
Virginia A Garner
Lonnie L. Lasman
Francis D. Lonergan
Mary J. Mader
Ed Takken
Joseph S. Toth
Clarence E. Vicroy, Jr.
Fredric J. Zerla

GEORGIA ALPHA, University of Georgia (January 30, 1959)

Marvin E. Atha
Ronald C. Bond
Robert O. Burdick
Tom W. Daniel
Thomas B. Dillard
George A Watson, Jr.

(May 13, 1959)

Edward T. Garner
Nancy L. Herner
Sylvia Randall
Emma H Thackston
Edgar R Yarn, Jr.
Britain J. Williams, III
Barry E. Wenzel

GEORGIA BETA, Georgia Institute of Technology (April 16, 1959)

Richard M. Crownover
Kenneth B. Dunham
David G. Herr
Charles M. Johnson, Jr.
Cuthbert L. McCarty
James M. Osborn
Vedene H Smith

(May 31, 1959)

Stanley S. Goldberg
Oscar V. Hefner
William M. Hubbard
Robert D. King
Michael C Mooney
Joseph L. Quarterman
Charles D. Roberts
Robert J. Sacker
Marvin B. Sledd
Howell K. Wilson

ILLINOIS ALPHA, University of Illinois (May 12, 1959)

Mudomo Sudigdomarto

(May 19, 1959)

Ansel C Anderson
Ella C. Arnold
Joan C. Arzbaecher
Robert C Arzbaechet
Richard Balsam
Robert C Banash
Beverly D Barr
Calvin H Besore
August J. Bethem
Mary E. Blewett
James L. Beottler
George W. Botbyl
Robert A. Brooks
James N Budwey
Donald Albert Calahan
Donald E. Carlson
John B. Clark
John T. Conley
John S. Cross
Arthur J. Daniel
John C. DeFries
Ruth O. Devney
Joseph L. Dorsett
Byron C Drachman
Francis G Droegemeier
Marilyn P. Earl
Edwin D Ecker
Garnet G. Ellis
Elton G Endebrock
Charles E. Enderby

ILLINOIS ALPHA (Continued)

Donald L. Epley
Osburn R Flener
Marilyn E. Fris
Hitendra N Ghosh
Reuben J. Goering
George C Graff
Dorothy Anne Gramer
Edward R Gray
Basil W Hakki
Japheth Hall
Alice Goodson Hart
Connor F. Haugh
Kenneth D Herr
Vern D Hiebert
Earl R. Hosler
Richard D Jenks
Albert S. Jacobson
Jerry J. Johnson
Kenneth A Johnson
Eugene F. Kalley
Mary A Kelling
Chung-yeh Liu
In Mao Liu
Ruey W. Liu
Thomas C Marshall
Mary-Dell hfatchett
Floyd O. McPhetres
Ronald J. Miech
Richard L. Monroe
Dale L. Mordue
Charles B Morris
Joseph A. Moyzis
Don L. Mueller
Joseph E. Mueller
Bobbie C Murphy
Joseph I. Maruishi
Tin N Ohn
Surendranath Patnaik
John O. Penhollow
Lucile C Puscheck
Kenneth A Retzer
Fannie E. Reynolds
Wells P. Rollins
Harry Sauerwein
Kin Sein
Harvey K. Shepard
Thomas W. Shilgalis
Peggy A Stamer
Charles R. Strom
Mary E. Tener
Noel A Thyson
Jeannine Timko
James S. Trefil
Margaret R Tregillus
Ching W Tseng
Hsue C Tung
Doris S. Ullman
Gloria M. Vanderbeck
Emmanuel J. Vourgourakis
David C Waae
Jay D Weaver
Sik S. Yau

ILLINOIS BETA, Northwestern University (May 27, 1959)

Henry L. Bertoni
Paul A Brown
Robert Burman
Frank Collins
Leroy W. Cooper, Jr.
James Cunningham
Leon Gilles
John Gosnell
David Guell
William Hough
Janice Levin
Gerald Malling
Lee Moffitt
John Newman
Judith A Perlow
George Platz
Fred Plotke
William G. Schaefer
Ronald Schwab
James A Thomas
Lloyd Zimmerman

ILLINOIS GAMMA, DePaul University (February 18, 1959)

James Arvia
Lorenz becker
Adam Czarnecki
Marshall Kitchen
Alfred Moretti
Robert Murawski

ILLINOIS DELTA, Southern Illinois University (April 9, 1959)

Marilyn S. Banks
Carl Bates
James Crenshaw
Lewis J. Crockett
Peggy J. Duckworth
Hosea K Elliott
Kenneth D Flowers
Michael D Groves
Donald K. Harriss
William E. Hayes
Gerald Hertweck
Ronald L. Kietzman
Vernon Marlin
William B. Millspaugh
Peter C Morris
Donald W. Schuchardt
Linda R Stevens
Charles G Wade
Jean M. Webb
William D Wiggins
Carl Willis

INDIANA ALPHA, Purdue University (February 17, 1959)

Florence H Ashby
Barbara A Connelly
Elisabeth M. Doehrman
Joseph A. Fromme
Norman H. Geary
David G. Graef
Elaine J. Hodson
Phillip C. Peters
Alan W. Severance

KANSAS ALPHA, University of Kansas (April 24, 1959)

Ellen E Bartley
Terrence Brown
James W. Cederberg
Marilyn S. Chapman
William T. Covert
Jane E Crow
Robert D Dancey
Roger T. Douglass
Donald B. Erwin
Peter Flusser
Charles B Frye, Jr.
Barbara J. Fugate
George C. Gastl
Myrna C Giles
Eugene R Grassler
Alfred Gray
Joanne Halderson
Frederick H Horne
Elaine L. Johnson
Howard M. Johnson
Neal M. Kendall
Shoichim Kobayashi
Lois Kuchenbecker
Dean W Lawrence
William D McIntosh
Patricia J. Minger
Nancy Parker
Raymond E. Pippert
A. Allan Richert
Charles H Roberman
Laurian Seeber
William C Smith
Richard L. Speers
Charles J. Stuth
Selmo Tauber
Ellen Veed
Janice A Wenger
Masanobu Yonaha
William J. Hudson
Alfred J. Shryock

KANSAS BETA, Kansas State University (May 4, 1959)

Robert D. Bechtel
Louis C. Burmeister
Shih-Chi Chang
Robert S. Cochran
Carol I. Faulconer
Rosa R. Garrett
Stephen R. Hilding
Ching Lai Hwang
Vincent Y. Hwang
William Tsu-Taw Kao
John E. Kipp
Harold L. Knight
George C. Leslie
William L. LeStourgeon
Tate F. Lindahl
Dale R. Lumb
Er-Chieh Ma
Francis R. Marvin
Carol M. McDonald
Roger F. Olson
Stanley L. Rieh
Garfield C. Schmidt
Kenneth J. Tiahrt
Hsun Tien
William H. Tobey
Willem van der Bijl
Arnold Wallender
Benton D. Weathers
Janet M. Weber
Yung Chia Yang

KANSAS GAMMA, University of Wichita (April 3, 1959)

Howard D. Backman
Josiah Beck
Robert D. Dobrott
Bana Kartasasmita
Luin L. Leisher
Charles Gordon McCarty
Paul A. Miller
Jack F. Morris
Kenneth T. Orr
Arthur J. Taylor
Derrick E. Tipping
Jack Walker

KENTUCKY ALPHA, University of Kentucky (Unknown)

Bobby R. Farris

(May 7, 1959)

Tracy D. Alexander
Jesse B. Allen
Max R. Harris
Rose U. Hawkins
William E. Kirwan
Ralph O. Meyer
Jady Ung

LOUISIANA ALPHA, Louisiana State University (May 14, 1959)

Byrd M. Ball
Harold M. Barnes, Jr.
William J. Beard
Maurice J. Bouvier, Jr.
John U. Callaghan
James H. Carruth
Gerard W. Daigre
George P. Distefano
Anthony J. Galli
Robert R. Gastmck
Richard A. Geiger
Patricia A. Haydel
Albert E. Hodapp
Sandy A. Hundley
Walter M. Langhart
Abel J. Legendre, Jr.
Jose A. Limonta
Richard P. Lowry
David J. McGill
Margaret J. McLaurin
Allen J. Pope
Stephen C. Pruyn
Bill E. Slade, Jr.
Jerry B. Swing
John C. Wiese

MARYLAND ALPHA, University of Maryland (May 15, 1959)

Fred J. Bellar, Jr.
Lan-keh Chi
Susan J. Curtis
Eileen Dalton
Robert J. Gauntt
Margaret Goldsborough
Petee Schwartz
David A. Sprecher
Eutiquio C. Young

MICHIGAN ALPHA, Michigan State University (January 1959)

Preston B. Britner
Robert T. Bush
Vincent L. Coates
Phiiip R. Humbaugh
John S. Kostoff
Carol A. Malan
Maxine H. Perkins
Peggy E. Prentice
Palma R. Richardson
Hazel S. Smith
Sandra M. Todd
Lois E. Vissering
Stephen A. Weller
Marilyn J. Wissner
Roger P. Grobe
Ronald J. Larsen

(May 14, 1959).

Thomas R. Allen
David A. Balzarini
Joseph C. Ferrar
Richard L. Gantos
Sidney Govons
Charles W. Hart
Harold K. Hodge
Dean C. Luehrs
Carolyn L. Premo
Walter P. Reid
David E. Stahl
Richard Wagner

MISSOURI ALPHA, University of Missouri (May 19, 1959)

Fakhruddin Abdulhadi
David C. Baker
Desta V. Baker
Jeay G. Colyer
Forest W. Crigler
Earl E. Deimund, II
James Coleman Dunn
Jon W. Durr
Wyman G. Fair
Obed D. Flandermeyer
Richard B. Frankel
William A. Gay
Robert L. Hiltenburg
John C. Huber
Henry L. Jackson
Richard E. Janitch
Harvey E. Jobson
Robert F. Kerwin
William A. Kirk
James R. Litzsinger
Vladi S. Malakhof
Joe A. Marlin
Robert A. Melter
Wayne L. McDaniel
David T. Pierce
Frederick M. Richardson
Charles A. Sigrist
James V. Smith
Lyman T. Smith
Billy L. Stout
Yozo Takeda
Norma J. White
Robert E. Williams
Edwin L. Woollett

MISSOURI GAMMA, St. Loais University (April 23, 1959)

Mohammed Ahmed
Arnold F. Barta
William C. Blecha
James L. Bledsoe
Jerome P. Brand
Judith R. Bruch
William Callen
John P. Carter
Mary Casey
Helen Connaughton
Thomas B. Dennis
Fred Drummond
Marise D. Eaton
King S. Eng
Margaret Fahey
Allen R. Fauke
Rev. James W. Felt, S.J.
James H. Ferrick
Barry B. Flachsbart
James R. Francoeur
Donald L. Franke
Margaret L. Forster
Bro. Augustine Furumoto
Louis A. Gibbons
Frank W. Greenway
John U. Hamm
Richard E. Hammer
Leonard F. Impellizzeri
Hubert C. Kennedy
Ronald J. Knight
Mary L. Kroner
Catherine M. Kuenz
John J. Lacey
Jerry J. Lavick
Arlene Lehde
Kathleen E. Lips
Richard J. Litschgi
Sam J. Lomonaco
Rev. John W. Milton, C.S.V.
Charles W. Mueller
Mohan L. Narchal
Daniel E. O'Connor
Daniel O'Toole
Kathleen A. O'Toole
Charles H. Riechmann
Joseph Sabella
Donald R. Schillermann
Glen H. Stadsklev
Thomas F. Sullivan
Harold B. Tinker
John J. Travalent
Larry L. Trimmer
Denis Tsao
Arif Turkeli
Marguerite M. Van Flandern
O. Decker Westerberg
Charles Weiss

MONTANA ALPHA, University of Montana (January 14, 1959)

John Anderson
Mary B. Billings
Roberta J. Chaffey
William Kirkpatrick
Merle E. Manis
David J. Parker
Robert J. Ruden
Irl K. Yale

NEBRASKA ALPHA, University of Nebraska (May 3, 1959)

Henry D. Berns
Richard R. Berns
Richard W. Carroll
Paul L. Dussere
James M. Eggers
Walter F. Gutschow, Jr.
John B. Hasch
Charles V. Heuer
William R. Holst
Fred J. Howlett
Gerald L. Kaes
Darrell H. Lau
John D. Nielson
Rolando E. Peinado
Earl K. Rudisil
Sanford L. Schuster
Alan J. Vennix
Robert A. Witte

NEVADA ALPHA, University of Nevada (May 12, 1959)

Robert F. Anderson
Richard W. Arden
Jean C. Best
John M. Brown
Glen H. Clark
William D. Dolan
Bobbie J. Jenkins
Robert C. Lyon
Robert M. Pearson
Virginia M. Pucci
Thomas A. Sloan

NEW JERSEY ALPHA, University of New Jersey

Harry B. Bethke, Jr.
Howard S. Daitz
Roland B. DiFranco
Edward P. Eardley
Robert M. Fesq, Jr.
Emery S. Fletcher
Kenneth A. Friedman
Donald J. Gallo
Ronald Edward Graf
Carl F. Grumet
Lars B. Hagen
Keith E. Hamilton
Richard B. Hieber
Masahim Iwata
Gerald B. Jaeger
John A. Kasuba
John G. Kennell
Richard Wilhelm Kopp
Joseph M. Landesberg
Barry S. Lowenstein
Robert E. Luna
Frank J. McMahon
Arthur H. O'Connor
Nicholas J. Passalaqua
Charles E. Pinkus
Sanford Platter
Conrad A. Schilling
William J. Schwatm
Neal F. Shepard
James A. Shissias
Martin Stempel
Robert C. Swiatek
Alfred G. Vassalotti
Erederic P. Weber

NEW JERSEY BETA, Rutgers University (April 17, 1959)

Margaret A. Boysen
Carolyn T. Cowan
Anne D'Amato
Vida Ray Hoskins
Laura A. Pernicone
Doris Pauline Scholl
Caroline Suchman

NEW YORK ALPHA, Syracuse University (March 23, 1959)

Albert C. McDowell
William G. Scheerer

(April 13, 1959)

Thomas C. Barkley
Yang S. Chun
Sandra N. Halleck
Donald A. Lutz
Robert Maisel
Richard F. Pavley
Elizabeth H. Saw
Kenneth Woosrer

(May 13, 1959)

David L. Austin
Jerrald B. Axelrod
Carol E. Bowerman
Richard P. Cook
Sheila A. Cruchley
Donald W. Dakin
David E. Dean
Oleg V. Fedoroff
Richard C. Flaherty
James B. Geyer
Gene P. Hammel
Clarise G. Lancaster
Barry Levitt
Theodore J. Nicolaides
James F. Pasto
Victor J. Persutti, Jr.
Katherine E. Price
Pearlann Rein
William H. Reynolds
Peter C. Rice
Nicholas J. Sterling, Jr.
Waldemar G. Szok
Vance D. Vanderburg
Dale W. Zeh

NEW YORK BETA, Hunter College (March 10, 1959)

Margaret Fong
Ada Peluse
Arthur Pfeiffer
Julia Polucci

NEW YORK GAMMA, Brooklyn College (April 17, 1959)

Solomon Braunstein
Leslie Eder
Beverly T. Forlager
Bruce Friedman
Ruth C. Guidone
Elaine R. Levy
Mark Mankoff
Alvin Michel
Diana J. Milgram
Frank Platt
Richard L. Pollak
Sallie A. Rhyne
Ari L. Rothstein
Barbara S. Rosen
Harold Schachter
Barbara H. Schwartz
Abraham Spiegel

NEW YORK ETA, University of Buffalo (May 20, 1959)

David R. Beuerman
Raoul Hailpern
Ruth E. Heintz
Virginia A. Snow
William Squire

NEW YORK EPSILON, St. Lawrence College (April 29, 1959 and Ocr. 27, 1958)

Ronald Daprano
Elaine S. Egnor
Augustin Martinez
Jeame T. Giffin
Virginia Mikus
Eleanor F. Terry
Joan Wagner
Margaret A. Wilcox
Arthur E. Williams
Carol M. Wintsch
Jon W. Wolfe
Zafirios N. Zafiris

NORTH CAROLINA ALPHA, Duke University (May 5, 1959)

William E. Baylis
Raymond L. Betrs
Robert P. Biggers
Linton F. Brooks
James R. Brown
Robert B. Burns
Ronald E. Busch
Terry S. Carlton
Jane E. Chaney
Dessie B. Davis
Lee Francis Day
Diane P. Dill
Burt S. Eldridge, III
Claudine E. Fields
Robert A. Garda
Ernest W. Hartman
James S. Humphrey, Jr.
Elisabeth H. Johnson
John A. Koskinen
Philip G. Little
Robert L. McNeely
Sheldon R. Pinnell
Mary J. Reinhardt
William R. Scott
John E. Sheats
Everrette V. Snotherly, Jr.
Charlene B. Sterba
Anne B. Thompson
Janice E. Turner
James Newman Walpole
William H. Wheater
John B. Whitsett

NORTH CAROLINA BETA, University of North Carolina (Not given)

Raymond H. Cox
Nelda McDermott
Donald Elliott
Ronald Fulp
John Gibson
Sarah Goodman
John U. Gwynn
Martha Lineberger
Edward J. Matulich
Lours T. Parker
Nathan T. Seely
Kermit Sigmon
Rebecca Slover
William R. R. Transue
Mai'hi Thanh Vu
Eugene W. Womble
Klaus Witz

OHIO ALPHA, Ohio State University (May 28, 1959)

Jane R. Andre
Frank E. Battocletti
Philip C. Benedict
Mulki R. Bhat
William S. Cariens
Robin W. Chaney
Robert D. Dixon
Barbara S. Eberlin
Bryce L. Elkins
Daniel P. Giesy
Richard A. Gill
Claude M. Gillespie
Noah I. Goldman
Flournoy L. Hardy
Edwin G. Hudspeth
David N. Keck
Edwin R. Lassetrre
Allen S. Lessem
Victor S. Levadi
Robert L. McFarland
William A. McWorter
Leonard M. Masiowski
M. Vijaya Menon
Earl L. Merryman
Gunars K. Neiders
David L. Outcalt
Charls R. Pearson
A. Alan Pritsker
Herbert B. Querido
Ramon A. Romanzi
Lawrence B. Shaffer
Jerry N. Shinkle
James U. Skaates
Bernard Steginsky
Edward J. Sturm
Earl H. Tharp

OHIO BETA, Ohio Wesleyan University (April 24, 1959)

Donald D. Allen
Carol F. Anderson
William G. Ball
Constance Bartram
Bruce G. Buchanan
Jack B. Carmichael
Alain G. Cavanie
Ngee P. Chang
William R. Gigax
Robert O. Ginaven
Thomas G. Grau
Sally A. Hyde
Melvyn D. Magree
Alice A. McAlllster
Elaine C. Petersen
Raymond B. Pond
Phillip G. Roos
Laurie A. Taylor
John C. Warren
Mary W. Welty
Earl D. Winters

OHIO GAMMA, University of Toledo (1959)

Dale W. Cooper
Donald E. King
Donald Schaarschmidr
Karen VanDrieson
Richard E. Webb

OHIO DELTA, Miami University (March 27, 1959)

Larry R. Brewer
James R. Clow
Virginia M. Dornbos
Alan L. Gilbert
Dennis T. Grantham
Paul S. Malcom
Kenneth D. Miller
Carol A. Purcell
Richard T. Stanley
Mary A. Weikel
John A. Young

OHIO EPSILON, Kent State University (February 4, 1959)

Jan R. Bauer
Donald Dimetry
William Etling
Joseph R. Galko
Dennis C. Gilliland
Leslie Gulrich
Gayle P. Hahn
Beth C. Horvath
Garerh R. Jones
William E. Kintz
Carole Kyser
Martha M. Lierhaus
Helen Medley
Carter D. Mehl
Paul R. Miller
William A. Monte
David J. Noll
Jimmey R. Petrit
Barbara G. Pleis
Udom Sriyotha
William L. Steele
Maureen Weber
Johanna V. Wirbel

OKLAHOMA ALPHA, University of Oklahoma (April 14, 1959)

Annabelle T. Comfort
Mary J. Gailey
Marvin Goldstein
William R. Gurley
Patrick E. Hensy
John W. Holtzclaw
Gary G. Jones
O. Maurice Joy
Lewis H. Watson

OKLAHOMA BETA, Oklahoma State University (Spring, 1959)

Roger C. Allen
Biruta Stakle
Larry B. Soucek
Jack Alexander
James W. Gentry
Vinson D. Henderson
Joel D. Hail
Kendall H. Johnson
Bill P. Clark
John W. Smith
Richard F. Baldwin
Allen A. Masters
James G. Houston
Marc E. Low
Robert P. Wakefield
Ronald R. Rowe
William R. Derrick
Jo J. Hicks
Marlys Anderson
Robby F. Tollison
John E. Allen
Lee J. Bain
Richard M. Lotspeich
Basil E. Lawson
Carol B. Ottinger
Mary E. Adams
Vivian Spurgeon
Robert G. Dean
Samuel H. Douglas
Jack B. Skelton
John T. Holland
Norman C. Hu
John D. Stark
Bill R. Grimes
Wilson E. Singletary
William Granet
Jerry G. Williams
Charles W. Mullins
Jerry L. Hodges
Louis C. Thomason

OREGON ALPHA, University of Oregon (May 13, 1959)

Arne Baarrz
Larry Blair
Ted Cannon
Joel Carroll
Edith Church
Donald Donohue
Ralph Gabrielson
Gene Gale
Narayan Giri
Dennis Gould
John Henderson
Elaine R. Jones
James Kennedy
Keith Leslie
Kenneth Liu
Thomas Marlow
Sonja Meyers
James A. Neideigh
Deborah Nelson
Kyusam Park
Atma Sangha
Raymond E. Smithson
Charles V. Eynden
Geraldine A. Jensen

OREGON BETA, Oregon State College (May 15, 1959)

Ibrahim T. Ayyoub
William A. Braun
Ward W. Carson
Leonard F. Chandler
Chung Chiang
Chung-Wei Chow
Clifford B. Cordy, Jr.
Jack D. Culbertson
Jean M. Defenbach
Homer Ding
Shirley Dow
Frederick N. Fritsch
Saul B. Gorski
Samuel E. Griffiths
Richard J. Hanson
Kent B. Harbinsky
Robert N. Harding
Calvin S. Henry
Charles E. Hull
Charles H. Journeay
A. L. Khidir
Michael E. Knips
Harold I. Laursen
Lin-Fa Lee
Teh-Hwei Lee
Gilbert R. Marguth, Jr.
Brian E. McIntosh
Charles O. Morris
Paul R. Schrammeck
Jan-son Shen
Stanton A. Shipley
H. Lowell Smith
James D. Merriam
Daniel G. Montague
Evangelos Moustakas
Robert L. Rettig
Frank A. Schmittroth
Bruce W. Schmitz
John E. Smathers
Wesley D. Spencer, Jr.
F. Dee Stevenson
David R. Thomas
Thomas L. Vincent
Wayne E. Woodmansee
Judith A. Yerian
Chia-ping Yu

PENNSYLVANIA ALPHA, University of Pennsylvania (April 10, 1959)

Robert N. Becker
William Beninghof
Maureen D. Brody
Daniel J. Davis
Joseph Giacoponello
Jon B. Goodblatt
David R. Gunderson
Eileen C. Haden
Kenneth J. Hertz
David C. Irving
Harvey Jauvtis
Marvin Katz
Arthur Klein
James Korsh
Richard G. Larson
Richard Levin
Melvyn Miller
Mark Nameroff
David Satinsky
Robert Secundy
Elizabeth Strekis
Elaine Sweital
Richard Swerdlow
David Y. Tseng
Michael Weinreb
Steven Weitz

PENNSYLVANIA GAMMA, Lehigh University (Not given)

Donald E. Bailey
Gordon W. Brown
George C. Burrell
William E. Clausen
William S. Connor
James Early
Robert K. Felter
Jack W. Fisch
W. Beall Fowler, Jr.
George C. Gotwalt
Paul J. Kunsman
Gerd N. LaMar
John H. Lane
Lowell Latshaw
Charles J. Long
Pearn C. Niiler
William F. Parks
Robert Scavuzzo
Richard Sigley
Peter S. Sheenfeld
Steven S. Shulman
Fred Soleiman
John S. Swartley
Gilfred B. Swartz
Robert G. Wagner
Eugene T. Walendziewicz
Ralph H. Weyer
Gary E. Whitehouse
Raymond W. Wolfgang

PENNSYLVANIA DELTA, Pennsylvania State University (January 13, 1959)

Robert H. Barlett
Dorothy G. Becker
Robert J. Bednar
David M. Brewer
Barron H. Cashdollar
Carl H. Dietrich
Patricia L. Downes
Michael P. Dutko
Harold L. Ergott
Edward U. Frymoyer
Bob Goosey
Danuta Hiz
Hai Sup Lee
Hillard C. Miller
James Percy
William J. Pervin
Robert Rutschow
Burton Squires
Bernard J. Waclawski

(May 25, 1959)

Robert M. Averill
E. Ray Bobo
Paul M. Canick
Joseph A. Cima
William D. Craven
Theodore K. Frutiger
Nevin B. Greninger
David W. Krautkopf
John F. Logue
Erika A. Mares
James McPherson
Guido Moeller
Robert A. Shaw
James L. Siebur
Ronald J. Slavecki
Ruth M. Thompson
Shih-hsiung Tung
Frank Warner

PENNSYLVANIA EPSILON, Carnegie Institute of Technology (May 19, 1959)

William N. Anderson, Jr.
James C. Becker
Kenneth R. Berk
Nicholas J. Bezak
Roger S. Fager
George P. Graham, Jr.
Arthur Evans, Jr.
David H. Hall
Maurice Hanan
Thomas F. Kimes
Gary J. Kurowski
John G. Moore
David L. Parnas
Donald L. Scharfetter
Herman Nils Seberg
Margaret Spock
Robert M. Wessely
David Winter
Pui-Kei Wong

TEXAS ALPHA, Texas Christian University (April 15, 1959)

Thomas C. Allen
Warner M. Bailey
Frederic R. Bamforth
Ina U. Bramblett
William C. Bush
AM L. Carter
Landon Colquitt
Ben T. Goldbeck, Jr.
Jane R. Harlan
Lee M. Hawthorne
Joyce J. Hubenak
Robert E. Huddleston
Michael P. Hughes
Charlie J. Jackson
Janet Lysaght
James N. Martin
June E. Massengale
Curtis L. Outlaw
James H. Peters
R. Miguel Peterson
Carroll A. Quarles
Mabel G. Reavis
Brown B. Rogers
James W. Rutledge
Charles R. Sherer
Gordon Shilling, Jr.
David P. Shore
Ann M. Swengel
Aubrey E. Taylor
Kelly A. Westlake
Walter Wesley
Fred A. Womack, Jr.
Cita U. Wright
Louise G. Yates

(May 22, 1959)

Doyle O. Curler
Kenneth Fulkerson
James R. Harvey
Orill F. Hicks, Jr.
Terry P. Kinney
Glenn D. Roe
Benjamin B. Udd

WASHINGTON BETA, University of Washington (June 3, 1959)

Bruno V. Boin
Ralph L. Carmichael
William E. Faris
James C. Ferguson
Victor W. Ingalls
J. David Kroon
James V. Michelow
Richard A. Michelson
Ray Mines, III
Shashanka S. Mitra
Nilmar L. Molvik
Edwin R. Newell
Peter H. Roosen-Runge
Donald M. Silberger
Kwong-Tin Tang

WISCONSIN ALPHA, Marquette University (May 7, 1959)

AM Bankofier
Thomas A. Bronikowski
Carl E. Edmund
Wm. B. Galles
Ronald L. Gassner
Gail Hamilton
Leo N. Heiting
James J. Hill
Ronald O. Hultgren
Joanne Kolasinski
Elizabeth Kunst
Jerrold J. Jacobson
Mary A. Leider
Robert E. Pesch
Larry T. Roth
Norbert C. Sinnott
Laura Sprengelmeyer
Gregor W. Swinsky
Phillip C. Tonne
Ruth Whitney

WISCONSIN BETA, University of Wisconsin (May 14, 1959)

Richard L. Andrews
William C. Bacher
Brenda Belsito
John Bray
Jean Chalk
Elizabeth Z. Chapman
Choong Yun Cho
Bruce A. Holum
Edward C. Ingraham
Eugene F. Kcause
Sandra Ladehoff
Walter W. Leffin
Tom L. McFarland
Marie T. McGovern
David L. Murray
Edith Robinson
Donald D. Rudie
Sister Mary St. Martin
Melvin R. Storm
Jane E. Thomas

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