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PLANE GEOMETRY WITHOUT POSTULATES[a]

LEONARD M. BLUMENTHAL[1]
Missouri Alpha, 1937

Introduction. In his famous ***Elements,*** Euclid made a distinction, usually neglected today, between certain assumptions that he called ***axioms,*** and others that he referred to as ***postulates.*** The former characterized those basic statements that expressed ***common notions*** **(e.g.,** If equals are added to equals, the sums are equal) while the latter term was used for those assumptions that are ***peculiar to geometry*** **(e.g.,** To describe a circle with any center and radius).

The title of this paper is to be understood in the light of that ancient discrimination. The basis for euclidean plane geometry presented here contains no postulates whatever (and consequently is free of ***primitive*** or ***undefined*** terms), but it is not devoid of common notions. The sole such notion that will be used is that of real number. This notion occurs in nearly all systems of postulates for geometry, usually without attention being called to its status as an axiom.

It was recently observed that "according to modern standards of rigor, each branch of pure mathematics must be founded in one of two ways: (1) either its basic concepts must be defined in terms of the concepts of some prior branch of mathematics (in which case its theorems are deduced from those of the prior branch of mathematics with the aid of these definitions) or else (2) its basic concepts are taken as ***undefined*** and its theorems are deduced from a set of postulates involving these undefined **terms."**[2] This foundation of plane geometry utilizes the first way; its basic concepts are defined in terms of that prior branch of mathematics - the real number **system.**

1. Points. Let $p_1p_2 = p_2p_1$, $p_2p_3 = p_3p_2$, $p_1p_3 = p_3p_1$ denote three ***positive real numbers*** (the reason for selecting this notation for the three numbers will be made clear later) such that the determinant

$$D(p_1,p_2,p_3) = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & p_1p_2^2 & p_1p_3^2 \\ 1 & p_2p_1^2 & 0 & p_2p_3^2 \\ 1 & p_3p_1^2 & p_3p_2^2 & 0 \end{vmatrix}$$

is ***negative.*** There are many such triples of numbers; indeed, any three positive numbers, each one of which is less than the sum of the other two, may be selected.

[a]**Received** by the editors June 23, 1960

[1]**See** footnotes.

Definition. A point is any ordered triple of *non-negative* real numbers (rp_1, rp_2, rp_3) such that

$$D(p_1,p_2,p_3,r) = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & p_1p_2^2 & p_1p_3^2 & p_1r^2 \\ 1 & p_2p_1^2 & 0 & p_2p_3^2 & p_2r^2 \\ 1 & p_3p_1^2 & p_3p_2^2 & 0 & p_3r^2 \\ 1 & rp_1^2 & rp_2^2 & rp_3^2 & 0 \end{vmatrix} = 0,$$

where $rp_i = p_ir$, $(i = 1,2,3)$.

If r denotes the point (rp_1,rp_2,rp_3) and s the point (sp_1,sp_2,sp_3), we define $r = s$ if and only if $p_ir = p_is$, $(i = 1,2,3)$.

Let P denote the set of all points.

Theorem 1. *If r is the point* $(0,rp_2,rp_3)$, *then* $rp_2 = p_1p_2$ *and* $rp_3 = p_1p_3$.

Proof. The determinant $D(p_1,p_2,r)$ may be written in factored form $D(p_1,p_2,r) = -(p_1p_2+rp_2+rp_1)(p_1p_2+rp_2-rp_1)(p_1p_2-rp_2+rp_1)(-p_1p_2+rp_2+rp_1)$, and since $rp_1 = 0$,

$$D(p_1,p_2,r) = (p_1p_2+rp_2)^2(p_1p_2-rp_2)^2 \geqq 0.$$

But since $D(p_1,p_2,p_3,r) = 0$, it follows that

$$D(p_1,p_2,p_3)\cdot D(p_1,p_2,r) - [rp_3^2]^2 = 0,$$

where $[rp_3^2]$ denotes the minor of rp_3^2 in $D(p_1p_2,p_3,r)$.[3]

This equality, together with $D(p_1,p_2,p_3) < 0$, implies that $D(p_1,p_2,r) \leqq 0$, and consequently

$$D(p_1,p_2,r) = 0.$$

Hence $rp_2 = p_1p_2$, and in a similar manner it is shown that $rp_3 = p_1p_3$.

Denote the point $(0,p_1p_2,p_1p_3)$ by p_1, the point $(p_2p_1,0,p_2p_3)$ by p_2, and the point $(p_3p_1,p_3p_2,0)$ by p_3. (It is easily shown that $(0,p_1p_2,p_1p_3)$, etc. *are* points).

2. **Distances.** We are now in position to define the important notion — *distance of two points.*

Definition. If $p = (pp_1,pp_2,pp_3)$ and $q = (qq_1,qq_2,qq_3)$ are two points of R, their distance is the unique non-negative real root of the equation $D(p_1,p_2,p_3,p,q;x) = 0$, where

$$D(p_1,p_2,p_3,p,q;x) = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & p_1p_2^2 & p_1p_3^2 & p_1p^2 & p_1q^2 \\ 1 & p_2p_1^2 & 0 & p_2p_3^2 & p_2p^2 & p_2q^2 \\ 1 & p_3p_1^2 & p_3p_2^2 & 0 & p_3p^2 & p_3q^2 \\ 1 & pp_1^2 & pp_2^2 & pp_3^2 & 0 & x^2 \\ 1 & qp_1^2 & qp_2^2 & qp_3^2 & x^2 & 0 \end{vmatrix}$$

This definition must be justified by showing that the equation has a unique non-negative real root.

Justification of distance definition. Since $D(p_1,p_2,p_3,p) = 0$ it follows (as in the proof of Theorem 1) that $D(p_1,p_2,p) \leqq 0$, $D(p_1,p_3,p) \leqq 0$, and $D(p_2,p_3,p) \leqq 0$. Now the equality sign cannot hold in all three relations for if it did, then $D(p_1,p) = D(p_2,p) = D(p_3,p) = 0$, and expansions of these vanishing bordered third-order determinants yields $pp_1 = pp_2 = pp_3 = 0$. But this is impossible, for then the determinant $D(p_1,p_2,p_3,p)$ reduces to $-2.p_1p_2^2.p_2p_3^2.p_1p_3^2$, which does *not* vanish. Hence we may assume $D(p_1,p_2,p) < 0$.

Denote a root of $D(p_1,p_2,p_3,p,q;x) = 0$ by pq. From $D(p_1,p_2,p_3) \neq 0$, while $D(p_1,p_2,p_3,p) = D(p_1,p_2,p_3,q) = D(p_1,p_2,p_3,p,q) = 0$, we conclude that the rank of $D(p_1,p_2,p_3,p,q)$ is 4, and consequently $D(p_1,p_2,p,q) = 0$. It follows that

$$D(p_1,p_2,p)\cdot D(p_1,p,q) - [pq^2]^2 = 0,$$

where $[pq^2]$ denotes the minor of pq^2 in the determinant $D(p_1,p_2,p,q)$, and since $D(p_1,p_2,p) < 0$, then $D(p_1,p,q) \leqq 0$.

Consider the function

$$y = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & p_1p^2 & p_1q^2 \\ 1 & pp_1^2 & 0 & x \\ 1 & qp_1^2 & x & 0 \end{vmatrix}$$

Its graph is a parabola whose axis is perpendicular to the x-axis, and which opens upward. It crosses the x-axis at the points $x = (pp_1 - qp_1)^2$ and $x = (pp_1 + qp_1)^2$. If pq^2 be substituted for x in the function, the corresponding functional value $y = D(p_1,p,q) \leqq 0$, and it follows that
$0 \leqq (p_1p - p_1q)^2 \leqq pq^2 \leqq (p_1p + p_1q)^2$. Hence $pq^2 \geqq 0$.

Since $D(p_1,p_2,p_3,p) = 0$, x is a root of $D(p_1,p_2,p_3,p,q;x) = 0$ if and only if x is a root of

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & p_1p_2^2 & p_1p_3^2 & p_1p^2 \\ 1 & p_2p_1^2 & 0 & p_2p_3^2 & p_2p^2 \\ 1 & p_3p_1^2 & p_3p_2^2 & 0 & p_3p^2 \\ 1 & qp_1^2 & qp_2^2 & qp_3^2 & x^2 \end{vmatrix} = 0.$$

This equation obviously has at most one non-negative **real** root, and the above argument shows that $x = pq \geqq 0$ is such a root.

Remarks. The distance of points $p_1 = (0,p_1p_2,p_1p_3)$ and $p_2 = (p_2p_1,0,p_2p_3)$ is p_1p_2; of p_2 and $p_3 = (p_3p_1,p_3p_2,0)$ is p_2p_3; and of p_1 and p_3 is p_1p_3. This explains why the notation p_1p_2, p_2p_3, p_1p_3 was adopted for the three positive numbers selected at the outset. They turn out to be the mutual distances of the points p_1,p_2,p_3.

The distances of a point $p = (pp_1,pp_2,pp_3)$ from points p_1,p_2,p_3 are pp_1,pp_2,pp_3, respectively. Hence we have a *tri-polar* coordinate system in P.

3. Congruence **of** P with E_2. With respect to the distance defined in Section 2, the **pointset** P is a metric space; that is, if **p**, q are any two points of **P**, distance $pq \geqq 0$, $pq = qp$, $pq = 0$ if and only if $p = q$, and if r is a third point of P, then $pq + qr \geqq pr$. (Perhaps the reader can supply the easy proof of this assertion).

Theorem 2. The metric space P is congruent with the euclidean plane E_2; that is, there is a one-to-one, distance-preserving correspondence between the points of **P** and the points of E_2.

Proof. Since P is a metric space, the plane contains three **points p_1',p_2',p_3' such that** $p_1'p_2' = p_1p_2$, $p_2'p_3' = p_2p_3$, $p_1'p_3' = p_1p_3$. We symbolize this by writing

$$p_1,p_2,p_3 \approx p_1',p_2',p_3' \quad ,$$

read "p_1,p_2,p_3 are congruent to p_1',p_2',p_3'".

Let p' denote any **point** of E_2, and consider the three non-negative numbers $pp_1 = p'p_1'$, $pp_2 = p'p_2'$, $pp_3 = p'p_3'$, where $p'p_i'$ denotes the distance in the plane of the points p' and p_i', (i = **1,2,3**). Clearly

$$D(p_1,p_2,p_3,p) = D(p_1',p_2',p_3',p') \quad ,$$

and since (as is well-known) the D-determinant vanishes for each quadruple of points of the plane, then $D(p_1,p_2,p_3,p) = 0$ and so $p = (pp_1,pp_2,pp_3)$ is a point of P. Hence, with each point p' **of** E_2 there is associated a unique point of P.

It is easily seen that each point q of P is the associate **of** exactly one point q' of E_2. For since p_1,p_2,p_3,q are a metric quadruple and $D(p_1,p_2,p_3,q) = 0$, it follows **that** the plane contains four points

contains four points $\bar{p}_1,\bar{p}_2,\bar{p}_3,\bar{q}$ congruent to that quadruple; that is,

$$p_1,p_2,p_3,q \approx \bar{p}_1,\bar{p}_2,\bar{p}_3,\bar{q}.^4$$

Then $\bar{p}_1,\bar{p}_2,\bar{p}_3 \approx p_1,p_2,p_3 \approx p_1',p_2',p_3'$ **, and there is a** congruence of the plane with itself that carries $\bar{p}_1,\bar{p}_2,\bar{p}_3$ into p_1',p_2',p_3', respectively. This congruence is unique (for since $D(p_1',p_2',p_3') = D(p_1,p_2,p_3) < 0$, **the points** p_1',p_2',p_3' **are not** collinear) and carries $\bar{q}$ into a point q'. Consequently

$$p_1,p_2,p_3,q \approx p_1',p_2',p_3',q'$$

and so $q = (qp_1,qp_2,qp_3) = (q'p_1',q'p_2',q'p_3')$; **that is, q is** associated with q'. Hence the association or mapping from E_2 to P is onto.

The mapping is a congruence. For if p, **q** are points of **P**, corresponding to p', q', respectively, of E_2 then by definition, distance pq is the unique non-negative real root of

$D(p_1,p_2,p_3,p,q;x) = 0$. **Since** $p_1',p_2',p_3',p',q' \in E_2$, $D(p_1',p_2',p_3',p',q') = 0$; that is, $p'q'$ is the unique non-negative root of the equation $D(p_1',p_2',p_3',p',q';x) = 0$.

From the congruences

$$p_1,p_2,p_3,p \approx p_1',p_2',p_3',p'$$

$$p_1,p_2,p_3,q \approx p_1',p_2',p_3',q'$$

it follows that $D(p_1,p_2,p_3,p,q;x) \equiv D(p_1',p_2',p_3',p',q';x)$, and consequently $pq = p'q'$, completing the proof of the theorem.

4. Logical identity of P and E_2. From the congruence of P with the euclidean plane E_2, established in Theorem 2, it is easy to show that P is logically identical with the euclidean plane. This is done by selecting any system of postulates for the plane (say, those of Hilbert) *defining* the primitive notions of that system (e.g., line, betweenness, etc.) and establishing the postulates as *theorems.*

This being done, a second question arises: is euclidean geometry easily developed on this basis? The reader might be interested in answering this second question for himself.

University of Missouri

FOOTNOTES

1. Presented by invitation at the Warrensburg meeting of the Mathematical Association of America, April 30, 1960.

2. Leon Henkin, *On mathematical induction,* Amer. Math. Monthly, 67 (No. 4), 1960, pp. 323-337.

3. See Corollary 3, page 33 of M. Bocher, *Introduction to higher algebra* Macmillan, 1922.

4. See pp. 99-101 of L. M. Blumenthal, *Theory and applications of distance geometry,* The Clarendon Press, Oxford, 1953.

AN APPRECIATION OF GIUSEPPE PEANO

HUBERT C. KENNEDY
Missouri Gamma, 1959

The purpose of this article is to recall some of the positive contributions of Giuseppe Peano in the field of mathematics and to point out some rather paradoxical reasons why only part of his work has gained general acceptance. In this short note we cannot do justice to the mathematician who, according to Bertrand Russell, "has a rare immunity from error." Giuseppe Peano was born at Spinetta (near Turin) on 27 August 1858. A graduate of the University of Turin, he was professor there from 1890 until his death on 20 April 1932. His interests were varied and he made important contributions in several fields. We consider here only his work in mathematics and begin with two contributions of the first rank which are known by his name. These are the Curve of Peano and Peano's Postulates for the Natural Numbers.

The first of these was the first (and by now classic) example of a "space-filling" curve. It is, in fact, a continuous function of the unit interval onto the unit square, such that the curve passes, in a continuous manner through every point of the unit square. In the critical revision of the foundations of mathematics, which began in the second half of the last century, the discovery of this curve was a landmark in the search for the characteristic distinguishing lines, surfaces, and solids. The first landmark was reached by Georg Cantor in 1878. Before then it was thought that the characteristic distinguishing lines, surfaces, and solids was in the number of points pertaining to each, i.e. that the number of points on a line was a good deal less than the number of points in a plane or solid. However, if we generalize for the moment the common notion of equality of number between two sets of physical objects, i.e. letting U and V be two sets of any objects whatever, we say that they have the same number of members when it is possible to establish a one-to-one correspondence, without exceptions, between U and V, then Cantor obtained the result – at first sight paradoxical – that the number of points of a linear segment is equal to the number of points of a plane or a solid.

Another property of the intuitive "line" is that which makes the concept of line depend on the concept of motion. Thus since Descartes and others had shown that a point in the plane could be represented by two numbers (Cartesian coordinates), it was believed that a final exact form could be given to the intuitive concept of line by identifying it with a continuous curve, i.e. by defining a (continuous) curve in a plane to be the set of points whose coordinates x,y satisfy the parametric equations:

$$x = f(t)$$
$$y = g(t)$$

where f and g represent continuous functions of t, and are defined on the same interval. In this definition the concept of continuous function has replaced that of intuitive motion. This, then, was the situation when **Peano** gave his sensational discovery in four short pages of the ***Mathematische Annalen*** of 1890 in an article entitled "**Sur** une courbe **qui** remplit toute une aire plane." **Peano** had found the parametric equations of a continuous curve in a plane which filled a square and of a continuous curve in space which filled a cube.

The Curve of **Peano** has been treated by various authors. We give it according to the geometrical construction of the fifth volume of the Formulaire de Mathematiques. Divide the interval $[0, 1]$ into nine equal parts, and number them in order as $1, 2, \ldots, 9$. Then divide the square into nine equal parts as in Figure 1, and number them $1, 2, \ldots, 9$ to correspond with the segments of the linear interval. Next divide each segment of the straight line into nine equal parts and each of the nine squares into equal parts as in Figure 2. The 81 squares so formed are then numbered in order, so that each square has one side

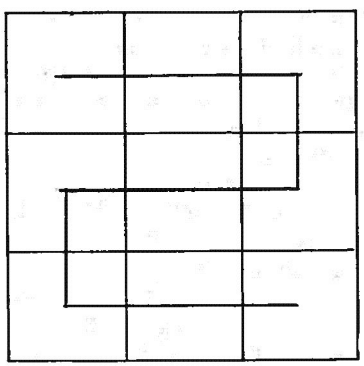

Fig. 1

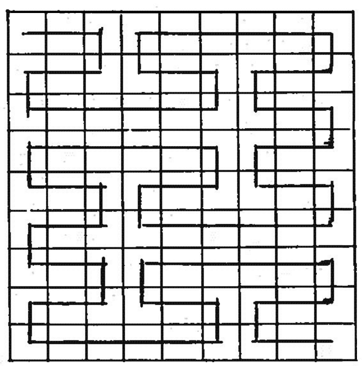

Fig. 2

in common with the one next in order; the squares then correspond with the segments numbered in the same way. Proceeding in this manner indefinitely, any point of $[0,1]$ is determined by the intervals of the successive sets of **sub-divisions** in which it lies. The corresponding point in the square area is determined by the succession of squares, each containing the next, in which it lies. The curve is thus determined as the limit of a sequence of broken lines denoted by the thickened lines in the figures. The curve thus obtained is continuous, but has no tangent. It should be noted that in contrast with Cantor's function, which is not continuous, **Peano's** curve is not one-to-one.

The year following the publication of "Sur une courbe qui remplit toute une aire plane," **Peano** purchased a small house in Cavoretto, where he had a copy of Figure 2 made on the balcony with small bits of cement, so that it showed up in black on the white tiles. (He could have had a copy of the ***completed*** curve by using only one black tile, since the completed curve looks like this: ■)

The Postulates for the Natural Numbers mark the last milestone in the process of axiomatization of arithmetic, which also began in the last century. H. Grassmann prepared the way when in his ***Lehrbuch der Arithmetik*** (1861) he showed that the commutative law can be derived from the associative law by means of the principle of complete induction. **Peano** first gave postulates for the natural numbers in 1889 in his ***Arithmetices Principia, Nova Methodo Exposita.*** Although much symbolism is used, the introduction and explanatory material of this short book of 36 pages are in Latin. Because of this, the later treatment of this matter in the early volumes of the ***Formulaire de Mathematiques,*** being written in French, probably had a wider reading audience.

The postulates, which are nine in number in the ***Principia,*** are reduced to five in the ***Formulaire.*** They may be stated as follows:

1. Zero is a number.
2. The successor of any number is another number.
3. There are no two numbers with the same successor.
4. Zero is not the successor of a number.
5. Every property of zero, which belongs to the successor of every number with this property, belongs to all numbers.

Peano showed that the whole structure of arithmetic can be derived from these five postulates, as purely logical conclusions, without any further reference to the intuitive significance of the arithmetical operations. They have been criticized on philosophical grounds as not completely characterizing the natural numbers. An example from B. Russell will illustrate this.

Let "O" be taken to mean 100 and let "number" be taken to mean numbers from 100 onward in the series of natural numbers. Then all our primitive propositions are satisfied, even the fourth, for, though **100** is the successor of 99, 99 is not a "number" in the sense which we are now giving to the word "number." It is obvious that any number may be substituted for 100 in this example.

The five postulates, then, do not characterize the natural numbers. **They** are characteristic of a much more general concept, that of a progression of any objects whatsoever which has a first member; contains no repetitions, and for each member of which there is an immediate successor.

In the face of this easy criticism two objects seem depreciated. These are **Peano's** Postulates and **Peano** himself. Because the postulates do not characterize the natural numbers, we tend to forget that they are nonetheless the completion of the process of reducing the number of basic laws of arithmetic by deriving them from fewer but **deeper**-lying propositions. They are the conclusion of the development begun by Grassmann, for, as F. Waismann remarks, "in the **Peano** axioms we actually have already reached the last starting point of arithmetical deductions." When we recognize this we are better able to appreciate **Peano,** for, it seems to me, if the postulates are criticized because they do not characterize the natural numbers, we criticize **Peano** for not having done what he in fact did not intend to **do. Peano** was aware from the beginning that the postulates do not characterize the natural numbers. He was familiar with R. Dedekind's essay ***Was sind und was sollen die Zahlen?*** (1888) in which this situation is noted. **Peano** shared with Dedekind the notion that the natural numbers are gained by some kind of abstraction from all systems satisfying the postulates. He recognized that with the Postulates he had completed the "**arithmetization**" of mathematics, **i.e.** the clarifying of the role of arithmetic as the sole and last base of the entire structure of analysis. **Peano** succeeded, as Prof. Geymonat says, "in defining all the highest concepts and the most difficult operations of mathematics starting from the arithmetic of the natural numbers." B. Russell wrote: "**Peano . . .** represents the last perfection of the 'arithmetization' of mathematics."

Much of the symbolism of modern symbolic logic and set theory was developed and clarified by G. **Peano.** In many cases the actual forms of the symbols were introduced by **Peano. If** typesetters had the deciding voice as to which symbols to **use,** or those who have to pay the costs of printing a book containing many formulas, as **Peano** remarks, it is very likely that we would today use a greater number of **Peano's** symbols, for he was not only concerned with giving them precise meanings for the maximum usefulness in mathematics, hut he even purchased printing equipment which he established in his home in Cavoretto and took an active part in the publication of the ***Rivista di Matematica,*** founded by him in 1891.

Let us take this year, 1891, as a vantage point in time from which to view the mathematical accomplishments of **Peano.** Already mentioned were the publication of the ***Principia*** in 1889 and the "**Sur** une courbe" in 1890. With regard to his studies in mathematical logic, he had demonstrated an equivalence between the calculus of propositions and the calculus of classes, and had made a complete analysis of the logical concepts which occur in mathematical reasoning. It would take too much space to adequately approach this subject. We may take as typical of his important, and pioneering work **Peano's** treatment of the symbols ϵ **and** ι in set theory. (In form, the first is the initial letter of the Greek ἐστί , "is", the second, the initial letter of ἴσος , "equal to.") The first of these, usually in the modified form $\in$, is in common use today with the same meaning given it by **Peano, i.e.** "is a member of." It was introduced in the ***Principia,*** where the necessity was noted of distinguishing the sign • from **the sign** $\supset$ (nowadays written $\subset$) indicating the relation "is contained in." Still lacking, however, was the distinction between a single object and the class made up of that object alone. This distinction was supplied the following year with the introduction of ι for "the set whose sole member is", **i.e.** ιa is the set whose sole member is a (We would write today "$\{a\}$.") This also allowed the decomposition of = (equality between classes) into the succession $\epsilon\iota$; thus, in place of a = b, we may write $a \epsilon \iota b$ As an instance of **Peano's** careful and conservative symbolism we may point out his use of inverted symbols to express the concept inverse to that of the original symbol, **e.g.** $\rotatebox{180}{\iota} a$ is the sole member of the set *a,* and $b \ni a$ can be used interchangably **with** $a \epsilon b$. We continue to follow **Peano** in the use of $\subset$ and $\supset$ to represent inverse concepts. It is unfortunate that adequate symbols for the concepts represented by $\ni$ and $\rotatebox{180}{\iota}$ are not in common use today.

Looking forward, now, from the year 1891, we see that **Peano,** with the symbolism he had developed for logic, began to treat logic mathematically and to apply his logical symbolism to the study of the foundations of mathematics. As a first consequence of this symbolism, he was able to develop a mathematical symbolism which allowed him to give a completely symbolic form to all mathematical propositions. The project of the ***Formulaire de Mathematiques*** furnished the immediate application of this mathematical symbolism. This monumental work, taken as a whole, constitutes the most important of **Peano's** mathematical publications from 1895 to 1908 and consists officially of five editions, or tomes.

This project was announced in 1892, when **Peano** gave the following statement of its purpose: "It would be extremely useful to publish a collection of all known theorems pertaining to the different branches of mathematics, so that the scholar would have only to consult such a collection in order to know how much has been done on a given point, and whether his research is new or not." It should be noted that the five volumes of the ***Formulaire*** do not treat separately different parts of mathematics. In general, each volume contains essentially the material of the proceeding volume while at the same time developing and enlarging it. We have seen, for example, that already in 1889 postulates were given for the natural numbers. They received definitive statement in Tome **II** (1898). How pioneering this work really was can be seen from the fact that **Peano** was able to say that year, in commenting on Tome **II,** that "the analysis of the ideas of arithmetic contained in it is the only one in existence."

The various volumes of the ***Formulaire*** are not solely the work of **Peano.** In fact, in the 1892 announcement he said: "We shall be grateful to readers who will help us in this work by collecting propositions (with or without proof) of other points in mathematics." Cooperating in the work were his university assistants F. **Castellano,** G. Vailati, and C. **BuraliForti,** and various others, including Bettazzi, **Fano,** Giudice, **Padova,** Pagliero, Vacca, Vivanti, Couturat, **D'Arcais,** Morera, Pieri, and Severi. The project, however, was and remained **Peano's.** The fact that the ***Formulaire*** is universally referred to **Peano** is indicative of the major part he played in its inception and continuance. This is in contrast to the **"Bourbaki"** group of a dozen or so modern French mathematicians, who are engaged in writing a comprehensive account of mathematics. Among them, no one person is of major importance, but all use the pseudonym "Nicholas Bourbaki" to disguise their individual identities.

The three sections of Tome **II** were issued separately in 1897, 1898 and 1899 respectively. Section 1, containing the part pertaining to mathematical logic, was presented by **Peano** at the First International Congress of Mathematicians, held in August 1897 at Zurich. The event is significant for English-speaking mathematicians, for the young **Bertrand** Russell was in the audience, and the interest aroused led him to devote several years to the study of mathematical logic and, eventually, with the collaboration of A. N. Whitehead, to the publication of the famous ***Principia Mathematica.*** Tome V (1908) has been described as **"a** classic work in the mathematical literature of all ages" and "of inestimable value for students of mathematics" and "an inexhaustible mine of science." We cannot begin here to describe the large number of important topics treated in Tome **V.** We can only be amazed by the man who succeeded in collecting together in a book of such moderate size so great a part of mathematical knowledge. Tome V contains about 4200 propositions, all written in a symbolic form which is complete, **i.e.** with the explicit statement of the conditions of validity and the meaning of the letters contained. In the majority of cases the sources are cited and often the original statements of the authors who discovered them.

The year 1908 was not the last year of important mathematical publication for **Peano,** but it did mark, with his nomination to the presidency of the Volapuk Academy, an important shift of interest. Even in this short account of **Peano's** activities, we cannot omit at least an outline of his contributions in the field of language. This will explain why the fifth volume of the ***Formulaire*** (entitled ***Formulario Mathematico)*** used Latino sine-flexione instead of French for explanation and comments. Hardly was Tome IV of the ***Formulaire*** completed when **Peano** turned his attention to the problem of an international auxiliary language. The result was the introduction in 1903 of Latino sine-flexione **(i.e.** classical Latin without grammatical inflections.)

Volapuk, which had a phenomenal but short lived success, was introduced in 1879 by the German priest J. M. Schleyer, who founded an academy for its promotion. **Peano,** as president of the academy, renamed it Academia pro Interlingua and transformed it into a scientific association open to every opinion, having as its fundamental principle the "internationality" of its vocabulary. (In practice Interlingua differs little from Latino sine-flexione.) Thereafter followed much philological research. We mention only his very important book ***Vocabulario commune ad latino-italiano-francais-english-deutsch*** (1915). It should be noted that interest in Interlingua as an international auxiliary language for science has not died out, but still is very much alive today.

This shift in interest, carrying with it **Peano's** usual thoroughness, probably had the effect of making his logical and mathematical writings less known than they would have been if he had used more conventional means, for he used Latino sine-flexione for the important Tome V of the ***Formulaire*** as well as for many later works. Another reason why **Peano's** writings are not better known is the perennial Italian problem of quickly becoming out of print. This situation has recently been helped by the publication, in three large volumes, of the ***Selected Works of Peano,*** sponsored by the Unione Matematica Italiana. Many thanks are due Prof. Ugo Cassina, of the University of Milan, editor of these volumes. We would like to add our wishes to his that the ***Selected Works*** "may be completed with the publication - if not of all the works of **Peano** - at least of the ***Formulario Mathematico."***

REFERENCES:

All quotations from **Peano** are from works found in the ***Selected Works of Peano*** mentioned above. ***(Giuseppe, Peano, Opere Scelte,*** Roma: Cremonese, 3 vol., 1957-1959.) Other quotations are from Prof. L. Geymonat's article in the volume ***In Memoria di Giuseppe Peano*** (Cuneo: **presso** il Liceo Scientifico, 1955) and Prof. U. **Cassina's** article in ***Collectione de scripto in honore de prof. G. Peano*** (Supplem. ad ***Schola et Vita, 27*** August 1928.) All translations are mine.

Saint Louis University

ON THE DIVISION OF ONE[1] POLYNOMIAL BY ANOTHER

by
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1. If a polynomial, $f(x)$, of degree n in x is divided by another polynomial, $g(x)$, of degree m in x,

$$\frac{f(x)}{g(x)} = j(x) + \frac{p(x)}{g(x)} \qquad n \geq m \tag{1}$$

it is possible to determine the coefficients of the quotient, $j(x)$, and of $p(x)$ in terms of the coefficients of $f(x)$ and $g(x)$.

2. Let

$$f(x) = a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-2} x^2 + a_{n-1} x + a_n , \tag{2}$$

and

$$g(x) = b_o x^m + b_1 x^{m-1} + b_2 x^{m-2} + \ldots + b_{m-2} x^2 + b_{m-1} x + b_m . \tag{3}$$

Then, $j(x)$ will be a polynomial of degree $(n-m)$ in x and $p(x)$ will be a polynomial of degree at most $(m-1)$ in x.

Again, let

$$j(x) = c_o x^{n-m} + c_1 x^{n-m-1} + c_2 x^{n-m-2} + \ldots + c_{n-m-2} x^2 + c_{n-m-1} x + c_{n-m}, \tag{4}$$

and

$$p(x) = d_o x^{m-1} + d_1 x^{m-2} + d_2 x^{m-3} + \ldots + d_{m-3} x^2 + d_{m-2} x + d_{m-1} . \tag{5}$$

3. Upon multiplying both sides of (1) by $g(x)$, we obtain

$$f(x) = g(x)\, j(x) + p(x) \tag{6}$$

which will enable an evaluation of the coefficients of $j(x)$ and $p(x)$.

After multiplying $g(x)$ by $j(x)$ and combining coefficients of like terms in x, the equality

$$g(x)\, j(x) = b_o c_o x^n + (b_o c_1 + b_1 c_o) x^{n-1} + (b_o c_2 + b_1 c_1 + b_2 c_o) x^{n-2}$$

$$+ (b_o c_3 + b_1 c_2 + b_2 c_1 + b_3 c_o) x^{n-3} + (b_o c_4 + b_1 c_3 + b_2 c_2 + b_3 c_1 + b_4 c_o) x^{n-4}$$

[1]Received by Editors, Nov. 16, 1959.

$$+ \ldots + (b_o c_{n-m} + b_1 c_{n-m-1} + b_2 c_{n-m-2} + \ldots + b_{n-m-1} c_1 + b_{n-m} c_o) x^m$$

$$+ (b_1 c_{n-m} + b_2 c_{n-m-1} + b_3 c_{n-m-2} + \ldots + b_{n-m-1} c_2 + b_{n-m} c_1) x^{m-1}$$

$$+ (b_2 c_{n-m} + b_3 c_{n-m-1} + b_4 c_{n-m-2} + \ldots + b_{n-m-1} c_3 + b_{n-m} c_2) x^{m-2}$$

$$+ (b_3 c_{n-m} + b_4 c_{n-m-1} + b_5 c_{n-m-2} + \ldots + b_{n-m-1} c_4 + b_{n-m} c_3) x^{m-3}$$

$$+ \ldots + b_{m-2} c_o x^{n-m+2} + (b_{m-2} c_1 + b_{m-1} c_o) x^{n-m+1}$$

$$+ (b_{m-2} c_2 + b_{m-1} c_1 + b_m c_o) x^{n-m} + \ldots + (b_{m-2} c_{n-m} + b_{m-1} c_{n-m-1} + b_m c_{n-m-2}) x^2$$

$$+ (b_{m-1} c_{n-m} + b_m c_{n-m-1}) x + b_m c_{n-m} \quad \text{is obtained.} \tag{7}$$

After combining coefficients in the expression $g(x)\, j(x) + p(x)$,

$$f(x) = b_o c_o x^n + (b_o c_1 + b_1 c_o) x^{n-1} + (b_o c_2 + b_1 c_1 + b_2 c_o) x^{n-2}$$

$$+ (b_0 c_3 + b_1 c_2 + b_2 c_1 + b_3 c_o) x^{n-3} + (b_o c_4 + b_1 c_3 + b_2 c_2 + b_3 c_1 + b_4 c_o) x^{n-4}$$

$$+ \ldots + (b_o c_{n-m} + b_1 c_{n-m-1} + b_2 c_{n-m-2} + \ldots + b_{n-m-1} c_1 + b_{n-m} c_o) x^m$$

$$+ (b_1 c_{n-m} + b_2 c_{n-m-1} + b_3 c_{n-m-2} + \ldots + b_{n-m-1} c_2 + b_{n-m} c_1 + d_o) x^{m-1}$$

$$+ (b_2 c_{n-m} + b_3 c_{n-m-1} + b_4 c_{n-m-2} + \ldots + b_{n-m-1} c_3 + b_{n-m} c_2 + d_1) x^{m-2}$$

$$+ (b_3 c_{n-m} + b_4 c_{n-m-1} + b_5 c_{n-m-2} + \ldots + b_{n-m-1} c_4 + b_{n-m} c_3 + d_2) x^{m-3}$$

$$+ \ldots + b_{m-2} c_o x^{n-m+2} + (b_{m-2} c_1 + b_{m-1} c_o) x^{n-m+1}$$

$$+ (b_{m-2} c_2 + b_{m-1} c_1 + b_m c_o) x^{n-m} + (b_{m-2} c_3 + b_{m-1} c_2 + b_m c_1) x^{n-m-1}$$

$$+ \ldots + (b_{m-2} c_{n-m} + b_{m-1} c_{n-m-1} + b_m c_{n-m-2} + d_{m-3}) x^2$$

$$+ (b_{m-1} c_{n-m} + b_m c_{n-m-1} + d_{m-2}) x + (b_m c_{n-m} + d_{m-1}) \quad \text{results.} \tag{8}$$

4. We now have two polynomials of degree n in x which are equal. Because two equal polynomials of equal degree in a single variable have equal coefficients for like terms of the variable [1], the coefficients of like terms of x in these two polynomials are equal.

Proceeding from this point, it may be easily seen that

$$a_o = b_o c_o$$

$$a_1 = b_o c_1 + b_1 c_o$$

$$a_2 = b_o c_2 + b_1 c_1 + b_2 c_o$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$a_{n-m} = b_o c_{n-m} + b_1 c_{n-m-1} + b_2 c_{n-m-2} + \cdots + b_{n-m-1} c_1 + b_{n-m} c_o$$

$$a_{n-m+1} = b_1 c_{n-m} + b_2 c_{n-m-1} + b_3 c_{n-m-2} + \cdots + b_{n-m-1} c_2 + b_{n-m} c_1 + d_o.$$

$$a_{n-m+2} = b_2 c_{n-m} + b_3 c_{n-m-1} + b_4 c_{n-m-2} + \cdots + b_{n-m-1} c_3 + b_{n-m} c_2 + d_1$$

$$a_{n-m+3} = b_3 c_{n-m} + b_4 c_{n-m-1} + b_5 c_{n-m-2} + \cdots + b_{n-m-1} c_4 + b_{n-m} c_3 + d_2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$a_{n-2} = b_{m-2} c_{n-m} + b_{m-1} c_{n-m-1} + b_m c_{n-m-2} + d_{m-3}$$

$$a_{n-1} = b_{m-1} c_{n-m} + b_m c_{n-m-1} + d_{m-2}$$

$$a_n = b_m c_{n-m} + d_{m-1}. \qquad (9)$$

After solving the equations (9) for the $(n-m+1)$ values of c and the m values of d, we have

$$c_o = \frac{a_o}{b_o}$$

$$c_1 = \frac{1}{b_o} (a_1 - b_1 c_o)$$

$$c_2 = \frac{1}{b_o} (a_2 - b_1 c_1 - b_2 c_o)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$c_{n-m} = \frac{1}{b_o} (a_{n-m} - b_1 c_{n-m-1} - b_2 c_{n-m-2} - \cdots - b_{n-m-1} c_1 - b_{n-m} c_o) \qquad (10)$$

and

$$d_o = a_{n-m+1} - b_1 c_{n-m} - b_2 c_{n-m-1} - b_3 c_{n-m-2} - \cdots - b_{n-m-1} c_2 - b_{n-m} c_1$$

$$d_1 = a_{n-m+2} - b_2 c_{n-m} - b_3 c_{n-m-1} - b_4 c_{n-m-2} - \cdots - b_{n-m-1} c_3 - b_{n-m} c_2$$

$$d_2 = a_{n-m+3} - b_3 c_{n-m} - b_4 c_{n-m-1} - b_5 c_{n-m-2} - \cdots - b_{n-m-1} c_4 - b_{n-m} c_3$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$d_{m-2} = a_{n-1} - b_{m-1} c_{n-m} - b_m c_{n-m-1}$$

$$d_{m-1} = a_n - b_m c_{n-m}. \qquad (11$$

It can be seen that all $(n+1)$ of these coefficients are in terms of the coefficients of $f(x)$ and $g(x)$.

Reference

1. Burnside, William Snow, and **Panton,** Arthur William; ***The Theory of Equations*** Vol. 1, pp. 23-24, Dublin University Press, 1928.

Note: This work of expressing the quotient of two polynomials in an easily computable form was done by Michael Mackey at the Midwest Research Institute between his Freshman and Sophomore years at the University of Kansas.

Edited by M. S. Klamkin,
Avco Research and Advanced Development Division

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication. Address all communications concerning problems to M. S. Klamkin, Avco Research and Advanced Development Division, T430, Wilmington, Massachusetts.

PROBLEMS FOR SOLUTION

122. ***Proposed by Paul Berman, New York City.***
What is the minimum number of queens which can be placed on an n x n board such that no queen is covered by any other queen and such that the entire board is covered by all the queens.

123. ***Proposed by John Selfridge, University of Washington and I.B.M.***
A set of n positive integers $\{a\}$ is said to be linearly independent if $\sum_{r=1}^{n} a_r c_r = 0$ implies that $c_r = 0$ where c_r are positive integers ≥ -2.

1. For every n, show that there exists a linearly independent set $\{a_r\}$.

2. Is the set

```
1 0 0 . . . 0,
1 1 0 . . . 0,
1 1 1 . . . 0,
. . .       .
. . . . . . . .,
. . .       .
1 1 1 . . . 1,
```

(to the base 2) linearly independent?

3. Given a linearly independent set of length n, show that its least element is at least 2^{n-1}.

4. It is conjectured that there is only one linearly independent set of length n with its largest element $\leq \sum_{i=1}^{n-1} 2^i$.

124. ***Proposed by H. Kaye, Brooklyn, N.Y.***
Construct the center of an ellipse with a straightedge only, given a chord and its midpoint.

125. ***Proposed by Leo Moser, University of Alberta.***
Find the largest number which can be obtained as the product of positive integers whose sum is S.

126. ***Proposed by M. S. Klamkin, AVCO RADD.***
Determine an n-digit number (denary system) such that the number formed by reversing the digits is ***nine*** times the original number. What other numbers besides ***nine*** are possible?

SOLUTIONS

112. ***Proposed by J. S. Frame, Michigan State University.***
Find all real analytic functions F such that

$$F(x+y)\,F(x-y) = [F(x)+F(y)]\,[F(x)-F(y)].$$

Solution by J. T. Singer, Chicago, Illinois.

By letting $x=y$, $F(0)=0$, or $F(x)=0$.

Differentiating the given equation by $\frac{\delta^2}{\delta x \delta y}$:

$$F''(x+y)\,F(x-y) = F(x+y)\,F''(x-y).$$

whence,

$$F''(x) = \pm a^2 F(x),$$

and

$$F(x) = c\sin ax,\ cx,\ \text{or } c\sinh ax.$$

Also solved by H. Kaye, P. Myers, J. Thomas, M. Wagner and the proposer.

113. ***Proposed by Leo Moser, University of Alberta.***
Prove that it is impossible to enter the integers 1,2,...,10, on the ten intersections of 5 lines of general position in such a way that the sum of the numbers on every line is the same (22).

Solution by C. W. Trigg, Los Angeles City College.
There are 4 points on a line and 2 lines on a point, so each number occurs twice in the sum of the sums of the numbers on the lines. Since the sum of the numbers along every line is the same, the sum must be $2\sum_{1}^{10} i/5$ or 22. Now there are 18 sets of 4 numbers (from 1 to 10) with sums equal to 22, namely:

10,9,2,1, – 10,7,3,2, – 10,5,4,3, – 9,7,5,1, – 9,6,4,3, – 8,7,4,3,
10,8,3,1, – 10,6,5,1, – 9,8,4,1, – 9,7,4,2, – 8,7,6,1, – 8,6,5,3,
10,7,4,1, – 10,6,4,2, – 9,8,3,2, – 9,6,5,2, – 8,7,5,2, – 7,6,5,4.

There are only three pairs of these sets containing 10 twice and no other duplications, i.e.,

(10,9,2,1,–10,5,4,3), (10,8,3,1,–10,6,4,2), (10,7,3,2,–10,6,5,1).

Then there must be a third set containing one number from each set of the pair and two other numbers not in either of the pairs. The only third sets possible are (2,5,7,8), (8,2,5,7), and (3,6,8,9), respectively. A fourth set in each case must contain the tenth digit, (6), (9), (4), respectively, and one digit from each of the three established sets which has not already appeared twice. No such set exists. Therefore, it is impossible to enter the numbers 1,2,...,10, so as to get equal sums on every line.

Also solved by P. Myers and *J.* T. Singer.

114. Proposed by D. *J. Newman,* Yeshiva University.

Solve the four simultaneous equations

$$\begin{aligned} x + y &= a, \\ ux + vy &= b, \\ u^2x + v^2y &= c, \\ u^3x + v^3y &= d, \end{aligned}$$

for x, y, u, and v.

Solution by M. *S. Klamkin,* AVCO RADD.

Eliminating x and y from the first three equations and then from the last three equations, we find that

$$\begin{vmatrix} 1 & 1 & a \\ u & v & b \\ u^2 & v^2 & c \end{vmatrix} = 0 = \begin{vmatrix} 1 & 1 & b \\ u & v & c \\ u^2 & v^2 & d \end{vmatrix}$$

It now follows immediately that u and v satisfy the quadratics

$$\begin{vmatrix} & a & b \\ u & b & c \\ u^2 & c & d \end{vmatrix} = 0 = \begin{vmatrix} 1 & a & b \\ v & b & c \\ v^2 & c & d \end{vmatrix}$$

x and y are now easily obtained.

This problem and its generalization have been given previously by **Ramanujan.**

Also solved by Dean Arthur Jackson, H. Kaye, *J.* T. Singer, *J.* Thomas and the proposer.

115. Proposed by Francis L. Miksa, Aurora Illinois.

What is the smallest integral set for which

$$(10a+b)^2 + (10b+a)^2 + (10c+d)^2 + (10d+c)^2 = R^2.$$

Solution *by C. W. Trigg,* Los *Angeles City* College.

The sixteen solutions for which a,b,c, and d are all less than 10 follow, with the seven primitive sets indicated by an asterisk.

*	$11^2 + 11^2 + 11^2 + 11^2 = 22^2$,
	$22^2 + 22^2 + 22^2 + 22^2 = 44^2$,
*	$03^2 + 30^2 + 24^2 + 42^2 = 57^2$,
	$33^2 + 33^2 + 33^2 + 33^2 = 66^2$,
	$44^2 + 44^2 + 44^2 + 44^2 = 88^2$,
*	$04^2 + 40^2 + 67^2 + 76^2 = 109^2$,
*	$26^2 + 62^2 + 56^2 + 65^2 = 109^2$,
*	$11^2 + 11^2 + 77^2 + 77^2 = 110^2$,
	$55^2 + 55^2 + 55^2 + 55^2 = 110^2$,
	$06^2 + 60^2 + 48^2 + 84^2 = 114^2$,
*	$15^2 + 51^2 + 39^2 + 93^2 = 114^2$,
	$66^2 + 66^2 + 66^2 + 66^2 = 132^2$,
*	$39^2 + 93^2 + 57^2 + 75^2 = 138^2$,
	$77^2 + 77^2 + 77^2 + 77^2 = 154^2$,
	$88^2 + 88^2 + 88^2 + 88^2 = 176^2$,
	$99^2 + 99^2 + 99^2 + 99^2 = 198^2$.

Also solved by *J. T.* Singer, M. Wagner and the proposer.

BOOK REVIEWS

Edited by
FRANZ E. HOHN, UNIVERSITY OF ILLINOIS

Measurement: Definition and Theories. Edited by C. W. Churchman and P. Ratoosh. New York, Wiley, 1959. viii + 274 pp., $7.95.

This book contains thirteen papers from five fields: physics, pyschology, economics, accounting and philosophy. All but one were presented at the symposium on measurement of the Dec. 1956 meetings of the AAAS.

Part I: "Some Meanings of Measurement", contains papers by P. Caws: "Definition and Measurement in Physics", S. S. Stevens: "Measurement, Psychophysics, and Utility", P. Kirchner: "Measurements and Managerial Decisions", and C. W. Churchman: "Why Measure?"

Caws defines measurement as "... the assignment of particular mathematical characteristics to conceptual entities in such a way as to permit (1) an unambiguous mathematical description of every situation involving the entity, and (2) the arrangement of all occurrences of it in a quasiaerial order." In the profounder sense, measurement is not, as is commonly be* lieved, the simple operation of counting repeated applications of a standard measuring rod to the object being measured since such process tells us nothing about measurement "... as it applies to the case in question that we did not know about it as it applied to the standard measuring rod that we used." As Caws sees it, "The true function of measurement is to link mathematics and physics not as a means to establish a connection between the empirical and the theoretical but to connect two parts of theoretical knowledge, the mathematical and the conceptual, imparting relevance to the one and precision to the other."

The paper by Stevens is the longest and also one of the most thought-provoking in the book. Stevens regards measurement as "... the assignment of numerals to objects or events according to rule — any rule, "... the process of measurement is the process of mapping empirical properties or relations into a formal model." He points out that the only relevant criterion to measurement in the modem view is that of *invariance*, and different scales of measurement arise from different permissible groups of transformations leaving the scales invariant.

The papers by Kirchner and Churchman may be summarized in one sentence: One author regards measurement as essential to the decision process and the other regards it as a decision making activity itself. Unfortunately both papers suffer from a lacuna of important things to say about the meanings of measurement as such.

Part II: "Some Theories of Measurement" consists of papers by K. Monger: "Mensuration and Other Mathematical Connections of Observable Material", P. Suppes: "Measurement, Empirical Meaningfulness, and Three-Valued Logic", and R. D. Luce: "A Probabilistic Theory of Utility and its Relationship to Fechnerian Scaling".

The first two authors attempt to clarify semantic issues involved in physical measurements. Menger seeks to clarify these by introducing "fluents" and "functors"; Suppes tries to resolve the same by formalizing a language* in which "meaninglessness" is accepted as a possible logical value in addition to truth and falsity. Both theories are theories on semantics in measurement rather than measurement as such.

Luce's paper presents a theory on scaling subjective values as well as subjective probabilities. He presents an axiomatic model for a scale of subjective values which bears resemblance to the Fechnerian scale in psychophysical studies. However, the model is not entirely compatible with certain intuitively reasonable empirical assumptions but such inconsistency may be removed by relaxing certain restrictions in the present model.

Part III: "Some Problems in Physical Sciences" comprises papers by H. Margenau, "Philosophical Problems Concerning the Meaning of Measurement in Physics", A. Pap: "Are Physical Magnitudes Operationally Definable?", J. L. McKnight: "The Quantum Theoretical Concept of Measurement", and E. J. Gumbel: "Measurement of Rare Events".

Margenau's main thesis is that measurements must form an aggregate to be of scientific significance and that physical laws should have validity independent of measurements taken of the physical system under consideration. He discredits the metaphysical conception that a measurement disturbs a physical system in a predetermined way as exemplified by von Neumann's projection postulate, the popularity of which owes in no small degree to a confusion between (1) the preparation of a state, and (2) a measurement. The way to resolve these difficulties is to sacrifice the theoretical system but not to place a restriction on possible observational experiences of the real world.

Pap's philosophical article is, in this reviewer's opinion, the most difficult to comprehend in the entire volume. He strives "... to rescue the analytic-synthetic distinction on the level of qualitative observation language ..." which distinction breaks down in a quantitative scientific theory. Since most significant scientific theories are presumably quantitative, it is not clear whether Pap has uttered a blessing or a curse on the entire scientific outlook.

McKnight's paper is largely expository but it is probably the best organized paper in the whole book. It provides a careful analysis of the uncertainty concept and of alternative interpretations of the quantum concept. Papers of equal competence in analysis and organization would be welcome in other parts of the volume.

Gumbel's paper presents a survey of the theory of extreme values or rare events, and applications to engineering and economic problems are given. Amusingly, the last section of the paper is devoted to a vehement attack against all occult sciences, which almost borders on comic opera.

Part IV: "Some Problems in the Social Sciences" contains papers by C. H. Coombs: "Inconsistency of Preferences as a Measure of Psychological Distance", and D. Davidson and J. Marschak: "Experimental Tests of a Stochastic Decision Theory".

The first paper reports an experiment which indicates that "inconsistency of preferential judgments is not monotonically related to psychological distance but is a function of two variables, one of which is psychological distance, and that the relation is monotone only if a second variable (... called laterality) is held constant." The second paper reports an experiment which shows "... the superior accuracy of a stochastic theory of decision in predicting certain choices as compared to two alternative theories."

The few minor errata will not disturb the experienced reader.

Systems Development Corporation — Richard Kao

Algebra Can Be Fun! By William R. Ransom. Portland, Maine; J. Weston Walch, 1958. ix + 195 pages, $2.50.

This is a collection of tricks, oddities, famous problems, and helpful comments. It is interestingly and clearly written and should be of tremendous value for the enrichment of high school classes and for material for mathematics clubs.

University of Illinois — Franz E. Hohn

Mathematical Programming and Electrical Networks. By J. B. Dennis. New York, Wiley (and the Technology Press of M.I.T.), 1959, vi + 186 pp., $5.50.

A very important problem, usually solved nowadays by using a digital computer, is the minimization or maximization of a function of n variables, subject to m constraints in the form of conditions that m other functions be non-negative. Typical examples of such problems occur in economics (minimize cost with certain standards of quality) and engineering (optimize performance with certain standards of safety.)

The author's main idea is to turn the above problem upside-down by constructing physical models which, in the manner of an analog computer, realize the functions of the inequalities as well as the one to be optimized. Since electrical models can be produced at low cost and are easily modified, the author focusses his attention on networks containing current and voltage sources, ideal diodes, and ideal transformers. He shows that such networks can represent all problems in linear programming, Le., all problems in which the functions mentioned above are linear with constant coefficients. One very nice feature of this approach is that abstract theorems, e.g., the so-called "complementary slackness principle" of programming theory become quasi-intuitive in the network formulation (in this example the corresponding requirement is that diodes deliver zero power).

The book then devotes quite some space to the "conservative capacitated network flow problems", Le., problems in which objects (cars, goods, messages) satisfying a conservation principle flow through given channels without overtaxing (or undertaxing) the capacity of these channels. The goal here is to determine flow patterns which maximize flow or minimize cost. It is shown here that the representative network does not have to contain transformers. Further insight into the transportation problem is gained by the tracing of the voltage vs. current characteristic of a source-diode network with two accessible leads ("breakpoint curve"). The methods are generalized to include quadratic programs and - in the last chapter - the general programming problem.

Although the reviewer has to take exception to a number of statements, like the one right at the beginning saying, "the electrical scientist, however, is merely looking for a distribution of currents and voltages which satisfies the conditions imposed by the circuit - he rarely thinks in terms of minimization, and may not even realize that an appropriate extremum principle exists," the book is undoubtedly an extremely valuable contribution to the field interrelating programming and networks. It contains a wealth of virtually unknown theorems and methods. It is felt that not only the engineering-minded mathematician will profit from it, but especially the more ambitious engineer who would like to have a good theoretical background for the optimization problems which become more and more common in his field.

University of Illinois W. J. Poppelbaum

The **Theory of Storage.** By P. A. P. Moran. Methuen's Monographs on Applied Probability and Statistics. New York, Wiley, 1960. 111 pp., $2.50.

This little book is an exposition of special aspects of the theory of storage. The inventory case, where output is random, and the case of a dam, where input is random, are compared and contrasted. The book is principally devoted to dam theory.

The treatment is mature and will be of interest primarily to specialists in the field. The literature of the subject is extensively quoted and there is a good bibliography.

University of Illinois Franz E. Hohn

Nomography. Second Edition. By A. S. Levens. New York: John Wiley & Sons, Inc., 1959. viii + 296 pages, $8.50.

This book is very thorough. It should appeal both to those who are curious as to what nomography is, and to those who actually design nomographic charts. The material is useful in many phases of industrial, business, and professional life, since "The fields to which nomography can be applied are many. Among these are . . . . statistics, electronics, ballistics, heat transfer, radioactivity, medicine, biomechanics, food technology, ... engineering, physical and biological sciences, and business." (From the Preface.) The text contains considerable quantities of sample data taken from actual practice - all presented in a clear fashion

In particular, there are several distinctive features. One is the grouping of the exercises according to various fields of interest. Another is the chapter on circular nomograms; these charts are not common knowledge, and are ignored by most nomography texts. Likewise, the chapter on projective transformations is quite timely. Practicing engineers should welcome the applications to experimental data contained in the last chapter. In fact, the abundance of examples found throughout the book is one of its most noteworthy assets.

I would offer the following suggestions for improving the presentation The author neglects to take note of certain practical considerations in nomographic design. For instance, in the discussion concerning the use of $f_1(u) = f_2(v)$ for the construction of adjacent scales, no mention is made of what effect various arrangements of the actual equation used will have on the reading of the chart. Also, the text tends to "lean" toward engineering, despite the author's statement (quoted above) concerning the versatility of nomography. The Summary of Type Forms, while in itself a most commendable feature, misleadingly implies that the text gives approximately equal weight to the geometric and determinant methods of chart design.

There is a minimum of proofreading errors. As is characteristic of Professor Levens' work, the Appendix is outstanding – of an excellence rivalling the text proper. No mathematics background beyond the college freshman level is necessary. In fact, the required mathematics is held to a minimum. This is exemplified by the approach used in the important area of empirical data, and by the subordination of determinant methods. No previous knowledge of nomography by the reader is presupposed; therefore the book is useful as an introductory text. At the same time, the subject is developed far enough for the book to be valuable as a direct aid in chart design work.

University of Detroit F. M. Woodworth

Lattice Theory. By L. R. Lieber and H. G. Lieber. New York, Galois Institute of Mathematics and Art, 1959. vii + 287 pp., $5.95.

This is an introduction to the basic ideas of partially ordered sets and lattices. It is accurate and clear and is presented in the authors' unique style: free verse illustrated with extraordinary drawings. In addition to the mathematics, the book presents philosophical ideas concerning the moral responsibilities of scientists.

The book is well suited for introducing the spirit of modern abstract algebra to able high school students and to undergraduates. It is therefore to be recommended highly as outside reading at these levels, but it will be found delightful even by those with more extensive formal training.

University of Illinois Franz E. Hohn

Axiomatic Set Theory. By Patrick Suppes. Princeton, Van Nostrand, **1960.** **xii + 265** pp., **$6.00.**

Professor Patrick **Suppes'** new book *Axiomatic Set Theory* is a well written exposition of Zermelo-Fraenkel set theory. This book is written in a clear and very informal style, and is well suited for self study by any serious student of mathematics. Numerous side **remarks occur** throughout the book, and these enhance the value of the book by giving interesting **information** concerning other systems of axiomatic **set theory** as well **as** the general historical development of set theory.

Axiomatic Set Theory is suitable for use as a text in graduate courses. It is a book which can be understood by reasonably sophisticated undergraduates who have **acquired** a working knowledge of intuitive set theory (such as one obtains from a sound course in real function theory). It is not an appropriate text for the "Set Theory for Teachers" type course which is now fashionable at many colleges and universities.

The chapter **headings** are: 1. Introduction; 2. General Developments; 3. Relations and Functions; 4. Equipollence, Finite Sets, and **Cardinal** Numbers; 5. Finite Ordinals and Denumerable Sets; 6. Rational Numbers **and** Real Numbers; 7. Transfinite Induction and Ordinal Arithmetic; 8. The Axiom of Choice.

Cardinal numbers are introduced by postulating that with each set is associated an object $\mathcal{K}(A)$, the cardinal number of A, such that $\mathcal{K}(A) = \mathcal{K}(B)$ if and only if there is a one-to-one correspondence between A and B. Real numbers are constructed by the Cauchy sequence method. Ordinals are introduced by means of the elegant **definition** due to **R.M. Robinson.**

In intuitive set theory many well known paradoxes, such as the Russell paradox and the **Burali-Forti** paradox, occur. These paradoxes appear when one makes use of the word "all" and describes an "extremely large" set such as "the set of all sets", "the set of all cardinal numbers" or perhaps "the set of all ordinal numbers". In order to be judged a success, it is necessary that an axiomatic system of set theory incorporate in its structure some technical device which will eliminate the classical paradoxes. The technical device which is utilized **in** Professor Suppes' book is a fairly complicated definition schema for the **"set builder"** notation $\{x: \psi(x)\}$. If ψ is a property, then $\{x: \psi(x)\}$ usually designates, just as in **intuitive** set theory, "the set of all objects having the property ψ". However, there is no universe in **Zermelo-Fraenkel** set theory, and if there are "too many" objects which have property ψ, then $\{x: \psi(x)\}$ is equal to the empty set. Thus, $\{x: x = x\}$ and $\{x: x$ is an ordinal$\}$ are both equal to the empty set.

Although von **Neumann-Bernays-Gödel** set theory (a close competitor of Zermelo-Fraenkel set theory) has some **unintuitive** features of its own, this reviewer is of the opinion that it comes slightly closer to intuitive set theory than does the Zermelo-Fraenkel system. For this reason he wishes that Professor Suppes had chosen a system of the former type as the basic system for his book.

The composition of the book is good, although there are a few obvious errors. For example, on page **114**, it is difficult to distinguish the symbol for zero (which was probably intended to appear in bold face type) from the symbol for the empty set, and on page **174**, part of Definition **40** seems to be missing.

University of Georgia — M. K. Fort, Jr.

Introduction to Analysis. By N. B. **Haaser,** J. P. **LaSalle** and J. A. **Sullivan.** Boston, **Ginn** and Company, **1959. xiv + 688 + xxxi pp., $8.50.**

The time of the old mathematics cookbook, with easy-going explanations and simple-minded exercises, is rapidly passing. College students are better prepared and more exacting than only a few years ago, and expect a mature, rigorous approach to mathematics. Curricula are undergoing rapid changes, and the enthusiasm of the more progressive schools carries along those which are reluctant to change.

In this atmosphere the appearance of a radically different book like this one can only be welcomed. The authors have taken upon themselves the pioneer's task of writing an elementary calculus textbook which adheres to the standards of modem mathematical **rigour.** Their contribution to expository mathematical literature, a field where incompetents have too often had a field day, should be fully acknowledged. This book is the outcome of many years of full-time work to which much of the energy usually devoted to research was probably sacrificed.

The main innovation consists in bringing down to the freshman-calculus level many of the fundamental notions and notations which make up the working tools of the mathematician. Such is the notion of function in its most general acception; the ideas and techniques connected with modern vector geometry, **inner** product, the geometry of quadratic forms, **linear** transformations of the plane. This requires a sweeping notational reform, a departure from the sloppy presentation of the old texts; in this the authors succeed admirably. To give a few examples, a function is always denoted by a single letter **f,** rather than by the logically incorrect $y = f(x)$. The definite integral, being an operation performed on functions, is written $\int_a^b f$ rather than $\int_a^b f(x)dx$. The composition of functions fog is studied in great **detail;** this leads to extremely simple expressions for the chain rule for differentiation and the rule for integration by **substitution.** Naturally, some students may find at first that such an abundance of notation is bewildering; however, once the use of a few fundamental symbols is mastered - and this should happen at the beginning of the course - the traditionally difficult parts, such as the technique of integration and the more recondite applications of the fundamental theorem of calculus, become strikingly simple to understand.

As in all pioneering work, the exposition is at times rather clumsy. There are pages of the book which are mazes of formulas, with few dry words of explanation in between. The student has a hard time coping with these, although once he does, the payoff is big. The authors do not always remember that their readers are not mature mathematicians, but beginners. The style is very often dry and elliptic. These are however minor defects in a work of this magnitude.

Massachusetts Institute of Technology — **Gian-Carlo** Rota

Gundlagen **der** Analysis, Third Edition (with a complete German-English vocabulary). By E. Landau. New York, Chelsea, **1960. 173** pp., **$1.95.**

This book was a pioneer in the detailed study of the structure of the number system. Because of its completeness and clarity it has become one of the classics that should be read and digested by every mathematics major in preparation for his study of analysis beyond the calculus. The modest price of this volume puts it **within** the reach of everyone.

University of Illinois — Franz E. **Hohn**

Mathematical Methods **for** Digital Computers, edited by Anthony Ralston and Herbert **S. Wilf.** New York, John Wiley, 1960. xi + 293 pp., $9.00.

Communication among experts, and between expert and novice, in the computing field, has been seriously hampered in the **past by** the lack of a suitable medium. The difficulty lies not in discussing the mathematics itself, for here the mathematical idiom serves as well as in any other area of mathematics. But the detailed description of the specific operations in sequence, required for the realization of a given method, must eventually **find** expression in the idiom of the machine that is to carry out these steps. And every year new machines are developed, each with an idiom of its own, or one that is common to only a relatively small class of machines in use. Among the earlier machines, those that were constructed in the 40's and early **50's,** each was, indeed, unique, and it was not **until** the appearance of several copies of **Univac** I that exchange of actual machine codes from one group to another (those groups using **Univac** I) became possible at all.

More recently, though, there have arisen a number of "problem oriented languages" in terms of which algorithms can be written and used directly on any machine possessing a translator. The translator is itself a machine code, written for a particular machine, and permitting the translation to be made into the language of that machine. Hence an algorithm written in this language is applicable to any machine possessing such a translator. Examples of these languages are Fortran, first developed for IBM machines of the 700 series, and, more recently, Algol.

Actually, flow charts provide a medium for exchanging information that is less complete than that contained in an algorithm written in Fortran or in Algol, but considerably more so than is easily possible in the standard mathematical idiom. Moreover, flow charts were described in some detail and used extensively already by von Neumann and Goldstine in connection with development of the machine at the Institute for Advanced Study. Curiously, though, these were used almost exclusively in planning, and have not been much exploited as a medium of communication. A flow chart, if constructed at all, was considered merely as a step toward the construction of the final code, and then was oriented toward a specific machine.

It is true that flow charts can be found in a few periodical articles, but this is the first book that attempts to use the flow chart systematically as a medium of communication The book is a handbook containing a number of articles (26, in fact), each describing a particular type of computation, and each, with a few exceptions, **containing** one or more flow charts. There is little theory, but enough explanatory material to make the method **understandable** to anyone with a reasonable background in mathematics. There are also critical comments, and indications as to speed, accuracy, storage requirements, and related items. The articles themselves are written by experienced programmers, and the book should fill a real need.

To indicate the contents, there are 6 parts, on elementary functions, matrices and linear equations, ordinary differential equations, partial differential equations, statistics, and a part entitled "miscellaneous". Under the last heading are collected six papers, on the solution of algebraic equations, numerical quadrature, multiple integration by Monte **Carlo** methods, Fourier analysis, linear programming, and network analysis.

Taken as a whole, the selection is natural and inevitable, but naturally and inevitably one can raise questions about particular items. Two papers on the use of Monte **Carlo** methods seem unnecessary, one of which is on inverting matrices, the other on solving elliptic partial differential equations. One can question whether these represent the more useful applications of Monte **Carlo** other than for simple integration. Only one paper deals with characteristic roots of matrices, and that gives only the **Jacobi** method for the real symmetric case. The coding of this method is **fairly** simple, but the method of Givens is widely used and far superior. Failure to include anything on **nonsymmetric** matrices may be due to the feeling that the treatment of these requires more art than science, but recent work by Wilkinson has shown how to automatize the calculations in any reasonably well conditioned case.

Nevertheless, the value of a book lies in what it does contain, and what someone thinks it should contain but does not is of secondary importance. **For what** it does contain, the book deserves a place on the shelves of novices and of experts alike.

Oak Ridge National Laboratory **A. S.** Householder

German-English Mathematics Dictionary. By Charles **Hyman.** New York, Intel-language Dictionaries Publishing Corporation, 1960. 131 pp. $8.00.

This dictionary will appeal most to translators of mathematical material who *are* not themselves mathematicians and to graduate students learning the language. It lists technical mathematical terms, many of which *are* not to be found in ordinary dictionaries, as well as mathematical meanings of words in colloquial use. Many of the terms listed come from applications of mathematics to physics and engineering, which is commendable.

The listing of technical terms ought to be more nearly complete. A quick check against some standard texts turned up these omissions among others:

Grundeck
Losungsstrahl
Maximalbedingung
Nulloperator
Nullstellensatz
Wurfelgutter

On the other hand, a composite word like **"Nulloperator"** should cause no one any difficulty.

Examples of colloquial words frequently used in mathematical writing but not included here are:

beziehungsweise
bezuglich
geringer

They are of course to be found in the standard dictionaries.

Some translations are awkward. For example, **"Inzidenzmatrix"** is translated "matrix of incidence" whereas it is usually called the **"incidence** matrix."

Some words have important mathematical meanings not listed here. For example, **"Verhaltnis"** is translated "ratio, proportion" whereas it frequently means "relationship" or "connection". The word **"Uberlagerung"** is translated only as "superposition" whereas in topology the customary meaning is "covering".

Finally, many cognates such as "Geometric", **"Tripel", "magnetisch",** etc. are listed. It is doubtful that anyone would ever need to consult a dictionary for the meaning of these.

To the present reviewer, it seems that the usefulness of the dictionary will be greatest to those whose knowledge of mathematical German is quite weak, but even they will not always find In it the translations they need. Since the author asks for suggestions from users, perhaps a second edition will be more adequate.

A list of errata is supplied with the volume.

University of Illinois Franz E. Holm

Handbook of Automation, Computation, and Control, **Vol.** 1 - Control Fundamentals. Edited by E. M **Grabbe,** S. Ramo, and D. **E.** Wooldridge. New York, John Wiley, 1958. $17.00.

This volume on Control Fundamentals is the first of **a trilogy** in a Handbook of Automation, Computation, and Control. The other two volumes will be Computers and Data Processing - Vol. 2, Systems and Components - Vol. 3.

In order to give an indication of the coverage in this volume, the following is a list of the chapter headings together with their length in pages: 1. Sets and Relations - **11** pages. 2. Algebraic Equations - 6. 3. Matrix Theory - 17. 4. Finite Difference Equations - 8. 5. Differential Equations - 23. 6. Integral Equations - 17. 7. Complex Variables - 28. 8. Operational Mathematics - 20. 9. **Laplace** Transforms - 21. 10. Conformal Mapping - **11.** 11. Boolean Algebra - 11. 12. Probability - 20. 13. Statistics - 21. 14. Numerical Analysis - 90. 15. Operations Research - 129. 16. Information Theory - 48. 17. Smoothing and Filtering - 34. 18. Data Transmission - 32. 19. Methodology of Feedback Control - 21. 20. Fundamentals of System Analysis - 86. 21. Stability - 83. 22. Relation between Transient and Frequency Response - 61. 23. Feedback System Compensation - 56. 24. Noise, Random Inputs, and Extraneous Signals - 20. 25. Nonlinear Systems - 68. 26. Sampled-Data Systems and Periodic Controllers - 32.

As can be seen from the listing, many topics (especially in General Mathematics) are treated briefly. However, the main points in each chapter are presented and each chapter is supplemented with an adequate bibliography.

According to the editors, "this Handbook is directed toward the problem solvers - the engineers, scientists, technicians, managers, and others from all walks of life who are concerned with applying technology to the mushrooming developments in automatic equipment and systems. It is our purpose to gather together in one place the available theory and information on general mathematics, feedback control, computers, data processing, and systems design. The emphasis has been on practical methods of applying theory, new techniques and components, and the ever broadening role of the electronic computer. Each chapter starts with definitions and descriptions aimed at providing perspective and moves on to more complicated theory, analysis, and applications. In general, the Handbook assumes some engineering training and will serve as an information source and refresher for practicing engineers. For management, it will provide a frame of reference and background material for understanding modem techniques of importance to business and industry. To others engaged in various ramifications of automation systems, the Handbook will provide a source of definitions and descriptive material about new areas of technology."

To give a comprehensive review of this Handbook would require a team of reviewers, especially since the number of contributors and their corresponding specialties is rather large. However, in the opinion of this reviewer, the editors have achieved their purpose as stated previously and have produced a well organized Handbook that should become a standard and very useful reference work for technologists for many years to come.

AVCO Research and Advanced Development Division — Murray S. **Klamkin**

Operations Research: Methods and Problems. By Maurice **Sasieni,** Arthur Yaspan, and Lawrence Friedman. New, John **Wiley,** 1959. xi + 316 **pp.,** $10.25.

Operations Research: Methods and Problems will be eagerly **greeted** by those teaching operations research to advanced undergraduates and first year graduates. The topics treated (Inventory, Replacement, Waiting **Lines,** Competitive Strategies, Allocation, Sequencing, Dynamic Programming) are those covered in ***Introduction to Operations Research*** by Churchman, **Ackoff,** and **Arnoff.** It seems clear that "Methods and Problems" was written to complement "Introduction" by providing the problem sets for each topic.

Each chapter contains a brief discussion of a topic, illustrative examples, a problem set, and a bibliography. The teacher and student may **well prefer the discussions in "Introduction" which are fuller both in description of the topic and in precision of its mathematical statement. The bibliographies list many books published since "Introduction" and so are quite useful. It is the examples and problems which are the real** worth of the book. They are well chosen for each topic. **Not only do** they give the student practice in formulating a problem **and solving it by** the appropriate methods, but they are taken from **various industrial and** commercial settings and point to the sort of operations **research that is** currently being practiced. There is no other such colle**ction of problems** now available.

In general, the tone of the book suggests that the **authors wished they** could get along without mathematics. For instance, **in developing the** simplex method, they never used the customary **matrix and vector notation.** On page 62 "settled down to a stable value" **is used rather than** "converge"; there is another instance of this on pa**ge 125. On page 222** occurs the expression **"variable** parameter". Since **the authors require of their readers a "working knowledge of the differential and integral calculus", they might well have made fuller use of the customary mathematical** notation to express mathematical ideas. None the less, this criticism is of notation and terminology. The mathematics is always quite adequate.

Jane Robertson

Classical Mathematics, A Concise History of the Classical Era in the History of Mathematics. By Joseph **Ehrenfried Hofmann.** New York, Philosophical Library, 1959. 159 pp., $9.75.

This book is a translation of the second and third volumes of the author's Geschichte der **Mathematik,** which were originally published in 1957 in the series, ***Sammlung Goschen.*** It covers roughly the period of the seventeenth and eighteenth centuries, which the author divides into the High Baroque Period (about **1625-1665),** the Late Baroque Period (about **1665-1730),** and the Age of Enlightenment (about 1700-1790). In spite of its small size, this volume covers the development of the calculus and the other mathematical advances of the time in great detail. The author does not waste words and gives us an enormous amount of authoritative Information in concise form. Although the uninitiated may find the mass of historical detail somewhat overwhelming on a first reading (as the reviewer did), the book is readable in the small and certainly makes a handy reference volume. It would be a worthwhile addition to **any** college library.

University of Illinois — Paul T. **Bateman**

Handbook of Automation and Control, Volume 2. Edited by M. Grabbe, S. Ramo, and D. E. Wooldridge. New York, Wiley, 1959. xxiii + 1031 pp., $18.50.

A handbook is a compendium of knowledge in a field which is so wide that no single person can cover it adequately. It is usually written by a staff of experts in each sub-field and by its very nature suffers from two faults: considerable overlap between chapters and varying depth of treatment in its different parts. Volume 2 of the Handbook of Automation and Control, which supplements the first volume - devoted to mathematics and the more abstract disciplines of feedback and information theory - is no exception to the rule enunciated above.

First of all, this reviewer feels strongly that there is a great lack of consistency in the method of approaching the subject. As an example of the degrees of depth found in the volume, let us compare Chapter 24 on "Analogs and Duals of Physical Systems" which presents in 13 pages a high-density introduction to the field, to Chapter 6 on "Facility Requirements" which - in the same number of pages - essentially states that a computer needs accessory space, power, cooling, and personnel. It is not this particular example that is important; it is the once-over-lightly attitude prevailing in many chapters that one has to object to. Many of these chapters seem to have been written primarily in order to increase the length of the index.

Secondly, it is felt that the subdivision into chapters is inadequate. The rather diffuse chapter on "Programming and Coding" is 260 pages long, while only 42 pages are devoted to "Logical Design". Then again it seems difficult to understand why such special cases as "Accounting Applications" (15 pp.), "Inventory and Scheduling Applications" (12 pp.), and "Scientific and Engineering Applications" (12 pp.) had to be treated in separate chapters.

The question comes up, "To whom is the Handbook of Automation and Control addressed?" Certainly not to the novice, for the chapters on programming or input-output equipment presuppose a good portion of initial knowledge. Not to the expert either, for the chapters on storage and circuit design give only rather elementary facts and often insist on obsolescent techniques.

In spite of these criticisms, which partially reflect the reviewer's dislike of handbooks in general, it must be admitted that the very fact that 41 authors have produced a book with a fair proportion of good chapters and a reasonably small amount of overlap is worthy of praise. To a large extent this praise must go to the editors: Ramo, Wooldridge, and Grabbe. Perhaps future editions could equalize the depth and redistribute the material; this could make an excellent compendium out of what is now a very average handbook.

University of Illinois — W. J. Poppelbaum

An Introduction to Mathematical Statistics. By H. D. Brunk. Boston, Ginn and Co., 1960. xi + 403 pp., $7.00.

This is the first of three new books to be reviewed as introductory works to the field of Mathematical Statistics.

This book is a one semester textbook with many starred sections which with slight additions from the instructor could be used in a year's course. The author gives a very fine modern introduction to probability theory consisting of about a fourth of the text. All the modern concepts are used in these sections and the author continues to use these modem notions throughout the book.

The usual topics are discussed quite thoroughly and many topics usually reserved for advanced courses are discussed where needed.

The author has very well succeeded in making this a teachable book by the use of many excellent illustrations, unique markings for theorems, discussions of difficulties of the student reader, summaries at the end of chapters, organizational chart for the course, and finally the addition of tables for the binomial and Poisson distributions to the usual set of tables.

Finally most excellent sets of problems are introduced, most of which are new and very interesting. The reviewer recommends the book most highly to all students, workers in the field, and those desiring introductory notions of probability and statistics.

St. Louis University — W. A. Vezeau

"An Introduction to Mathematical Statistics". By Robert V. Hogg and Alien T. Craig. New York, Macmillan, 1959. ix + 245 pp., $6.50.

The authors have presented a very fine textbook for a year's course in introductory mathematical statistics. The authors suggest a selection of certain chapters and sections for a one semester course.

Typical topics are presented along with such fine discussions as a chapter on transformations of variables, and sections on point estimation not usually given in introductory textbooks. The first chapter gives an excellent introduction to modern concepts and discusses probability notions very well. There are a sufficient number of practice problems presented, some demanding extensions of the theory presented.

The reviewer feels that the list of references should be extended to include the standard statistical textbooks and other Journal works. The number of statistical tables and the content in the tables is much less than usually presented in statistical textbooks.

This textbook is the first in a series of mathematics textbooks under the general editorship of Carl B. Allendoerfer. We agree with the publishers' note "It was invited to become the first publication in the series because of its underlying philosophy and its general mathematical excellence."

St. Louis University — W. A. Vezeau

Elements of Mathematical Statistics. By D. Ransom Whitney. New York, Henry Holt and Co., 1959. ix + 148 pp., $4.75.

The author gives a summary of elements of Mathematical Statistics in this textbook suitable for a quarter course or with additional topics supplied by the instructor for a semester course. The reviewer has used the textbook this last summer as a companion textbook in statistics to the Commission's book on Probability in a National Science Foundation Summer Institute Course for High School Teachers.

The book summarizes very well the usual topics in statistics. The typical distribution functions are especially well presented graphically. The author prefers to use the characteristic function instead of the usual moment generating function used in other introductory mathematical textbooks. This is particularly of advantage to engineers who need to use statistics, particularly in Communication Theory.

In Chapter II the author is following the classical method of introducing statistics to students that most teachers have used in the past. The notation used in the probability theory follows that presented in Hoel's book and others.

For many short courses in statistics such as are prevalent now for electrical engineers, computers, industrial engineers, etc., the reviewer thinks this book would very quickly present them with the basic concepts of statistics for use in their fields and as background for further study.

St. Louis University — W. A. Vezeau

The Analysis of Variance. By Henry **Scheffé.** New York, John Wiley, **1959.** xvi + 477 pp., $14.00.

This is the first book devoted to analysis of variance, predecessors being only Snedecor's monograph in 1934 and Jackson's in 1940. The gap between them and the present volume is wide, Scheffé's book giving a thorough and rigorous mathematical treatment, as well as considerable explanation and comment on applications, some sound practical advice, and numerous problems many of which include real data. A wryly humorous example of the practical advice is that "A statistician. . . may discredit himself by thoughtlessly offering 7.32179 0.05248 instead of 7.32 ± 0.05. . .

The book is intended for a semester course for graduates and advanced undergraduates, with mathematical background including calculus. Vector and matrix algebras are used, and the needed definitions and theorems are given in two appendices.

Though the author's effort to be precise is evident, still there are occasional statements which use terms unconventionally (mentioning "five main effects" of a single factor) or are curiously inefficient ("an increasing function", with a footnote saying "I mean strictly increasing"), or are weak or inadequate (saying that a justification of randomization is that . . . there may be other uncontrolled factors . . ." when in fact there are always many others, thus missing the opportunity to emphasize for the student the fact that randomization not only deals effectively with recognized extraneous factors but also with those of which the investigator is entirely unaware). However, these are small points, and the book deserves a warm recommendation. It is well produced, and there are few misprints.

University of Illinois — Horace **W.** Norton

String Figures, by W. W. **R.** Ball, 72 pp.; Geometrical Construction, by J. **Petersen,** 102 pp.; Noh-Euclidean Geometry, by H. **S. Carslaw,** 179 pp.; A History of the Logarithmic Slide-Rule, by **F. Cajori,** 135 pp. Bound as. a single volume. New York, Chelsea, 1960, $3.95.

Ball's String Figures describes various patterns and representations which may be accomplished by looping a length of string over the fingers. There is good recreation here for those who are interested.

Petersen's Geometrical Construction is a famous classic on principles and techniques of ruler and compass constructions.

Carslaw's Non-Euclidean Plane Geometry and Trigonometry was first published in 1914. It begins with an extensive historical summary, treats the hyperbolic and elliptic plane geometries, and concludes with a discussion of the parallel postulate.

Cajori's History of the Logarithmic Slide Rule carries the subject up to 1909 and includes a list of rules designed and used from 1800 to 1909.

Those interested in Euclidean and Non-Euclidean geometry will find the second and third monographs well worth the price of the book.

University of Illinois — **Franz E. Hohn**

L. J. Adams: Intermediate *Algebra.* New York, Holt, 1960. $4.50.
S. L. **Altwerger:** Modem Mathematics. New York, Macmillan. 1960. $6.75.
R. H. **Atkin:** Classical Dynamics. New York, Wiley, 1960. $5.25.
R. P. Boas, Jr.: *A* Primer of Real Functions *(Carus* Monograph *# 13).* New York, Wiley, 1960. $4.00.
*H. D. **Brunk:** *An* Introduction to Mathematical Statistics. Boston, **Ginn,** 1960. $7.00.
E. A. Cameron: Algebra and Trigonometry. New York. Holt, 1960. $5.00.
T. Fort: Differential Equations. New York, Holt, 1960. $4.75.
*N. B. Haaser, J. P. **LaSalle,** and J. A. Sullivan: A Course in Mathematical Analysis. **Boston,** Ginn, 1959. $8.50.
M. A. Hill, Jr. and J. B. Linker: Brief Course in Analytics, Second Edition. New York, Holt, 1960. $3.90.
*J. E. **Hofmann:** Classical Mathematics: A Concise History of the Classical Era in Mathematics. New York. Philosophical Library, 1960. $4.75.
*R. V. Hogg and A. T. Craig: Introduction to Mathematical Statistics. New York, Macmillan, 1959. $6.50
R A. Howard: Dynamic Programming and Markov Processes. New York, Technology Press and Wiley, 1960. $5.75.
*C. Hyman: German-English Dictionary of Mathematics. New York, Interlanguage Dictionaries Publishing **Corp.,** 1960. $8.00.
A. Jaeger: Introduction to Analytic Geometry and Linear Algebra New **York,** Holt, 1960. $6.00.
S. L. **Karlin:** Mathematical Methods and Theory in Games, Programming, and Economics. *(2 vols.)* Reading, Mass., Addison-Wesley, 1959. $12.50 each vol.
*E. Landau: *Grundlagen* der Analysis, Third Edition (with a complete *German-*English vocabulary). New York. Chelsea, 1960. Paperback, $1.95.
*L. R. Lieber and H. G. Lieber: Lattice Theory: the Atomic Age in Mathematics. Brooklyn, New York, Galois Institute of Mathematics and Art, 1959. $5.95.
S. F. Mack: Elementary Statistics. New York, Holt, 1960. $4.50.
J. D. **Mancill** and M. O. **Gonzalez:** Modem College Algebra. Boston, **Allyn** and Bacon, 1960. $6.25.
N. H. McCoy: Introduction to Modern Algebra. Boston, **Allyn** and Bacon, 1960. $7.50.
*P. A. P. Moran: The Th*eory of Stora* e. New York, Wiley, 1960. $2.50.
G. M. Murphy: Ordinary *Differential* Equations and Their Solutions. Princeton, Van-Nostrand, 1960. $8.50.
J. H M. Olmsted: Real Variables. New York, Appleton-Century-Crofts, 1959. $9.00.
E D. **Rainville:** Special Functions. New York, **Macmillan,** 1960. $11.75.
*A Ralston and H. S. Wilf: Mathematical Methods for Digital Computers. New York, Wiley, 1960. $9.00.
J. B. Rosenbach, **E. A. Whitman,** B. **E. Meserve,** and P. M. **Whitman:** Intermediate Algebra for Colleges, Second Edition. Boston, Ginn, 1960. $5.00.
W. L. Schaaf: Basic Concepts of Elementary Mathematics. New York, Wiley, 1960. $5.50.
*H. Scheffe: The Analysis of Variance. New York, Wiley, 1959. $14.00.
A. **Schwartz:** Analytic Geometry and Calculus. New York, Holt, 1960. $9.00.
F. W. Sparks: A Survey of Basic Mathematics (a text and workbook for *college* students). New York. **McGraw-Hill,** 1960. $3.95.
*P Suppes: Axiomatic Set Theory. Princeton, Van Nostrand, 1960. $6.00.

*See review, this issue.

NOTE: All correspondence concerning reviews and all books for review should be sent to PROF. FRANZ E. HOHN, 374 ALTGELD HALL, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

OPERATIONS UNLIMITED

Science needs you

You need science

This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essential as today.

Your election as members of Pi Mu Epsilon Fraternity is an indication of scientific potential. Can you pursue advanced study in your field of specialization?

To point out the need of advanced study, the self-satisfaction of scientific achievement, the rewards for advanced preparation, the assistance available for qualified students, etc., it is planned to publish editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

Through these and future editorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

In this issue this section is devoted to the need for advanced study as experienced in the teaching of mathematics and science. The National Science Foundation, the Woodrow Wilson National Fellowship Foundation, and the Mathematics Teachers College of Columbia University have assisted with editorials emphasizing this need. Well qualified students will find assistance, if need be, for graduate study from many sources. The American Mathematical Society published last December and plans to publish again this December a "Special Issue" of the "Notices" that list Assistantships and Fellowships in Mathematics.

Certainly this is not a time to rest on our laurels, but a time f a continued achievement in one's profession. With permission I quote President Eisenhower:

'Recent events have brought renewed emphasis upon education, particularly in the fields of Mathematics and Science. It is my earnest hope that this increased interest be translated into greater support for education of our children, and a greater realization of the key role of the teachers in our society."

The following lists contributing corporations with the issue in which their editorials appeared.

Aeronutronics	Vol. 3, No. 2
Army Ballistic Missile Agency	Vol. 2, No. 10
AVCO, Research and Advanced Development	Vol. 2, No. 10
Bell Telephone Laboratories	Vol. 2, No. 10
Bendix Aviation Corporation	Vol. 2, No. 8
E. I. du Pont de Nemours and Company	Vol. 3, No. 2
Emerson Electric Company	Vol. 2, No. 7
General American Life Insurance Company	Vol. 2, No. 9
Hughes Aircraft Corporation	Vol. 2, No. 9
International Business Machines Corporation	Vol. 2, No. 8
Eli Lilly and Company	Vol. 3, No. 2
Mathematics Teachers College, Columbia U.	Vol. 3, No. 3
McDonnell Aircraft Corporation	Vol. 2, No. 7
Monsanto Chemical Company	Vol. 2, No. 7
National Science Foundation	Vol. 3, No. 3
North American Aviation, Inc.	Vol. 2, No. 9
Olin Mathieson Corporation	Vol. 2, No. 7
Shell Development Company	Vol. 3, No. 1
Woodrow Wilson Foundation	Vol. 3, No. 3

NATIONAL SCIENCE FOUNDATION

TEACHING AS A CAREER

C. Russel Phelps

By

C. RUSSELL PHELPS

The function of the teacher - at any level - is to assure that replenishment and development of the intellectual capability of society take place. In replenishing the manpower resources in his field, the teacher provides inspiration to attract students, and then provides them appropriate instruction and guidance. For the intellectual development of mankind the teacher sets an example through his own continual search for new knowledge and understanding and enables his students to follow in his footsteps and beyond by critical attention to the sharpening of their thought processes as well as to their clear understanding of the current state of knowledge.

Who should teach? The person who wants to share his ideas and his intellectual pleasures, who can work patiently with others for the enjoyment of observing ideas taking root - such a person should consider teaching as at least a part-time occupation. This is the true basis of being a "born" teacher.

The college undergraduate majoring in mathematics who is interested in becoming a teacher, either in secondary school or in college, can anticipate at the present time that there will be no difficulty in getting a job, that his compensation will be reasonable and regular, and that in a very few years he can achieve a job security and permanence unequalled in industrial or business occupations. Along with this he is assured of valuable fringe benefits - holidays and vacations during the academic year, together with an extended summer period for study and travel.

Secondary School Teaching

The requirements for a teaching position in secondary school consist normally of a major in mathematics, consisting of at least 15 hours of mathematics more advanced than the calculus, together with basic courses in educational theory (18 or more hours), and practice teaching. For those undergraduates who have completed all, or most, of the mathematics requirements but not the educational ones, a number of universities offer attractive opportunities for study in a Master of Arts in Teaching program. Typical programs provide internship in teaching - approximately half-time - accompanied by related classwork. The "intern" completes the work for the master's degree in about two years, earning while he learns.

Current salaries for secondary school teaching begin from $4,000 to $4,800 in many city and suburban school districts and are higher for those holding a master's degree. Salary schedules provide for annual increments, and in many school districts the competent teacher who has the master's degree - frequently earned while he is teaching - will reach a salary of $7,500 - $10,000 in about 15 years. It should be noted that these salaries are for nine or ten months; additional income is available for summer activities.

Through its extensive program of Summer Institutes, Academic Year Institutes, and Summer Fellowships, the National Science Foundation makes it possible for teachers of mathematics and science to take advanced studies, leading often to a master's degree. Teachers participating in these programs are relieved from the payment of tuition, and receive in addition stipends of $75 per week plus allowances for dependents and travel, so that professional improvement is easy to achieve.

College Teaching

The college teacher, on the other hand, is normally expected to have taken extensive graduate work in mathematics, and usually to have obtained a Ph.D. This extended period of study is rewarded by higher salaries; mathematicians who have just received their doctorates are currently being offered $6,500 - $7,500 by many universities and colleges. These salaries are likely to be even higher in the next few years because of the extreme shortage of trained mathematicians.

In preparation for college teaching, the college graduate is aided by numerous fellowships and teaching assistantships. The number of fellowships and assistantships available is such that the *cum laude* graduate in mathematics will have little difficulty in obtaining one paying typically $1,500 - $2,500. A list of graduate fellowships and assistantships is compiled annually by the American Mathematical Society and is available from most mathematics department chairmen.

A fellowship - such as the Graduate or Cooperative Fellowships awarded by the National Science Foundation, or similar fellowships available in large numbers - enables a graduate student to complete the requirements for a doctorate in a minimum amount of time, normally about three years. On the other hand, with a teaching assistantship or part-time instructorship one can get valuable teaching experience as well as determine through trial whether teaching would be a congenial career. And whether or not one eventually decides on teaching, graduate study immediately after college is an ideal preparation for a future career in mathematics.

Women in Teaching

Women majoring in mathematics should give serious consideration to teaching as a career. At the present time, over one-third of our high school teachers of mathematics and more than ten percent of our college teachers are women. For women in teaching, the salaries and advancement opportunities are the same as for men. In this age, when most women college graduates will be seeking employment directly after college and again after their children are in school or grown up, those who have qualified to teach in either secondary school or college have job opportunities open to them at any time. Teaching is especially suitable as an occupation for women with children in school, since the working hours and vacation periods are consistent with those of her children. College teaching, in particular, can be done on a part-time basis as an adjunct to home-making. But, whatever the situation, early preparation for teaching will provide women an ever-open door to a professional status in society.

Teaching and Scholarship

An interest in mathematics is typically manifested in a desire to solve intriguing problems and to explore logical, numerical, and structural relationships. It is both the privilege and the obligation of the teacher to carry on scholarly activity in mathematics as the important background for what he teaches; at the same time he has the real freedom to investigate those pockets of the unknown which excite him most.

Subject-matter insights serve also in combination with classroom experience to form the training ground for participation in general educational planning and development. Our future leaders in mathematics and mathematical education must necessarily rise from the ranks of great teachers to meet the challenges of the times in the dynamic progress of the world.

TEACHERS COLLEGE, COLUMBIA UNIVERSITY

MATHEMATICS: ITS PLACE IN TEACHER TRAINING

By
MYRON F. ROSSKOPF,
Professor of Mathematics

Myron F. Rosskopf

At no time in the academic life of the writer, extending back to the late twenties, has teaching mathematics been so exciting as at present. During the depression years of the twenties, few persons proposed new programs, or new approaches to topics. Teachers were on the defensive, busily engaged in a bitter fight to hold a place for mathematics in the curriculum. The war years saw teaching devoted to essentials - or what at that time were thought to be the essentials - of mathematics. Other matters than what was happening in the field of mathematics took all our attention.

The expansion in the types of positions open to mathematically trained men and women in the post-war years and the rapidity with which applications of mathematics were made in industrial and government research served to underscore the need for changes in the collegiate, secondary school, and elementary school mathematics programs. The initial work was done in connection with the first two years of college. Soon, however, realization came that few changes at that level would be effective without some work being done on the secondary school program. Although the arithmetic curriculum for the elementary school is sound, still there are portions of it that can be strengthened. Thus, there is work going on subsidied by foundations, by states, and by the Federal government through the National Science Foundation. Activity has progressed to the point where individuals in schools, or even whole school systems, are trying out new programs. It is exciting.

But there is one drawback to all this. In order to present well any one of the suggested new programs, a teacher has to know more mathematics. And not just any mathematics, but so-called modern mathematics. The language of mathematics is changing; old concepts have a different emphasis; new concepts are introduced. The result is a great increase in the number of in-service and summer programs in mathematics for teachers at all levels of instruction.

The effect is felt at Teachers College, Columbia University. All of our students have already earned a Bachelor's degree. They are working toward a Master's, a Doctor of Education, or a Doctor of Philosophy degree. No matter at what level they study, they must earn credits in mathematics courses. It is impossible to earn a degree in the department through professional education and methods courses alone.

Old courses have been changed. New courses have been introduced. But all have the flavor of contemporary mathematics and reflect the changes suggested for the secondary school program. Even the methods courses are no longer devoted to the old topics in algebra and geometry and how to teach them effectively. Nowadays a member of such a class is exposed to the language of sets, how to develop related concepts, the logic underlying mathematics, and the latest in geometry programs.

Mathematics teachers are finding that their chosen field is indeed an advancing one. To be sure this makes some uncomfortable, and there are objections, criticisms, and dragging of feet. But they form a minority. The larger group is enthusiastic in their study; in their discovery that mathematics involves ideas as well as skills; and in their growing realization that there are patterns that serve to relate large parts of mathematics. One of the most encouraging observations that can be made, so far as the future of mathematics and its teaching are concerned, is the growing number of capable students who are finding their way into the profession. Anyone considering secondary school teaching who is hesitant because of questioning whether he, or she, will find someone to talk to who is truly interested in mathematics among his future colleagues can put his mind at ease. There are many such and, we are pleased to say, their number is increasing.

WOODROW WILSON NATIONAL FELLOWSHIP FOUNDATION

COLLEGE TEACHERS FOR TOMORROW

By
HANS ROSENHAUPT
National Director
Princeton, New Jersey

Hans Rosenhaupt

The general need for teachers has been widely publicized, but not many facts about the critical shortage of qualified college teachers are known. In the Sixties we will need 25,000 new college teachers a year, whereas today the total annual production of Ph.D's has not quite reached the 10,000 mark. Remember that less than half of all Ph.D.'s enter the profession of college teaching, and you have a good picture of the critical shortage anticipated.

Inevitably, heightened demand for new faculty brings on a lowering of education requirements. For example, the ratio of Ph.D.'s among newly hired mathematics instructors diminished from 34.2 per cent in 1953-1954 to 20.0 per cent in 1958-1959. While the shortage of college teachers has thus impaired quality, it has had a desirable concomitant: under the pressure for manpower the employers - presidents and deans - have to an increasing extent become receptive to the idea of hiring women: the percentage of women among newly hired faculty in mathematics rose from 17.8 in 1957-1958 to 18.7 in 1958-1959; 77 per cent of over 900 college officials polled in 1958-1959 indicated a willingness to explore the possibilities of hiring more women faculty members. I cannot prove that racial, religious, and regional prejudices among academic employers have been similarly affected, but I believe so. In my travels around the country college deans often ask for names of candidates for faculty positions, and it seems that the more or less veiled reservations against certain minorities - not uncommon in the past - have virtually disappeared.

I wish I could say that young **Ph.D.'s** have become equally enlightened. Placing good men and women in good jobs is sometimes frustrating because many future colleagues will consider only a few choice regions of our beautiful and varied country and only a few colleges. The residents of big cities, **particularly** New York, often think that all life outside their city is unbearable, and too many think that all academic happiness is contained in a few prestige colleges and universities. Because they have narrowed their choices so drastically, opportunities for these young scholars are so reduced that from their own subjective viewpoint they can't see the abundance of choices which in fact exists.

The most tangible result of the teacher shortage, of course, has been the spectacular improvement in faculty salaries. In the two years from 1956 to 1958 the median salary for all academic grades rose almost 15 per cent. The rise has been most noticeable in full professors' salaries – where increases affect the total budget less than raises in the lower ranks. However, the median salary for instructors too has been favorably affected. For the entire country the median in 1955-1956 was $4,087 whereas by 1957-1958 it had risen to $4,562.

Finally the college teacher shortage has improved the chances of young college teachers for more elevated academic positions. During the depression I served many years as an instructor – at an annual salary of $1000 – before being promoted to assistant professor, and I can't recall one instance of a newly appointed faculty member fresh out of graduate school being offered anything better than an instructorship. Today chairmen in some fields, in order to attract desirable young faculty, frequently offer assistant professorships to young men and women who have only recently **earned** their **Ph.D.'s.**

The need for college teachers is particularly intense in the sciences, especially physics and mathematics; in economics; in foreign languages, in business administration, and in engineering. College teaching, particularly in a field such as mathematics, offers good opportunities for the immediate future and great prospects for the years to come.

Don't even give a thought to teaching if your principal interest in life is money. Anyone who chooses his future occupation mainly for the steak dinners it will provide is destined to be a dissatisfied teacher. The pay is adequate and will become better than that, but it will rarely equal salaries offered in industry. The truly great prospects for any teaching career – secondary or college – consist in the growing realization among our fellow citizens that the life of the mind has dignity. The teacher of the future will enjoy greater social respect than teachers in the past. He will instruct students whose eagerness for what he has to offer will be keener than ever **before.** And the prospect of working with young people eager to profit from his knowledge is one which every true teacher will find exciting.

Pi Mu Epsilon members who are at this stage weighing the advantages of graduate school against immediate employment need not decide on a teaching career yet. But they should keep in mind that there are many good job opportunities for mathematicians with graduate training, be it as teachers or in industrial or government employment. The income in a job, to be sure, is higher initially. But in the long run the mathematician with advanced training is bound to have a more challenging – and generally better paying – position.

One last thought: to an ever growing extent graduate students are able to count on financial support either from the federal or a state government, from the graduate school itself, or from a foundation such as the one with which I am connected. Many Pi Mu Epsilon members have won Woodrow Wilson Fellowships, and my colleagues in other fellowship programs, particularly those of the National Science Foundation and of the National Defense Education Act, must be as pleased as I am that they have a small share in helping outstanding young men and women to enter graduate schools.

The **fall** of your senior year is the time to inquire about fellowship and assistantship opportunities for next fall. Consult your Woodrow Wilson Campus Representative, your dean, your departmental chairman, or your teachers for information about the numerous opportunities to finance graduate studies.

NOTICE TO INITIATES

On initiation into Pi Mu Epsilon Fraternity, you are entitled to two copies of the Journal. It is your responsibility to keep the business office informed of your correct address, at which delivery will be assured. When you change address, please advise the business office of the Journal.

NEWS AND NOTICES

Edited by
Mary L. Cummings, University of Missouri

Charles C. Dillio, associate professor of mechanical engineering, and Dr. Isador Sheffer, professor of mathematics, were two of 11 teachers cited for excellence in teaching: by The Pennsylvania State University this year.

Both are members of Pi Mu Epsilon. They received citations and $100 stipends at June commencement exercises for their teaching excellence.

University President Eric A. Walker noted that the awards "recognize outstanding service, encourage superior teaching, and advance the cause of higher education generally."

Dr. J. Sutherland Frame, Director-general of Pi Mu Epsilon, and for the past seventeen years head of the mathematics department at Michigan State University, will step down from the chairmanship, at his own request, in order to devote more time to writing and research.

WINNERS OF AWARDS AND FELLOWSHIPS

ARIZONA STATE UNIVERSITY

Jonathan Wexler has been at the University of Chicago under a Woodrow Wilson Fellowship during 1959-60. He was awarded a National Science Foundation Fellowship for next year, but declined it because he was offered a more attractive fellowship in Meteorology by the University of Chicago.

AUBURN UNIVERSITY

George Dezenberg was awarded a National Defense Education Act Fellowship in Electrical Engineering at the University of Arkansas.

John T. Ellis III was awarded a Fulbright Fellowship in Mathematics for advanced study in Paris, France.

Jimmie D. Gilbert was awarded a National Science Foundation Summer Fellowship at Auburn.

Paul D. Hill was awarded a National Science Foundation Fellowship at the Princeton Institute of Advanced Studies.

Paul Major, Stanley Lukawecki and Porter Webster were awarded non-teaching fellowships at Auburn.

Joe B. Smith was awarded a National Defense Education Act Fellowship in Mathematics at Florida State University.

Kenneth E. Whipple was awarded a National Defense Education Act Fellowship in Mathematics at the University of South Carolina.

BROOKLYNCOLLEGE

Lawrence Freundlich won the Sol Cohen Memorial Award of $25 for graduating senior distinguished in mathematics.

Jay R. Goldman won the $200 Adele Bildersee Undergraduate Scholarship for outstanding scholarship.

David Goodstein was awarded the Interfraternity Council Award, a $50 bond, for outstanding scholarship.

Hannah Wolfson Rosenblum was given the Lorraine Levine Memorial Award of $50 as the outstanding woman student in mathematics.

Robert Shloming won the James W. Park Memorial Scholarship of $150, given to a student in the School of General Studies for outstanding scholastic record.

CARNEGIE INSTITUTE OF TECHNOLOGY

Lincoln E. Bragg and Melvin Hinich had National Science Foundation Cooperative Fellowships for 1959-60. Lincoln E. Bragg has another fellowship of the same kind for 1960-61.

DUKE UNIVERSITY

Gail Elizabeth Foster has a National Science Foundation Cooperative Graduate Fellowship for study at Virginia Polytechnic Institute.

William H. Halliday, Jr. was first place winner of the Julia Dale Prize.

Deborah Pike was second place winner of the Julia Dale Prize.

The Julia Dale winners receive cash awards of $25 for first and $20 for second place. The awards are determined by a special examination in mathematics.

FLORIDA STATE UNIVERSITY

William M. Boyce, Sidney D. Calkins, John D. Grow, Guy W. Johnson, and Donald Vander Jagt all have National Defense Education Act Fellowships in Mathematics, beginning the program in 1959.

John Cobb and Stanley W. Harbour! were recipients of Southern Maid Scholarships for 1959-60.

Forrest E. Dristy was awarded a Nuclear Science Fellowship for 1959-60 and again in 1960-61.

Walter J. Koss had a University Corporation on Atmospheric Research Fellowship for 1959-60.

Stephen Peyton was awarded a Bayshore (Miami) Exchange Club Scholarship for 1959-60.

Bobby L. Sanders received fellowships from the Southern Fellowship Fund both for 1959-60 and 1960-61.

James Shiver had a Gilchrist Memorial Scholarship for 1959-60, and received a Ruge Memorial Scholarship for 1960-61.

D. Bodsford Smith received the Sara Levy Scholarship for 1959-60, and again for 1960-61.

Keith P. Smith and Fredric J. Zerla were awarded National Science Foundation Cooperative Fellowships.

Sandra D. Stewart had a Florida State University Graduate School Fellowship for 1959-60.

Edward M. Takken had a Ruge Memorial Scholarship for 1959-60, and received a Davis Brothers Scholarship for 1960-61.

HUNTER COLLEGE

Lorraine Fu was awarded a fellowship from New York University.

Susan Koppelman received a New York State College Teaching Fellowship.

Diana Li received a fellowship from Lehigh University.

Stanley Mamangakis was awarded a Woodrow Wilson Fellowship, a fellowship from Cornell University, and the Gillette Memorial Scholarship Award (Hunter College).

Ellen Rosenfield received a New York State College Teaching Fellowship.

IOWA STATE UNIVERSITY

Robert C. Bueker and William H. Richardson were awarded National Science Foundation Summer Fellowships.

Harry Coonce and Kenneth Deckert have National Science Foundation Cooperative Graduate Fellowships.

Susan Jane Dobson won the Pi Mu Epsilon Prize.

Albert W. Zechmann received an I.B.M. Fellowship.

KENT STATE UNIVERSITY

Victor Ch'iu was the recipient of the Pi Mu Epsilon Award. He was given a $25 cash gift and a plaque from the Ohio Epsilon chapter.

MIAMI UNIVERSITY

Wayne Kimmel won the **Corwin** Smith Prize for being the best junior in mathematics.

Mary Jane Oring received a no-duties fellowship to Louisiana State University.

Beverly **Quanstrom** won the **McFarland** Prize for being the best senior in mathematics.

MONTANA STATE COLLEGE

John Ellefson, Ellen **Missall** and Charles Thompson were winners of National Defense Fellowships.

MONTANA STATE UNIVERSITY

Morgan Long, George McRae and Merle Morris were awarded National Science Foundation Cooperative Graduate Fellowships at Montana State University.

George McRae also won the **Lennes** Senior **Scholarship** of $100.

Jack Silver received the $100 Richard B. Wood Scholarship.

Keith Yale received a National Science Foundation Cooperative Graduate Fellowship at the University of California.

OHIO STATE UNIVERSITY

Robin Chaney, Daniel Giesy and Frank Williams were awarded National Science Foundation and Woodrow Wilson Fellowships.

OHIO WESLEYAN UNIVERSITY

Robert L. Wilson received first prize in the sophomore division of the Leas Prize competition The Leas Prize competition is a local test given in two sessions to freshmen and sophomores and is intended to encourage early interest in mathematics.

POLYTECHNIC INSTITUTE OF BROOKLYN

L Martin Isaacs, Donald **Passman** and Gerald Stoller constitute the team that won first prize in this year's William Lowell Putnam Mathematical Competition Mr. Isaacs received a Woodrow Wilson Fellowship at Harvard University, while Mr. **Passman** and Mr. Stoller were awarded National Science Foundation Fellowships at Harvard.

ST. LOUIS UNIVERSITY

James G. **Broerman,** Willard **J. Hannon,** Sr. Gregory **M.** Meyer, Robert Rutledge, Richard F. David, David **M.** Detchmendy and Bro. Augustine **S. Furumoto** have won National Science Foundation Fellowships for 1960-61.

Daniel **J.** Troy has a National Science Summer Fellowship.

Thomas **Volkmann** was appointed to a Woodrow Wilson Fellowship and will study at the University of Michigan

TEMPLE UNIVERSITY

Eli Mandelbaum, who was president of the Temple Mathematics Society and initiated the application to Pi Mu Epsilon, received both a Woodrow Wilson Fellowship and a National Science Foundation Fellowship. He expects to take his graduate work in mathematics at the University of Pennsylvania.

TEXAS CHRISTIAN UNIVERSITY

Robert Edward Huddleston was awarded a Woodrow Wilson Fellowship to the University of Arizona.

Curtis Outlaw received a National Defense Education Act Fellowship to the University of North Carolina.

Aubrey E. Taylor will study under a U.S. Public Health Fellowship at the University of Mississippi Medical School.

Fred A. Womack, Jr. has a Woodrow Wilson Fellowship to the University of Kansas. He was also offered a Fulbright grant.

UNIVERSITY OF ALABAMA

Julia Brown has a summer fellowship from the National Science Foundation.

Peggy **Mullins** has been granted a National Science Foundation Cooperative Graduate Fellowship.

Wilford Dell **Raburn** and Billy Don Weaver have been granted National Defense Education Act Fellowships.

Betty Sheffield has been given the Thomas Waverly Palmer Award, which is a yearly cash award to an outstanding undergraduate. It's value is between **$100** and $150.

UNIVERSITY OF BUFFALO

Bruce **Chilton,** Howard Humphrey and Eugene **Rozycki** were awarded National Science Foundation Summer Fellowships.

Alexander Bednarek and Richard Meyers were granted Cooperative Fellowships from the same source.

UNIVERSITY OF GEORGIA

Charles L. Christmas was granted a fellowship by the National Science Foundation for study at a summer institute for high school teachers.

Britain **J.** Williams has a **National Science** Foundation Summer Fellowship.

Roy E. Worth has a Cooperative Graduate Fellowship from the same source.

UNIVERSITY OF ILLINOIS

Harvey Kenneth **Shepard** has been awarded a Woodrow **Wilson** Fellowship for graduate study.

UNIVERSITY OF KANSAS

Spencer Dickson and Alfred Gray each won a $25 prize for the highest junior-senior score on The University of Kansas mathematics contest.

Roger T. Douglass and Richard Speers were awarded Woodrow Wilson Fellowships. Mr. Douglass will study at the University of Michigan Mr. Speers will be at Yale University.

George C. Gastl, Alfred Gray. Martin Lang and Raymond **Pippert** were granted National Defense Education Act Fellowships. All will continue their work at the University of Kansas.

Edward Gaughan was awarded a National Science Foundation Summer Fellowship.

Harold Hanes received a National Science Foundation Predoctoral Fellowship, while Thomas Kezlan and Charles **J.** Stuth were awarded Cooperative Fellowships from the same source.

Ann Marsh has a University of Kansas Exchange Scholarship and will study at the Swiss Federal Institute of Technology, Zurich.

UNIVERSITY OF KENTUCKY

Jerry P. King and William T. Sledd have National Science Summer Fellowships for Teaching Assistants. Mr. Sledd will continue for the year on a Cooperative Fellowship.

Thomas S. **Bagby** won the Elementary Physics Achievement Award (book award).

Lael F. **Kinch** and William E. **Kirwan** II were given Omicron Delta Kappa Book Awards.

UNIVERSITY OF MARYLAND

Diana Clark was granted a Woodrow Wilson Fellowship.

Howard **Rawlings** received a National Science Foundation Cooperative Fellowship.

Herbert **B. Putz** received a National Science Foundation Summer Fellowship.

George **Blakely** has a Research Associateship from the Air Force Office of Scientific Research.

UNIVERSITY OF MISSOURI

Edward Andalafte and Raymond Freese were granted National Science Foundation Cooperative Fellowships. Both are doctoral candidates.

William Kirk, Carol Sexton and Eugene Steiner have Summer Fellowships from the same source.

William F. Brinkman, Jr. was granted a National Defense Education Act Fellowship. He will work in physics.

Robert Fairbanks, Virgil **Hein** and Charles Kuehnel were winners, in a **three-way** tie, of the Pi Mu Epsilon Calculus Competition Each r eceived a $20 cash award. The competition is sponsored by the **Missoui** Alpha chapter.

UNIVERSITY OF NEVADA

Margot **Berney** won the Mary Elizabeth Talbot Scholarship in Mathematics, administered by the Department of Mathematics at the University of Nevada.

Eugene Isaeff received a grant from the **Marye** Williams Butler Fund, also administered by the Department of Mathematics.

UNIVERSITY OF SOUTH CAROLINA

Elleanor Crown was swarded the Rion Honorary Scholarship in Mathematics at the University Awards Day.

UNIVERSITY OF UTAH

John Ronald Jones was the winner of the Gibson Award, $25 given by the Gibson **family** in memory of Professor J. L. Gibson, who was for many years chairman of the Department of Mathematicsat the University of Utah.

UNIVERSITY OF WASHINGTON

William **Faris** received both a Woodrow Wilson Fellowship and a National Science Foundation **Predoctoral** Fellowship. He will enroll in Princeton University.

UNIVERSITY OF WICHITA

Ellen Jean Kolde won the Pi Mu Epsilon Mathematical Scholarship Award furnished by the Kansas Gamma Chapter of Pi Mu Epsilon.

UNIVERSITY OF WISCONSIN

Robin Ward **Chaney,** Daniel Perry Giesy, Alfred B. Manaster, Andrew Peter Soms, Edgar Lee Stout, William Harold Row, Jr., and Joseph M. Weinstein were awarded Woodrow Wilson Fellowships.

Kenneth P. Casey, John L Cobb, John R. **Dowdle,** Charles **Ehrenpreis,** Edward C. Ingraham, John **T. McCall,** Jr., David G. Moursund, Andrew M Olson, Richard **D. Sinkhorn** and Francis **D.** Williams received National Science Foundation Graduate Fellowships.

Howard Bell, Lawrence Cannon, Paul Dussere, Donald **Gignac,** David **Gillman,** Martin **Hanna,** John **Hempel,** Eugene **Krause,** Joan Rand, Charles **Seguin,** Richard Sinkhorn, Maynard Thompson and Pat Tucker were granted National Science Foundation Cooperative **Fellowships.**

David H. Carlson has a **National** Science Foundation Summer Fellowship.

NATIONAL MEETING OF PI MU EPSILON - FALL, 1960

The national meeting of Pi Mu Epsilon was held on the afternoon and evening of August 30 at Michigan State University, East Lansing. This meeting was held in conjunction with the joint meetings of the Mathematical Association of America and the American Mathematical Society.

Registration for this meeting was held the morning of August 30. A luncheon was held at noon which was followed by a business meeting. There were eight papers presented by members of Pi Mu Epsilon at three sessions. Two sessions were in the afternoon and one in the evening. After the evening meeting a social hour for members, guests and delegates was held at the home of Professor J. S. Frame, Director-General.

On the morning of August 31, the present officers and former officers who were present attended a breakfast followed by a discussion of the state of the fraternity during which future policy was outlined.

Papers presented at the three sessions were as follows:

(1) Konigsberg Bridge Problem
Fred Howlett, Nebraska Alpha - Universit**y of Nebraska**

(2) Filters and Ultrafilters
John E. Allen, Oklahoma Beta - Oklahoma State University

(3) A geometric Interpretation of the Solution of Some 3x3 Games
Mrs. Virginia S. Thrasher, New York Eta - University of Buffalo

(4) On the Number of Representations by a Cubic Polynomial Molulo **p**
Stanley **Mamangakis,** New York Beta - Hunter College

(5) Bounds for **Waring's** Problem, Modulo p
Sam Lomonaco, Missouri Gamma - St. Louis University

(6) The Construction of the Affine Plane and its Associated Group **in** Terms of the **Barycentric** Calculus
James **V. Herod,** Alabama Alpha - University of Alabama

(7) Characterizations of Certain Lattices
Geraldine A. **Jensen,** Oregon Alpha - University of Oregon

(8) Some Properties of Prime Numbers
Andrew **P. Soms,** Michigan Alpha - Michigan State University

MEETING NEXT SUMMER - WITH PRIZES

The East Lansing Michigan meeting of Pi Mu Epsilon proved such a success that the national council voted to have a meeting in August, 1961, in conjunction with the mathematical meetings in **Stillwater,** Oklahoma, and in August, 1962, in conjunction with the mathematical meetings in Vancouver, Canada. Speakers will again receive full transportation up to a maximum of $150 and official delegates, half the above amount. Each chapter may nominate either a speaker or a delegate, not both. Nominees must ***not*** have received their Master's Degree by May 1 of the year in which the meeting is held to be eligible for the travel stipend.

Two special awards will be made at these meetings:

$100.00 for the best paper presented at the meeting by an undergraduate (as of May 1 of that year) student.

$100.00 for the best paper presented by a beginning graduate student.

To be eligible for these awards, five copies of the paper must be submitted at least two weeks before the meeting. Judging will be on the basis of both written and oral presentation, as judged by the scholarship committee. Begin now to consider who should represent your chapter at these meetings. Applications must be in the hands of the secretary-treasurer in early spring. It is hoped that those attending the Pi Mu Epsilon meetings will also plan to attend the concurrent meetings of the Mathematical Association of America and other mathematical meetings.

AWARD BOOK PLATES AVAILABLE

Special book plates suitable for book awards given by local chapters will be designed and printed this fall. These book plates will be available, without charge, from the national secretary-treasurer.

SCHOLARSHIP PLAQUES

The national office is arranging to have scholarship plaques with a large Pi Mu Epsilon insignia on them available to chapters who wish to have them either for presentation or for a permanent display giving names of former winners or officers. These will be available directly from the manufacturer sometime in the spring if there is sufficient demand. Write to the national secretary-treasurer if you are interested.

PRIZE MONEY FOR YOUR CHAPTER

Local chapters giving undergraduate prizes will be delighted to know that the Pi Mu Epsilon Council voted to match local funds up to a maximum of $20.00 per chapter. The national council suggests that the prizes be mathematical books and journals of the winner's choice and/or membership in the Mathematical Association of America rather than cash. To be eligible for funds from the national treasury, the chapter must:

1. File a request on the proper form by December 1, 1960.

   (Form available from R. V. Andree, The University of Oklahoma, Norman, Oklahoma)

2. Stipulate that the chapter will at least match the funds supplied from the national treasury, and that the combined amount will be used for ***undergraduate*** prizes for mathematical excellence.

3. Agree to send a list of winners and a short description of the basis of selection to the editor of the journal, Francis Regan, St. Louis University, St. Louis 3, Missouri.

DEPARTMENT DEVOTED TO CHAPTER ACTIVITIES

Edited by

Houston T. Karnes, Louisiana State University

EDITOR'S NOTE: According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an annual report of the chapter activities including programs, results of elections, etc." The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the Pi Mu Epsilon Journal. Besides the information listed above, we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Chapter Activities Editor Houston T. Kames, Department of Mathematics, Louisiana State University, Baton Rouge 3, Louisiana. These reports will be published in the chronological order in which they are received.

REPORTS OF THE CHAPTERS

ALPHA OF MICHIGAN, Michigan State University.

The Michigan Alpha Chapter held thirteen meetings during the academic year 1959-60. The following papers were presented:

'Farey Fractions" by Dr. Heinrich Larcher
"Philosophy and Mathematics" by Dr. Harold Walsh
'Integration Over the Set of Integers" by Dr. Robert Oehmke
"Steller Evolution and Origin of the Elements" by Dr. John Mathis
"Non-Associative Algebra" by Dr. Marvin Tomber
"Problems: Old and New in Egyptian Fractions" by Dr. B. L. Stewart
"The Four Color Problem" by Joseph Ferrar
"Continued Fractions" by Dr. J. S. Frame
'A Historical Look at the Gamma Function" by Gretchen Brown

Officers for 1959-60 were: Director, Gretchen Brown; Vice-Director, Thomas McIlrath; Secretary, Maxine Perkins; Treasurer, Carolyn Premo; Faculty Advisor, Dr. Howard Campbell.

Social activities of the chapter included a fall picnic and the annual banquet at which Dr. R. L. Wilder of the University of Michigan was the guest speaker. The L. C. Plant awards were presented to the outstanding mathematics students on the occasion of the banquet.

BETA OF PENNSYLVANIA, Bucknell University

The Pennsylvania Beta Chapter held five meetings during the academic year of 1959-60. The following papers were presented:

"Modem Mathematics Curricula" by Mr. Donald G. OM
"Fortran - Programming the 704 IBM Computer" by Miss Dorothy Bell
"Life Testing" by Dr. William Mendenhall
"Some Elementary Functions from an Elementary Standpoint" by Dr. William L. Miller
'Geometric Solutions to Quadratic and Cubic Equations" by Mr. Robert Crovelli.

Officers for 1959-60 were: Director, Donald G. OM; Vice-Director, Norman Edgett; Secretary, Joan Piersol; Treasurer, Sherry Rhone; Corresponding Secretary, Dr. William L. Miller.

Social activities during the year included the annual banquet following initiation on December 2, 1959 at which time Dr. William L Miller was the guest speaker.

Officers for 1960-61 are: Director, Dr. Stanley Dice; Vice-Director, Priscilla **Teleky;** Secretary, Doris **Bryson;** Treasurer, George **Kenyon;** Corresponding Secretary, Dr. William L Miller.

ALPHA OF NEW JERSEY, Rutgers University

The New Jersey Alpha Chapter held five meetings during the 1959-60 academic year. The following papers were presented:

'Use of Modern Algebra in Genetics" by Professor Harold Gaushor

"The Computer and **Mathematics"** by Professor Fredrick Fender

Other activities included the showing of three films dealing with calculus and hosting many high school students who visited the campus on New Jersey Mathematics Day.

Officers for 1959-60 were: Director, John **Kasuba;** Vice-Director, Donald **Gallo;** Secretary-Treasurer, Richard **Hieber; Corresponding** Secretary, Philip Hirschfield.

Social activities included the annual initiation banquet at which Professor Louis F. **Nanni,** guest speaker, spoke on "Operations Research". Also the New Jersey Alpha enjoyed its annual picnic at which time all interested mathematics students and faculty members were invited.

Officers for 1960-61 are: Director, Clemens Thoennes; Vice-Director, William **Bisignani;** Secretary-Treasurer, Arthur Kagan; Corresponding Secretary, Philip Hirschfield; Faculty Advisor is Dr. Harold Grant.

GAMMA OF MISSOURI, St. Louis University

The Missouri Gamma Chapter held four meetings during the 1959-60 academic year. The following papers were presented:

"Curve Tracing" by Grattan P. Murphy

"Two Interesting Functions" by Daniel **D.** Cronin

"Peano's Postulates for the Real Numbers" by Hubert **C.** Kennedy

"Continued Fractions" by Professor **J. S.** Frame

Activities during the year included the twenty-third annual banquet at which time Mr. James D. Thomas was toastmaster. Dr. Waldo **Vezeau** presented the Pi Mu Epsilon Junior Problem Contest award to Mr. Michael Mahon and the Senior Problem Contest award to Mr. Sam Lomonaco. Miss Annette **Krygiel** received the Mathematical Award of the Chemical Rubber Company. Miss Barbara **Resnik,** Mr. Simon P. Cassens and Mr. Thomas **Volkman** received the annual James W. Gameau award for being the highest ranking seniors in mathematics.

Mr. Daniel **D. Cronin** was elected director for 1960-61 and Dr. Francis **Regan** again accepted the position of Faculty Advisor and permanent Secretary-Treasurer of the chapter.

GAMMA OF ILLINOIS, DePaul University

The Illinois Gamma Chapter **held** nine meetings during the 1959-60 academic year. The following papers were presented:

"Differential Geometry" by Louis **Aquila**

"Statistical Mechanics" by **Marylyn Prost**

"Topology" by Frances Kutt

'Fractional Derivatives" by Dr. **O'Neill**

"Fourier Series" by John Hinds

"Existance Theorem in Partial Differential Equations" by Dr. **Caton**

"The **LaPlace** Transform" by Michael Matkovich

"Algebra of Finite Sets" by Dr. O'Neill

Officers for 1959-60 were: Director, Louis **Aquila;** Vice-Director and Treasurer, Thomas Cook; Secretary, Michael Matkovich.

Officers for 1960-61 are: Director, John Hinds; Vice-Director and Treasurer, Marylyn **Prost;** Secretary, Frances Kutt.

ALPHA OF LOUISIANA, Louisiana State University

The Louisiana Alpha Chapter held eight meetings during the 1959-60 academic year. The following papers were presented:

"The Cantor Set" by Dr. R. J. Koch

"What is a Point?" by Dr. Louis **McAuley**

"The Projective Plane" by Dr. **R. D.** Anderson

"Purpose of Pi Mu **Epsilon"** by Dr. Houston T. Kames

"The Five Color Problem" by Dr. L S. **Krule**

"Special Problems in Number Theory" by Dr. Hubert S. Butts

"Differential and Integral Equations" by Dr. Joseph S. **Levinger**

"The Role of Mathematics in the Present Day World" by Dr. **Pasquale** Orocelli

Louisiana Alpha gives two annual awards based upon an honors examination One is for the graduating senior and the other is for the member of the freshman class. Mr. John Michael Callaghan received the Senior award and Mr. Jules **William** Delambre received the Freshman award. In addition the Freshman award winner also received the book award donated by the Chemical Rubber Company. Both award winners' names were engraved on a permanent plaque displayed in the Department of Mathematics.

Harvey **Carruth** represented Louisiana Alpha as its delegate to the National meeting in East Lansing, Michigan during August.

Officers for 1959-60 were: Director, Harvey **Carruth;** Secretary, Sandra **Hundley;** Treasurer, John **Weise;** Faculty Advisor, Dr. Haskell Cohen; Corresponding Secretary, Dr. Houston T. Kames.

Officers for 1960-61 are: Director, Dave Evans; Vice Director, Kenneth Freeman; Secretary, Elizabeth Sloan; Treasurer, John M. Callaghan; Faculty Advisor, Dr. Haskell Cohen; Corresponding Secretary, Dr. Houston T. Kames.

BETA OF OKLAHOMA, Oklahoma State University

The Oklahoma Beta Chapter held fifteen meetings during the 1959-60 academic year. At a joint meeting with the honorary fraternities of His**tory,** Physics and Electrical Engineering, Dr. T. L. **Agnew** was the guest speaker. His topic was "The Intellectual Development of the Twentieth Century."

Social activities included the Spring Banquet at which Prof. Nathan A. Cort was the guest speaker.

Officers for 1960-61 are: Director, John Ed Allen; Vice-Director, Roger Allen; Secretary, **Biruta** Stakle; Treasurer, George **Dysinger;** Faculty Advisor, Prof. John E. Hoffman

ALPHA OF NEBRASKA, University of Nebraska

The Nebraska Alpha Chapter held eight meetings during the 1959-60 academic year. The following programs were presented:

"Mathematical Fallacies" by Paul Dussere

"Euler" by Al **Vennix**

"Non-Associative Arithmetic" by Prof. Trevor Evans

"Regular Polyhedra" by James E. Kellogg

"Opportunities in Mathematics" by Prof. **R. E. Larson**

"Differential Equation of Damped Harmonic Motion" by Prof. Hunzeker

"Euler's Formula" by Prof. Rebeiro

"Exhaustion and Eudoxes" by Prof. Guy

The Freshman Award was given to Larry **Dornhoff.** The Prize Exam winners were: Fall, 1959: David Gustavson and Henry Pollock; Spring, 1960: David Bliss and Dave Roberts.

Officers for 1960-61 are: Director, Thomas F. **Eason;** Vice-Director, Dennis Nelson; Secretary, James E. Kellogg; Treasurer, John A. **Byram;** Faculty Advisor, Prof. **H. L. Hunzeker.**

BETA OF OREGON, Oregon State College

The Oregon Beta Chapter held its annual Pi Mu Epsilon Mathematics Competition for Freshmen and Sophomores. The winners were as follows: First Prize, Paul Hurdlik; Second Prize, Botan Eross; Honorable Mentions, Marilyn McLennan, Walker Powell and Linda S. George.

The chapter also added copies of German-English, French-English and Russian-English dictionaries to the chapter library.

Officers for 1960-61 are: Director, Don Marsh; Vice-Director, L Ayyoub; Secretary, F. D. Stevenson; Corresponding Secretary and Treasurer, Dr. A. R. Poole.

ALPHA OF NORTH CAROLINA, Duke University

The North Carolina Alpha Chapter held four meetings during the academic year 1959-60. The following papers were presented:

'Job Opportunities for Mathematics Majors" by Mrs. Eugene Smith

"Knot Theory" by Dr. H. H. Debrunner

Officers for 1960-61 are: Director, John A. Koskinen; Vice-Director, Lawrence S. Williams; Secretary, Carol Ann Wilson; Treasurer, Sarah Core.

ALPHA OF KENTUCKY, University of Kentucky

The Kentucky Alpha Chapter held six meetings during the academic year 1959-60. The following papers were presented:

"Computers Write Their Own Programs" by Dr. J. Hamblen

"A Short History of Geometry Stressing Euclid" by Mrs. Marion J. Ball

"The Fundamental Theorem of Arithmetic" by Mr. L Presson

Other activities included the annual banquet at which time Dr. Karl Lange spoke on "History of the Meteorological Study of the Upper Atmosphere."

Officers for 1960-61 are: Director, William Sledd; Vice Director, John Pfaltzgraff; Librarian, Steadman Bagby, Jr.; Secretary, Cecily Sparks; Treasurer, Clifford Swauger; Faculty Advisor, Dr. T. J. Pignani. Dr. John B. Wells was elected to replace Dr. H. H. Downing as Permanent Corresponding Secretary.

ALPHA OF NEW YORK, Syracuse University

The New York Alpha Chapter held eight meetings during the academic year 1959-60. To promote the purpose of the Honorary and to establish prestige for Pi Mu Epsilon, a mathematics contest was held in April 1960 for high school seniors in Onodaga County in New York State. A full-tuition scholarship was given to the highest person on this examination (four years at Syracuse University). This turned out excellent and the idea is to be expanded in 1960-61.

Officers for 1960-61 are: Director, James F. Pasto; Vice-Chairman, Richard C. Flaherty; Secretary, Pearl Reim; Treasurer, Albert Vosburg.

ETA OF NEW YORK, University of Buffalo

The New York Eta Chapter held six meetings during the academic year 1959-60. The following papers were presented:

"Linear Programming" by Fred Miller

"Cubism" by Bruce Chilton

"Linear Transformations of Vector Spaces" by Stephen Graczyk

"The Carden-Tartaglia Dispute" by Richard Feldmann

"Exhaustively Describing the Communication Process" by B. Checkman

"Some Intuitive Ideas in Measure Theory" by Dr. Albert Fadell

Officers for 1960-61 are: Director, Richard Feldmann; Vice-Director, James Lowerre; Secretary-Treasurer, Ruth Heintz.

ALPHA OF GEORGIA, University of Georgia

Georgia Alpha Chapter held ten meetings during the academic year 1959-60, of which six were program meetings. The following papers were presented:

"Spirals" by Dr. B. J. Ball

"Multiplications" by Dr. J. G. Home

"On the Theory of Primes" by Dr. John W. Jewett

"The Combinatorial Problem" by Dr. M. K. Fort, Jr.

"The Measure of False Statements" by Dr. T. R. Brahana

"Nets" by Curtis P. Bell

Other activities included two initiations, a party during the fall session and a banquet during the spring session

Officers for 1960-61 are: Director, Britain J. Williams; Vice-Director, Roy E. Worth; Secretary, Susan Johnson; Treasurer, Mike Donahue; Faculty Advisor, Dr. J. G. Horne.

GAMMA OF CALIFORNIA, Sacramento State College

The California Gamma Chapter held three meetings during the academic year 1959-60. The following papers were presented:

"Baronov's Theorem on Vibrating Systems" by Dr. O'Callaghan

"Physical Theory" by Dr. Melvin O. Fuller

BETA OF KANSAS, Kansas State University

The Kansas Beta Chapter held five meetings during the academic year 1959-60. The following papers were presented:

"Rank Order Statistics" by Dr. Stanley Wearden

"The Contributions of Measure Theory to Axiomatic Problem Theory" by Dr. A. M. Feyerherm

"Almost Separating Points" by Dr. J. M. Marr

The annual initiation banquet was held on May 16, 1960 with eighty members and guests present. The banquet speaker was Dr. Lowell Schipper. The title of his address was "What Makes Johnny Gamble?"

Officers for 1960-61 are: Director, F. J. McCormick; Vice-Director, Grace Woldt; Secretary, Evelyn Kinney; Treasurer, S. T. Parker.

BETA OF GEORGIA, Georgia Institute of Technology

The Georgia Beta Chapter held four meetings during the academic year 1959-60. The following papers were presented:

"Group Theory" by Charles Roberts

"Some Remarks on the Heat Equation" by Dr. John A. Noel

"Infinite Series" by Dr. Tomlinson Fort

"Proof of Picard's Theorm Using the Principle of Contraction Mappings" by Stanley Wertheimer

Officers for 1960-61 will be elected at the first fall meeting.

EPSILON OF OHIO, Kent State University

The Ohio Epsilon Chapter held seven meetings during the academic year 1959-60. The following papers were presented:

"Contrapositive, Reductio ad Absurdum, and Direct Methods of Proof in Geometry" by Miss Maureen Weber

"The Abacus and Its Use" by Miss Young Kim

"A Problem on the Gyrocompass - Experience in Industry" by Mr. Dennis Gilliland

"Computor Mathematics" by Dr. Magnus Hestenes

"A Topic in Algebra" by Prof. L. Earle Bush

"Solution of Simultaneous Linear Equations" by Mr. Jan Bauer

Films shown during the year included "Big Numbers", "New Numbers", "Earliest Numbers" and "Fractions".

The guest speaker at the annual banquet was Professor Russell Iwanchuk who spoke on "Education in Mathematics in Eastern Europe".

Social activities during the year included the annual Student-Faculty Tea at which time Dr. Magnus Hestenes was the honored guest.

The Pi Mu Epsilon award was presented to Mr. Victor Ch'iu at the university Honors Day assembly.

Officers for 1960-61 are: Director, Carol Pay; Vice-Director, Mary Deisman; Treasurer, Elizabeth Ryan; Faculty sponsor, Professor John Kaiser.

ALPHA OF CALIFORNIA, university of California at Los Angeles

The Alpha Chapter at U.C.L.A. held twelve meetings during the academic year 1959-60. The following papers were presented:

"Advice to a Student of Mathematics" by Prof. R. S. Phillips
"To Continue or Not to Continue . . .?" by Prof. R. Steinberg
"The Rational Gambler" by Prof. P. G. Hoel
"Mathematical Expectation" by Prof. R. M. Redheffer
"The Least" by Prof. M. R. Hestenes
"More or Less" by Prof. E. F. Beckenbach
"Under Constant Surveillance" by Prof. E. A. Valentine
"Game Theory" by Prof. B. Gordon
"Who Knocked Out Round Robin?" by Prof. P. B. Johnson
"The Greatest and the Least" by Prof. E. F. Beckenbach
"On Number Theory" by Prof. K. Rogers
"The Lion and the Martyr" by Prof. A. Gleason

During the year a Graduate Colloquium Lectures Series was planned on an experimental basis for the Fall Semester. These Lectures were held once a week and were given by members of the mathematics faculty. It was considered highly satisfactory, but was discontinued during the Spring Semester in order to avoid duplicating existing departmental seminars.

Social activities during the year included the annual picnic and two initiation ceremonies.

Officers for 1959-60 were: Director, J. W. Lindsay; Vice-Director, J. F. Mount; secretary, E. A. Sallin; Treasurer, Prof. R. Blattner; Faculty Advisor, Prof. R. Redheffer.

Officers for 1960-61 are: Director, E. A. Sallin; Vice-Director, T. McLaughlin; Secretary, Barbara Ames; Treasurer, Prof. B. Gordon; Faculty Advisor, Prof. E. Beckenbach.

ALPHA OF PENNSYLVANIA, university of Pennsylvania

The Pennsylvania Alpha Chapter held eight meetings during the academic year 1959-60. The following papers were presented:

"An Affine Transformation" by Dr. Perry A. Caris
"Axioms of Set Theory" by Dr. Smbat Abian
"Quadratic Forms" by Dr. Bernard Epstein
"Symmetry" by Dr. David Harrison
"Curvilinear Coordinates" by Mr. Richard Larson
"Archimedes Died Thrice" by Dr. Pincus Schub
"How Not to Trisect an Angle" by Mr. Murray Eisenberg
"General Economic Equilibrium" by Mr. David Ostroff

Officers for 1960-61 are: Director, Kenneth Hertz; Vice-Director, Richard Larson; Secretary, Howard Wasserman; Treasurer, Alan Gart; Faculty Advisor, Dr. Pincus Schub.

GAMMA OF NEW YORK, Brooklyn College

The New York Gamma Chapter held seventeen meetings during the academic year 1959-60. The following papers were presented:

"Real Number System" by Harry P. Allen
"Cardinality" by Richard Pollak
"Symbolic Logic and Electric Circuits" by Norman Bleistein
"Theory of Incidence Geometry" by Prof. Walter Prenowitz
"Introduction to Modem Algebra" by Harry P. Allen
"The Axiom of Choice" by Jay Goldman
"Theory of Knots" by Prof. J. Singer
"Mathematicians of the French Revolution" by Prof. C. Boyer
"A Higher View of Determinants" by Mr. L. Saremsky

Officers for 1960-61 are: Director, Saul Zaveler; Vice-Director, Jay Goldman; Secretary, Ann Fassler; Treasurer, Joan Leventhal.

ALPHA OF MONTANA, Montana State University

The Montana Alpha Chapter held nine meetings during the academic year 1959-60. The following papers were presented:

"Method of Projections" by Dr. Joseph Hashisaki
"Transcendental Numbers" by Dr. William Myers
"Remarks on Finite Differences" by Dr. W. R. Ballard
"How a Statistician Makes Up His Mind" by Dr. Howard Reinhardt
"Remarks on a Unifying Set of Axioms" by Dr. Paul Rygg
"A History of Geometry" by Dr. T. G. Ostrom
"The Foundations of Mathematics" by Dr. Frederick Young
"How to Make Two Unit Spheres Out of One" by Mr. Jack Silver
"The Axiomatization of Mechanics" by Mr. Keith Yale

The Pi Mu Epsilon awards for outstanding achievement in mathematics and physics were presented to Jack Silver and Harry Bauer, respectively.

ALPHA OF KANSAS, University of Kansas

The Kansas Alpha Chapter held five meetings during the academic year 1959-60. The following paper was presented:

"Some problems in Number Theory" by Fred Morrison

Other papers were presented at the series of meetings known as "Colloquia". These meetings were in conjunction with the Mathematics Club for which Pi Mu Epsilon supplied half the speakers.

Officers for 1960-61 are: Director, DeWayne S. Nymann; Vice-Director, Charles Stuth; Recording Secretary, Thomas Kezlan; Treasurer, Katheleen O'Donnell; Corresponding Secretary, Wealthy Babcock; Librarian, Gilbert Ulmer.

LEHIGH UNIVERSITY BRANCH OF PI MU EPSILON

Lehigh University Branch of Pi Mu Epsilon held four meetings during the academic year 1959-60. The following papers were presented:

"A Numberic Value of π as Found in the Bible" by Dr. Wilansky
"The Familar Pouring Problem" by Dr. Wilansky
"Can a Scientific Hypothesis be Refuted?" by Dr. Grumbaum

Officers for 1959-60 were: Director, William Parks; Secretary, Ralph Weyer; Treasurer, Peter Shoenfeld.

Officers for 1960-61 are: Director, William Freed; Secretary, John Buchanan; Treasurer, Richard Moll.

ALABAMA ALPHA, University of Alabama, (April 30, 1960)

Roberta Jane Adams
John Tilmon Bagwell, Jr.
Lee Pershing Dodd, Jr.
Marion Kenneth Etheredge
William Hull Forster
Victoria E. Gonzalez
Walter LeVaughn Hales
Phillip Glenn Harris
Lester Katz
Glenn Thomas Kimbrough
Edward Perry Phillips
Wilford D. Rabum
Jarvis DeVaughn Ryals
Susan Agnes Schembs
Samuel Paul Shramko

ALABAMA BETA, Alabama Polytechnic Institute, (Spring, 1960)

William Wayne Bailey
Lucian F. Bloodworth
Russell M. Brengelman
Royal E. Colson
Bobby B. Edwards
Judity Kay Farkas
Olga Naomi Hamilton
Robert Allen Hardekopf
Grady R. Harmon
Martha Blanche Hodges
Thomas R. Horn
James Nestor Issos
Ernest Casey Jones
Max Killingsworth
Frank B. Lockridge, Jr.
Charles B. Mathews
James D. McMillan
Martha Helen Moseley
Alton Benaja Overstreet
Walter T. Pease
Herman H. Plott
William E. Reynolds
Frederick J. Richmond
James C. Rogers, Jr.
Herbert G. Sanders
Deward V. Sloan, Jr.
Fred W. smith
Paul L. Speckmann
D. Reginald Traylor
Thomas H. Vanderver
Roland L. Waters.
Terry Blaine Watson
Kenneth E. Whipple
Kenneth N. Williams

ARKANSAS ALPHA, University of Arkansas, (April 13, 1960)

Phillip Lee Almond
Carroll Fairfax Blakemore
Charles C. Bodishbaugh
Robert James Byers
Margaret Elizabeth Bates
Walter L. Graves
Melvin Reed Greenwood
Nell Rose Greer
Robert Lester Hall
David Benjamin Holt
Jimmy Wyatt Ivey
Robert Mar
Ann Martin
Kazuo Oishi
Joseph Albert Plunkett
Glenn D. Sandlin
James Buford Sivley
John Arthur Sparks
Pat Throneberry
Frances Wilson

ARIZONA ALPHA, University of Arizona, (May 23, 1960)

William P. Bennett
Harry E. von Bergen
Donald J. Collins
Charles O. Ford
Horace B. Gardner
Barney D. Hunts
Kathleen A. Langford
Roy A. Lippman
Shelby G. McCauley
George S. McLain
Dan W. Patterson
Harry E. Ruhsam
Pat L. Swanson

CALIFORNIA ALPHA, U.C.L.A., (January 9, 1960)

Barbara Ames
Martin Barmatz
Manuel Berri
Theodore Clarke
Daniel Gallin
Thomas L. Humphrey
Gerald Hutchison
Robert Inman
John R. Klugh
Marvin Lubofsky
Gilbert M. Masters
William J. Reilly
Edwin B. Stear
Maxine Stem
Froylan Tiscareno
Kenneth K. Warner
Walter Zimmerman

(June, 1960)

Leonard Asimow
Nancy Ault
George Chapline
Edward Fairbrother
Edward Fletcher
Stanley Franklin
Daniel Gottlieb
Gabriel Broner
Charles Howard
Trygve Lerwick
Irwin Levin
John McGhee
David Osteyee
Gene Potter
Peter Saecker
Alfred Schainblatt
Abraham Silvers

CALIFORNIA GAMMA, Sacramento State College, (May 27, 1960)

William Arthur Abolt
David Clarence Barnes
Marilyn Louise Carlson
Chung Fong
Marino Giustino
John Stanley Gray
Dennis Leigh Hobde
Kazuo Masai
Susanne M. Shelley
Mary Wyat
Marian Yoko Yoshikawa

FLORIDA BETA, Florida State University, (May 11, 1960)

Margaret Ann h o l d
Mary Frances Betts
Robert Michael Brush
Phung Lien Doan
Stewart B. Fox, Jr.
Neil L. Frank
Timothy C. Galvin
Clifton Averette Johnson
Guy W. Johnson
William Robert Johnson
Armand Monaco
Philip Jonathan Owena
D. Bodsford Smith, Jr.
Robert L. Stallard
Donald Wayne VanderJagt
John Calhoun Wells, Jr.

GEORGIA ALPHA, University of Georgia, (May 10, 1960)

Stanley C. Beard
Charles Lee Christmas
Robert Lee Franklin
Susan Kaye Johnson
James David Lifsey
Anne Walton
Roy Eugene Worth

GEORGIA BETA, Georgia Institute of Technology, Uune 5, 1960)

William Y.S. Chen
Bertram M. Drucker
Albert C. Holt
Charles Rhind Newman
John A. Nohel
Wilbur Janes Stiles

ILLINOIS ALPHA, University of Illinois, (June 1, 1960)

Richard Keith Ahrenkiel
Mehdi Nejad Bahadori
Peter Joseph Bertoncini
John Williams Burton
Henry Grady Campbell, Jr.
Leslie Muir Cooper
Robert Edward Crawford
Peter Kuanghsun Dai
Geraldine C. Darden
Richard Earl Dennis
Cutberto Diaz-Gomez
Phillip Harris Doppelt
James Ray Downing
Rachel Ragle Dyal
Harvey Lynn Elder
William Ray Everett
Chien Fan
Robert Clinton Fay
Morris W. Firebaugh
John Francis Fitzgerald
Vernon Eugene Friesen
Virgie Herrin Puller
Mildred Mary Gausman
Alice Graeber
Marvin Willard Grossman
Hubert Lee Grush
John Leo Gubser
Stanley Phillip Gudder
Kenneth George Harbison
John Gerard Harrlngton
Carl Lane Hanman
Marjorie Chelsea Horton
Robert Leo Johnson
Kay Keiji Kanazawa
Evangelos Kostogiannis
Toshiro Kunihiro
Virginia Moser Latshaw
Denny Joseph Laughhunn
John Richard Lehmann
Jane Ellen Lemme
Richard Anthony Lutz
Douglas John Malewicki
Leo Wesley Manuel
Andre Robert Marguinaud
William L. McMillan
James John Meaders
Charlotte Milstein
Kenneth Francis Morman
Virginia W. Mullins
Thomas Earl Murley
Katherine A. O'Brien
Henry Chuenhsien Pao
Philip Irvin Pavlik
Theodore R. Portis
Robert Walter Prielipp
Joseph Anton Raab
James Albert Resh
Joe Rios
Joan Lange Ross
Paul L. Sadagursky
Mohammed Safiuddin
Edwin T. Schulz, Jr.
William M. Schuyler, Jr.
Glenn T. Sincerbox
Darbari Lal Sharma
Amarjit H. Slngh
James Edward Skeath
Judith R. Slotnikoff
Sigmund h o l d Smith
Robert James Spry
Robert William Stafford
John Joseph Stein, Jr.
Sydney Stephens, Jr.
Robert Joseph Strain
Edward Clayton Straub
Anne Penfold Street
Robert Earl Swarthwout
Robert Lawrence Thomas
Paul Hanson Tingleff
Carl John Twkstra
Robert Jerome Valek
Joseph Earl Valentine
Masami Wakae
Ru-Liang Wang
Daniel James Weintraub
Stephen Hughes Williams
Howard Leroy Wilson
Paul Eugene Wilson
Ruth Eiko Wong
Anthony T. Y. Wu
Edward Leon Yellin

ILLINOIS BETA, Northwestern University, (June 6, 1960)

Elaine Cosley
William P. Cleveland, Jr.
Samuel A. Culbert
Nancy Duff
James H. Feit
Jay W. Feldmann
William A. Goodwin
Robert D. Gustafson
Samuel A. Haubold
David Hector
Walter Johannes
Bemard Lefkowitz
Robert Newhoff
Terry A. Taebel
Judy Reinach
Pat R. Roach
Paul D. Roach
Bruce Schimming
Alexander Sachs
Richard Selden
Charles J. Schwiedergoll
Patricia F. Theisen
Richard A. Volz
Frederick A. Waldmann
H. Lee Watson
William White
Ronald J. Yuill
Kenneth Zanid

ILLINOIS GAMMA, DePaul University, (October 20, 1959)

Alice Catherine Halpin

ILLINOIS DELTA, Southern Illinois University, (May 19, 1960)

Charles T. Baker
Phyllis Jean Brown
Thomas I. Brown
Lary Kent Burns
Jerry K. cline
James P. Conrad
Charlotte Michal Foster
Claude Ray Gunter
Joanna Hampton
Lewis Owen Hicks
Anita Angiin Howell
Joel W. Jennings
Ruth Ann Lavelle
Chi-hang Lee
Patricia Lee
Kendall Lee McDonald
David McIntyre
David Lee Rector
Glenda R. Smith
Virginia Mae Stewart
Rsymond E. Stockton
Joseph E. Tste
Jeanne Vine
Stephen Matthew Williams
Robert Eugene Winters

INDIANA ALPHA* Purdue University, (March 22, 1960)

Anita Louise Allison
Lowell Wayne Beineke
Charles E. Christian
Marilyn Coopersmith
David H. Fritz
Ronald Eugene Harris
Ann Elizabeth Hopkins
Carol Annis Houser
Robert Mane Jewett
Harold J. Linnerud
Joyce Mansfield
William John Maybury
James Robert Rahfeldt
Roger William Rollins
James Schmidt
Robert B. Underhill
Maxine E. Willman
Norma Jean Wright
James W. Yost

KANSAS ALPHA, University of Kansas, (April 22, 1960)

Marilyn E. Alpert
Robert E. Barnhill
Clair J. Becker
Jean Irland Challinor
Norman Clark
Thomas C. Clark
James Stanley Dombek
Joan A. Dunkin
Alice Forssberg
K. Stanley Gale
Kenneth M. Graham
Robert L. Gray
William J. Halm
Nancy G. Haskin
Douglas David Kuper
Donna Lee Lamb
Martin T. Lang
Loren Charles Larson
Yourn J. Lee
Ann Marsh
Thomas W. Mason
Fred L. Morrison
John C. Musgrave
Damon Lee Patton
Rollin Dean Quinn
James Rice
Kent D. Richert
Richard C. Rinkel
John Albert Rupf, Jr.
Nancy Suellentrop
David Earl Sutherland
B. Alan Taylor
Donald B. Tillotson

KANSAS BETA, Kansas State College, (May 16, 1960)

Rochelle Abend
David A. Adams
Gordon E. Carlson
Donald O. Christy
Shih Chi Chang
Glenn F. Cochrane
Norman D. Collins
Herbert P. Cormack
John C. Crawford
Sylvan Dawson
Marilyn McCord Dillinger
Roscoe Ellis, Jr.
Joel R. Erickson
Ping Liong Ho
Arthur S. Hobson
Han Min Hung
Alphia E. Knapp
Bong L. Koh
Gary Fredrick Krause
Donald E. Jones
Helmer B. Junghans
Ming Min Lei
Er Chieh Ma
Uma Rani Mathur
Ru-Hsin Mo
Irvin L. Reis
Joyce M. Rogers
Richard H. Schelp
Roland H. Sundberg
James E. Swain
Virginia Irene Taylor
Billy J. Thorne
Rolland D. Turner
En Shiun Ueng
Kenneth R. Veraska
Yung Kuang Wu
Chen Nuo Yu

KANSAS GAMMA, University of Wichita, (April 8, 1960)

Otto K. Boothe
Clyde M. Dubbs
Larry Raymond Hebert
Ralph S. Hoagland
Jeanne Kolde
Robert Edward Martin
D. Joe Moore
Lawrence J. Smith

KENTUCKY ALPHA, University of Kentucky, (May 12, 1960)

William David Arnett
Steadman Thomas Bagby
David James Caveny
William Anderson LaBach
Glenda Doyle Merhoff
James Edward Miller
Barbour Lee Perry
Evelyn Frances Rupard
Kusno Kronodihardso Sudagaran
Clifford Joseph Swauger, Jr.
Mary Jane Templin
Claudette Stivers Thompson

LOUISIANA ALPHA, Louisiana State University, (May 11, 1960)

Edward W. Ashford
Jeff W. Baird
William J. Bernard
Margaret Y. Cowsar
Goldwyn R. Dillard
David J. Evans
Kenneth Glenn Freeman
Lorice Gill
Mama J. Goodrich
Tybe D. Haas
Robert J. Matherne
William K. Owens
James C. Plnac
Charles Sparks Rees
Wynn Patterson Rickey
Hayes E. Ross, Jr.
Louis Larry Rost
David F. Schnebelen
Elizabeth Sloan
Alfred Lloyd Stoessell
Carvey Allen Streeter
Jean Alfred Tennant
Morgan M. Zimmerle

MARYLAND ALPHA, University of Maryland, (May 20, 1960)

Earl Carpenter
Theodore L. Felsentreger
Donald Flanders
Bernard Alexander Fusaro
Jane Conoley Gager
Jung Soo Kim
William A. Losaw
Miles D. MacMahon
James Harvard Reed
Dean Myers Reily
Nathan Rubinstein
Frank Wilson

MICHIGAN ALPHA, Michigan State University, (May 1960)

John R. Adams
George G. Bankeroff
Paul W. Beukema
Evelyn H. Brayton
Calvin K. Burge
Donna J. Decker
Doris C. DeHardt
Nancy L. Hogan
Karla K. Hoover
Joanne M. Jasper
Arvydas J. Kliore
Juang-ming Lin
Caroline L. Matto
Ricardo S. Pascual
Judith A. Peobles
A. Duane Porter
John G. Richardson
Stanly L. Steinberg
Merlin L. Wheeler
Rita B. Zemach

MISSOURI ALPHA, University of Missouri, (May 10, 1960)

Richard C. Allen, Jr.
Herbert Eugene Black
Wilson Edward Brunley
Carol Ann Buchmueller
~~George Burke~~
John Robert Cochran, Jr.
William G. Copenhaver
John Howard Cupp
Wayne Francis DeVilbiss
Nancy Elizabeth Ely
Robert William Fairbanks
Kenneth M. Foster
James Charles Frank
Harold Kenton Frisbee
Jerry Dale Garrett
Anthony A. Gioia
Leon D. Godfrey
~~Patrick~~ Donald Harris
Virgil Louis Hein
William Smith Hendrick
Pad Ray Henley
Russel F. Himmelsbach
Morman Urton Huffmaster
Charles George Kuehnel
David Norman Martin
Wilfred Jerry Mattes
Robert William Northrop
William Phillip Novinger
Carl Dean Pence
Timothy Joel Robertson
John Robert Rogers
Nancy Joe Rose
M. David Saferstein
Carol Ann Sexton
Daniel Lee Steele
Eugene Francis Steiner
Jesusa Torato Taleon
Frederick Walter Wilke
Frank Stephen Gillespie

MISSOURI GAMMA, St. Louis University, (April 28, 1960)

Charles M. Ankenbrandt
Afework Atlabachew
Richard G. Bergmann
Albert J. Bevolo
Alvin J. Birsinger
Dale A. Brelje
John W. Brodak
Charles A. Bucher
Joseph J. Bullmer
Patricia A. Bylebyl
Simon P. Cassens
Mary A. Clark
Carole Coleman
Richard Conlon
Teresa Conway
James S. Deitering
Martha J. Dues
Patricia L. Ellis
Richard R. Ensminger
Egerico Expuivel, S.J.
Albert J. Gegen
Ronald Beist
Kerry E. Grant
Marceline C. Gratias
Doris I. Halbert
Gerald Harshany
Paul Heigold
Rosemary A. Hines
Robert Hippler
Gerald R. Hodge
Thomas D. Hritz
Thomas F. Jerrick
Louis C. Kappel
Daniel A. Klingesmith
Stephen J. Koob
Susan E. Kribs
James A. Kulik
Edward F. Lenzi
John Alan Love, III
Roger P. Main
John Louis Martin
James P. McCarthy
Thomas M. McGee
Edward M. Meyer, Jr.
Lawrence A. Morgan
Lawrence J. Mueller
Richard M. Reiter
Edwin L. Schmidt
Richard H. Schmidt, S.J.
John T. Sprehe, S.J.
Veronika Stancius
William J. Steinmetz
Ruth Swenson
Richard H. Travers
Rudolph F. Trost
Paul A. Westhaus
Thomas Wood, S.J.
Thomas F. Wulfers
Dr. Lester I. Zimmerman

MONTANA ALPHA, Univeraity of Montana, (May **11,** 1960)

Duane Albert **Adams**
Harry Bauer
Michael **Billings**
Morgan A. Long
Daniel George **McRae**
Richard Warren Peterson
Anthony Howe Provost
Edward Michael **Risse**
Lee Wallace **Shrock**
George Walter **Trickey**
Cyril Welch

NEBRASKA ALPHA, Univeraity of Nebraska, (May 8, 1960)

Mary Dale Alexander
Richard **Charles** Altrock
Donald Alfred Anderson
Kenneth Paul Bartos
Marjorie Jo **Brickman**
Alex **Chuen-mou Cheng**
Edward Carl Collett
Larry Lee **Dornhoff**
William Duane **Fish**
Donald Lee **Hagrman**
Ronald L. **Knauber**
Paul Henry **Koenig**
Lloyd Edward **Krivanek**
Donald Herbert **Larson**
Lawrence Henry **Luehr**
Norman Eugene Pace
Frederick Robert **Rickers**
Robert **Thomas Savely**
Paul Dean Schaudt
Edward **Miles Steele**
Winston Jay Wade
Larry Alvin **Weitzenkamp**
William Thomas **White**

NEVADA ALPHA, University of Nevada, (June 3, 1960)

Belinda Alta Adams
Peter **Starkey Aldrich**
Margot Ruth **Berney**
Boyce William **Burge**
John P. **Dirksen**
Leonard **Gilmore**
Roland P. **Hebert**
Jon Allen **Huntsman**
Eugene **Isaeff**
J. **Lewis** Morrison

NEW HAMPSHIRE ALPHA, Univeraity of New Hampshire, (May 5, 1960)

Charles R. Caatellano
Galen R. Courtney
John M. **Hodsdon**
Carroll E. **Johnson**
Norman R. **Turgeon**
Harold C. Wing

NEW JERSEY ALPHA, Rutgera University, (Spring, 1960)

Hisham Musa

NEW JERSEY BETA, Douglaas **College, Rutgers** Univeraity, (May **12,** 1960)

Jeannette C. Andrea
Rhoda **Chaiet**
Susan **Goldman**
Judith Gutterman
Lillian **Kaplovsky**
Susan Gordan **Marchand**
Marilyn F. **Robbins**
Toby B. **Weissbraten**

NEW YORK BETA. Hunter College, (March 10, 1960)

Susan Berger
Marion **Drescher**
Carrol Greenbaum
Paul **Hess**
Camille Wise
Marcia **Zess**

NEW YORK GAMMA, Brooklyn College, (April **25,** 1960)

Gerald **Bierman**
Claire **Goldman**
Martin **Guterman**
Eli **Hirsch**
Joan Leventhal
Seymour H. **Nussbaum**
Richard D. Pollack
Eugene **Spiegel**
Frances **Wilson**

NEW YORK DELTA, New York University,

Gabriel **Atamian**
Gloria Carter
Elizabeth **Chang**
Jack Cohen
Yvette **Kaminsky**
Paul **Rabinowitz**

NEW YORK ETA, University of Buffalo, (May 18, **1960)**

Jacqueline Anderson
Louise Burbid
Ira Hinden
Robert Hofer
Lawrence Huber
James Lowerre
Ralph Marshall
Richard Meyer
Rudolf **Meyer**
Albert D. **Polimeni**
William **Schotz**

NEW YORK IOTA, Polytechnic Institute of Brooklyn, (April, **1960)**.

Alan Braver
Burton I. **Fein**
George Glauberman
I. Martin **Isaacs**
Donald **S. Passman**
Gerald **S. Stoller**

NORTH CAROLINA ALPHA, Duke Univeraity, (May, 1960)

Barbara Lee Burton
Sarah Core
James Albert **Kennedy**
Robert Jay Maxaon
Charles W. Rose
James Richard **Sawers,** Jr.

NORTH CAROLINA ALPHA, Continued

Albert Sidney
Charles Leo **Fincher**
Marvin Hill **Greene**
Dwight **Hillard Harrelson**
Douglas Paul **Nuetzman**
John Stoakes **O'Neall**
Deborah Pike
Roxanne Dora Smathera
Laura Hormine Turner
Francis Edgar Walker, Jr.
Carol Ann Wilson

OHIO ALPHA, Ohio State University, (May 13, 1960)

Floyd R. Banbwy
Robert A. **Barnes**
Randall Franklin **Barron**
Virool Boonyasombut
John R. **Brannan**
Arnold L. Cooper
William E. **Coppage**
Norman **Cornish**
David E. **Cummins**
Joseph **K. Davidson**
Robert V. **DeVore**
Vir A. Dhaka
Kenneth Duchamp
Paul E. Fehlau
J. Richard Garhham
Phillip H. **Gifford, II**
Dexter **Girton**
Frederic M. Glaser
Ralph B. Hoffman
David L. **Hutchins**
William B. Joyce
Stephen **Kanler**
Eugene M. **Klimko**
Jason R. Lemon
Jade Lin
Hugh W. Long, **III**
Gerald Lew Payne
William I. Powell, Jr.
Rebecca Claire Prather
William C. Ramaley
Richard **Henry** Rapp
Ernesto T. Roland
Roger **Carroll** Rudduck
David Arthur Sealer
John Arthur **Seaton**
Charles W. **Skinn**
Michael T. **Skubiak**
Hasel J. **Slone,** Jr.
Joan E. Smith
James Porter Spencer

OHIO GAMMA* Univeraity of Toledo, **(Spring, 1960)**

Thomas J. Couny
John D. **Haller**
Richard M. **Heinz**
Steve W. **Kormanyos**
Leonard Frank **Malec**
Robert I. **O'Desky**
John R. **Smith**
Jon Albert **Staib**

OHIO ZETA, University of Dayton, (March 21, 1960)

Joseph George Asbury
Louis Ira Boehman
Joseph Edward Bosshart
Brian Thomas Brady
Franklin D. **Demana**
Paul Francis **Dierker**
Herbert John Dietrich
Thomas Eugene **Doerfler**
William **Fass**
Richard **Joseph Feldmann**
Theodore George Flach
George Oscar Geissler
Jerome X. Goldschmidt
Thomas Norbert Grogean
Lawrence **LeRoy Gutman**
Carol B. Henneberger
James Joseph Hogan
Lawrence Andrew John
Richard Thomas Kleiner
Charles William Koeller
Morris James Kreider
Jacqueline Mary Kuhn
Peter **Joseph Liotino**
John William **McCloskey**
Henry Theodore **Mohlman**
William Leo **Nighan**
Albert Y. K. Pang
Richard Edward Peterson
Alberta Ginstie Prather
Lawrence Carl Raiff
Thomas Anton Schoen, S.M.
Robert Bernard Schwartz
Allan John Sieradski
Armand Verne Smith
Jay Vincent Smith
Jerome Allen Smith
Lawrence Herbert Smith
Ralph Cletus Steinlage
Samuel Ralph Thompson, III
Timothy Noel Trick
James Allen Utz
John Maurice Wells
Dorothy Ann White
Donald Francis Wiedemer

OKLAHOMA ALPHA, University of **Oklahoma, (April 19**, 1960)

Edward F. Blick
John L. Brodkbank
Norman R. Clark
Tom J. Love
James D. Palmer
Edward L. Shreve
Gene **Slaughter**
Don R. **Smith**
Albert P. **Weeks**

OKLAHOMA BETA, Oklahoma State Univeraity (Spring, 1960)

Charles A. **Codding**
Billy E. **Gillett**
Joseph M. **Hawkins**
Dale L. **Keairns**
William C. **McCormick,** Jr.
Rosetta Schmidt
Gene Paul **Sturm,** Jr.

OREGON ALPHA. University of Oregon, (June **1,** 1960)

Robert L. Backatrom
Seth **Catlin**
Richard B. **Crittenden**
Roger J. **Dillan**
Robert **D. Dyson**
Frederick **Easton**
Robert **Ferguson**
Robert **Guderjohn**
John **K. Harris**
Robert Jewett
John B. Lane
Robert J. **Lindahl**
Lee J. **Mahoney**
Mary Lou **McCarthy**
Forrest McMains
Arthur J. Nadaa
Joyce Palmer
Alan D. **Peterson**
Carol **S.** Pratt
Janet L. **Stevenson**
Helen J. **Terzaghi**
Peter C. **Trenholme**
Nan **K.** Wood
Burke **Zane**

OREGON BETA, Oregon State College, (Spring, **1960)**

Charles D. Baker
Richard Allen Baker
Allen H. Brady
Maurice L. **Bregel**
Clayton H. **Chisum**
Chris A. Clark
Don R. **Conant,** Jr.
William G. Conn
Gunards R. **Drusts**
Botond G. **Eross**
Gerald I. **Findley**
Ken O. Gamon
Linda George
William E. **Greene**
Raymond D. **Haertel**
David O. Harris
Don E. Heard
Paul F. **Hudrlik**
Richard A. Jaenicke
Bruce R. Johnson
Gary L. Kvammen
Jay A. **Mackie**
Kenneth C. **Marx**
Stuart J. **McAlpine**
James L. **McCormick**
Marilyn L. **McLennan**
Lawrence H. **Merk**
John L. Miller
Victor A. **Mullett**
Harold Y. Okamoto
John R. Phillips
Michael G. Poole
Walter Lee Powell, Jr.
Chung-Yi Shen
Edward H. Stockwell
Ralph M Toms
James L. **Unger**
David **M.** Ward
Hwai N. Yang

PENNSYLVANIA ALPHA, University of **Pennsylvania,** (March **25, 1960)**

E. David **Abrams**
Krishan Ahuja
Sheila Auerbach
Robert Cantor
Frank **Desort**
Deward Fwnalont
Mervyn Friedman
Alan Gart
Jane **Golubitsky**
William **Klepcyznski**
Roman Kowalchuck
Gerson Levin
Thomas Lindsay
Leon **Maimud**
Ruben **Meyer**
Anthony Mucci
Phillip Radoff
Saul **Rosenberg**
Lawrence Rothenberg
Gerald **Russakoff**
Frederic Shirk
Robert **Silverman**
Elliot **Slutsky**
Howard **Wasserman**
Frederic **A.** Wyle
Cleon **Yohe**

PENNSYLVANIA ALPHA, University of Pennsylvania, **(July 6, 1960)**

Martin **Caplan**
Fu Kiong Chan
Mark Cohen
Spyridon Diamessis
Marvin Gelblatt
Fred H. Kauffman
Nelson R. **Lipshutz**
George W. **Mebus**
Joanne **K. Moliver**
Rade **Pejic**
Arthur J. Schatz
Ronald Sherman
Marlene **Stillman**
Myra C. **Weisgold**

PENNSYLVANIA GAMMA, Lehigh University, (November **12, 1959)**

Brian K. Bauknight
John B. Buchanan
Stephen **Burrick,** Jr.
Thomas R. Downs
William M Freed
Roy W. **Grabner**
Nicholas M Guydosh
Harold S. **Haller,** Jr.
Joel **Heisler**
Theodore U. **Horger**
Peter **M. Jeffers**
Richard K. Martin
John H. **Mindker,** Jr.
Richard T. Moll
Jerry A. Nolen, Jr.
Robert Paternoster
James P. **Perneski**
David A. Polefka
Donald L. **Ritter**
Alan K. **Stiffler**
Thomas **D. Swartz**

PENNSYLVANIA DELTA, Pennsylvania State University, (April **29, 1960)**

Josian P. Alford
Eugene A. Francis
Richard E. **Llorens**
James A. Miller
Lauren Lee **Pryor**
Samuel **D.** Shore
Dorothy Jane **Smeal**
Robert **R. Sproule**
James J. **Tietjen**

PENNSYLVANIA EPSILON, Carnegie Institute of Technology, (May **18, 1960)**

Judith **Binstock**
Edward Howard **Blum**
Lincoln Ellsworth **Bragg**
Philip B. James
Glenn **Marcenia Julian**
Les A. Karlovitz
Richard Carl Lehman
Ralph Leslie London
Margery Ruth **Morgenstern**
David **'Tipton** Thomas
James Bryan Turner
Richard A. Uner
John **Vance Weaver**

PENNSYLVANIA ZETA, Temple University, **(April 1, 1960)**

Irving J. **Brand**
Marilyn **Dinter**
Zandria A. **Dunchak**
Henry **Friedman**
Marvin Gelman
Robert **A. Gramp**
Walter **S. Lawton**
Ethel M **Logue**
Eli M Mandelbaum
Sheila S. **Shein**
Thomas H. **Slook**

(May **6, 1960)**

Saul **Axelrod**
Carol **Bernhard**
Arlene Bobroff
Leonard Brattman
Harry L. **Brano**
Gerson H. Cohen
David Drasin
Mary **A. Dunphy**
Lilyan Fireman
Irwin Goodman
Barbara Gordon
Sheila L. Machinton
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Phyllis **Taksey**
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TEXAS ALPHA, Texas Christian University, **(January 29, 1960)**

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(May **18, 1960)**

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(June **12, 1960)**

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