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THE HONORARY MATHEMATICAL FRATERNITY

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COMPUTERS AND TODAY'S MATHEMATICS[1]

RICHARD V. ANDREE

If you ask people to name the most important industrial development of the last thousand years, you will receive many different answers. The thousand-year time limit eliminates the wheel, roads, writing, and many such obvious developments. Some people will list specific inventions, such as the automobile, the airplane, or the telephone; others will give more general concepts, like electricity and the assembly line. About 170 years ago, a really startling development took place. History books refer to it as the ***Industrial Revolution:*** the beginning of the use of power-driven machinery to supplement hand labor. Most of the inventions that people advance as the most important industrial development are merely examples of the application of power-driven machinery to industry.

Man has made almost unbelievable progress since 1790, but with that progress a new curse has arisen- that of record-keeping, or "paperwork", as we are apt to scornfully term it. This involves large quantities of routine thinking. Today we find ourselves in the first few years of a second revolution; the use of electronic equipment to supplement thinking. This revolution is similar in scope and in magnitude to the one which occurred when power-driven machinery was applied to routine labor. Now electronic equipment is being used to replace routine thinking.

As man developed new skills, he extended the use of power-driven machinery to non-routine manual labor; and his progress has gone far beyond the wildest dreams of the people of 1790. I expect that, as man makes similar developments in the use of computers, his progress will astound the world today. If you are a science-fiction fan, you probably realize that the wildest dreams of people today are actually pretty wild; and yet, I fully expect that many of the younger people in the audience today will be alive to see things far beyond these dreams. I recently saw a series of cartoons drawn in 1900 which protrayed the artists' conception of what the world of 2000 might be like. It is interesting to note that in every case, we have already not only achieved but gone far beyond what this series of artists envisioned as the "brave new world".

Engineering has changed a great deal. Fifteen years ago, if one wished to design a new airplane or rocket, it took eight months to build a model. Then the model was put into a wind tunnel and as much information as possible was learned about it. When the model

[1]This paper was presented as the James E. Case Mathematics Lecture, presents by the Missouri Gamma Chapter of St. Louis University on April 23, 1959.

was finally fired, during the few minutes while it was in the air, as much data as possible was gathered. When it crashed, a hundred thousand dollar model was almost invariably destroyed. Now, one simply programs a computer and the computer gives the height, velocity, acceleration, yaw, pitch, roll, and everything else one can think of, in tenth-of-a-second intervals for the entire flight of the rocket. What's more, once the computer is programmed, it will fly twenty-five such rockets in an eight minute period.

Fifteen years ago, if one wished to determine the fin angle on a rocket, two or possibly three models with the fin angles placed at various degrees, perhaps 8°, 10°, and 15 °, were made; then the models were tested. On the basis of these three experiments, and a good deal of intuition, it was decided that perhaps the proper angle was 11°. Now, a computer is programmed to simulate all the possible fin angles between 0° and 90° in one half degree jumps; a button is pushed and the operator goes out for a coffee-break. When the operator returns, all the answers are waiting. Now there is a catch to all this, of course. It may take three months or longer to write and debug the program that is used to "fly" this computer model. Even so, this costs less than the cost of one model. Speaking of costs, how much would you charge me to add up 60,000 ten-digit numbers and guarantee the correct total? I'll furnish not only the numbers, but also a pencil and paper. Now, how much do you think a fair charge would be? An IBM 650 computer will do it for $1.00. A bigger and faster machine like the IBM 704 will do the same task for 24 cents. When the University of Oklahoma research computer is completed in 1960, the cost will be only 7 cents. The higher the original cost of a computer, the less expensive it is per job, providing you can keep it busy. This is not at all surprising. Computers are built for hard, common-sense industry, where cost-per-job and speed are important factors. For example, the flutter analysis in an airplane wing member requires about two hours on a modern computer and costs $600. Some of you might remember CPC, the Card Program Calculator, which was the best computer generally available during World War II. The CPC would take eight months to do this same computation that a modern computer can do in two hours, and it would cost $12,000 instead of $600.

Returning to an engineering problem, let us discuss present day rockets. When one of the Vanguard rockets is launched, it has already flown hundreds of times on a computer. It is interesting to note that when these rockets are launched and some difficulty arises, it is usually some fault in a valve or the fuel or some mechanical part; but so far, there has been no trouble with the most complicated part of the rocket, the computation of the actual trajectory.

An interesting problem which you might like to propose to some of your freshmen is this: "Why was the first satellite spherical?" Remember that the satellite with its solar battery is far above the earth. Its path is between 400 miles and 45,000 miles above the earth, depending on where it is in its elliptical orbit. If there were air resistance up there, a spherical shape would not be the best to combat it. However, there is no air resistance at that altitude. The satellite is protected by the nose-cone on the way up so it is not spherical to reduce air resistance. If the satellite were designed to pack as much into it as possible, the shape of the nose-cone certainly would have been more efficient and involved less weight than a spherical shape. So why, was the shape spherical? The answer is a mathematical one. Mathematical computations are much easier for a spherical satellite than for any other shape. For, no matter how it yaws or pitches or rolls, it is still exactly the same mathematically. The center of gravity does not change. Therefore, if an aberration from our expected path occurs, it is quite reasonable to believe that it is caused by some factor not taken into account which is indigenous to the satellite's environment; that is, the aberration results from the forces acting upon the satellite, rather than from some error in calculation.

Perhaps we should now turn our attention to mathematics, or at least to mathematicians. Today a college or university is not really considered up to date unless it has a computer of some type. Not only are engineers and physicists and chemists interested, but research workers in psychology and anthropology and economics also use the computer. At the University of Oklahoma there are people in government, music, history, and library science, all of whom are making use of the computer. Many places are using the computer for language translation. Some of the best work that is currently being done in this field is being done by Dr. Ida Rhodes of the National Bureau of Standards. Dr. Rhodes is not a professional linguist; she has a Ph. D. in mathematics. Her attack on languages is most promising. She has done a very careful analysis of the structure of language, and is using this to translate Russian into English.

Almost all the departments on a university campus are vitally interested in its computer, but usually there is one department on the campus which gives the computer the cold shoulder. Unfortunately. that is the Department of Mathematics. The general attitude is, "all a computer can do is arithmetic and that isn't mathematics". But, they are wrong. They are wrong on three counts. (1) It is true that when a computer takes over, the mathematics is all done; but there is a tremendous amount of theoretical mathematics to be done before the computer ever starts. This is good, respectable, and, I might add, remunerative mathematics. Let me give one example: not long ago, a major pharmaceutical company wished to increase the yield of its biologicals – penicillin,, terramyacin, and similar drugs. A mathematician was engaged as a consultant, and using a medium-speed computer made a statistical analysis of some of their processes. On the basis of his results, that company is now saving more than $50,000 every year. Even if he never works for them again, the company can pay that mathematician a consulting fee of $20,000 a year *for the rest of his life* and still be way ahead. It is not surprising that industry now employs more mathematicians than ever before. (2) When a person says that a computer can do nothing but arithmetic, he doesn't know what he is talking about. Today's computers are far better symbol manipulators than they are arithmetical engines. Symbol manipulation is what is needed to assist in structure analysis. After all, what is mathematics today? A hundred years ago, one might have gotten away with saying that mathematics is a study of number and form; but not so today. Today, mathematics is a study of structure; the structure of a number system, the structure of an economy, the structure of a topology, the structure of a language, the structure of a social system, the structure of almost anything. Mathematicians owe it to society to put their knowledge of structure at the disposal of people in other fields who are just beginning to study structure. Mathematics has no corner on the brain market; there are intelligent people in psychology and economics and the behavioral sciences and other fields which are now beginning to study structure. If we allow each of them to develop their own studies, they will have to go through much of the same development that mathematics has already gone through. Furthermore, they will each develop a language of their own, so that people in one field will be unable to communicate with people of another field, even though they may be studying the same basic structure. Therefore, it is imperative that mathematicians make their knowledge available to all of these sciences. In this way mathematics will also benefit from it, as the great minds in these many fields can apply their intellect to solving some of the advanced structure problems which mathematicians have thus far been unable to conquer. (3) Perhaps the most interesting error of the people who say "all a computer can do is arithmetic and that isn't mathematics" is that good research mathematicians are now using computers to do some of the drudgery involved in respectable mathematical research. Professor Marshall Hall of Ohio State University has recently made a major breakthrough by proving that there exists one and only one projective plane of order eight. That is, the projective plane of order eight is unique! This was done by comparing sets of 72 by 72 matrices. The set contains a large number of such matrices, 25,184 of them. There is no computer built today which in any reasonable amount of time can test the similarity of these matrices and group them into equivalence classes. Professor Hall, using a computer to do the drudgery, was able to alternate between his great mathematical ingenuity and the computer's ability to do the coolie labor and thus complete this breakthrough.

Dr. Morris Newman of the National Bureau of Standards is also doing some interesting work with a computer in pure mathematics. A typical number theory theorem will state: "There exists a number K such that for all N greater than K, thus-and-such is true." The trouble with theorems of this nature is that we have no idea how big K is. By the use of his own mathematical ingenuity and a computer,

Dr. Newman has been able to establish a K in several cases. Once K is established, it is quite possible to use a computer to examine all the values of N less than K. Then the final therem can be of a form, "Thus-and-such is true for all N except for N equal to the following values". This is certainly a much stronger theorem.

Another typical number theory theorem pattern is: "If there exists a number R such that so-and-so is true, then thus-and-such follows". For years mathematicians have delighted in proving theorems of this type. Dr. Newman, however, has used the computer to examine cases and determine if such an R actually exists. In this way, a much stronger theorem can be formulated.

Professors Parker, Bose, and Shirkhande have announced another mathematical breakthrough. They have proved that for every integer greater than 6, there exists a pair of N by N orthogonal Latin squares, thus completely disproving Euler's conjecture that pairs of N by N orthogonal Latin squares for N of the form 4K + 2 do not exist.

Perhaps even more interesting is the fact that today's computer actually creates new mathematical problems. For example, in most of the algebraic systems which mathematicians study, it is assumed that the distributive law holds; that is, that $a(b + c) = ab + ac$. Generally, this is not true (hat $a\cdot(b/c)$ is the same as $(a/c)\cdot b$. Now, in general a computer works in three different modes. It works in an integral mode which is a modular system. Although modular systems are fairly well known, there are some subtle variations in a computer which must be looked into. It also works in an approximate system in which there is an • such that if two numbers differ by less than • they are called equal. It also works in a third so-called floating-point mode, which also carries an ϵ; but the ϵ itself varies not only with the size of the numbers involved, but also with the operations. These systems must be studied in more detail.

There are also some well known mathematical problems. For example, in general matrices do not commute; that is, AB does not equal BA. However, if for a given square matrix A, a matrix B can be found such that AB = I, the identity matrix, then BA is also equal to I. However, it is a somewhat horrifying fact that it is possible to find a matrix B such that AB is arbitrarily close to the identity matrix, and yet such that BA differs considerably from I. Since on a computer there is always an ϵ such that one cannot tell AB from I if they differ by less than ϵ in their elements, this is a real problem.

Last year I attended a meeting at which one of the main topics of discussion was matrix inversion. During the question period afterward, one of the members of the audience arose and said, "I have been wondering just how large matrices people invert, as a matter of regular routine." One of the men from a Naval Ordnance Test Station promptly got up and said, "Well, we invert 36 x 36 matrices quite regularly." So the chairman of the group said, "Well, can anyone here beat 36 x 36?" Someone else admitted having inverted 72 x 72 matrices. Finally a young lady from one of the aircraft companies **got** up and said, "We need to invert 400 x 400 matrices, but we are not at all satisfied with our results." The chairman looked around for other volunteers. Finally a little man got up. (Matrices all of whose elements are zeros and ones are called "sparse", however, their inverses can have very nasty decimals all through them.) He said apologetically, "Well now, of course, you must remember that we are dealing with sparse matrices; however, we recently had to invert several **1000** x **1000** matrices." A hush came over the audience. Any of you students who can invent a method of inverting a matrix which is better than those we have now, will be making a teal contribution not only to mathematics but to the whole society in which we live. Someone is going to do it. It might be one of you.

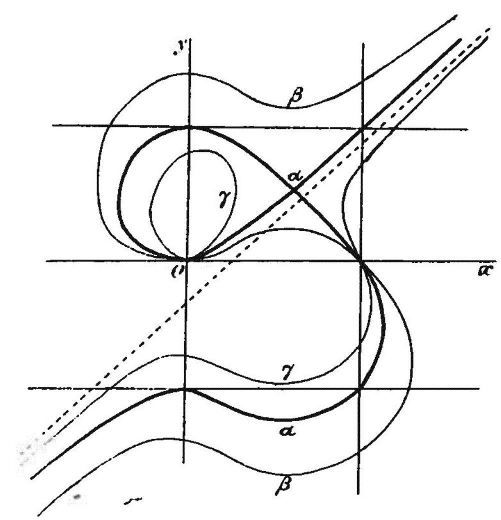
There is still another contribution that somebody needs to make. You see, the world we live in is not a continuous world; it is composed of discrete particles. This paper is actually composed of small particles of matter separated by large quantities of empty space. Light consists of minute particles of mass or energy, and all the physical phenomena with which we deal are believed to be discrete phenomena. However, discrete mathematics is so difficult that a hundred years or so ago men decided to use continuous mathematics to approximate these discrete phenomena because the **mathematics was** easier. The rub is that continuous mathematics is not particularly applicable to computers, so discrete difference equations are used to approximate our continuous differential equations. What are we doing? We are using a discrete representation of a continuous approximation of a discrete phenomenon. Actually, the discrete representation used is quite different from the original discrete **phenomenon.** Mathematicians will surely investigate discrete mathematics at considerable length, now that there is reason to do so, and eventually we may be able to describe discrete phenomena in terms of discrete mathematics without having to make the double approximation of going through a continuous representation.

Allow me to reiterate: **It** is vitally important that mathematicians get in on the work which is now being done on computing machines. If they don't, mathematics **may find** itself in the same mess that statistics is in today on many campuses. Don't look so shocked. Perhaps you are fortunate enough to be on a campus that has one strong statistics group. However, on many campuses you will find statistics is being taught in the college of education, in the psychology department, the business school, the medical school, the engineering school, and many others. This is not a desirable situation. Some of these courses are just plain **rubbish.** Why are we in this mess? Frankly, it is because 25 years ago mathematicians refused to have anything to do with statistics. If those who have the knowledge and the ability and the training refuse to do a job which needs to **be done,** then someone else has to take over and do their best. Look

at Education (with a capital E) today. Because the subject matter fields refused to undertake the job, we now fight to get some subject matter into the Education curriculum in many colleges.

If you are a mathematician today, you must be interested in the study of structure, because today mathematics is the study of structure. At present the computer is the most important tool available for examining and simulating structure. My first plea to fellow mathematicians is: Learn to ***use*** the computer! **I** do not mean just read about it; **I** mean actually get **you.** hands dirty and use it. The computer is a powerful tool which can do a good deal of the coolie labor involved in mathematical research. My final plea is that you be generous and share your knowledge of mathematics with scholars who are interested in studying structure in other fields. **I** assure you, that you will have fun doing it.

University of Oklahoma



$$y(b^2 - y^2) = x^2(a - x)$$

If: $b = a$, $b\sqrt{3} = a$, $2a^3 = 3\sqrt{3}\,b^3$ then β, γ, a

A ROLE FOR COMPLEX NUMBERS IN ANALYTIC GEOMETRY[1]

V. C. HARRIS

Most students, I believe, are introduced to the idea of imaginary number when they are studying the quadratic equation in algebra. At that time they are given the definition of complex number and shown how to perform some manipulations with it. The symbol i is then used in writing the roots of the quadratic when needed. None of the so-called word problems has answers involving imaginaries, and it is little wonder that few students develop any appreciation of or fondness for complex numbers.

After this the students may or may not have presented to them the trigonometric form of complex number in their algebra or in their trigonometry. At any rate, the material is available and complex numbers are treated as numbers, although the utility of them may not be apparent to the students.

When we come to analytic geometry, however, the imaginary is excluded. In algebra the introduction of the imaginary has been made — the issue has been met. But in analytic geometry the issue is avoided or ignored. Since the students have probably never had a careful presentation of the number system, they are usually very much confused about the place in mathematics to be occupied by complex numbers.

Now how do authors of analytic geometry texts eliminate complex numbers?

A prevalent method, and probably the worst, is to ignore complex numbers. Thus, a mote or less typical text on page 1 has a paragraph entitled ***Graphical Representation*** of ***the Number System.*** Obviously ***the number system*** means ***the real number system.*** The first mention of real numbers or of complex numbers occurs where the text states: 'Every pair of ***real*** coordinates corresponds to one and only one point, and conversely every point has one and only one pair of coordinates". Note the confusing use of the word ***real*** in one place only.

The more careful authors merely dismiss the subject without discussion. One says: "This book deals with real numbers." Another says: "In our analytic geometry, imaginary numbers are not coordinates of points."

Frequently, misleading statements are made such as: Imaginary values of rectangular coordinates cannot be plotted. Only a sentence would be needed to explain that in more advanced work, graphing of complex numbers is customary.

But even more, there is an easy way of employing complex numbers

[1] Received by the editors May 31, 1958.

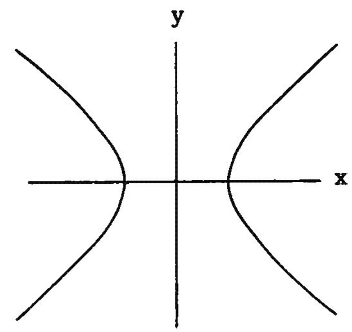
in analytic geometry, a method which involves what I shall call a combined plane.

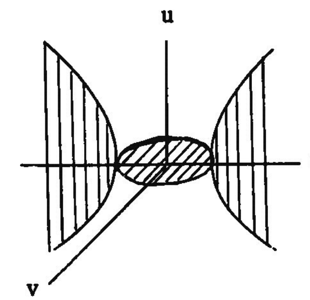
Four or five years ago in an article in the Monthly, J. A. Ward called attention to the method of graphing imaginaries by using a complex plane perpendicular to the axis of reals. Given the function $y = f(x)$, he set $y = u + iv$ and plotted the complex value of y in a complex plane perpendicular to the x-axis for each real value of x. He gave two examples, which I am repeating with the addition of the ordinary analytic geometry graphs as well.

Plane Analytics

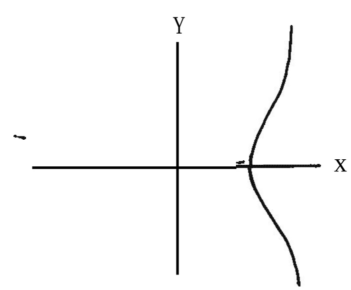
Ward

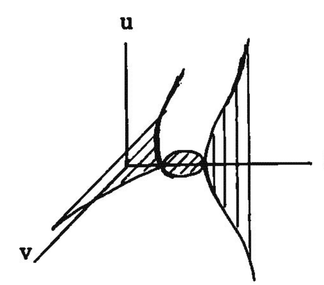
$y^2 = x^2 \cdot a^2$





$y^2 = (x-1)^2(x-2)$





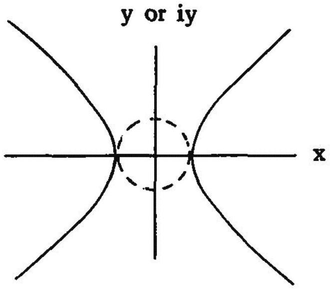
The shading in the figures is intended to indicate the planes in which the curves lie. Ward's graphs showed how a disjointed curve becomes one piece and how an isolated real point may arise.

In the preceding examples, the dependent variable $y = u + iv$ is either real or pure imaginary. The method in more general situations would involve the use of space curves. Not only are these frequently difficult to draw and to interpret, but the student will not have studied space curves when the problem of imaginary values first arises in the analytic geometry courses.

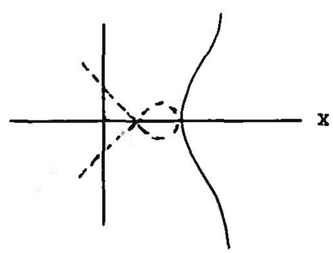
Another method of exhibiting these functions is by letting both variables be complex and by considering the functions as mapping one complex plane on the other. In our examples we would have say $w^2 = z^2 - a^2$ and $w^2 = (z-1)^2(z-2)$, However we are considering only the real axis in the z plane - that is, $z = x$ only - and our figures would give little detailed information. If we wished to construct the Riemann surfaces, we would need two sheets to take the places of the z-planes, and one or two cuts. Evidentally this would be too complicated for an elementary course in analytic geometry.

As an alternative, we could plot the real part and the imaginary part on separate planes. However, by using one plane as a real plane and a complex plane both, that is, by using a "combined plane" where the x-axis is real but the vertical axis represents both (or either) y and iy, we have for the above examples the graphs:

$y^2 = x^2 - a^2$



$y^2 = (x-1)^2(x-2)$



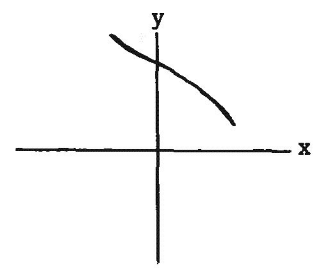
Note that here we are designating the real parts with solid lines and the imaginary parts with broken lines. If one of the parts is zero over an interval, it is omitted.

Graphing in this manner is within the capabilities of the class in analytics. Very little additional technique, if any, would be needed.

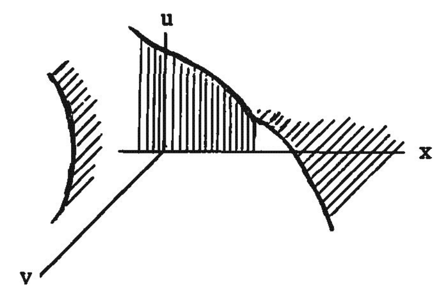
As a further example, let us take $y = \sqrt{2-x} + \dfrac{1}{\sqrt{1+x}}$.

The various graphs are:

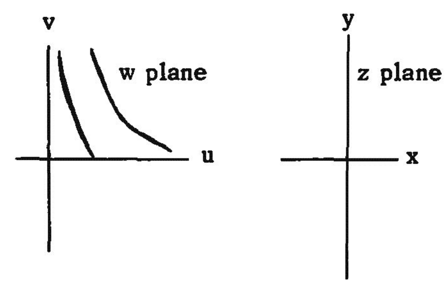
Plane Analytics



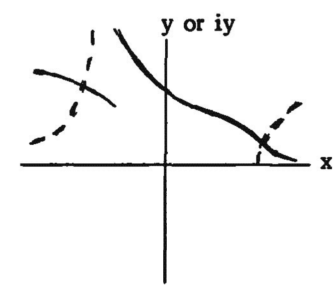
Ward



Complex Planes

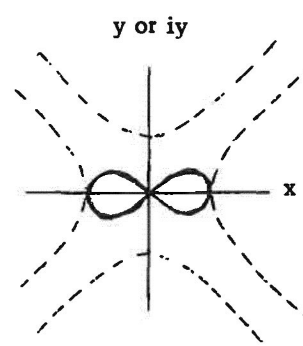


Combined Plane



This example differs from the preceding since the function is not everywhere either real or pure imaginary.

As another example, consider $(x^2 + y^2)^2 - (x^2 - y^2) = 0$. This is the lemniscate with imaginary parts added:



One advantage of sketching imaginary parts of graphs is that it throws light on various obscurities. Text writers have trouble in distinguishing between the meaning of curve, or path of a moving point, and pairs of numbers satisfying an equation. They have trouble in distinguishing between intercepts, symmetry and asymptotes, and tests for these properties. In particular, some texts define intercepts as points where the graph crosses or touches an axis. Is an isolated point on an axis an intercept? Again, is the line x = 0 an asymptote for the curve which is the graph of

$$y = 1/(x \sqrt{x^2 - 1})?$$

These and other ideas treated by enumerating special cases or overlooked would be taken care of automatically if the combined plane were used.

A more important result would be the continued use of complex numbers by the students. Since no complex numbers are employed in the present calculus courses, about two years go by in the students' training with no use of complex numbers. It is small wonder that many students who major in mathematics, including many who will teach in high school, have a distorted impression of the number system. The scheme of graphing presented here might help correct this undesirable situation, especially if some application is made of it in the calculus.

San Diego State College

PROBLEM DEPARTMENT

Edited by M. S. Klamkin,
Avco Research and Advanced Development Division

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication. Address all communications concerning problems to M. S. Klamkin, Avco Research and Advanced Development Division, T430, Wilmington, Massachusetts.

117. Proposed by Michael *J. Pascaul*, Siena College

If the lengths of two sides x and y of a triangle and the angle θ opposite one of them are chosen at random in the intervals

$$0 \leqslant \theta \leqslant \pi, \quad 0 \leqslant x \leqslant L, \quad 0 \leqslant y \leqslant L$$

(x, y, and θ are assumed to be uniformly distributed), find the probability that

a) there is no triangle possible,
b) there is exactly one triangle possible,
c) there are two triangles possible.

118. *Proposed* by Leo *Moser*, University of Alberta

Split the integers 1, 2, 3, ... 16 into two classes of eight numbers each such that the $\binom{8}{2} = 28$ sums formed by taking the sums of pairs is the same for both classes.

119. Proposed by Maurice *Eisenstein*, AVCO RAD

An infinite sequence of points on a line have coordinates given by the R progressions

$$\{a_r n + b_r\} \quad r = 1, 2, \ldots\ldots R, \quad n = \ldots\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\ldots .$$

Find the average distance between contiguous points.

120.* Proposed by Michael Coldberg, Washington, D. C.

1. All the orthogonal projections of a surface of constant width have the same perimeter. Does any other surface have this property?

2. A sphere may be turned through all orientations while remaining tangent to the three lateral faces of a regular triangular prism. Does any other surface have this property?

Note that a solution to 2. is also a solution to 1.

*No solution sent in by proposer.

121. Proposed by *M. S.* Klamkin, AVCO Rad and D. *J. Newman*, Brown University

Three circular arcs of fixed total length are constructed, each passing through two different vertices of a given triangle, so that they enclose the maximum area. Show that the three radii are equal.

Solutions

107. Proposed by Michael J. Pascual, Bwbank, California

Professor Pushover being a sympathetic soul wishes to make it as easy as possible for his class on an analytic geometry examination, so he decides to set up second degree equations with integral coefficients such that the rotation transformations necessary to eliminate the cross-product term involve only rational numbers. What set of formulas could the kind professor use?

Solution by M. Wagner, Boston, Massachusetts

In order that the xy term be eliminated from

$$AX^2 + BXY + CY^2 + DX + EY + F = 0,$$

the coordinate system has to be rotated through an angle θ where

$$\operatorname{Tan} 2\theta = \frac{B}{A - C}.$$

In order for the rotation equations to involve only rational numbers, sin 6 and $\cos\theta$ have to be rational. Consequently

$$\cos\theta = \frac{2mn}{m^2+n^2}, \quad \sin\theta = \frac{m^2 - n^2}{m^2 + n^2}.$$

It then follows that

$$\frac{B}{A - C} = \frac{4mn(m^2 - n^2)}{4m^2n^2 - (m^2 - n^2)^2}$$

and that

$$B = 4\rho mn(m^2 - n^2), \quad A - C = \rho[m^2n^2 - (m^2 - n^2)^2].$$

Also solved by H. Kaye, J. Thomas and the proposer.

108. Proposed by Daniel Block, Yeshiva University

Evaluate the n^{tn} order determinant P where

$$\begin{vmatrix} 1 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 1 & -3 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 3 & -3 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & -1 & 3 & -3 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & -1 & 3 & -3 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ \vdots & \vdots & \vdots & & & & & & & & \vdots \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & -1 & 3 & -3 \end{vmatrix}$$

Solution by Paul **Yearout,** University of Washington

$D_n = (-1)^{n+1} n^2.$ Proof is by induction.

First, $D_1 = 1$, $D_2 = -4$, $D_3 = 9$. Then for $n > 4$

$$D_n = -3D_{n-1} - 3D_{n-2} - D_{n-3},$$

by expanding by minors of the last column.

Also solved by H. Kaye, Paul Myers, J. Thomas, M. Wagner and the proposer.

109. Proposed by Leo ***Moser,*** University of Alberta

Fifteen professors attend regular committee meetings. At the end of each meeting every professor leaves with the hat brought by the man he most admires. Assume no professor is more popular than any other and that their relative admiration for one another does not change. After the first meeting, to which everyone brings his own hat, they agree to disband their committee as soon as everyone recovers his own hat. They find that after 100 meetings they are still going strong. When will they disband.

Solution by Jack Silver, Montana State University

Since each professor gets exactly one hat, the problem is that of finding a permutation P over fifteen elements with order $n > 100$. Write P as a product of disjoint cycles; let the number of cycles be s and their lengths $L_1, L_2, \ldots L_s$. If P is a solution, then $\Sigma L_i = 15$ and the **l.c.m.** $[L_1, L_2, \ldots L_s] > 100$.

The only solution is $L_1 = 3$, $L_2 = 5$, $L_3 = 7$ and the professors disband after $3 \cdot 5 \cdot 7 = 105$ meetings.

Also solved by H. Kaye, Paul Meyers, J. Thomas, M. Wagner, T. Zetto and the proposer.

110. Proposed by Lawrence Shepp, Princeton University

If $P(z)$ is a polynomial of degree n, then $P(z)/z^{n+1}$ is real and positive for some z on the unit circle $|z| = 1$.

Solution H. Kaye, Brooklyn, New York

If $P(z) = 0$ on $|z| = 1$, the result follows. If not, then consider the mapping of $|z| = 1$ by the function $w = P(z)/z^{n+1}$. By the principle of the argument, the w curve must include the origin and consequently it crosses the positive real axis.

Also solved by Paul Meyers, D. J. **Newman,** L. Shepp, and M. Wagner.

111. Proposed by M. ***S.*** Klamkin, AVCO RAD, and D. ***J. Newman,*** Brown University

It is conjectured that at most N-2 super-queens can be placed on an $N \times N$ $(N > 2)$ chessboard so that none can take each other. A **super-**queen can move like an ordinary queen or a knight.

It should have been stipulated that N is even. For N = 5, Michael J. Pascual shows that one can place 4 super-queens.

BOOK REVIEWS

Edited by
FRANZ E. HOHN, UNIVERSITY OF ILLINOIS

Ramanujan, Twelve Lectures on Subjects Suggested by His Life and Work. By G. H. Hardy. Originally published by Cambridge University Press in 1940. Photographic reprint edition by Chelsea Publishing Company, New York, 1959, $3.95.

"I have set myself a task in these lectures which is genuinely difficult and which, if I were determined to begin by making every excuse for failure, I might represent as almost impossible. I have to form myself, as I have never really formed before, and to try to help you to form, some sort of reasoned estimate of the most romantic figure in the recent history of mathematics; a man whose career seems full of paradoxes and contradictions, who defies almost all the canons by which we are accustomed to judge one another, and about whom all of us will probably agree in one judgment only, that he was in some sense a very great mathematician." - From the first lecture.

The celebrated Indian mathematician Srinivasa Ramanujan was born in December. 1887. He had no university education, and worked unaided in India until he was twenty-seven. In **1913,** however, letters from Ramanujan to the late G. H. Hardy in England gave unmistakable evidence of his powers, and he was brought to Trinity College, Cambridge, in April, 1914. There he had several years of very fruitful research activity in the analytic theory of numbers, much of it in collaboration with Hardy. He was ill (of tuberculosis) from May, 1917, onwards, returned to India in February, 1919, and died in April, 1920.

"**Ramanujan** was, in a way, my discovery. I did not invent him—like other great men, he invented himself—but I was the first really competent person who had the chance to see some of his work, and I can still remember with satisfaction that I could **recognise** at once what a treasure I had found **. . . .** My association with him is the one romantic incident in my life." - From the first lecture.

Hardy was obviously proud of his role in the Ramanujan story, and he had every right to be. Great as Ramanujan's gifts were, it has to be admitted that without Hardy's interest, encouragement, assistance, teaching, and collaboration, Ramanujan's name might well be comparatively unknown today. And not only did Hardy help Ramanujan directly while the latter was alive, but after Ramanujan's death Hardy did much to spread Ramanujan's reputation and further the appreciation of his work.

'The contents of the book are described quite accurately by its title. It is not a systematic account of Ramanujan's work (though most of his more important discoveries are mentioned somewhere), but a series of essays suggested by it. In each essay I have taken some part of his work as my text, and have said what occurred to me about its relations to that of earlier and later writers. But even when I digress furthest, when I am **writing,** for example, about Rademacher's work **in** Lecture VIII, or about **Rankin's** in Lecture XI, 'Ramanujan' is the thread which holds the whole together." - From the preface.

This book is probably the outstanding example of an exposition of the work of one great mathematician by another. Hardy's **writing** skill is justly famous, and he applies it admirably to the task of **explaining Ramanujan's** mathematical triumphs—and also his failures. While there is much hard mathematics in this book when the author gets down to cases and while very few readers can expect to follow all the discussions in detail, nonetheless there is plenty that is accessible to the less expert reader. On the whole the book makes fascinating reading for anyone with some interest either in the theory of numbers or in the recent history of mathematics.

The typographic excellence of the original edition comes through well in this unaltered reproduction. Needless to say, the historical comments do not cover the many advances made in the last twenty years in the subjects which this book takes up. The following misprints occur both in the original edition and in this one. On page 65, line 4 from below, replace 10 by 8. On page 142, line 16, replace $(\ell m)^{-s}$ by q^{-s}. On page 149, line 6 from below, interchange $\vartheta_3(0|\tau)$ and $\vartheta_4(0|\tau)$. On page 165, formula **(10.4.3),** delete 23. On page 168, formula **(10.6.3),** replace $(\log x)^{1-\kappa}$ by $(\log x)^{\kappa}$. On page 174, formula **(10.9.6),** replace $\mathfrak{F}(s)$ by $2^{-1/2}\mathfrak{F}(2s)$. On page 226, line 12, replace m by $m\prod_{p|m}(1+p^{-1})$.

University of Illinois P. T. **Bateman**

Information Theory and Statistics. By Solomon Kullback. New York, John Wiley, 1959, **xvii** + 395 pp., 512.50.

In 1951 a new measure of information was introduced into mathematical statistics by Solomon Kullback and R. A. Leibler. The Kullback-Leibler information statistic is related to the measure of information introduced by R. A. Fisher in 1925, which was the first attempt to express mathematically the amount of information supplied by data about an unknown parameter. Both of these measures are logarithmic in nature, and consequently it is not surprising that they are additive for independent events.

Fisher's concept of information has been applied mainly in the theory of statistical estimation. The Kullback-Leibler statistic, however, in addition to the theory of estimation, has many applications in the theory of statistical hypotheses. In fact, the present volume is largely devoted to describing properties of this statistic and its applications to testing hypotheses in experiments with fixed sample sizes. Applications to the sequential case have also been worked out, but they are beyond the scope of this book.

The Kullback-Leibler information statistic is shown to have a simple relationship to sufficient statistics, and likewise to be related to type I and type II errors. The value of the Kullback-Leibler statistic for differing parameter values of certain well-known distributions, such as the normal and Poisson, usually turns out to be a surprisingly simple function of the respective parameters.

The book includes chapters on the applications of information theory to contingency tables, multivariate analysis, and the multivariate linear hypothesis.

The reader is expected to have a background in the theory of probability and mathematical statistics. A knowledge of measure theory would also be most helpful, although a casual reader can learn facts of statistical interest by applying his knowledge of the calculus.

There is an extensive problem section at the end of each chapter, and the book also contains an extensive bibliography and a glossary. This book is highly recommended as a text for graduate students, as well as for the mature statistician. The broad generality with which the concepts of information theory are used to unify many diverse aspects of statistical theory provides a most rewarding insight to the reader.

Since the emphasis' in this book is on statistical theory, the application of information theory to communications problems is not extensively treated. **A** few problems in the text deal with these applications. Also, much of the voluminous literature on this subject, which has appeared since the original publications of Shannon and Wiener in 1948, is referenced in the bibliography-

Problems similar to some of those considered in this book have been investigated in the USSR by I. N. Sanov (Mat. **Sbornik,** Moscow, 1957, **Vol.** 42, No. **1(84),** pp. 11-44.)

Department of Defense William H. Martin

Real Analysis. By E. J. McShane and T. A. Botts. D. Van Nostrand, Princeton, New Jersey, 1959. ix + 272 pages, $6.60.

The authors state in the preface that "the aim of this book is to present, in a form accessible to the mature senior or beginning graduate student, some widely useful parts of real function theory, of general topology, and of functional analysis." The reviewer believes that they have achieved this aim. The exposition is careful and clear, and students at the indicated level should be able to benefit greatly from a study of this book.

The authors start with a brief and informal discussion of sets and functions and introduce the real numbers as the (essentially unique) complete ordered field. They next turn to a discussion of topological spaces, which they approach via a novel theory of convergence. Continuous functions are then introduced as functions which preserve convergence and are discussed to some extent. They next give a brief discussion of functions of bounded variation and absolutely continuous functions, and then a thorough development of the Lebesgue-Stieltjes integral for functions defined on a finite dimensional Euclidean space. This is approached via Daniell's "up-down" method. Next follows a chapter on the L-spaces, which discusses briefly Fourier series and the Fourier integral in L_2 and gives a proof of the spectral theorem for **bounded Hermitian** operators. In addition, there are appendices on inductive definition, **maximality** principles, and the **Tychonoff** theorem on compactness.

Some people may object to the non-standard convergence approach to topology and continuity. Some will feel that the book is too abstract, while I am sure that others will argue that the restriction to Euclidean spaces in the integration theory is not desirable. Probably all will be surprised at some topics included as well as some excluded. Certainly the list of exercises could be enriched. The present reviewer has his own opinions on these matters and is not without reservations. However, this book is not intended to be a complete treatise on analysis. What it does, it does well and clearly. All in all, we feel that it is a useful new book that will prove to be very valuable to students at this level. If they go through this book they will still have a lot to learn, but they will certainly not have anything to unlearn.

University of Illinois — Robert G. Bartle

Linear Programming: Methods and Applications. By Saul I. Gass. McGraw-Hill, New York, 1958. xii + 223 pp., $6.50.

There are currently being published many books dealing with applications of algebra to the social sciences. "Linear Programming" is one such book: it is an introduction to the subject.

The first part of the book is a summary of the theory and computational techniques which underlie linear programming. It is useful to refresh one's memory of mathematics already mastered, but a student meeting this material for the first time will probably find it too condensed to learn in a semester.

The last portion of the book contains applications of linear programming and is quite interesting. Some examples are game theory, the diet problem, inter-industry problems.

The book is written for use as a text book for first year graduate students. However, many others will enjoy browsing through the last portion of the book.

Urbana, Illinois — Jane L Robertson

Analysis of Linear Systems. By David K. Cheng. Addison-Wesley, Reading, Massachusetts, 1959. xiii + 431 pp., $8.50.

In the past fifty years there has been a concentrated effort to generalize the techniques for analyzing linear systems, i.e., essentially those systems whose behaviour can be best described by a linear differential equation. The operational techniques (Fourier transforms, Laplace transforms, etc.) generally have been accepted as the most useful tools for such purposes.

Professor Cheng has written a well-rounded book on this subject. This book covers almost all important applications of the operational techniques of solving differential equations to the analysis of linear systems. Although Professor Cheng is an electrical engineer, he has made an effort to include some consideration of the linear mechanical systems.

This book is primarily intended as a text for the advanced undergraduate and the first-year graduate students in electrical engineering. It serves this purpose very nicely. Most topics are covered clearly and with sufficient rigour for this level. However, for more advanced students in electrical engineering and for students in applied mathematics the coverage on the Fourier transforms and the Laplace transforms is inadequate. The sufficient number of examples in the text, and the well-suited problems at the end of each chapter, help make this book one of the best books available on this subject.

University of Illinois — S. L. Hakimi

Elementary Decision Theory. By Herman Chernoff and Lincoln E. Moses. John Wiley, New York, 1959. xv + 364 pp., $7.50.

Designed primarily as a text for a beginning course in statistics offered to undergraduate students in social science whose only mathematical preparation is high school mathematics, this book succeeds remarkably well in introducing the basic concepts of modem decision theory in a very readable yet quite rigorous manner. Moreover, the addition of selected mathematical derivations in the appendix makes the book well suited to the more mathematically sophisticated student who would like to learn something about the theory of decision making under uncertainty.

The style of the book is very informal and much use is made of simple examples to introduce new concepts. The many exercises are well chosen and provide an aid in understanding the material. Also helpful is the summary appended to the end of each chapter as well as a glossary containing an explanation of the notation and symbols used.

Following an introductory chapter and a chapter devoted to the methods of data handling, the reader is introduced to some fundamental notions of probability theory, including a description of such mathematically useful concepts as functions and sets. Utility theory is then discussed and used in the following two chapters together with the notion of convexity in order to develop such principles of choice as Bayes and minimax strategies. Three of the final four chapters, the fourth being devoted to a discussion of mathematical models, then apply the principles developed to such standard statistical problems as testing hypotheses, estimation, and confidence intervals.

Considering the scope and need for careful development of the material, it is to be expected that many of the standard statistical techniques useful to social scientists are left untouched. The authors promise a second volume to cover this area—but in the meantime this book deserves the careful attention of those who are interested in learning about the fundamental notions of statistics.

University of Illinois — Donald M. Roberts

Applications of the Theory of Matrices. **By F. R. Gantmacher,** translated by J. L. Brenner. Interscience Publishers, New York, **1959.** ix **+ 317** pp., **$9.00.**

This volume is a revised translation of the second part of "A Theory of Matrices" by the same author, published (but not translated) in **1954.** Since the usual introductory material appears in the first part, this book deals with more advanced and specialized developments of the general theory. Therefore, in spite of the heroic attempts of the translator to make the book self-contained, it cannot be recommended to the uninitiated.

Beyond this, however, it must immediately be stated that the topics have been chosen with great care, are lucidly presented, and the whole has been impeccably rendered into smoothly flowing English.

The first chapter deals with decompositions of complex matrices of certain special types, and will be of great utility, for example, to physicists working with S matrix scattering theory. Chapter two is concerned with matrix pencils and applications to differential equations. Matrices with **non-negative** elements are the subject of the third chapter, and the complete classical **Perron-Frobenius** theory with generalizations will be found here. Such matrices are of extreme importance in many applications: Markov chains, and neutron transport theory, to mention only two.

Chapter IV applies matrix theory to systems of ordinary differential equations, with a particularly complete discussion of the handling of various types of singular points.

Finally, the last chapter could be made into a book itself, consisting, as it does, in an entirely self-contained and minutely complete discussion of one of the fundamental problems of stability theory, namely the determination of necessary and sufficient conditions that a prescribed number of roots of a given **polynominal** lie in the left half plane. Included are the classical Routh-Hurwitz theory, the Lienhard-Chipart simplifications, and some later generalizations and applications.

The quality of the typography is excellent, and only a few minor misprints were noted. The book is unreservedly recommended to applied mathematicians and physical scientists who need to "speak the language" of matrices.

University of Illinois — Herbert S. Wilf

Statistics: An Introduction. By D. A. S. Fraser, John Wiley, New York, **1958.** ix **+ 398** pp., **$6.75.**

This book may quite possibly come to serve widely as a course text for students who have studied calculus for at least a year. It deals with mathematical rather than applied statistics, and a course in engineering and science curricula based upon it would be generally useful. The list of topics is familiar, including elementary probability, distribution theory, **point** and interval estimation, hypothesis testing, regression analysis, and design of experiments. As an introduction to the theory it is well balanced, with **more** material on experimental design than such books formerly had, but no more than is now desirable for the intended audience.

It contains some minor irritants. It is one more text confusing variable with value. The theorems on maximum likelihood estimates could have all the conditions more explicitly stated. The choice between confidence and fiducial limits is left to the student.

The basic reservation about the book concerns its succinctness. Mathematics undergraduates are more likely to complete it in one year with understanding than are those in engineering. Furthermore, in classes for the latter the instructor will have to supply any illustrative material.

Lehigh University — Sutton Monro

Introduction to the **Laplace** Transform. By D. L. Holl, C. **G.** Maple, and **B. Vinograde.** Appleton-Century-Crofts, New York, **1959. viii + 174** pp., **$4.25.**

This little book (the text and exercises occupy **143** pages) is designed for use as a text in an introductory course in **Laplace** transform. The authors assume only calculus and elementary differential equations as prerequisites for such a course. Within this limitation they have succeeded very well in producing a clearly written exposition.

The development of the properties of the transform occupies the first three chapters. It is carried out with a fair degree of rigor, though some results of advanced calculus must be borrowed without proof. The fourth and fifth chapters contain applications to ordinary and **partial differential** equations respectively. A **final** chapter presents extensions of some transform properties to functions with infinite discontinuities. There are several appendices, including a table of operations and a table of transforms.

Though the authors have accomplished their main purpose, the book is perhaps too short. The amount of material is sufficient for a two hour semester course, but it leaves little leeway for the instructor. The student may also miss applications in his particular field However, the good organization and presentation of the subject *go* far to offset this disadvantage.

University of Illinois — R. P. Jerrard

Scientific Russian. By G. E. Condoyannis. John **Wiley,** New York, **1959. xii + 225** pp., **$3.50.**

This spirally bound, **5½" X 7"** volume is a companion to **Condoyannis'** *Scientific* German and Locke's Scientific French, reviewed earlier in this *Journal.* This book presents the basic structure of the Russian language in such a way that the student can learn to read Russian rapidly with a dictionary. To facilitate this **learning process.** an **analytic** rather than a **synthetic** approach is employed. **Formal** grammatical **terminology** is kept to a minimum and is fully explained when first introduced.

The explanations are **easy** to follow and main facts are well summarized in tables wherever possible. Illustrations are numerous and provide extensive early practice in reading Russian. The exercises include many scientific selections, which must be read with a dictionary since there is no vocabulary. For the serious student this is probably a good arrangement.

This reviewer has carried this pocket-size volume around for some time, studying it when opportunity arose. He found the volume helpful and an excellent travelling companion. His only wish is that more of the exercise material came from the literature of mathematics.

University of Illinois — Franz E. Hohn

American Institute of Physics Handbook. D. E. Gray, Editor. **McGraw-Hill**, New York, **1957.** ix **+ 1524** pp., **$15.00.**

This volume is "**the** first handbook specifically on physics to be published in America". It provides basic theoretical outlines as well as charts and tables of essential data in the fields of mechanics, acoustics, heat, electricity and magnetism, optics, atomic and molecular physics, and nuclear physics. There is also a list of references to mathematical volumes of use to the physicist. The volume has been carefully organized and written and will be of use to research workers in both physics and engineering, particularly since it takes into account so many of the recent discoveries and advances in the various fields.

University of Illinois — Franz E. Hohn

Analytic Function Theory, Volume 1. By Einar Hille. **Ginn,** Boston, 1959. xi + 308 pp., $6.50.

Professor Hille's treatment of the most elegant of undergraduate subjects appears to be one of the best available. It is simultaneously precise and unhurried, logically and pedagogically **satisifying,** reasonably modem yet historically correct, and it is likely to appeal to a wide audience.

The scope of the book is similar to that of Part I of **Knopp's** familiar text, except that enough new material is interleaved with the old to double the size. The new material is especially welcome since it consists of careful expositions of the appropriate algebra and topology, several excursions into closely related subjects, many problems, and many very interesting historical notes. The leisurely pace necessarily postpones the definition of a line **integral** until page 160 and the Laurent expansion until page 209, which should be a handicap only to those students who object to a thorough development of the equally interesting and important subject matter of the earlier portions of the book.

The first chapter begins with the definition of a field and contains clear statements of the properties of real numbers upon which complex analysis depends, with proofs of nearly everything. The second chapter introduces the complex field with a rather strong geometric flavor which includes much detail about stereographic projections, and the third chapter discusses Mobius transformations and others with regard to their properties as conformal maps, along with a heuristic description of their Riemann surfaces. It is possible that a more sophisticated treatment might have served better in the latter two chapters, regarding the complex sphere as a projective 1-space and using homogeneus coordinates to smooth out some familiar wrinkles, but it is hard to quarrel with the immediate appeal of the classical approach.

The fourth chapter defines **holomorphic** functions, concluding with **such** material as the interior mapping theorem and a quick look at Banach spaces. From this point one passes through chapters on power series, elementary functions, and complex integration in a standard fashion which is distinguished, however, by Hille's very clear style. The eighth chapter is labelled "Representation Theorems", which means the Taylor theorem, the principle of analytic continuation, the double series theorem, the maximum modulus theorem (!), the "fundamental theorem of algebra", and finally the Laurent expansion and the representation theorems of **Weierstrass** and **Mittag-Leffler** for entire and meromorphic functions, illustrated by the gamma function; the most familiar representation theorem of all, the Cauchy formula, occurs in an earlier chapter. The classical calculus of residues appears in the final chapter.

There are three appendices, on compactness, the polygonal version of the Jordan curve theorem, and the **Riemann** integral, respectively. There are many **interesting** exercises at the end of each section, which are designed to make the student ask his own questions; none of the exercises could be described as routine, but they all appear to be of uniformly moderate difficulty for an undergraduate course. A short bibliography follows each section, and a somewhat longer one appears at the end of the book. Finally, there is an especially detailed and useful index.

In summary, the book contains a mellowed but lively account of complex function theory which can reasonably be expected to be a standard text for a generation of undergraduate mathematicians.

University of Illinois — Howard Osbom

Reading German, For Scientists. By H. Eichner and H. **Hein.** John Wiley, New York, 1959. xi + 207 pp., $5.25.

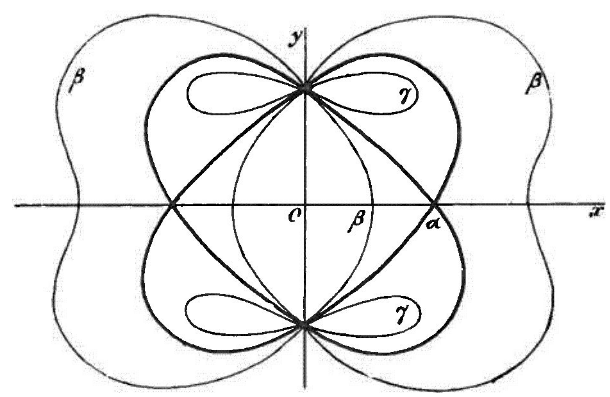
This book is designed "to serve the needs of the student who wants to be able to read German scientific texts and wishes to reach this limited aim by the shortest route."

Part I emphasizes those apsects of grammar which are needed for rapid learning to read, while those aspects required only for accurate speaking and writing are given minimum attention. Syntactical problems peculiar to German are given special attention.

Part **II** contains selected readings from chemistry and physics. Mathematical selections should have been included also in order to widen the appeal of the book, particularly since there exists a wealth of material suitable for the general reader.

The grammar section is nicely organized for rapid learning and there is excellent exercise material. As a consequence, the book should be widely acceptable.

University of Illinois — **Franz** E. Hohn



$$(x^2 - a^2)^2 + (y^2 - b^2)^2 = a^4$$

If: $a = b$, $a = b\sqrt{2}$, $a = b/\sqrt{2}$, then α, β, γ

BOOKS RECEIVED FOR REVIEW

E. Bodewig: *Matrix Calculus,* 2nd Edition. New York, Interscience, 1959. $9.50.
C. W. Churchman and P. Ratoosh: *Measurement.* New York, John Wiley, 1959. $7.95.
*G. E. Condoyannis: *Scientific Russian.* New York, John Wiley, 1959. $3.50.
J. B. Dennis: *Mathematical Programming and Electrical Networks.* New York, John Wiley and The Technology Press of M.I.T., 1959. $4.50.
*H. Eichner and H. Hein: *Reading German, for Scientists.* New York, John Wiley, 1959. $5.25.
*F. R. Gantmacher: *Applications of the Theory of Matrices.* Translated from the Russian by J. L. Brenner. New York, Interscience, 1959. $9.00.
F. R. Gantmacher: *The Theory of Matrices.* Translated from the Russian by K. A. Hirsch. New York, Chelsea, 1959. Vol. I: $6.00; Vol. II: $6.00.
F. H. George: *Automation, Cybernetics, and Society.* New York, Philosophical Library, 1959. $12.00.
E. M Grabbe, S. Ramo, and D. E. Wooldridge (Editors): *Handbook of Automation, computation, and Control, Vol. II.* New York, John Wiley, 1959. $17.50.
*D. E. Gray (Editor): *American Institute of Physics Handbook.* New York, McGraw-Hill, 1957. $15.00.
*G. H. Hardy: *Ramanujan, Twelve Lectures on Subjects Suggested by His Life and Work.* New York, Chelsea, 1959. $3.95.
*E. Hille: *Analytic Function Theory.* Boston, Ginn, 1959. $6.50.
*D. L. Holl, C. G. Maple, and B. Vinograde: *Introduction to the Laplace Transform.* New York, Appleton-Century-Crofts, 1959. $4.25.
M. Kac: *Statistical Independence in Probability, Analysis, and Number Theory (Carus Monograph #12).* New York, John Wiley, 1959. $3.00.
E. L. Lehmann: *Testing Statistical Hypotheses.* New York, John Wiley, 1959. $11.00.
A. S. Levens: *Nomography (Second Edition).* New York, John Wiley, 1959. $8.50.
*E. J. McShane and T. A. Botts: *Real Analysis.* Princeton, Van Nostrand, 1959. $6.60.
M Sasieni, A. Yaspan, and L. Friedman: *Operations Research, Methods and Problems.* New York, John Wiley, 1959. $10.25.
D. R. Whitney: *Mathematical Statistics.* New York. Henry Holt, 1959. $4.75.
E. J. Williams: *Regression Analysis.* New York, John Wiley, 1959. $7.50.

*See review, this issue.

NOTE: ALL CORRESPONDENCE CONCERNING REVIEWS AND ALL BOOKS FOR REVIEW SHOULD BE SENT TO PROFESSOR FRANZ E. HOHN, 374 ALTGELD HALL, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

OPERATIONS UNLIMITED

Science needs you

You need science

This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essential as today.

Your election as members of Pi Mu Epsilon Fraternity is an indication of scientific potential. Can you pursue advanced study in your field of specialization?

To point out the need of advanced study, the self-satisfaction of scientific achievement, the rewards for advanced preparation, the assistance available for qualified students, etc., it is planned to publish editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

Through these and future editorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

The following is a brief introduction to the men and their companies whose articles appear in this issue.

Aeronutronic (a division of Ford Motor Company) has a $22 million Research Center at Newport Beach, Southern California, where Doctor Montgomery H. Johnson is General Operations Manager, Space Technology Operations. There he directs research programs in space electronics, aerodynamics, propulsion, materials and other aspects of vehicle technology and missile defense. Dr. Johnson has had 29 years of experience in the fields of atomic energy and theoretical physics, nuclear propulsion and nuclear weapons. He is the author of some 25 papers on quantum theory and on nuclear and ionosphere physics. In addition, he holds five patents covering work in microwaves and gyroscopes.

Eli Lilly and Company, one of the leading pharmaceutical manufacturers in the United States was founded in 1876 by Colonel Eli Lilly. Today they produce 900 pharmaceuticals and biologicals vital to the health and life of people all over the world. Dr. Edgar P. King is head of their statistical research department. He is a member of the American Statistical Association, Institute of Mathematical Statistics, Operations Research Society of America, Society for General Systems Research, and the American Association for the Advancement of Science.

E. I. du Pont de Nemours and Company, established in 1802, is a leader in America's Chemical Industry. Last year they spent approximately $90 million on fundamental research. They are assisting education by grants for the academic year 1960-1961 totaling $1.3 million made to 143 Colleges and Universities. Doctor Sigurd L. Andersen, Statistical Engineer from their research division at Wilmington, Delaware has cooperated with this program to encourage advanced study.

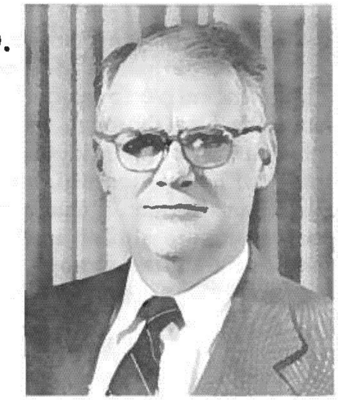
The following lists contributing corporations with the issue in which their editorials appeared.

Aeronutronics	Vol. 3, No. 2
Army Ballistic Missile Agency	Vol. 2, No. 10
AVCO, Research and Advanced Development	Vol. 2, No. 10
Bell Telephone Laboratories	Vol. 2, No. 10
Bendix Aviation Corporation	Vol. 2, No. 8
E. I. du Pont de Nemours and Company	Vol. 3, No. 2
Emerson Electric Company	Vol. 2, No. 7
General American Life Insurance Company	Vol. 2, No. 9
Hughes Aircraft Corporation	Vol. 2, No. 9
International Business Machines Corporation	Vol. 2, No. 8
Eli Lilly and Company	Vol. 3, No. 2
McDonnell Aircraft Corporation	Vol. 2, No. 7
Monsanto Chemical Company	Vol. 2, No. 7
North American Aviation, Inc.	Vol. 2, No. 9
Olin Mathieson Corporation	Vol. 2, No. 7
Shell Development Company	Vol. 3, No. 1

AERONUTRONICS,
DIVISION FORD MOTOR CO.

SCIENCE AND INDUSTRY

by
MONTGOMERY H. JOHNSON
General Operations Manager,
Space Technology Operations



Dr. Montgomery H. Johnson

Science has revolutionized the material world around us. We are affected every minute of the day from awakening by the clock radio to snapping off the electric lights before going to sleep at night. Science has created industries which shape our economic life, with one happy result that scientists receive increasingly large economic rewards. The atomic bomb dominates the political scene. Relativity and the uncertainty principle underlie much of our intellectual life. Biological and psychological discoveries are modifying our understanding of religion and ethics. Science has changed every aspect of our life in the past fifty years and promises even more drastic changes in the near future.

The growth of science roughly approximates an exponential law since progress now depends upon the totality of discoveries made up to this time. Not only is the rate of scientific discovery rapidly increasing but also the time necessary to translate scientific discovery into practical consequences is becoming shorter and shorter. Compare, for example, electrical and nuclear energy. Faraday discovered the basic induction laws upon which the practical use of electrical energy rests in 1840. Some seventy years elapsed before electrical power was used in any substantial way. Meitner and Hahn discovered nuclear fission in 1939. Six years later devastating nuclear explosions over Hiroshimo and Nagasaki heralded the dawn of the nuclear age.

The rapid expansion of science and the speed with which scientific discoveries are now-being applied create a great need for scientifically trained people. Moreover the historical distinction between engineering and science is disappearing in many areas. The translation of discovery into applications today is made on the basis of far less experience. Direct application of scientific principles, rather than empirical experience, is the basis for the solution of many development problems. Engineering is truly becoming applied science. In my organization, devoted to space vehicle systems, this tendency is enhanced by the great difficulty in obtaining direct experience in a space environment. Moreover in several areas; **e.g.,** large propulsion systems, experiments are costly and difficult. Consequently there is a large incentive to make the best possible use of experimental information by a thorough understanding of the phenomena involved. Other areas; **e.g.,** trajectories, use fields of knowledge such as celestial mechanics which have been largely developed at the hands of mathematicians. Consequently we have a continuing need for people who are skilled in the disciplines of science and mathematics and are interested in applying such disciplines to a wide variety of technical and scientific problems.

ELI LILLY AND COMPANY

MATHEMATICAL APPLICATIONS IN THE DESIGN OF EXPERIMENTS

by

EDGAR P. KING

Head, Statistical Research Department

Dr. Edgar P. King

Careful experiment design is vital to investigate work in **all** branches of science. Even the most conscientious and systematic scrutiny of data does not always reveal information on aspects that were not taken into account when the experiment was planned. The determination of the general purpose and scope of the investigation, the choice of experimental treatments and equipment – basic plans such as these must of course be made by the research worker because of his intimate knowledge of the subject under study. In an ideal, completely controlled experiment, unhampered by external influences and experimental error, these plans alone might assure the success of the undertaking. Unfortunately, an ideal situation seldom exists. Uncontrolled factors are present; factors that can magnify the variation in the results lead to misinterpretations and in some instances render the experiment useless. The contribution of mathematics to experiment design is to help furnish logical schedules for taking measurements that will hold potential sources of bias in check and increase the accuracy of the comparisons to be made.

For example, suppose four different methods of chemical analysis are to be intercompared by four different analysts using four different types of material. One possibility would be to make **16** determinations in accordance with the following scheme:

Material

Analyst	1	2	3	4
1	A	B	C	D
2	B	A	D	C
3	C	D	A	B
4	D	C	B	A

where the letters A, **B,** C, and D denote the four methods. This arrangement has the property that each method occurs once and only once in each row and each column. Such an arrangement is called a Latin square. Any comparison of methods is unaffected by average differences which exist between rows or between columns. Such differences will not affect the errors of method comparisons, hence these arrangements can lead to greatly increased precision.

Suppose the **16** determinations in the above example are run on 4 consecutive days. A more complicated design of the Latin square type could be constructed as follows:

Material

Analyst	1	2	3	4
1	A α	B β	C γ	D δ
2	B γ	A δ	D α	C β
3	C δ	D γ	A β	B α
4	D β	C α	B δ	A γ

where $\alpha, \beta, \gamma, \delta$, denote the four days. In this arrangement, called a Graeco-Latin square, every Latin letter occurs once in each row and once in each column, every Greek letter occurs once in each row and once in each column, and each Greek letter occurs once with each Latin letter. In effect, this design has been constructed by superimposing one Latin square upon another in such a way that the two are "orthogonal" to each other. Two Latin squares are termed orthogonal if, when one is superimposed upon the other, every ordered pair of symbols occurs exactly once in the resulting square.

The problem of constructing a set of r orthogonal Latin squares of side m is still unsolved for many combinations of r and m. However, within the range useful in the design of experiments the solution has been obtained for most cases. One of the most important results is contained in the following theorem:

When m is a prime or a power of a prime, there exists a set of (m-1) orthogonal Latin squares of side m. For example, when m = 4 we have

Aα1	Bβ2	Cγ3	Dδ4
Bγ4	Aδ3	Dα2	Cβ1
Cδ2	Dγ1	Aβ4	Bα3
Dβ3	Cα4	Bδ1	Aγ2

where the Latin letters, the Greek letters, and the numerals form the three orthogonal squares. The proof of this theorem follows from the existence of Galois fields of p^n elements where p is a prime and n is an integer. It has also been proved that no orthogonal Latin squares of side 6 exist. Quite recently it was shown that Graeco-Latin squares of side m exist for all values of m of the form $m = 4k + 2$ (k an integer) provided m is greater than 6.

The above discussion describes the construction of only one of many types of practically useful designs. The construction process frequently hinges on interesting and sophisticated combinatorial problems, in which such advanced concepts as Galois fields and finite projective geometries have proved extremely useful.

Rapid recent developments in science and technology have brought an increased need for efficient experimentation - hence a need for many new types of experiment **designs.** This will undoubtedly mean greater demands for individuals with advanced mathematical training - as mathematicians, as research workers in the biological and physical sciences, and as engineers in industry.

E. I. du PONT** de **NEMOURS AND COMPANY

MATHEMATICS IN THE CHEMICAL INDUSTRY

by

SIGURD L. ANDERSEN
Engineering Service Division,
Engineering Department
Wilmington, Delaware

One hundred and fifty years ago Humphry Davy asserted that, "Nothing tends so much to the advancement of knowledge as the application of a new instrument." While this comment on the impact of instruments on scientific inquiry was concerned with much simpler devices than the modern electronic computer - the "new instrument" of this decade - he did, nevertheless, state a principle which is still true.

A more contemporary comment on the effect of computers on our way of life was given in a recent article, "Industrial Dynamics", by Jay Forrester of M.I.T. in which the author noted that: "The performance of electronic computers has increased annually by a factor of nearly ten per year over the last decade; in almost every year we have seen a ten-fold increase in speed, memory capacity or reliability. This represents a technological change greater than that effected in going from chemical to atomic explosives. Society cannot absorb so big a change in a mere ten years. We therefore have a tremendous untapped backlog of potential devices and applications."

These two quotations are cited not because of the importance of computers in the use of mathematics in the chemical industry; rather, they are **relevant** because the lack of computational facilities in this industry five years ago was the limiting factor in a large per cent of potential applications of mathematics and statistics to practical problems. Therefore, (here was considerable reticence to an analytical approach to a problem where there was a good chance that the solution would require calculations beyond practical bounds. With this restriction eliminated, a psychological as well as real barrier has disappeared, and a rapidly expanding range of problems is being undertaken in a more analytical manner.

To give some indication of the breadth of this trend, consider some of the following examples:

1. Preliminary design of a chemical reactor requiring analysis of flow rates, heat transfer and chemical kinetics.

2. Specification of proper automatic controls and correct operating procedures for an intricate and new plant refrigeration system through computer simulation of this system.

3. More efficient use of nylon cord in the construction of a tire by a study of the visco-elastic behavior of the cord in various constructions under different stresses.

4. Determination of improved "construction" of "Orion" sweaters to give improved aesthetics by the use of statistically designed experiments, regression analysis and subjective comparisons of sweater performance.

5. Statistical study of plant operating data by large-scale regression analysis to determine more economic plant operating and maintenance schedules.

6. Derivation of formal production scheduling rules for a multi-product plant to obtain a greater stability in labor scheduling with minimum inventory compatible with adequate customer service.

7. Selection of most profitable product mix from a plant with several types of capacity limitations - in the face of fluctuating prices for the various products.

The major facets of **du Pont's** business are included in these few examples. In (1), the preliminary reactor design, we are concerned with process research; in (2) we are in the area of system design and operation; in (3) and (4) we are using theoretical and empirical methods for aiding our customer to use our product more effectively; in (5) we are assisting the plant manager in specifying detailed instructions for scheduling operations; in (6) we are balancing the needs for working capital, efficient manufacturing and adequate sales service; in (7) we are assisting sales management in planning his strategy within the restrictions of the market place and available manufacturing facilities.

The role of the skilled mathematician or statistician in each of these problems varies. The several steps of getting the problem solved are:

1. Tentative mathematical formulation.
2. Specification of desired data - experimental design.
3. Verification of model and estimation of unknown constants.
4. Deduction of appropriate process or product design, production scheduling rule, etc.

Carrying out these steps involves skill in:

a. Relating physical situations to abstract ideas
b. Applying principles of statistical inference

c. Manipulation of mathematical symbols
d. Specification and application of appropriate algorithms

The relative importance of these four abilities in any given situation depend upon the problem and the other technical people involved In a research problem, the mathematician may be assisting a **highly-**trained physical chemist. Here his task may be the selecting of an efficient algorithm for a specific computer to compute the yields at various temperatures from a completely specified set of differential equations. In the case of a problem involving assistance to sales management, such as (7), the mathematician may be required to carry out all four steps himself, requiring a much wider range of skills.

It should be clear from these brief examples that very little depth or breadth acquired in college training need be wasted in working on problems arising in the chemical industry. Neither will any inclination or experience in teaching be lost by the industrial mathematician. There is an unmistakable revival of interest among engineers of five, ten, even twenty years' experience in reviewing long forgotten principles of elementary calculus as well as learning for the first time Boolean or matrix algebra, numerical analysis or elementary set theory. Du Pont has sponsored courses ranging from a ten-lecture orientation to mathematics for operations research to a 300 lecture hours course in modern applied mathematics and statistics.

All of these comments have been on the positive and optimistic side. One brief comment on the other side of the coin - the chemical industry has prospered without us for some time. It has made much progress in the last quarter century with heavy reliance on empirical reasoning and engineering ingenuity. In a rapidly expanding segment of industry, there has been limited need and less time for defining in mathematical terms the micro-behavior of chemical reactions and production-inventory systems. The burden of proof has been on us to **prove** our economic worth to the corporation. Fortunately, this is coming to an end. Problems requiring more refined mathematical treatment are being suggested more and more by the chemist, engineer and manager, furnishing an increasing backlog of challenging problems for the mathematician at work in this industry.

NEWS AND NOTICES

Edited by
Mary L. Cummings, University of Missouri

NEW YORK ETA, UNIVERSITY OF BUFFALO

Harvey Selib was awarded one year's subscription to The American Mathematical Monthly for receiving the highest grade point average among the undergraduate mathematics majors at the University.

University of California, Los Angeles

The following students were awarded scholarships:

Arnold Allen - University Fellowship
Judith Bruckner - National Science Foundation Fellow
Joseph McCarty - National Science Foundation Fellow
Thomas McLaughlin - National Defense Act Scholarship
Arthur Sagle - National Science Foundation Fellow

Montana State University

The annual Pi Mu Epsilon prize for the outstanding exhibit at the 1959 Montana State Science Fair at Montana State University at Missoula, Montana, was awarded to Mr. Eugene Lalonde of Park County High School, Livingston, Montana.

Mr. George McCrae received the annual N.J. Lennes prize for the outstanding paper written in competition during his sophomore year.

University of Pennsylvania

The following students were scholarship recipients in June, 1959:

Howard Byer - Columbia University
William Eaton - Brandt Fellowship, University of Berlin
Donald Goldsmith – Assistantship, University of Pennsylvania
Diane Splaver - University of Pennsylvania Fellowship

Eldon McKee, Kansas Gamma, of the University of Wichita after having been elected to Pi Mu Epsilon died before being initiated. However, he was awarded membership in Kansas Gamma posthumously.

Kansas Gamma Chapter of Pi Mu Epsilon was saddened by the fact that one of its newly elected members – Eldon Lynn McKee passed away, after a brief illness, just a few days prior to the formal initiation ceremony. In recognition of an honor well earned, at the formal initiation, Mr. McKee's name was entered on the Chapter rolls with the names of the other initiates and a letter of sympathy, together with the membership certificate, was sent to his parents.

ATTENTION ALL CHAPTERS

A national meeting of Pi Mu Epsilon will be held at Michigan State University, East Lansing Michigan on Tuesday, August 30, 1960 as a part of the summer mathematical meetings of MAA, AMS, SIAM, and other mathematical groups. *You* are invited to attend. Each chapter is encouraged to send as many members to these meetings as possible. To encourage participation, the national council has authorized partial payment of traveling expenses.

Each chapter may ***either*** elect one official delegate ***or*** nominate one speaker to represent that chapter at the meeting. Partial expenses will be paid as follows:

Speaker: Must be a duly registered student who is a member of P.M.E. and who will not have received a master's degree by March 15, 1960. Actual

travel expense (not meals or room) will be paid at a rate not to exceed first class rail fare. The maximum payment is $150.00 per chapter, irrespective of distance. Speakers ***must*** submit abstracts of their papers to the secretary treasurer by April 25, 1960.

Delegate: Must also be a duly registered student who is a member of P.M.E. and who will not have received a master's degree by March 15, 1960. One-half of the actual travel expenses (not meals or room) will be paid at the rate not to exceed one-half of the first class rail fare. The maximum payment is $75.00 per chapter, irrespective of distance.

A chapter may nominate ***either*** a speaker ***or*** an official delegate, (but ***not*** both) for partial travel remuneration. Chapters may send as many additional delegates as they wish, but only one person per chapter can receive travel funds from the national treasury.

Send in your nomination NOW. All nominations for speakers and for delegates must be in the hands of the Secretary-Treasurer, R. V. Andree, The University of Oklahoma, Norman, Oklahoma, ***by April*** 25 to be eligible for a travel grant.

INSTALLATION OF NEW CHAPTER

Professor J. S. Frame, Director General, presents the charter of South Carolina Alpha to chapter Director Joe D. Bickley in installation ceremonies conducted December 15, 1959. Looking on (on the right) is Dr. Raymond Lytle, Faculty Advisor and Donna J. McCay, Secretary.

The South Carolina Alpha chapter of Pi Mu Epsilon was installed at the University of South Carolina, Columbia, on Tuesday, December 15, 1959. Professor J. S. Frame of Michigan State University conducted the installation and initiated the fourteen charter members. After the presentation of the charter, he also Initiated fourteen more members into the chapter. (The names of the initiates may be found elsewhere in this issue.)

In conducting the installation, Professor Frame gave a short account of the history and purposes of Pi Mu Epsilon, led the initiates in the Pi Mu Epsilon pledge and, as Director General of the fraternity, declared South Carolina Alpha to be a regular chapter of Pi Mu Epsilon. Prior to the installation ceremonies. Dr. Frame gave a talk entitled "A Bridge to Relativity."

After the installation a banquet was held in the Union Building on the campus of the University of South Carolina. Guests at the banquet included Professor Fort of Emory University and Professors Dye and Reves of the Citadel.

St. Louis University Joseph J. Malone, Jr.

DEPARTMENT DEVOTED TO CHAPTER ACTIVITIES

Edited by

Houston T. Karnes, Louisiana State University

EDITOR'S NOTE: According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an annual report of the chapter activities including programs, results of elections, etc." The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the Pi Mu Epsilon Journal. Besides the information listed above, we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Chapter Activities Editor Houston T. Kames, Department of Mathematics, Louisiana State University, Baton Rouge 3, Louisiana. These reports will be published in the chronological order in which they are received.

REPORTS OF THE CHAPTERS

GAMMA OF ILLINOIS, DePaul University.

The Illinois Gamma Chapter held eight meetings during the academic year of 1958-59. The following papers were presented:

"Theory of Sets" by Mr. Louis Aquila

"Introduction to Logic" by Dr. Willis Caton

"Applications of Group Theory to Molecules" by Dr. J. Ziomek (Department of Chemistry)

"Introduction to Non-Euclidean Geometry" by Mr. Robert Murawski and Mr. Richard Schiff.

"The Theory of Limits" by Mr. Thomas Cook

"The Derivative" by Mr. Constantine Georgakis

"Power Series" by Mr. Adam Czarnecki and Mr. Lorenz Becker

"Beyond the Googol" by Mr. Louis Aquila

To attract new members, several students who were not members of Pi Mu Epsilon were invited to give lectures. These lectures were well developed and received.

The Officers for 1959-60 are to be elected in October.

ALPHA OF MONTANA, University of Montana

The Montana Alpha Chapter held five program meetings during 1958-59. The following papers were presented:

"Monte Carlo and Russian Roulette" by Mr. Howard Reinhardt

"Finite Geometry" by Dr. T. G. Ostrom

"Group Structure" by Dr. W. R. Ballard

"Topology" by Dr. Joseph Hashisaki

"Steiner Symmetrization" by Dr. Wolfgang Schmidt.

The Pi Mu Epsilon award for the best entry in the mathematics section of the Montana Science Fair went to Eugene Lalonde of Park County High School of Livingston. The Pi Mu Epsilon awards to outstanding undergraduate students in mathematics and physics went to Jack Silver and Keith Yale, respectively.

Officers for 1958-59 were: Director, Jack McGuire; Vice-Director, Larry Newell; Secretary-Treasurer, Jack Silver. At the end of the fall quarter, William Kirkpatrick was elected Director to succeed Jack McGuire who was leaving school.

BETA OF WISCONSIN, University of Wisconsin

During the year 1958-59, the Wisconsin Beta Chapter held six meetings. Two initiation meetings were held at which forty. new members were inducted. At the first initiation, Dr. R. H. Bing spoke on "The Charm of Creating Mathematics." The second initiation was held at the annual spring banquet.

One of the most interesting programs of the year was a medieval disputation on Intuitionistic Mathematics conducted by Dr. E. C. Posner and Mr. S. J. Doorman. The following papers were presented during the year.

"Rings and Modules" by Hiram Paley

"The Case Against the Continuum Hypothesis" by James Kister

"A Nominalistic Interpretation of Mathematics" by S. J. Doorman

"Imbedding Sets in Euclidean Space" by Ronald Rosen

Social activities of the Chapter included a fall picnic, a Christmas banquet, and the annual spring banquet at which Dr. C. C. MacDuffee gave a talk entitled "The Ammonities Revisited."

Officers for 1958-59 were: Director, Jules B. Vieaux; Vice-Director, Simon J. Doorman; Secretary-Treasurer, Patricia A. Tucker and Faculty Advisor, J. Marshall Osbom.

Officers for 1959-60 are: Director, Douglas A. Clarke; Vice-Director, Charles P. Seguin; Secretary-Treasurer, Sandra Ladehoff.

DELTA OF OHIO, Miami University

The Ohio Delta Chapter held three program meetings and two initiations during 1958-59. The spring initiation was followed by a banquet. The guest speaker on this occasion was Professor Leung who spoke on "The Construction of Geometric Shapes With Given Data." The following papers were presented during the year:

"The Varied Uses of Mathematics in the Field of Physics" by Dr. David Griffing of the Physics Department.

"A Problem in Counting Isomers" by Dr. Howard Ritter of the Chemistry Department.

"Inversions" by Mr. Paul E. Dohnke

Officers for 1958-59 were: Director, Waldo Patton; Vice-Director, Carol Argus; Secretary, Claudette Cook; Treasurer, Edward Fries.

BETA OF WASHINGTON, University of Washington

The Washington Beta Chapter held a bi-weekly lecture series throughout the year. The following papers were presented:

"Training of Mathematicians in England" by Dr. C. B. Allendoerfer

"Picture Writing" by Dr. J. M. Kingston

"Trigonometry for the Sophisticate" by Dr. E. Hewitt

"Boolean Algebras" by Mr. R. Mayer

"Projective Geometry - Mathematics' Sleeping Beauty" by Mr. P. Yearout

"Generalized Nim" by Dr. J. M. G. Fell

"Waring's Problem" by Mr. J. Reid

"An Unusual Number System" by Dr. J. P. Jans

"The Infinity of Primes" by Mr. R. Peterson

"Some Strange Inhabitants of the Function Zoo" by Dr. M. G. Arsove

"The Real Numbers" by Mr. J. G. Ceder

"The Distributive Law" by Dr. R. A. Beaumont

During the Spring Quarter a picnic was held for members and faculty. A competitive examination for freshmen and sophomores was sponsored by the chapter. The winners were: Jack V. Miller (first prize) and Carl W. Vemon (second prize).

Officers for 1958-59 were: Director, James W. Armstrong; Vice-Director, Kenneth A. Ross; Secretary, Gloria C. Hewitt; Treasurer, Theodore C. Mueller. Mr. Laurence Hoye replaced Mr. Ross as Vice-Director for Spring Quarter.

Officers for 1959-60 are: Director, Charles W. Austin; Vice-Director, Gloria C. Hewitt; Secretary-Treasurer, Lawrence Wold.

BETA OF MONTANA, Montana State College

The Montana Beta Chapter was installed on January **26, 1959. Secretary**-Treasurer General R. V. Andree was the installing officer. The ceremonies were followed by a banquet. Two program meetings were held during the spring semester. The following papers were presented:

"Certain Aspects of Modern Trigonometry" by Mr. William H. **Jamison**

"Geometrical Constructions" by Mr. David L. Carlstrom

Officers for **1959-60** are: Director, Robert Motsch; Vice-Director, David Cromely; Secretary, Richard **Frey.**

ALPHA OF VIRGINIA, University of Richmond

Officers for the academic year **1958-59** were: Director, James T. Carr; Vice-Director, John M. Smith; Secretary, Eleanor Dickson; Treasurer, Nancy Kipps.

During the course of the year, the following papers were presented by student members of the Virginia Alpha Chapter:

"My Summer Job in the Actuarial Department of the Traveler's Insurance Company" by James T. **Carr**

"My Summer Job in the Computing Section of the National Bureau of Standards" by John M. Smith

"The Theory of Games" by Alice Jo Barker

"An Application of Eigen-values of Matrices" by John M. Smith

"Some Applications of Higher Mathematics in Psychology" by Jolien Edwards

"Some Aspects of Linear Programming" by John R. Cummins

One of the highlights of the year's program was provided by the visit of Professor Richard E. Johnson, of Smith College. Traveling under the Visiting Lectureship Program of the Mathematical Association of America, Professor Johnson gave four formal lectures before audiences totaling approximately **500** students and visitors. Members of Pi Mu Epsilon worked with the Mathematics Department in arranging for these lectures.

Winners of the annual prize examinations sponsored by the Chapter for students in elementary courses were:

Freshman Mathematics: First Prize, Russell B. **Wayland**
Second Prize. Sandra **Atkinson**

Sophomore Mathematics: First Prize, Reginald T. Puckett
Second Prize, Claude Gravatt

Officers for **1959-60** are: Director, Paula Williams; Vice-Director, William E. Seward; Secretary, Ann Loving; Treasurer, William J. Bugg.

ALPHA OF ARKANSAS, University of Arkansas

During the academic year of **1958-59,** the Arkansas Alpha Chapter conducted a tutoring program for students who were having difficulties in mathematics. This service was conducted daily. In the spring, a picnic was held and also a banquet following the initiation ceremonies.

Officers for **1959-60** are: Director, Jon Bamette; Vice-Director, Roger **Dennett;** Secretary, Ann Yancey; Treasurer, Stuart Thomas; Publicity Director, Carol **McCartney;** Tutoring Chairman, Joe **Nosari.**

ALPHA OF PENNSYLVANIA. University of Pennsylvania

The Pennsylvania Alpha Chapter held six program meetings during the academic year of **1958-59. The** following papers were presented:

"This is the Limit" by Dr. W. H. Gottschalk

"Theories of the Creation of the Universe" by Dr. William Protheroe

"The Man and the Lion" - a problem in geometry by Dr. A. S. Besicovitch

"Integration Theory" by Michael **Barr**

"Boolean Algebra" by Murray Eisenberg

"The Laurent-Heaviside Functions" by Mr. Harry Heskett

"The Binominal Symphony in D Minus" by Dr. Pincus Schub

GAMMA OF NEW YORK, Brooklyn College

The New York Gamma Chapter held eight meetings during **1958-59.** The first meeting was devoted to a study of the new by-laws which were adopted. On this occasion it was decided that most of the lectures for the year would be given by student members using "What is Mathematics" by Courant and **Robbins** as a guide for the lectures. In addition to the student lectures, Mr. Ronald Abramoff spoke on one occasion. The title was "The Solution of a Differential Equation."

Officers for Fall, **1958** were: Director, Harold Engelsohn; Vice-Director, Myra Cohen; Secretary, Rochelle **Fuhr;** Treasurer, Alfred Brandstein; Faculty Advisor, Harriet Griffin.

Officers for Spring, **1959** were: Director, Carol Hochman, Vice-Director, Terry Schuster; Secretary, Harry Allen; Treasurer, Donald **Gelman;** Faculty Advisor, Harriet Griffin.

Officers for Fall, **1959** are: Director, Hannah **Wolfson;** Vice-Director, Harry Allen; Secretary, Linda Lerman; Treasurer, Richard Pollak; Faculty Advisor, Harriet Griffin.

EVENDALE AFFILIATE CHAPTER OF OHIO

The **Evendale** Affiliate Chapter of Ohio held two meetings during **1958-59.** These meetings were both business and program meetings. Among the business items discussed were the following: plans for a scholarship committee, a chapter constitution and by-laws, and a standard coloring scheme for the fraternity.

The following papers were presented during the year.

"Use of Series in Psychological Testing" by Mr. Harold Moore

"Zeros of **Polynominals"** by Mr. Gamett Stephens

Five new members were initiated during the year.

Officers for **1960** are: Director, **Garnett** Stephens; Vice-Director, Martha Creal; Secretary-Treasurer, Edward Brooks.

NOTICE TO INITIATES

On initiation into Pi Mu Epsilon Fraternity, you are entitled to two copies of the Journal. It is your responsibility to keep the business office Informed of your correct address, at which delivery will be assured. When you change address, please advise the business office of the Journal.

ALABAMA ALPHA, University of Alabama (December 15. 1959)

Earl Clyde Anderson
Donald S. Asquith
Charles Bernard Barfoot
Patricia P. Barker
Julia D. Brown
Thetford H. Callahan
Wilton Causey
John D. Cloud
Murray J. Dinkins
Charles L. Dumas
Isaac P. Espy
Max L. Guthrie
Paul E. Haug Jr.
Harwell L. Holmes
Betty J. Kushan
Still Hunter
Floyd D. Jury
Mark F. Lasker
Jerome H. LeVan
J. Bennett Lewis
Frank Little
Ellen McInnis
Marijo O'Conner
William J. Phares
Vernon W Pidgeon
Samuel W Powell
George H. Prigge
Bennie F. Shelley
Neel B. Shepard
Earl Tillman
Kenneth S. Williamson
Paul W Womack
Paton L. Woodham

ALABAMA BETA, Auburn University, (January 25, 1960)

Mary Carolyn Arant
Thomas G. Avant
James Taylor Beard
Joe S. Boland III
Carl Michael Bowie
David Allen Conner
James W Cook
Walter G. DeWitt
William E. Faust
Joseph Gera
John Crowell Henry III
Joel C. Hosea
Claude M Kilgore
Henry L. McElreath
Carl L. Perth
Cecil A. Ponder, Jr.
David Macklin Porter
David K. Price
John P. Scheiwe
Joseph A. Self
Clyde Lynn Sharpe
Robert George Springle
Earl H. Weaver
James R. Whitley, Jr.
Charles E. Woodrow, III

ARKANSAS ALPHA, University of Arkansas (November 6, 1958)

Phillip Atterberry
James Edward Bryant
Joyce Caraway
Frankie L. Crutcher
Jack Hammett
Day Mary Hampton
Howard L. Hodges
Wray Henry Jones
Dwain L. Kitchell
Richard L. Lanford
Paul Elo Michaelis
Dorothy W. Ragland
Louis Alden Rose
Elizabeth Ann Simpson
Glenn Everett Spence
Stuart P. Thomas
Mary Eugenia Wray

(November 11, 1958)

Richard O. Ellis
James Oliver Holcomb
Janet Sue Jerome
Fay Ann Jew
Gilbert A. Kane
Edwin Lewis Keith
Arthur Don Kenniker
Elizabeth Marion Logan
Murl Wayne Parker
Robert T. Patton
Billie Frank Rider
Alice Marion Rothert
Ramon Satterwhite
Mary Sue Thornton

(April 9. 1959)

Franklin A. Albey
George Morgan Barnwell
Charles Ronald Bush
Kelly M Carter
Joe Franklin Marlar
Gary L. McClain
Larry Jeff Newkirk
Eldon Joe Nosari
William Roy Tooley
John William Watson

(April 22, 1959)

David Randall Dewitt
Terry Demott Hunt
William Wallace Phillips
James Thomas Taylo

(December 3, 1958)

Jon Hall Barnette
Donald Murray Dallas
Roger D. Dennett
James Louis Dennis
Carolyn Dille Dietrich
Carol Jean McCartney
Calvin Mace Mitchell
David A. Mulkey
Patricia Ann Springer
Danny Bryan Stephens
Ann Yancey

FLORIDA BETA, Florida State University, (January 6, 1960)

Wilson Jean Boaz
Elaine M Bogue
Thomas P. Bower
B. J. Boyer
Daniel J. Cotter
Kenneth H. Crawford
Ewell Thomas Denmark Jr.
Forrest E. Dristy
Harry T. Gaines
Stanley W Harbourt
Donald Charles Martin
Ralph David McWilliams
James W Pegram Jr.
Frank Darwin Reeder
James Ronald Retherford
Frank P. Scruggs Jr.
Robert F. Sturgeon
Gene Watson

FLORIDA BETA (Continued)

Francis Edwin Dean
Dorothy Louise Morton
Linda Lee Potter
Robert F. Woodnal

GEORGIA ALPHA, University of Georgia, (November 11, 1959)

Marion J. Blanchard
Syrus E. Box
Suzanne Chapman
Michael Alan Donahue
Harvey Ralph Durham
William Hoyle Gibbons
Dewey S. Jones
Thomas W. Kethley Jr.
Jerry Michael Kortes
Chattey R. Pittman
Janis Roberson
Leonard M. Scruggs
James Jefferson Sneed Jr.
Chung-Lie Wang
Dianne Wolfe
David Yeeda

GEORGIA BETA, Georgia Institute of Technology (January 31, 1960)

John V. Baxley
James Joseph Buckley
Aubrey Marvin Bush
Ollie Brown Francis, Jr.
William Moore Graves
Ellis Lane Johnson
Louis Truitt Wells, Jr.
Stanley J. Wertheimer
Jack W. Whatley

ILLINOIS GAMMA, DePaul University, (August 19, 1959)

Thomas Trager Cook
Constantine Georgakis
John Knox Hinds
Michael Matkovich
Marilyn Prost

(October 14, 1959)

John J. Kroepfl
Charles Marquardt
John Matese
Natalie Ann Satunas
James G. Voss
Edward Walsh
Warren Wolfe
Edward Zalewski

KANSAS GAMMA, University of Wichita, (December 10, 1959)

F. Kenneth Atnip
Gerald E. Bergmann
John Kirkwood Bonner
Paul A Carver
Dolores Yvonne Covey
Lawrence Gore
Loren Dale Hull
William Allen Keeler
Wayne Riley Maple
Eldon Lynn McKee
Vernal Miller
Richard Earl Monical
Roy Howard Norris
Glenn Eugene Rudder
Larry Kelso Thomas

KENTUCKY ALPHA, University of Kentucky, (January 1960)

Marion Jikl Ball
Richard Andrew Fleck
Charles Ronald Marcus
John Andrew Pfaltzgraff
Cecily Ann Sparks
Shirley Weihe
Diane Yonkos

LOUISIANA ALPHA, Louisiana State University, (December 9, 1959)

Ronald J. Allain
James B. Allen
Alex Begrowicz Jr.
Elizabeth Blewer
Anna Ruth H. Brumfield
Charles W Chapaton Jr.
Gertrude Creaghan
Wasil Curtioff
David M. Egle
Jack Goods
Pete Gracia
Wayne A. Hall
Thomas Hatchett
Jack Hays Jr.
Frances Hightower
Linda Hill
Charles R. Hoffpauir
William P. Keiser
Eric Lane
Morris J. McRae
Grace Peerson
Teddy M. Pledger
Donald L. Porter
Marian Rooks
James Sawyer
Thomas Scannicchio
Charles Steib
Josephine Strong
Joseph L. Templet Jr.
Marvin Trask
John C. Wilson
D. Wayne Wilson Jr.
Adril L. Wright

MICHIGAN ALPHA, Michigan State University, (May 14, 1959)

Richard W. Wagner

(January 1960)

Gerald K. Austin
William Barker
Nancy Bruce
Fred Gilman
David W. Halsted
William Hudec
Douglas B. Ostien
Francis E. Paris
James Rossi

MICHIGAN ALPHA (Continued)

Donald K. Creyts
James R. Elmleaf
Ron V. Estes
Sharon L. Kron
Merrit Mallory
Richard F. Melka
James J. Murdock
Peter Schaldenbrand
William Welsh
Elizabeth L. Woodward

MISSOURI ALPHA, University of Missouri, (December 9, 1959)

Kenneth George Aubuchon
Rita Rose Boston
John Arthur Connor
William R. Covington
Larry Dean Diehl
Carol Ann Dundley
Barbara Ann Eckley
Eugene Merle Engle
Brian Colyer Hanrahan
Robert Francis Haukap
Vernon Henry Horton
Billy Gail Kay
Albert Kuroyama
John Russell Michel
James Clesie Moore
Stanley Alan Myers
Tom Wilmer Myers
George Ewell Osborne
John D. Scobee
Thompson N. Tate Jr.
Ellen Louise Thomas
Alexander Bixby Willis

MONTANA BETA, Montana Slate University, (April, 1959)

Loren W. Acton
Lloyd G. Becraft
David Carlstrom
David A. Cromley
Robert Currie
Richard Hamm
Adrien L. Hess
Allen L. Hess
Raymond P. Hitchock
John W. Hurst
William H. Jamison
Donald J. Kastella
Robert E. Lowney
Richard A. Mahugh
Vinnie Miller
Ellen Missal
Charles J. Mode
Robert Motsch
Raymond Mountain
Jerry Sanders
Fred T. Schilling
Gary Schrieber
Eugene Schumacher
James L. Simpson
Colin W. Skinner
George E. Uhrich
Douglas S. Wattier
J. Eldon Whitesitt
Kenneth D. Wiegand
Marvin J. Woodring

NEBRASKA ALPHA, University of Nebraska (December 1, 1959)

John P. Andersen
Robert Charles Anderson
Sonia Ruth Anderson
John Arnold Byram
Thomas F. Eason
Duane B. Eickhoff
James Austin Glathar
Jerry Dell Harris
Lane Cameron Isaacson
Robert Charles Johnson
James Eugene Kellogg
Dennis Roger Krause
Bruce Alfred Marts
Dennis B. Nelson
Russell Lee Rasmussen
Mary Eileen Schmelzer
Gene Arnold Schriber
Donald Frank Torczon
Lawrence Scott Tucker

NEW JERSEY ALPHA, Rutgers University, (November 30, 1959)

William L. Augustine
William T. Bisignani
Hugh Graven Brady
Claude Allan Bugg
Milo Burnham
Arthur Philip Daley
Robert De Laurentis
Ralph Peter Doelling
Leroy William Dubeck
Charles B. Greenburg
Lewis M. Greenburg
James V. Griepenburg
Jerome Golub
Edward P. Helpert
Walter C. Hersman
Philip Hirschfield
Thomas Hollinger
Robert Emil Holle
Arthur Barry Kagan
Kenneth T. Keller
Tbomas Jacob Kessler
Harold Henry Klug, Jr.
Louis John Koczela
Robert Jon Kraus
Raymond H. Kurland
Fred Fouse Lange
George Henry Lenz
Robert John Marsala
David P. Martin
Richard G. Murdock
Anthony M. Passalaqua
Howard M. Phillips
Kurt A. Pocsi
Thomas Joel Presby
Howard M. Relies
Montreville Shinn, III
Walter C. Tetschner
Clemens M. Thoennes
Felix Thomas Ullrich
John A. Vernon
John V. Wierzbicki
Barry Arnold Wittman

NEW JERSEY BETA, Douglass College, (November 30, 1959)

Joan Alyn Rasken

NEW YORK BETA, Hunter College, (November 18, 1959)

Geneva Butts
Harvey Carroll
Marcia Deghan
Lorraine Fu
Mary Garguilo
Sarah Goldberg
Susan Koppelman
Betty Levine
Mary Minchilli
Maureen O'Donnell
Marilyn Scharf
Betty Sotirakis
Walter Staubi
Alice Weiss

NEW YORK GAMMA, Brooklyn College, (November 23, 1959)

Virginia Berlin
Elaine C. Blatt
Sydelle Baron
Ann Fassler
Jay Goldman
David Goodstein
Marcus Horowitz
Amy Lassner
Robert MacCormack
Bernard Rosenbacb
Joseph B. Seif
Claire Sommers
Joel Spiro
Robert Shloming
Saul A. Zaveler
Carl Kirshen

NORTH CAROLINA ALPHA, Duke University, (December 9. 1959)

Norma Sue Barnes
Mark Brownlow Edwards
William H. Halliday Jr.
Harriet Joan Naviasky
Philip Cox Smith
Samuel Sheung-lok So
Diana Tilley Strange
Bobbye K. Whitenton
Lawrence S. Williams
Warren Hoyle Young Jr.

NORTH CAROLINA BETA, University of North Carollina, (October 22. 1959)

Willard Ray Bagwell
Frederick W. Blackwell
George Donald Brock
Robert L. Cannon Jr.
Lonnie Irving Carey
Nancy Cleveland
Robert K. Coe
Isabella Jeffreys Cole
Penelope Carter Crockett
Mary Diniwiddie Crow
Patricia DeLashmutt
George Marvin Eargle
David Quitman Garrison
Jacob F. Golightly
Richard Lamar Hastings
Rose Hayes Hawk
Clifford Heindel
Henry G. Howell
Judith G. Huntress
Peter H. Jessner
Robert Stanley Johnson
Joseph Donald Jones
Ruth Ellen Kurlzwiel
Paul M. LeVasseur
Edmund P. Lively
Albert Margolis
Thomas Lowell Markham
Thomas Hawkes Nash Jr.
Cordelia R. Robinson
Frank Gerhard Schafstedde
William David Smith
William Henry Somerville
Robert E. Spencer
Douglas Spiegel
John L. Stone III
James A. Ward Jr.
Charlie Thomas Whitley
Gabrella Palmer Wilson

OHIO BETA, Ohio Wesleyan University, (March 10, 1960)

Wm Andrew Boag, Jr.
James Thomas Campbell
George H. Comrades
Edwin J. Elton
Tbomas F. Fitzsimons
Richard E. Gillespie
John Harry Ginaven
Karen Elizabeth Gram
Arnold A. Johnson
E. Jane Johnson
Robert Bruce Keller
Peter Miroslav Mrdjen
Phyllis Marie Patterson
Gloria Wurst Priest
Paul Frederick Richards
Ruth Ellen Smith
Margaret G. Tbomas
John Herman Welch
Robert Lee Wilson. Jr.

OHIO DELTA, Miami University (Ohio) (November 5, 1959)

Paul Faust
Kenneth M. Glover
Donald F. Grether
Wayne Edward Kimmel
Lerene B. Lambert
Leo F. Lightner
John William Mandt
Sandra J. Merry
Jobn Robert Miles
Joseph O. Reebel
Mary Frances Ritter
Rob Roy Rogers
John Marshall Rose
Charles Tracy Smith

OKLAHOMA ALPHA, University of Oklahoma (December 8, 1959)

Archie D. Brock
Joe M. Egar
Michael L. Howe
Neal F. Lane
Mickel A. Lynch
Lloyd L. Koontz
James E. Mantooth
Marvin L. McDaniel
Charles A. Nicol
Jean D. Reed
James H. Rice
Marilyn K. Smith
Eddie C. Tacker
Natalie R. Tarter
Wallace H. Tucker

OKLAHOMA BETA, Oklahoma State University, (January 13, 1960)

Jerry Borocboff
W. Louis Byerly
Ernest Chace
H. Allen Evans
Sr. Mary Firmina
Glen Haddock
Melvin Roy Hagan
Bob A. Hardage
Margaret Rose Horn
Reza Kordbacheh
Wayne Lanier, Jr.
Joe Alex McKenzie
William R. Pogue
George S. Reeves
Jahangeer Shafa
Esber Shaheen
Winston G. Shindell
James D. Simpson
Thomas Wayne Ulmer
Robert E. Winters

PENNSYLVANIA BETA, **Bucknell** University, (December 2, 1959)

Margrete Joyce Anderson
Robert Arthur **Artman**
Robert Wm. **Bartlett**
Ira Mark Brim
Robert **Merrill** Brodrick
Robert Hugh Brown
Doris Jean Bryson
Kai **Leung** Chu
Wm. Franklin Caul, Jr.
Marlyn Robert **Etzweiler**
Sally Strode **Fackler**
Suzanne Carol **Friedman**
Roger Marshall Goodman
Barbara Ann Hall
Linda Hardy
George Lommel **Kenyon**
Carl **Dowell** Keefer
Susan **Amelia** Kilgore
Wilma Louise **Schatzle**
Jon Lowell Teeter
Priscilla Madeline Teleky
Richard Louis Werner

(February 25, 1960)

Robert Ray Medley

PENNSYLVANIA DELTA, Pennsylvania State Unive**rsity,** (December 15, 1959)

Nancy Jean **Balogh**
Vernon D. Barger
Carlton J. Bates
Arleen Joyce Bickel
Walter H. **D'Ardenne**
Elizabeth Ellen Dowling
Daniel **Eastman**
Catherine L. **Engel**
George A. **Etzweilter**
Robert **S.** Fine
John Albert Fridy
Gilbert Howard **Friedman**
Jack Gaudinsky
Gary **L. Gentzler**
Stanley **S. Glickstein**
Anthony W. Hager
Jack I. Hanoka
Lothar **O.** Hoeft
Alan M. Jacobs
John J. **Marley**
Joseph **E. Martine**
David **McGrew**
Robert Dale Moyer
Joseph Francis Mulson
Glenn W. **Olsen**
Susan **E. Reen**
Edward **J.** Rodgers
Allan R. Sattler
Paul Howard **Schoener**
Harry Eugene **Shull**
Theodore **R. Sykes**
William L. Young

SOUTH CAROLINA ALPHA, University of South Carolina (December 15. 1959)

Charter **Members**

Joe D. **Bickley**
Wade T. Cathy, Jr.
James **R.** Faulkner
Marlin **R. Gillette**
Hsiu **Huang**
Raymond A. **Lytle**
Donna **McCay**
Joseph D. Novak
John H. **Overton**
Robert M. **Siegmann**
William D. Stanley
David D. **Strebe**
Kenneth **R. Symonds, Jr.**
Alfred A. Y. Tang
Wyman L. Williams

Non-Charter Members

L. Carol Cobb
Martin Duszynski, Jr.
Hugh B. Easter
Wai-Kit Leung
Charles F. Martin
Karly H. **Matthies**
Frances **McFadden**
Richard **Dowling** Nix
Lillian **Perkins**
C. Ecebia Shuler
Hans **Sonner**
Chin-Sha Wang
W. Wilson Weber

VIRGINIA ALPHA, University of Richmond, (November 23, 1959)

Nancy Lois Adams
Hilton Robinson Almond
Betty Wade **Blanton**
Elizabeth Lyons Bond
Anne Louise Cunningham
Virginia Dix **Hargrave**
Anna **Rudik** Jones
Robert **Landon** Meredith
Betty M. Pritchett
Reginald Turner Puckett
Barbara Lee **Randlett**
William Carter Roberts
Jessica B. Scarborough
Joyce Elizabeth Steed
Carey Elliott **Stronach**
Raoul Louis Weinstein
David **Allan** West
James Edwin Williams

WISCONSIN BETA, University of Wisconsin, (October 16, 1959)

Kenneth P. Casey
Robert A. **DiPaola**
Robert M. **Gasper**
Leslie C. Glaser
Charles A. Green
Martin **S. Hanna**
Ann Marie Peters
Tadepalli V.S.L.N. **Sastry**
Robert **J. Sliben**
Daniel J. Sterling
Kenneth Wm. **Weston**
Leslie **E. Whitford**