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THE CONSTRUCTION OF THE AFFINE PLANE AND ITS ASSOCIATED GROUP IN TERMS OF THE BARYCENTRIC CALCULUS[1]

By JAMES V. HEROD
Alabama Alpha

Introduction: The classical method of studying the affine plane is by means of the cartesian coordinate axes. This differs essentially from the study of euclidean coordinate geometry by the absence of any restrictions on the angle connecting the coordinate axes and hence the absence of a metric. After defining an affine transformation to be one that transforms a point (x,y) in the cartesian plane into the point (x',y') such that $x'=ax+by+c$ and $y'=dx+ey+f$, then the affine group and its definable subgroups may be studied.

A second method of studying the affine plane is through the set of all linear combinations $\alpha X+\beta Y+\gamma Z$ where α, β, and γ are scalars with a non-zero sum and X,Y, and Z are basis vectors of a three dimensional vector space. The affine transformations are those transformations which carry a point, or vector, W into the point $S(W)$ where S is a transformation that leaves the sum of the scalars invariant.

It is the purpose of this paper to expand a third method of studying the affine plane and its associated group which is closely related to the second of the two listed above. This will be done via the barycentric calculus. We shall include a method of studying lines and conic sections as scalar-valued functions of points.

THE AFFINE PLANE: Two sets of objects are assumed. One is the set H of objects called points and denoted by capital letters. The other is the set R of real numbers which will be denoted by Greek letters. The elements of the set R will be referred to as scalars. We shall consider elements of $R\times H$ denoted by αA.

To every set of n elements of $R\times H$ such that the sum of the scalars is not zero, there corresponds a unique element of $R\times H$ denoted by $\sum_{i=1}^{n} \alpha_i P$. We say that P is the centroid of the finite set. This correspondence between the set of elements in $R\times H$ and the single element is denoted thus:

$\alpha_1 A_1 + \alpha_2 A_2 + \dots + \alpha_n A_n = \sum_{i=1}^{n} \alpha_i P$. For simplicity $1P$ shall be denoted by P.

We assume the calculus to have the following properties:

$$(\alpha A+\beta B)+\gamma C=\alpha A+(\beta B+\gamma C)$$
$$\alpha A+\beta A=(\alpha+\beta)A$$
$$\alpha(A+B)=\alpha A+\alpha B$$
$$0P+\alpha A=\alpha A$$

Furthermore, if $\alpha A+\beta B=\gamma C$, $\gamma \neq 0$, then $\alpha/\gamma\, A + \beta/\gamma\, B = C$.

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We shall define the points $B_1, B_2, \ldots B_n$ to be linearly independent if and only if $\alpha_1 B_1 + \alpha_2 B_2 + \ldots + \alpha_n B_n = O P$ implies $\alpha_i = 0$ for $i = 1, 2, \ldots n$. The points are linearly dependent if and only if there exist scalars $\alpha_1, \alpha_2, \ldots \alpha_n$, not all zero, such that $\alpha_1 B_1 + \alpha_2 B_2 + \ldots + \alpha_n B_n = OP$.

To determine the dimension of the space, we postulate that any four points are linearly dependent and that there exist three linearly independent points.

Every point may be expressed as a linear combination of three linearly independent points. In fact:

Theorem 1: If B_1, B_2, and B_3 are linearly independent, then for every point P there exist unique scalars that $P = \lambda B_1 + \mu B_2 + \nu B_3$.

In case the sum of the scalars of two elements of RxH is zero, we shall define these elements as a point difference.

Definition: The point difference B-A is equivalent to the point difference D-C if and only if $B+C = D+A$. We shall denote this equivalence relation by $B-A \sim D-C$. In a similar manner we may define $\lambda(B-A) \sim \mu(D-C)$ if and only if $\lambda B + \mu C = \mu D + \lambda A$. We are justified in calling this an equivalence relation since every point difference is equivalent to itself, and if $(B-A) \sim (F-E)$ and $(D-C) \sim (F-E)$, then $(B-A) \sim (D-C)$. The equivalence classes of point differences are free vectors.

Definition: The set L is a line if $L = \{C: A + \lambda(B-A) = C,\ B \neq A,\ \lambda \in R\}$.

Theorem 2: If A and B are points in a line, there exist unique scalars α and β such that $\alpha A + \beta B = C$ if and only if C is a point in the line.

Definition: If L_1 and L_2 are lines such that $L_1 = \{C: A + \lambda(P-Q) = C\}$ and $L_2 = \{D: B + \mu(R-S) = D\}$, then L_1 is parallel to L_2 if and only if there exists α and β such that $\alpha(P-Q) \sim \beta(R-S)$.

Theorem 3: The relation of parallelism is an equivalence relation.

Proof: As a result of the properties of point differences, every line is parallel to itself and, if two lines are parallel to the same line, they are parallel to each other.

Theorem 4: Parallel lines are either coincident or have no points in common.

THE AFFINE GROUP: In their discussion of barycentric coordinates, Birkhoff and MacLane show that an affine transformation carries centroids into centroids. This follows in accordance with their previous definition of an affine transformation in a vector space. They restrict the transformations to be non-singular; that is, $S(A) = S(B)$ implies $A = B$. However, we shall define the affine transformations as the endomorphisms of the barycentric calculus. More explicitly, S is an affine transformation if $S(\alpha A + \beta B) = \alpha S(A) + \beta S(B)$. Hence, we allow a projection to be an affine transformation.

Theorem 5: The set of automorphisms of the barycentric calculus form a group with respect to transformation multiplication. This group is called the affine group.

The reader's attention is called to the fact that the automorphisms, as opposed to the endomorphisms, compose the affine group since the inverse of every element must be in the group.

Definition: A transformation is a translation if there exists (P-Q) such that $T(X) = X + (P-Q)$.

Theorem 6: The set of translations forms a normal subgroup of the affine group.

Proof: We see that a translation is an affine transformation since $T(\alpha A + \beta B) = \alpha T(A) + \beta T(B)$, $\alpha + \beta = 1$. Furthermore, the set of translations forms a subgroup of the affine group; for if T_1 and T_2 are translations, then

$T_1 T_2(X) = T_1(X + (P-Q)) = X + (P-Q) + (R-S) = X + 2(B-A)$ where $2B = P + R$ and $2A = Q + S$; and if $T_1(X) = X + (P-Q)$ then $T_1^{-1}(X) = X + (Q-P)$.

Since $(S^{-1}TS)(x) = X + S^{-1}(P) - S^{-1}(Q)$, then the group of translations is a normal subgroup of the affine group.

Before studying invariants of the affine group and of this normal subgroup, we must look at some properties of a *cell function.*

Definition: A n-simplex is an ordered set of n+1 points $\{A_0 A_1 \ldots A_n\}$. Two simplices are equivalent if one may be obtained from the other by an even permutation of the points. Using the fact that the identity permutation is even and that the product of two even permutations is even, we see that this relation is an equivalence relation.

Under this definition, there are two equivalence classes that we shall call cells. This follows from the fact that every permutation is even or odd. We shall denote the cell of which $\{A_0 A_1 \ldots A_n\}$ is an element by σ_n and the cell of which $\{A_1\ A,\ A,. \ldots A\}$ is an element by $-\sigma_n$.

Definition: A cell function is a scalar-valued function satisfying the following postulates:

1) $f_r(\sigma_r) = \Sigma_{i=0}^{r} (-1)^i f_r(P\ A_0\ A_1 \ldots \hat{A}_i \ldots A_4)$ where $\hat{A}_i$ is deleted and P is arbitrary.
2) $f_r(\sigma_r) = 0$ if $A_i = A_j$, $i \neq j$.
3) $f_r(\mu A + \nu B\ \ A_1 \ldots A_r) = \mu f_r(A\ \ A,\ A,\ \ldots A_r) + \nu f_r(B\ A_1\ A_2 \ldots A_r)$.

Theorem 10: If $r > 2$, the $f_r(\sigma_r) = 0$.

This theorem follows from the application of postulate three for the cell function plus the fact that any four points are linearly dependent.

Theorem 11: $f_r(\sigma_r) = -f_r(-\sigma_r)$.

Proof of this theorem follows from the expansion of $f_r(\sigma_r)$ with the condition that $P=A_1$.

Because of the length of the algebraic operations needed to supply proofs of the following theorems, let it suffice to say that they follow from the definitions of the transformations together with repeated application of postulate three for the cell function.

Theorem 12: The function f_r is invariant under the set of translations; that is, $f_r[T(A_0) \dots T(A_r)] = f_r[A_0 \dots A_r]$.

Theorem 13: There exists χ such that $f_2[S(A_0)\,S(A_1)\,S(A_2)] = \chi f_2(A_0\,A_1\,A_2)$.

Theorem 14: If P, A_0, and A_1 are collinear, then $\frac{f_r(P\,A_0)}{f_r(A_0A_1)}$ is invariant under the affine group.

The significance of the last five theorems becomes apparent if we interpret $f_1(A_0A_1)$ as the length of the line segment A_0A_1 and interpret $f_2(A_0A_1A_2)$ as the signed area of the triangle of $A_0A_1A_2$. We see that it is the ratio of lengths of segments on a given line or on parallel lines and the ratio of areas of triangles that are invariant under the affine group. It is of interest to note that Theorem 12 implies that length and area are invariant under the set of translations.

In the *Erlanger Program,* Felix Klein showed that we may obtain a subgroup of a group of transformations from the set of all transformations that leave some property invariant. Consequently, Theorems 15 and 16 follow:

Theorem 15: The set of transformations that leave f_r invariant forms a normal subgroup of the affine group which we shall call the equi-affine group.

Theorem 16: The set of transformations that leave a particular point invariant forms a (non-normal) subgroup of the affine group. (This is the so-called full-linear group of the vector space of the same dimension as the affine space.)

LINEAR AND QUADRATIC FUNCTIONS: As has been suggested in the introduction, we can study lines and conic sections in terms of linear and quadratic functions. We begin with the properties of the linear function.

Definition: Let the scalar-valued function $\mathcal{L}$ be defined on the set of all points. The function $\mathcal{L}$ is linear if $\mathcal{L}(\alpha A + \beta B) = \alpha\mathcal{L}(A) + \beta\mathcal{L}(B)$.

Theorem 17: Let $\mathcal{L}$ be a linear function, not a constant, such that $\mathcal{L}(A) = \mathcal{L}(B) = \delta$. Then $\{X: \mathcal{L}(X) = \delta\} = \{C: A + \lambda(B-A) = C,\ \lambda \in R\}$.

The proof of this theorem results from considering elements in each set and seeing that they can be expressed as an element of the other set. We have, then, a new definition of a line in terms of a linear function of points.

The following theorems show the effect of a general affine transformation, and of a translation, upon a line.

Theorem 18: If S is an affine transformation and $L_1 = \{X: \mathcal{L}_1(X) = \delta\}$ then $S(L_1) = \{Y: \mathcal{L}_2(Y) = \epsilon\}$ where $\mathcal{L}_2$ is a linear function defined as follows: for A and B, elements of L_1, $\mathcal{L}_2(S(A)) = \mathcal{L}_2(S(B)) = \epsilon$

Theorem 19: Let $L_1 = \{X: \mathcal{L}_1(X) = \delta\}$. Then $L_2 = \{Y: \mathcal{L}_1(Y) = \epsilon\}$ if and only if there exists an element T of the group of translations such that $T(L_1) = L_2$.

Theorem 20: The lines L_1 and L_2 of Theorem 19 are parallel.

Thus we see if S is an element of the affine group, S carries lines into lines. Translations carry lines into parallel lines. Furthermore, the linear functional value of points in parallel lines differs by a constant.

We now turn to properties of the quadratic function.

Definition: The scalar-valued function θ defined on the set of all points is homogeneous and quadratic if 1) $\theta(A+B+C) = \theta(A+B) + \theta(A+C) + \theta(B+C) - \theta(B) - \theta(C)$. 2) $\theta(\alpha A) = \alpha^2\,\theta(A)$.

Definition: Let Λ be an arbitrary scalar-valued function defined on the set of all points. We define a polarization operator on the function Λ such that the operator $\begin{bmatrix}A\\B\end{bmatrix}$ satisfies the following properties:

1) $\begin{bmatrix}A\\B\end{bmatrix}\Lambda(A) = \Lambda(A+B) - \Lambda(A) - \Lambda(B)$.

2) $\begin{bmatrix}A\\\lambda B\end{bmatrix}\Lambda(A) = \lambda\begin{bmatrix}A\\B\end{bmatrix}\Lambda(A)$.

Theorem 21: a) $\begin{bmatrix}A\\B\end{bmatrix}\Lambda(A) = \begin{bmatrix}B\\A\end{bmatrix}\Lambda(B)$.

b) $\begin{bmatrix}A\\B\end{bmatrix}\mathcal{L}(A) = 0$.

c) $\begin{bmatrix}A\\A\end{bmatrix}\theta(A) = 2\theta(A)$.

d) $\begin{bmatrix}A\\B+C\end{bmatrix}\theta(A) = \begin{bmatrix}A\\B\end{bmatrix}\theta(A) + \begin{bmatrix}A\\C\end{bmatrix}\theta(A)$.

Theorem 22: Let $\phi(X) = \theta(X) + \mathcal{L}(X)$. Then ϕ is quadratic but not homogeneous. That is,

$\phi(A+B+C) = \phi(A+B) + \phi(A+C) + \phi(B+C) - \phi(A) - \phi(B) - \phi(C)$

but $\phi(\alpha A) \neq \alpha^2\phi(A)$ for all α.

Before proving the next theorem, we need to know that $\theta(P-Q)$ is well defined.

Lemma: $\theta(P-Q)=2\theta(P)+2\theta(Q)-\theta(P+Q)$.
This follows from expanding $\theta(X+P-Q)$ and then letting $X=Q$.

Theorem 23: Let $\phi_1(X)+\delta = \theta_1(X) + \mathcal{L}_1(X) + \delta$. Then the quadratic part of $\phi_1(X)$ is invariant under the set of translations.

Proof: Let T be an element of the set of translations such that for some P and Q, then $T(X) = X + \lambda(P-Q)$. Then it must be shown that

$$\phi_1(T(X)) + \delta = \theta_1(X) + \mathcal{L}_2(X) + \epsilon.$$

We know that $\phi_1(T(X)) + \delta = \theta_1(T(X)) + \mathcal{L}_1(T(X)) + \delta$.

Therefore,
$$\phi_1(T(X)) + \delta = \theta_1(X+\lambda(P-Q)) + \mathcal{L}_1(X+\lambda(P-Q))+\delta$$
$$= \theta_1(X) + \begin{bmatrix}X\\ \lambda(P-Q)\end{bmatrix}\theta_1(X) + \mathcal{L}_1(X) + \lambda^2\theta_1(P-Q)+\lambda\mathcal{L}_1(P-Q)+\delta.$$

Let $\mathcal{L}_2(X)=\mathcal{L}_1(X)+\begin{bmatrix}X\\ \lambda(P-Q)\end{bmatrix}\theta_1(X)$ and define the scalar ϵ such that $= \lambda^2\theta_1(P-Q)+\lambda\mathcal{L}_1(P-Q)+\delta$. Then $\phi_1(T(X))+\delta = \theta_1(X)+\mathcal{L}_2(X)+\epsilon$ and the quadratic part of $\phi_1(X)$ is invariant under the set of translations.

It is recalled that for three linearly independent points B_1, B_2, and B_3, called basis points, there exist three unique scalars α, β, and γ such that $\alpha+\beta+\gamma=1$ and $P=\alpha B_1+\beta B_2+\gamma B_3$ for any point P. Hence we may denote P by an ordered triple of scalars (α,β,γ) where $\alpha+\beta+\gamma=1$. In view of the last equality, the final element in the triple is actually excess notation. Hence for each ordered pair (α,β), there exists a unique point $P=\alpha B_1+\beta B_2+(1-\alpha-\beta)B_3$ and conversely. Using this notation, B_1, B_2, and B_3 take on the pairs (1,0), (0,1), and (0,0) respectively.

A line has been shown to be the set $L=\{X: \mathcal{L}(X)=\Delta\}$ where $\mathcal{L}$ is a linear function. Since any point may be expressed as a linear combination of the three basis points B_1, B_2, and B_3, then $L = \{\alpha B_1+\beta B_2+\gamma B_3: \mathcal{L}(\alpha B_1+\beta B_2+\gamma B_3)=\Delta\}$ where α, β, and γ are variables in R subject to the restriction that $\alpha+\beta+\gamma=1$. If $\mathcal{L}(B_1)=\Pi$, $\mathcal{L}(B_2)=\Psi$, and $\mathcal{L}(B_3)=\Omega$, then $\Delta = \alpha\Pi+\beta\Psi+\gamma\Omega$. It follows that $0=\alpha(\Pi-\Omega)+\beta(\Psi-\Omega)+(\Omega-\Delta)$. Then a line may be defined as the set of all points such that $P=\alpha B_1+\beta B_2+\gamma B_3$ and $\alpha+\beta+\gamma=1$ where there is a linear relation between α and β. In terms of the ordered pair notation, $L=\{(\alpha,\beta): \alpha\mathcal{K}+\beta\mathcal{M}+\mathcal{N}=0\}$ where $\mathcal{K}$, $\mathcal{M}$, and $\mathcal{N}$ are constants in R.

A conic section may be defined as the set $C=\{X: \phi(X)=\Delta\}$ where $\phi(X)=\theta(X)+\mathcal{L}(X)$. Again, since any point may be expressed as a linear combination of the basis points B_1, B_2, and B_3, then $C=\{\alpha B_1+\beta B_2+\gamma B_3: \phi(\alpha B_1+\beta B_2+\gamma B_3)=\Delta\}$ where α, β, and γ are variables in R subject to the restriction that $\alpha+\beta+\gamma=1$.

Then $\Delta = \phi(\alpha B_1)+\phi(\beta B_2)+\phi(\gamma B_3)+\alpha\beta\begin{bmatrix}B_1\\B_2\end{bmatrix}\phi(B_1)+\beta\gamma\begin{bmatrix}B_2\\B_3\end{bmatrix}\phi(B_2)+\gamma\alpha\begin{bmatrix}B_3\\B_1\end{bmatrix}\phi(B_3)$.

Then $\Delta = \phi(\alpha B_1)+\phi(\beta B_2)+\phi(\gamma B_3)+\alpha\beta\begin{bmatrix}B_1\\B_2\end{bmatrix}\phi(B_1)+\beta\gamma\begin{bmatrix}B_2\\B_3\end{bmatrix}\phi(B_2)+\gamma\alpha\begin{bmatrix}B_3\\B_1\end{bmatrix}\phi(B_3)$. Using the definition of $\phi(X)$, the last equation simplifies to:

$0=\alpha^2\xi_1+\alpha\beta\xi_2+\beta^2\xi_3+\alpha\xi_4+\beta\xi_5+\xi_6$ where $\xi_1=\theta(B_1)+\theta(B_3)-\begin{bmatrix}B_1\\B_3\end{bmatrix}\phi(B_3)$, $\xi_2=2\theta(B_3)+\begin{bmatrix}B_1\\B_2\end{bmatrix}\phi(B_1)-\begin{bmatrix}B_2\\B_3\end{bmatrix}\phi(B_2)-\begin{bmatrix}B_3\\B_1\end{bmatrix}\phi(B_3)$, $\xi_3=\theta(B_2)+\theta(B_3)-\begin{bmatrix}B_2\\B_3\end{bmatrix}\phi(B_2)$,

$\xi_4=\mathcal{L}(B_1)-2\theta(B_3)-\mathcal{L}(B_3)+\begin{bmatrix}B_3\\B_1\end{bmatrix}\phi(B_3)$, $\xi_5=\mathcal{L}(B_2)-2\theta(B_3)-\mathcal{L}B_3+\begin{bmatrix}B_2\\B_3\end{bmatrix}\phi(B_3)$, and finally $\xi_6=\theta(B_3)+\mathcal{L}(B_3)-\Delta$.

In terms of the ordered pair notation, a conic section is the set

$$C=\{(\alpha,\beta): \alpha^2\xi_1+\alpha\beta\xi_2+\beta^2\xi_3+\alpha\xi_4+\beta\xi_5+\xi_6=0\}.$$

We shall define C to be a ellipse, parabola, or hyperbola according as $\xi_2^2-4\xi_1\xi$, is less than, equal to, or greater than zero.

Theorem 24: Ellipses, parabolas, and hyperbolas are invariant under the affine group.

Proof: For an element **S** of the affine group, let $S(B_1) = \lambda_1 B_1 + \mu_1 B_2 + \nu_1 B_3$; $S(B_2) = \lambda_2 B_1 + \mu_2 B_2 + \nu_2 B_3$; $S(B_3) = \lambda_3 B_1 + \mu_3 B_2 + \nu_3 B_3$ where $\lambda_i + \mu_i + \nu_i = 1$. Then if $\alpha + \beta + \gamma = 1$, $\theta(S(\alpha B_1 + \beta B_2 + \gamma B_3)) = \theta[\alpha(\lambda_1 + \lambda_2 + \lambda_3)B_1 + \beta(\mu_1 + \mu_2 + \mu_3)B_2 + \gamma(\nu_1 + \nu_2 + \nu_3)B_3]$. This may be simplified to an expression of the form

$$\alpha^2 \eta_1 + \alpha\beta \eta_2 + \beta^2 \eta_3 + \alpha \eta_4 + \beta \eta_5 + \eta_6 = 0.$$

Then it may be shown that

$$\eta_2^2 - 4 \eta_1 \eta_3 = \begin{vmatrix} \lambda_1 & \mu_1 & \nu_1 \\ \lambda_2 & \mu_2 & \nu_2 \\ \lambda_3 & \mu_3 & \nu_3 \end{vmatrix} (\xi_1^2 - 4 \xi_1 \xi_3)$$

Theorem 25: The discriminant $\xi_2^2 - 4 \xi_1 \xi_3$ is invariant under the group of translations.

Proof: This follows from Theorem 23. However, we now see that an alternate proof may be supplied thus: $T(B_1) = (1+\alpha)B_1 + \beta B_2 + \gamma B_3$, $T(B_2) = \alpha B_1 + (1+\beta)B_2 + \gamma B_3$, and $T(B_3) = \alpha B_1 + \beta B_2 + (1+\gamma)B_3$ where $\alpha + \beta + \gamma = 0$.

But
$$\begin{vmatrix} 1+\alpha & \beta & \gamma \\ \alpha & 1+\beta & \gamma \\ \alpha & \beta & 1+\gamma \end{vmatrix} = 1.$$

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FILTERS AND ULTRAFILTERS[1]

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The purpose of this paper is to extablish the following theorem as proposed by N. Bourbaki: Every filter is the intersection of all the ultrafilters finer than it. Before we are able to prove this statement, however, it is necessary that we set up the definitions and preliminary results that we will use.

Then we begin with the definition of a filter. ***A filter*** $\mathfrak{F}$ on a set E is a collection of subsets of E satisfying the following properties: (F_I) Any set containing a set of $\mathfrak{F}$ belongs to $\mathfrak{F}$. (F_{II}) Any finite intersection of sets of $\mathfrak{F}$ belongs to $\mathfrak{F}$. (F_{III}) The empty subset of E does not belong to $\mathfrak{F}$. In order that those unfamiliar with the concept of filters may get a better feel for the idea, we give a few examples. A very trivial example is to let $\mathfrak{F} = \{E\}$, where E is some non-empty set. Then $\{E\}$ is seen to satisfy all the conditions of the definition. For a second example, let E be the set of positive integers. Then the collection $\mathfrak{F} = \{A \mid C(A) \text{ is finite}\}$ is a filter on E. This particular filter is called the **Fréchet** filter. For a third example, let E be a topological space; then the collection of neighborhoods of a subset A of E is a filter on E called ***the filter of neighborhoods of A.***

Let Φ be the collection of all filters on a set E and let us introduce an order relation on Φ. Let $\mathfrak{F}$ and $\mathfrak{F}'$ be two filters on E. Then $\mathfrak{F}$ is said to be ***finer than*** $\mathfrak{F}'$ if $\mathfrak{F} \subset \mathfrak{F}'$, **i.e.,** every set in $\mathfrak{F}'$ is also in $\mathfrak{F}$. The relation of order opposite to "finer than" is "coarser than" and we say that $\mathfrak{F}'$ is coarser than $\mathfrak{F}$ if $\mathfrak{F} \supset \mathfrak{F}'$. Furthermore, if $\mathfrak{F} \supset \mathfrak{F}'$ and $\mathfrak{F} \neq \mathfrak{F}'$, then we say that $\mathfrak{F}$ is strictly finer than $\mathfrak{F}'$.

Now let Φ' be an arbitrary collection of filters on a set E, say $\Phi' = \{\mathfrak{F}_\alpha\}_{\alpha \in \Lambda}$, where Λ is the index set. (A) ***Then it can easily be shown that*** $\mathfrak{F} = \bigcap \mathfrak{F}_\alpha$ ***is a filter on*** E. Certainly $\mathfrak{F} \neq \phi$ **since** for every filter $\mathfrak{F}_\alpha$ on **E,** E belongs to $\mathfrak{F}_\alpha$ by (F). Then it is simply a matter of verifying that the conditions of the definition are satisfied.

Let us establish the concept of a system of generators of a filter. Let $\mathfrak{A}$ by any collection of subsets of a set E. (B) ***Then in order that there exist a filter on E containing*** $\mathfrak{A}$ ***, it is necessary and sufficient that any finite intersection of sets of*** $\mathfrak{A}$ ***be non-empty.***

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The condition is obviously necessary in order to satisfy (F_{III}). To show that this condition is sufficient, we construct a filter $\mathcal{F}$ on E containing $\mathcal{G}$. First let $\mathcal{G}'$ be the set of all finite intersections of sets of $\mathcal{G}$. Then it can be shown that $\mathcal{F}$, the collection of all subsets of E containing a set of $\mathcal{G}'$, is a filter on E. The set $\mathcal{G}$ satisfying the conditions of sentence (B) is said to be a ***system of generators*** of the filter $\mathcal{F}$

A partially ordered set E is said to be inductive if it satisfies the following condition: any totally ordered subset of E has a least upper bound. The Theorem of **Zorn** states that every ordered inductive set has at least one maximal element. The set Φ of all filters on E is ordered by the relation "is finer than" and can be shown to be inductive. In fact, any arbitrary collection Φ' of filters on E is inductive. This idea of maximal element motivates the definition of ultrafilter. An ***ultrafilter*** on a set E is a filter $\mathcal{U}$ on E such that there does not exist any filter on E strictly finer than $\mathcal{U}$. (C) ***An immediate consequence of the Theorem of Zorn is that for every filter*** $\mathcal{F}$ ***on*** E, ***there exists an ultrafilter*** $\mathcal{U}$ ***on*** E ***finer than*** $\mathcal{F}$, ***i.e.,*** $\mathcal{F} \subset \mathcal{U}$

Another theorem which may be obtained is that if $\mathcal{G}$ is a system of generators of a filter, such that for every $X \subset$ **E,** either X belongs to $\mathcal{G}$ or **C(X)** belongs to $\mathcal{G}$, then $\mathcal{G}$ is an ultrafilter on E. We use this theorem to give the following example of an ultrafilter: Let $\mathcal{G}$ be the collection of subsets of a set E containing the point p belonging to E. Then for every subset X of **E,** either p belongs to X or p belongs to **C(X),** and it thus follows that $\mathcal{G}$ is an ultrafilter on E.

We now have the tools for proving the following theorem: ***Every filter is the intersection of all the ultrafilters finer than it.*** Let $\{\mathcal{U}_\alpha\}_{\alpha \in \Lambda}$ be the collection of ultrafilters on a set E finer than a filter $\mathcal{F}$ on the set E. Let $\mathcal{D} = \bigcap_\alpha \mathcal{U}_\alpha$. Then $\mathcal{D}$ is a filter on E by sentence (A). We must show that $\mathcal{D} = \mathcal{F}$. $\mathcal{F} \subset \mathcal{U}_\alpha$ for every a in Λ which implies $\mathcal{F} \subset \mathcal{D}$. Now let us show that $\mathcal{D} \subset \mathcal{F}$. Assume that there exists a set $A \subset$ **E** such that A belongs to $\mathcal{D}$ but A does not belong to $\mathcal{F}$, and attempt to lead to a contradiction. A intersects all the sets of $\mathcal{F}$, since $\mathcal{F} \subset \mathcal{D}$ and A belonging to $\mathcal{D}$ and F belonging to $\mathcal{F}$ implies F belongs to $\mathcal{D}$ and thus $A \cap F \neq \phi$ by definition. **C(A)** intersects all the sets of

since if there exists a set F in $\mathcal{F}$ such that $C(A) \cap F = \phi$, then $F \subset A$ which implies that A belongs to $\mathcal{F}$ contradicting the assumption. $\mathcal{F} \cup \{C(A)\}$ generates a filter $\mathcal{F}'$ on E by sentence (B). Then there exists an ultrafilter $\mathcal{U}$ finer than $\mathcal{F}'$ by sentence (C). $\mathcal{F} \subset \mathcal{F}' \subset \mathcal{U}$ implies that $\mathcal{U}$ is an ultrafilter finer than $\mathcal{F}$. But A does not belong to $\mathcal{U}$ contradicting that A belongs to the intersection of all the ultrafilters finer than $\mathcal{F}$. Therefore, A belongs to $\mathcal{F}$ and it follows that $\mathcal{D} = \mathcal{F}$.

Reference

N. Bourbaki, ***Topologie Générale,*** Chapter *I*, Actualities Scientifiques et Industrielles 858-1142, Paris, 1951, pp. 9-46.

Oklahoma State University

PROBLEM DEPARTMENT

Edited by
M. S. KLAMKIN,
AVCO Research and
Advanced Development Division

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication. Address all communications concerning problems to M. S. Klamkin, Avco Research and Advanced Development Division, T-430, Wilmington, Massachusetts.

PROBLEMS FOR SOLUTION

127. Proposed by Harry Furstenberg, M. I. T.

Show that

$$\{\text{Rank } \|A_{rs}\|\}^2 \geq \text{Rank } \|A_{rs}^2\|$$

128. Proposed by Robert P. Rudis and Christopher Sherman, AVCO RAD

Given 2n unit resistors, show how they may be connected using n single pole single throw (SPST) and n single pole double throw (SPDT) (the latter with off position) switches to obtain, between a single fixed pair of terminals, the values of resistance of i and i^{-1} where $i = 1, 2, 3, \cdots, 2n$.

Editorial Note: Two more difficult related problems would be to obtain i and i^{-1} using the least number of only one of the above type of switches.

129. Proposed by Leo Moser, University of Alberta

If R be a regular polyhedron and P a variable point inside or on R, show that the sum of the perpendicular distances from P to the faces of R, extended if necessary, is a constant.

Editorial Note: Also, consider the case when P lies outside of R by assigning proper signs to the various perpendiculars.

130. Proposed by H. Kaye, Brooklyn, N. Y.

If P is a variable point on the circular arc $\widehat{AB}$, show that $\overline{AP} + \overline{PB}$ is a maximum when P is the mid-point of the arc $\widehat{AB}$.

131. Proposed by M. S. Klamkin, AVCO RAD

Solve the following system of equations:

$$\begin{aligned}
AX_1 + bX_2 + bX_3 + \cdots + bX_n &= C_1,\\
bX_1 + AX_2 + bX_3 + \cdots + bX_n &= C_2,\\
bX_1 + bX_2 + AX_3 + \cdots + bX_n &= C_3,\\
\vdots \qquad &\\
bX_1 + bX_2 + bX_3 + \cdots + AX_n &= C_n.
\end{aligned}$$

SOLUTIONS

117. Proposed by Michael J. Pascual, Siena College

If the lengths of two sides x and y of a triangle and the angle θ opposite one of them are chosen at random in the intervals

$$0 \leq \theta \leq \pi, \quad 0 \leq x \leq L, \quad 0 \leq y \leq L,$$

(x, y, and θ are assumed to be uniformly distributed), find the probability that

a) there is no triangle possible,
b) there is exactly one triangle possible,
c) there are two triangles possible.

Solution by M. Wagner, Boston, Massachusetts

No triangle can be formed if

$$y < x \sin\theta, \quad 0 \leq \theta \leq \pi/2,$$

or

$$y < x, \quad \theta > \pi/2.$$

Only one triangle can be formed if

$$y > x.$$

Two triangles can be formed if

$$x \sin\theta < y < x, \quad 0 \leq \theta \leq \pi/2.$$

Consequently,

$$P_0 = \frac{\int_{\theta=0}^{\pi/2} d\theta \int_{x=0}^{L} dx \int_{y=0}^{x\sin\theta} dy + \int_{\pi/2}^{\pi} d\theta \int_{x=0}^{L} dx \int_{y=0}^{x} dy}{\int_{\theta=0}^{\pi} d\theta \int_{x=0}^{L} dx \int_{y=0}^{L} dy},$$

or

$$P_0 = \frac{\pi + 2}{4\pi},$$

$$P_1 = \frac{1}{\pi L^2} \int_{\theta=0}^{\pi} d\theta \int_{x=0}^{L} dx \int_{y=x}^{L} dy = \frac{1}{2},$$

$$P_2 = \frac{1}{\pi L^2} \int_{\theta=0}^{\pi/2} d\theta \int_{x=0}^{L} dx \int_{y=x\sin\theta}^{x} dy = \frac{\pi - 2}{4\pi}.$$

Check: $P_0 + P_1 + P_2 = 1$.

Also solved by H. Kaye, Paul Meyers, J. Thomas and F. Zetto.

118. Proposed by Leo Moser, University of Alberta

Split the integers 1, 2, 3, ⋯, 16 into two classes of eight numbers each such that the $\binom{8}{2} = 28$ sums formed by taking the sums of pairs is the same for both classes.

Solution by C. W. Trigg, Los Angeles City College

When all possible pairs of the integers, 1, 2, ⋯, 16 are taken, there is only one sum equal to 3 and only one sum equal to 4. Hence, if 1 is in class A, then 2 and 3 must be in class B. Now $2 + 3 = 5$, so 4 must be in A. If 5 were in A, there could be no matching sum equal to six in B, so 5 is in B. But $2 + 5 = 7$, so 6 is in A; and $3 + 5 = 8$, so 7 is in A. Now $4 + 6 = 10$, so 8 is in B.

By similar reasoning beginning with 16, 15, 14 (or by subtracting each member of classes A and B from 17) we establish two sets, C: 16, 13, 11, 10 and D: 15, 14, 12, 9. Then the unique solution to the problem is:

AC:	**1**	4	6	7	10	11	13	16,
BD:	**2**	3	5	8	9	12	14	15.

The 28 sums of pairs are: 5, **7**, 8, 10, **11(2)**, 12, 13, **14(2)**, 15, 16, **17(4)**, 18, 19, **20(2)**, 21, 22, **23(2)**, 24, 26, 27, and 29.

Also solved by H. Kaye, Paul Meyers, J. Thomas, M. Wagner, F. Zetto and the proposer.

119. Proposed by Maurice Eisenstein, AVCO RAD

An infinite sequence of points on a line have coordinates given by the R progressions

$\{a_r n + b_r\}$, $r = 1, 2, \cdots, R$, $n = \cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$.

Find the average distance between contiguous points.

Solution by F. Zetto, Chicago, Illinois

The number of points of the progression $\{a_r n + b\}$ lying in the interval $-N \leq n \leq N$ is given by

$$2N/a_r + \epsilon_r$$

where

$$\lim_{N\to\infty} \epsilon_r/N \to 0.$$

Consequently, the total number of points in the interval $(-N, N)$ is

$$\Sigma\, \{2N/a_r + \epsilon_r\}.$$

The average distance $\bar{D}_N$ between contiguous points is then

$$\bar{D}_N = \frac{2N}{\sum_r \{2N/a_r + \epsilon_r\}}$$

Finally,

$$\lim_{N\to\infty} \bar{D}_N = \{\sum_r 1/a_r\}^{-1}.$$

Also solved by Merwyn M. Friedman, H. Kaye, Paul Meyers, J. Thomas and the proposer.

Errata:

124. Proposed by H. Kaye, Brooklyn, N. Y. Construct the center of an ellipse with a straightedge only, given a chord and its midpoint.

The editor regrets that he did not examine this problem more carefully when it was sent in because on subsequent examination of the problem he finds that the construction is indeed impossible. Consequently, the problem should be ch'nged to read:

Prove the impossibility of constructing the center of a circle with a straightedge only, given a chord and its midpoint.

BOOK REVIEWS

Edited by
FRANZ E. HOHN, UNIVERSITY OF ILLINOIS

Rings of Continuous Functions. By Leonard Gillman and Meyer Jerison. Princeton, Van Nostrand, 1960. ix + 300 pp., $8.75.

General Theory of Banach Algebras. By Charles E. Richart. Princeton, Van Nostrand, 1960. xi + 394 pp., $10.50.

Both of these excellent new books deal with subjects which are above the level attained by the ordinary undergraduate mathematics student Hence most of the readers of this Journal are not advised to attempt to read them with only the traditional undergraduate background. However, some general remarks concerning the contents of these books may be of value to the reader, for it is likely, if he continues with graduate study of mathematics, that he will at some future date wish to look into these attractive volumes.

The book of Gillman and Jerison assumes that the reader has some background in point-set topology and abstract algebra (especially ring theory). One semester of each would probably suffice if this book were used as a text in a course or with suitableguidance, but more background would be required for independentstudy. In addition to these prerequisites, Rickart supposes a knowledge of the basic facts concerning Banach and Hilbert spaces and for a real understanding of some of his examples more analysis would be required (e.g., measure theory, topological groups, etc.). Both books advance the reader to the forefront of knowledge in their fields and would provide adequate background for reading research papers.

The subject matter of these two books is about as similar as possible without actually overlapping, which they scarcely do. They both treat the same type of objects and with similar tools but from different points of view. We shall now try to describe briefly what their contents deal with. Let X be a nice topological space; for example, the closed interval $0 \leq x \leq 1$ or the whole real line. Iff and g are two functions defined on X we define their sum, which we donote by $f + g$, to be the function whose value at the point x in X is given by

$$(f+g)(x) = f(x) + g(x), \qquad x \text{ in } X.$$

Similarly we define their product fg by

$$(fg)(x) = f(x)\, g(x), \qquad x \text{ in } X,$$

and we can define the product of a constant c with the function f by

$$(cf)(x) = c\, f(x), \qquad x \text{ in } X.$$

Under the first two operations, the collection $C(X)$ of all continuous real-valued functions on X form a ring in the sense of modem algebra. Under all three operations, this collection or the similar collection of complex-valued continuous functions forms a (commutative) algebra (over the real or complex field).

The object of study of Gillman and Jerison's book is the ring $C(X)$ of real-valued continuous functions on a topological space X. The main questions are to relate the algebraic properties of $C(X)$ and the topological properties of X. For example: What topological properties of X are determined by algebraic properties of $C(X)$ and vice versa? If every element in $C(X)$ is automatically bounded, what does this imply about X? What sets of functions on X form an ideal in the ring $C(X)$? When can the subring of all bounded functions in $C(X)$ be realized as a space $C(Y)$ and when this is possible what is the connection between X and Y? A considerable interplay between algebra and topology enters; it is difficult to imagine a better way to review and re-examine the basic notions of these two fields and to refine one's understanding and intuition than to work through this book. In addition to their careful exposition, the authors have provided a substantial list of problems of varying difficulty which augment and exemplify the main development.

In contrast to Gillman and Jerison, where $C(X)$ is the object of study, in Rickart's book it is one of the main examples, but not the whole thing. Here the primary interest is centered on algebras over the complex field in which there is also a notion of distance satisfying certain additional properties. These algebras are called Banach algebras (or *normed* rings) and they have been extensively studied since 1940. The space $C(X)$ forms a Banach algebra if we take as the distance between f and g

$$\|f - g\| = \sup |f(x) - g(x)|, \qquad x \text{ in } X.$$

In fact $C(X)$ is a commutative Banach algebra. There are many other Banach algebras (both commutative and not) which arise in analysis. The purpose of Rickart's book is to examine the general properties of Ranach algebras, to classify them in types, to analyze the structure of these different types of algebras and to represent abstractly-presented Banach algebras in terms of simpler and more concrete constructs. For example, the important Gelfand-Naimark theorem gives a necessary and sufficient condition for a commutative Banach algebra to be an algebra of the form $C(X)$ for a suitable topological space X. While Rickart does not provide any exercises he does provide an appendix full of examples, a guide to the literature, and a fifty page bibliography of research papers. Unquestionably the theory of Banach algebras is of great importance for certain areas of functional analysis and highly enlightening for many others. It is this reviewer's opinion that any student will benefit by a study of this theory which, in Rickart's words, has its "feet in analysis and its head in algebra." This book gives the most comprehensive treatment of the algebraic aspect of this theory that is available.

University of Illinois — Robert G. Bartle

Commutative Algebra, Vol. II. By Oscar Zariski and Pierre Samuel. Princeton, Van Nostrand, 1960. x + 414 pp., $7.75.

With publication of the above volume the authors have completed their two-volume text on commutative algebra. The purpose of the text is to present, in organized form, the basic known results on commutative rings and their modules with particular emphasis on those parts of the subject which are needed in algebraic geometry.

Volume I deals with field theory, theory of ideals (particularly prime and primary) in commutative rings, theory of noetherian rings, and theory of dedekind domains. Volume II deals with the theory of valuations, polynomial and power series rings, and local algebra. The two-volume set is self-contained, but the second volume draws on results developed in the first volume. The authors state, in the preface to the first volume, that they hope to follow this set with a text on algebraic geometry.

The presentation followed by the authors is modern in the sense that the results are presented in ascending order of logical rather than intuitive complexity. The result is that a great deal of subject matter can be treated in a relatively small amount of space, and the proofs can still be done in considerable detail. For some of the more important results more than one proof is given.

A student can gain a great deal from reading these volumes provided that he is working under the guidance of an instructor who can explain the ideas behind the logical concepts discussed in the text, and who can provide problems dealing with the subject matter.

University of Illinois — Lawrence Levy

special Functions. By Earl D. Rainville. New York, Macmillan, 1960. xu + 365 pp., $11.75.

It is an unusual pleasure to read Prof. Rainville's book on special functions, for it is very clear from the beginning that he has included only topics which he loves and enjoys teaching, and has carefully woven these together into a smoothly unfolding fabric. The major theme is really **polynomials,** with a strong minor in hypergeometric functions. All the classical sets of polynomials are included, and dozens of other interesting sets besides. Similarly, there is an extensive treatment of the classical hypergeometric functions, with emphasis on the transformations, but there is much additional material on generalized hypergeometric functions which is not to be found elsewhere in an equally accessible form.

There are many unusual and delightful features to the way in which the polynomials are handled, and it is worth mentioning a few of these. General properties of whole classes of generating functions, many of them collected for the first time, are developed together. General techniques for obtaining recursion relations, and for expanding one set of polynomials in terms of another, are emphasized. A unified treatment is thus achieved. Furthermore, a variety of analytic techniques is stressed without prolonged use of any one. Happily, there are only two complicated contour integral arguments in the whole book; they are enough to give the feeling for that kind of development, and analytic ingenuity replaces a number of others.

Since the book (348 pages of text) is finite, it follows that the treatment of a number of other special functions cannot be more than cursory. This applies particularly to those which have been the subject of extensive exposition elsewhere, namely Bessel functions and elliptic functions. After the definition of Bessel functions, only the simplest recursions and the Neumann polynomials are taken up. On the other hand, there is an excellent chapter, with the most readable exposition I have seen, of the Jacobi Theta functions. And speaking of good developments, the introductory chapter on Gamma Functions is equally delightful.

The book deserves to be used as a classroom text, for self-study, and as a reference work. The teacher will find many excellent exercises which have obviously been designed with as much care as the text; he may, however, wish to supplement the almost total lack of any connection between the mathematics presented and any physical or engineering applications. For self-study, the book is remarkably self-contained, with only a few outside references to the theory of ordinary differential equations. And the exposition is exceptional. Finally, as a reference work, the book contains a considerable component orthogonal to any of the other standard collections on special functions, and, without question, deserves to become one of them.

Bell Telephone Laboratories — H. O. Pollak

A Modem Introduction to College Mathematics. By Israel H. Rose. New York, John Wiley, 1959. xxi + 530 pp., $6.50.

The topics included in this book are about the same as those in text by Altwerger, plus Statistics and Probability; the major exception being the exclusion of symbolic logic, although informal deductive and inductive logic are included. Generally, the work is well written, clear, and well suited for a variety of purposes. Concepts are sharply defined, and many interesting examples and exercises usually follow. The reading of this text should prove valuable to any college freshman.

Los Angeles State College — T. J. Cullen

Ordinary Differential Equations and Their Solutions. By George M. Murphy. Princeton, Van Nostrand, 1960. ix + 451 pp., $8.50.

This is not a textbook but combines a collection of methods for solving ordinary differential equations and a collection of solutions of 2315 given equations in a single volume. A similar book is Kamke: **Differentialgleichungen, Lösungsmethoden** und **Lösungen,** Band 1, **Gewöhnliche Differentialgleichungen,** which lists solutions of 1530 equations, including 83 systems of equations and 15 functional differential equations.

The equations whose solutions are listed in Murphy's book are broadly classified thus:

- A: First Order
  - Al: First Degree
  - A2: Second or Higher Degree
- B: Second Order
  - Bl: Linear
  - B2: Nonlinear
- C: Order Higher Than Two
  - Cl: Linear
  - C2: Nonlinear

These classifications are further broken down into the familiar types.

Part I lists familiar and unfamiliar devices for solving given types of ordinary differential equations. Part II lists specific equations together with their solutions. There is a rather complete treatment of singular points. However, the book does not contain an extensive treatment of boundary value and eigenvalue problems such as is found in Kamke.

This book will be useful to workers in applied fields who are obliged to solve differential equations which have solutions in closed form. (Numerical methods are not treated.) It should be in every mathematical library.

University of Illinois — Franz E. Hohn

Finite-Difference Methods for Partial Differential Equations. By George E. Forsythe and Wolfgang Wasow. New York, Wiley, 1960. x + 444 pp., $11.50.

This book is a presentation of finite-difference methods for solving partial differential equations, particularly with the use of high-speed computing machines. It is a valuable addition to the literature, presenting an exposition of results that are essential for anyone who is entering this field. The authors have kept the material at an intermediate level, their only stated prerequisites being a knowledge of advanced calculus and some matrix theory. Though no knowledge of partial differential equations is presupposed, the book would be difficult for a reader who is entirely new to the subject.

The text consists of a brief introduction followed by four chapters. These are on hyperbolic equations in two independent variables, parabolic equations, elliptic equations (this chapter forms more than half of the book), and initial-value problems in more than two independent variables. There is an exceptionally complete bibliography. The exposition is very clear, though there are few numerical examples and no exercises.

The authors have tried to give the basic finite-difference methods in this field together with their theoretical foundation, and have succeeded very well in this aim. Their book is an admirable introduction for anyone who is new to the subject, and contains much that is of interest to those who are concerned with problems of this kind.

University of Illinois — Richard Jerrard

Classical Dynamics. By R. H. **Atkin.** New York, Wiley, **1959. ix + 273** pp., **$5.25.**

Professor **Atkin** has written a book with the primary purpose of preparing students to pass examinations, such as the Cambridge Tripos papers, in Classical Dynamics. The format of the text is well designed for this purpose. A principle is introduced in simple terms followed by many worked examples. The reader is then given the **opportunity** to test his **new** found knowledge on a great number of exercises. A careful reading, with abundant use of paper and oencil. should **give** the reader a mastery in the intricacies of this phase of classical physics.

The author assumes a good mathematical background as well as an elementary knowledge of physics. The mathematical background should include a thorough knowledge of algebra, a mastery of elementary calculus and analytic geometry, an introduction to differential equations, and some familiarity with matrix algebra. Although a great amount of detail is presented in the working of examples, many of the mathematical operations are omitted or briefly indicated. Professor **Atkin** does provide a short introduction to the **theory** of vectors. The rapid pace of the text is indicated by the fact that this **eighteen** page **section begins** with a definition of vectors and proceeds to the concept of second order tensors.

Classical Dynamics is restricted to the usual undergraduate material. There is much discussion of the motions of particles and rigid bodies. The Lagrangian function is discussed while topics such as Hamilton's equations are left for advanced work.

The book has several defects which seriously detract from its usefulness. Although one would expect some errors in the first printing of any book, the many errors in this volume cause one to come to the conclusion that the book was thrown together in great haste. Some of these errors are obvious misprints while others will not be so obvious to the **beginning** student. Examples of these errors are: p. **60,** the omission of a constant, k; p. **78,** cot p should read **cot**φ; and p. **99,** (k^2/f) should be (f/k^2). Another type **of** annoyance is shown by **Figure 44** on page **49,** where two labelling letters are omitted. Furthermore, a very special position is illustrated which does not show the general aspects of the accompanying derivation. As is usual with English texts, there is no index. The repeated references to examinations and examiners (**e.g.,** pp. **48, 70, 202,** etc.) appear to be out of place considering the maturity of the readers to whom the book is aimed.

The book should prove quite useful to the serious student who desires an intimate knowledge of the problems of classical dynamics and who is willing to accept the deficiencies mentioned in the previous paragraph. It should also prove useful to workers outside the field of dynamics who need some background in the subject. In this respect, the text is an excellent companion for the author's previous volume: MATHEMATICS AND WAVE MECHANICS. Finally, the book should prove of interest to mathematicians who take pleasure in seeing the many applications of their science.

Southern Illinois University — Boris Musulin

Statistical **Theory** and Methodology in Science and Engineering. By K. A. Brownlee. New York, John Wiley, **1960.** xv + **570** pp., **$16.75.**

Although the author in the preface almost apologizes for writing another elementary text in statistics, this is not just another elementary text. It is an excellent presentation of some concepts which one would encounter in the field of statistics. The presentation of the material is made very clear with good explanation and, where needed, numerical examples of the theory are added to further clarify matters.

Among others, such concepts as Testing of Hypothesis, Power of a Test, Nonparametric Tests, Linear and Multiple Regression, Analysis of Variance, Experimental Design, Frequency Functions, and Confidence Limits, are presented. Rather than define the bivariate normal distribution the author develops the distribution. The order of the subject matter is quite good except that this reviewer thinks that Chapter **10,** "One-way Analysis of Variance" should directly precede Chapter **13,** "Two-Way Analysis of Variance."

Every chapter in most cases is followed by a sufficient number of problems, related to various fields of engineering and science, which add understanding to the **material.** Also, each chapter has references so that one may easily probe further into the material presented. The book has an appendix **which** contains the Cumulative Standarized **Normal** Distribution Function for both negative and positive values of the variable, Percentage Points of the **t-Distribution,** and others.

The author intended this book to serve students who have a mathematical background of roughly college algebra and the material should be covered in three quarters. This reviewer feels that that task can only be accomplished with either very good or excellent students. He recommends that students have at least one semester of calculus and that the time element be extended.

This reviewer recommends the book highly.

San Diego State College — Joseph M. Moser

Differential Equations. **By T.** Fort. **Holt,** Rinehart, and Winston, New York, **1960.** viii + **185** pp., **$4.75.**

This book is designed as a text book for a first course in differential equations which is to follow calculus. The text introduces the reader to the simpler concepts of rigorous mathematics, as well as the manipulations required to solve differential equations. All definitions are carefully stated, and many of the simpler theorems are proved. Applications to physical problems are kept to a minimum.

A weakness of the book is that it has very few references for further study. For the better student, it does not have many challenging problems on the extension of the theory. A strong point is that an unusually large amount of space is devoted to the classical theory of **linear** differential equations. This gives the reader a good feel for the subject. Later, however, matrix methods are introduced. A noteworthy feature is the symmetrical treatment in x and y for equations of the first order and first degree. This avoids many of the assumptions about implicit functions.

Following the first chapter, which is a careful statement of what constitutes a solution, the author covers various techniques of solving equations of the first order and degree. The next two chapters deal with an existence theorem and particular solutions. Chapter **6** gives various techniques of solving equations of the first order and higher degree. This is followed by a chapter on the general theory of linear equations. The next several chapters are devoted to the methods of undetermined coefficients, operators, power series, and **Laplace** transform. The book ends with a study of boundary value problems, and partial differential equations.

The book is well written. The explanations and proofs are easy to follow, and there are numerous illustrations. This plus the author's approach to the subject should see the book in wide use.

University of Illinois — George L. Kvitek

Introduction to Modern **Algebra.** By Neal H. McCoy. Boston, Allyn and Bacon, **1960.** xi + **304** pp., **$7.50.**

This book is the best the reviewer has seen for a first course in modem algebra. It begins with a chapter on sets, mappings, equivalence relations, etc., but offers good motivation for abstract concepts throughout, Generally, the author proceeds from the concrete to the abstract at a comfortable rate. The chapter titles, beginning with the second, are: Rings, Integral Domains, Some Properties of the Integers, Fields and the Rational Numbers, The Field of Real Numbers, The Field of Complex Numbers, Polynomials, Groups, Vector Spaces, Systems of Linear Equations, Linear Transformations and Matrices. The book is solid, and has the beautiful feature that the exercises are properly placed and should all be assigned with the feeling that they are sufficient. There is enough material for six semester-hours of work.

Southern Illinois University — James R. **Boen**

Algebra and **Trigonometry.** By Edward A. Cameron. New York, **Holt,** Rinehart, and Winston, **1960.** xi + **290** pp., **$5.00.**

This book, designed for college freshmen, is a notable improvement on the traditional "College Algebra" text. The exposition is careful and clear and the material is in keeping with the modem trend without going off the deep end of abstraction. Most college freshmen will find much to be learned between its covers.

The text begins with an axiomatic approach to the properties of the real number system. This chapter includes many rigorous proofs of simple theorems. An introduction to elementary set theory is then given and used to define the function concept. Some basic concepts of Analytic Geometry are introduced. Next, a brief chapter on equations deals with the logical aspects of solving equations while assuming the ability to handle basic algebraic operations. Inequalities are given due attention in a chapter which begins with the order properties of the real numbers and includes a fine discussion of absolute values. Systems of linear equations and determinants are treated extensively; matrices are introduced in a wholesome manner and used as **an example** of a non-commutative algebraic system. Mathematical induction, the binomial theorem, progressions, exponents and logarithms are treated in the traditional manner in which Professor Cameron is a past master. Trigonometry is presented in two chapters. The first deals with trigonometric functions **of** angles and their applications. The second treats the circular functions of real numbers and shows clearly the relation between these and the trigonometric functions. Worthy of special mention is the presentation of the inverse trigonometric functions. Following the trigonometry is an excellent chapter on the complex number system. The complex numbers are introduced as a system of numbers represented by ordered pairs and later **shown** in the **traditional** notation. Geometric representation of complex numbers and the **polar** coordinate system are included in this discussion. The final chapter **of** the book covers the theory of equations. In most sections, lists of exercises are more than ample although not too imaginative.

This reviewer believes this book is highly suitable for a five semester **hour** course for beginning college students. Professor Cameron's method of presenting this subject matter clearly, but without pedantry, makes this text one which both students and instructors should appreciate.

University of Illinois — Henry Frandsen

Modern College Algebra. By J. D. Mancill and M. O. Gonzales. Boston, **Allyn** and Bacon, **1960.** xi + **386** pp., **$6.25.**

Contrary to the **view-all** too prevalent today-that freshman algebra is primarily a techniques course, the authors feel that all essential topics and concepts can be **correctly** and fully discussed. To this end, careful proofs are introduced immediately and employed throughout. When more advanced concepts are brought in, they are clearly explained and always used in what follows. For example, the method of proof by mathematical induction is introduced early as a useful tool in proving various properties of real numbers and is not relegated to a separate chapter with its own peculiar set of exercises. Thus, the book shows a remarkable continuity in spite of the diversity of some of its topics.

The first few chapters are concerned with enlarging upon the **student's** prior knowledge of the real number system followed by a careful axiomatic development of it. Next the authors discuss extensively polynomials and the solution of polynomial equations. Matrices are defined and used as one of the methods of handling systems of linear equations. A very careful discussion of permutations and probabilities is given. The logarithmic function is defined in an axiomatic manner and its properties as a function, rather than only its use in numerical computations are stressed. The exercises, while not too numerous, avoid repetition and are thus definitely in keeping with the spirit of the book.

In summary, this reviewer feels the book represents a significant improvement in the standard of elementary mathematics texts.

University of Illinois — Charles F. Koch

Real Variables, An Introduction to the Theory of Functions. **By** John M. H. Olmsted New York, Appleton-Century-Crofts, **1959.** xvi + **621** pp., **$9.00.**

The purpose of this book is to present the basic ideas and techniques of analysis, for real-valued functions, in such a way that it can be studied by students who have had only a standard calculus background, as well as by those with extended backgrounds. The chapters on **functions** of a single variable are essentially those of Intermediate **Analysis,** published in **1956** by the same author and publisher. As in the earlier book, great flexibility is achieved by stamng sections and exercises. Very few of the **2200** exercises are trivial and students who work all of them would have an excellent command of the theory of functions of a real variable. Many of the more complicated and advanced ideas occur only in the doubly-starred exercises.

Because of the clearness and detail in proofs, the book is most suitable for an undergraduate or first year graduate course, especially where such a course replaces advanced calculus. Although the author mentions notations and definitions coming into current use, the style and flavor is traditional. The teacher will have to select exercises carefully in order to keep from spending too much time on each chapter.

The book is an excellent addition to the undergraduate **library,** and, with the increasing emphasis on rigor, should prove to be a popular text.

Knox College — Rothwell Stephens

Matrix Calculus. By E. Bodewig. New York, Interscience, 1959. xi + 452 pp., $9.50.

After a preliminary introduction to the Algebra *of Matrices,* the author assembles in compact form a large amount of information concerning the computational aspects of the solution of linear equations, the inversion of matrices, and the solution of eigenproblems: more than is contained in any other single volume known to this reviewer.

The book is not for beginners. The style is extremely compact and at times the material is made unnecessarily difficult to understand. A great deal of prior knowledge of matrices and determinants is assumed. Statements are at times imprecise and definitions do not always define, so that the reader must know from past experience what is intended in order to follow the text. (This may be in part a language difficulty since the author's native tongue is not English.) Frequently use is made of material that follows later in the book. For such reasons, those who seek an introduction to matrix calculus should probably turn elsewhere.

On the other hand, those who have a great deal to do with matric computation and who are already well-acquainted in the field will find the book useful for its critical evaluation of the advantages and disadvantages of the various procedures discussed and for its encyclopedic character.

The compactness of the treatment and the elegance of some of the proofs are largely attained through the effective use of special notations for the rows and columns of a matrix and for certain specific types of matrices. However, the special notation seems to make some of the manipulations more complex rather than simpler.

There are some misprints and some mistakes, but neither will cause difficulty for the mature user of the book.

University of Illinois — Franz E. Hohn

Basic Concepts of Elementary Mathematics. By William L. Schaaf. New York, John Wiley, 1960. xvii + 386 pp., $5.50.

This is one of the many books appearing recently which try to present elementary mathematics from the modem viewpoint. The volume here does not attempt to go into calculus, as most others do, but concentrates on the rudiments of mathematics: logic and set theory, geometry, number systems, exponents and logarithms, measurement and mensuration, functions and graphs, interest, probability, and insurance. The book is intended primarily for teacher education, and as such, it is well done.

Los Angeles State College — T. J. Cullen

The Theory of Matrices.. By F. R. Gantmacher. New York, Chelsea, 1959. Vol. I, x +374 pp., Vol. II, x + 276 pp., $6.00 each volume. (Translated from the Russian by K. A. Hirsch.)

Hardly a month passes during which the reviewer does not receive notice of a new book dealing with matrices. In fact, the writing of such books now seems to be as popular pastime among mathematicians as the production of books at the freshman level dealing with the fundamentals of mathematics. Unfortunately, the word "pastime" is regrettably appropriate, and many recent books on matrix theory are poorly written and hastily contrived, owing their existence merely to the economics of the sudden and current popularity of the subject in our engineering schools and industrial engineering laboratories.

Thus, the reviewer started his inspection of this new addition to the collection of such books without much enthusiasm, expecting to find little to distinguish these two volumes from their many predecessors. A glance at the table of contents gave the first indication that perhaps these two volumes are somewhat unique. The contents indicated that the author covers thoroughly every conceivable aspect of matrix theory, and in a mere 650 pages. Indeed, a subsequent closer inspection of the book itself disclosed that my appraisal gleaned from the table of contents was overly conservative.

After familiarizing myself with both volumes I would pounce upon an unsuspecting colleague and say,

"Name a topic in matrix theory which you have yet to see given a satisfactory exposition in a text."

A variety of answers were received. For example:

(1) A discussion of multiple eigen-values and linearly independent eigen-vectors.
(2) Lyapunov's work in stability in connection with matrices and differential equations.
(3) A complete exposition of the Jordan normal form of a matrix.
(4) Hankel forms.

In these and many other cases, I dragged my reluctant and skeptical colleague into my office and produced the requested material in Gantmacher's book, carefully and thoroughly discussed and explained.

Furthermore, the exposition of the book under review is above criticism. Too many other mathematics texts are written like novels, and after the innocent reader has been led through several pages of flowing prose, the author announces that he has just proved the following six theorems. However, in Gantmacher's book each term is precisely defined, and each theorem is carefully stated and then meticulously proved.

Unfortunately no problems are included. Thus, when used alone these books are not suitable as a text. Rut if augmented with suitable problems, sufficient material is included in both volumes to cover at least three, or perhaps four, one-semester courses in matrix theory. Moreover, as complete reference books in matrix theory these two volumes are unsurpassed; this will probably be the situation for some years to come.

University of Arizona — Paul Slepian

Dynamic Programming and Markov Processes. By R. A. Howard. New York, Wiley, 1960. viii + 136 pp., $5.75.

It is frequently necessary to make a sequence of decisions, the object of which is to maximize a return or minimize a cost. Only rarely will the decisions be made on the basis of complete information concerning cause and effect, the object of the process, and so on, so that a theory for decision making under conditions of uncertainty is of paramount importance in many branches of applied mathematics. These remarks obviously apply in operations research and industrial engineering, where human beings serve as the decision makers. They also apply in the field of automatic control, where presently control devices that have the capability to "learn" to improve their own capabilities based on experience are contemplated. One of the great challenges to the creative mathematician working in these areas is that many of the significant problems have not yet even been formulated in mathematical language. Still other fascinating problems exist at the analytical and computational levels.

This monograph – essentially the author's doctoral thesis – provides a delightful introduction to the subject of multi-stage decision processes, a field pioneered and extensively cultivated by Richard Bellman, who coined the term "dynamic programming."

The reader is first introduced to the concept of discrete-time Markov processes and their treatment via matrix analysis and generating functions. Then the expected return from a Markov process is discussed, and finally a decision aspect is introduced which leads to the question of the determination of optimal policies. The emphasis is upon determining optimal policies for processes of infinite duration, and a solution based upon successive approximations in policy-space is proposed. The remaining chapters are devoted to processes in which there is discounting of future returns and the treatment of continuous time Markovian decision processes. Throughout, Bellman's principle of optimality plays a key role.

Some of the carefully worked-out examples, concerning baseball, operation of a taxicab fleet, and maintenance of equipment, are interesting and suggestive. One, concerned with the operation of a business concern, demonstrates the advisability of engaging in research and development activities, no doubt intended for subliminal effects on managerial readers.

Concepts from probability theory, matrix theory, differential and difference equations, Laplace transform theory, and numerical analysis, as well as the functional equation technique of dynamic programming are skillfully blended together to provide a framework for the formulation and treatment of a variety of significant and topical conundrums.

The Rand Corporation — Robert Kalaba

Mathematical Methods and Theory in Games, Programming, and Economics. By Samuel Karlin. Reading, Massachusetts, Addison-Wesley, 1959. Vol. I x+433 pp., $12.50. Vol. II, xi384 pp., $12.50.

Students of management problems, economics, military tactics, and general operations research recognize the importance of game theory and linear programming as a tool in solving many classes of decision problems. However, it is certainly equally important that the student of mathematics be made aware of the exciting potentialities these and allied disciplines within the general framework of decision theory offer both for applying existing mathematics and for suggesting avenues for the development of a new mathematical structures.

In these twovolumes Professor Karlin has presented a clear and penetrating analysis of the structure of game theory and programming, both linear and non-linear, together with applications to mathematical economics. Volume I is devoted to the study of matrix games, programming, and mathematical economics, all of which draw on the tools of vector spaces; convex sets and convex functions; and Volume 11 is concerned with the mathematically more difficult subject of infinite games.

Throughout both volumes, emphasis is placed on the exposition of the underlying mathematical structure. Certain topics, such as vector spaces, convex sets, and convex functions are developed in the appendices while others, such as positive operators, conjugate functions, and the generalized Neyman-Pearson lemma are introduced where needed in the text. The more advanced topics are treated in starred sections which may be omitted on a first reading, while notes and references at the end of each chapter give some historical background as well as suggesting material for further study. Problems form an integral part of the text, and a discussion of the more difficult ones is included. The two volumes have been made independent by reproducing the introductory chapter and the appendices in each.

In summary, we feel that the mathematically mature reader will profit much from studying these two volumes though he will probably need guidance to comprehend the more advanced sections. The work should also prove useful as a text at the graduate level and also as a reference for those engaged in research in the many fields of application.

University of Illinois — Donald M. Roberts

Modern Mathematics. An Introduction. By Samuel I. Altwerger. New York, Macmillan, 1960. xii + 462 pp., $6.75.

There is certainly enough material included in this work to cover a full year's program of general education in mathematics. Most of the topics of the previous book reviewed are covered, together with the more elementary parts of the calculus; probability, interest, and insurance do not appear. As in many mathematics texts of recent vintage, even advanced ones, the author has included an index of symbols, which would prove quite helpful to students. In criticism, I feel I must say that the definitions given are, at times, somewhat fuzzy, a serious complaint if the text is to be used as for general education. The author seems somewhat hesitant to develop some of the concepts in the spirit of "modem mathematics ". The arrangement of topics is sometimes questionable; e.g., "Symbolic Logic" does not appear until page 170.

Los Angeles State College — T. J. Cullen

BOOKS RECEIVED FOR REVIEW

*K. A. Brownlee: *Statistical Theory and Methodology in Science and Engineering.* New York, Wiley, 1960. $16.75.

*E. A. Cameron: *Algebra and Trigonometry.* New York, Holt, Rinehart, and Winston, 1960. $5.00.

*G. E. Forsythe and W. R. Wasow: *Finite-Difference Methods for Partial Differential Equations.* New York, Wiley, 1960. $11.50.

*T. Fort: *Differential Equations.* New York, Holt, Rinehart, and Winston, 1960. $4.75.

W. W. Garvin: *Introduction to Linear Programming.* New York, McGraw-Hill, 1960. $8.75.

*L. Gillman and M. Jerison: *Rings of continuous Functions.* Princeton, Van Nostrand, 1960. $8.75.

E. J. Hannan: *Time Series Analysis.* New York, Wiley, 1960. $3.50,

I. Niven and H. S. Zuckerman: *An Introduction to the Theory of Numbers.* New York, Wiley, 1960. $6.25.

*C. Rickart: *General Theory of Banach Algebras.* Princeton, Van Nostrand, 1960. $10.50.

L. Takacs: *Stochastic Processes.* New York, Wiley, 1960. $2.75.

*O. Zariski and P. Samuel: *Commutative Algebra,* Vol. II., Princeton, Van Nostrand, 1960. $7.75.

*See review, this issue.

NOTE: All correspondence concerning reviews and all books for review should be sent to PROFESSOR FRANZ E. HOHN, 374 ALTGELD HALL, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essential as today.

Your election as members of Pi Mu Epsilon Fraternity is an indication of scientific potential. Can you pursue advanced study in your field of specialization?

To point out the need of advanced study, the self-satisfaction of scientific achievement, the rewards for advanced preparation, the assistance available for qualified students, etc., it is planned to publish editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

Through these and future editorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

We are pleased in this issue to have an editorial from the Union Electric Company. This is one of the nation's major utility systems. They have participated in the Nuclear Power Group, Inc., a non profit corporation which was formed to conduct research studies in the field of nuclear power. They produce electric power for use in three states and their 6,500 employees include many mathematicians and research scientists.

The article "Mathematics Today and Tomorrow" is reprinted from the Mathematical Association of America's "Recommendations for the Training of Teachers of Mathematics". Information and recommended undergraduate curriculum may be requested from the Executive Director, Dr. Robert J. Wisner, Michigan State University Oakland, Rochester, Michigan.

The following lists contributing corporations with the issue in which their editorials appeared.

Aeronutronics	Vol. 3, No. 2
Army Ballistic Missile Agency	Vol. 2, No. 10
AVCO, Research and Advanced Development	Vol. 2, No. 10
Bell Telephone Laboratories	Vol. 2, No. 10
Bendix Aviation Corporation	Vol. 2, No. 8
E. I. du Pont de Nemours and Company	Vol. 3, No. 2
Emerson Electric Company	Vol. 2, No. 7
General American Life Insurance Company	Vol. 2, No. 9
Hughes Aircraft Corporation	Vol. 2, No. 9
International Business Machines Corporation	Vol. 2, No. 8
Eli Lilly and Company	Vol. 3, No. 2
Mathematics Teachers College, Columbia U.	Vol. 3, No. 3
McDonnell Aircraft Corporation	Vol. 2, No. 7
Monsanto Chemical Company	Vol. 2, No. 7
National Science Foundation	Vol. 3, No. 3
North American Aviation, Inc.	Vol. 2, No. 9
Olin Mathieson Corporation	Vol. 2, No. 7
Shell Development Company	Vol. 3, No. 1
Union Electric Company	Vol. 3, No. 4
Woodrow Wilson Foundation	Vol. 3, No. 3

UNION ELECTRIC

MATHEMATICS IN THE ELECTRIC UTILITY INDUSTRY

by
G. W. GERELL
Manager
Transmission & Distribution
Engineering Staff

Gordon W. Gerell

The Electric **Utility** Industry, like many others, in its earlier years operated in a quite successful manner, without benefit of unusually involved mathematical concepts. However, it should hastily be added that this is not to imply that the basic design formulae of electrical equipment did not resort to mathematical procedures of a very high order. It most certainly did - but the reference here is, of course, to utility engineering, operating and statistical personnel whose main function was concerned with the application of available equipment, its operation and maintenance.

It is **axiomatic** that the many facets of our technological society must advance hand in hand, — witness for example the tremendous impetus given to the development of the physical sciences by the invention of the steam engine and the electric telegraph in the early nineteenth century - giving us transportation and communication.

The Electric Power Industry has moved along with the tide, and today it is a highly specialized and complex technology, wherein mathematics of most any order and the most involved of the physical sciences lends a hand in the solution of its many problems.

Electric power is generated, transmitted and distributed to our homes and factories throughout a great range of voltages - from the 120 volts necessary to operate our lights and appliances, up to 460,000 volts used in the transport of large blocks of power from generating sources over great distances to points of power utilization.

The problems associated with the necessity to confine this voltage to its designed environment have always been challenging ones, and have given rise to mathematical concepts involving dielectric and corona losses, insulation voltage gradient and stresses, arcing phenomena and others of a highly technical nature, in which the use of integral calculus and other forms of mathematics are most essential.

Frequently, however, these voltages do "get out of hand" and take a short-cut path back to the power sources, usually because of failure of some insulating medium; by mechanical damage to electrical structures; by excessively high transient power surges and in some instances, not too infrequent, because lightning may strike the power circuits.

When these things happen, it is essential to isolate the faulted circuit from the system, and to do it fast to prevent damage and loss of service continuity to other parts of the system. Protection of the electric system through the medium of automated control devices and circuit interrupting devices capable of opening circuits carrying as much as ten million kilowatts of power in a tenth of a second and less, is another specialized branch of utility system engineering.

The precise determination of fault current magnitude and location within a complex arrangement of circuit interconnections necessitates a broad appreciation of the principles of network analysis and application of such specialized mathematical tools as the Theory of Symmetrical Components.

Like any other business, Electric Utility Companies are profit-making organizations — they exist specifically for the sole purpose of supplying electric service under conditions that will insure reasonable return on system investment. As an aside, it is of interest to note that capital investment per customer served is approximately $900.00 - much higher than any other type of business.

Plant investment per dollar of sales is close to $5.00 — ten times as much as the average of all of the manufacturing industries in our country.

Be that as it may, the important aspect here, is that electric power is not sold over the counter, like a pack of cigarettes, instead it is brought into your homes and factories over wires, and at that point, it must be metered with great precision, not only for your satisfaction but also to insure that the utility stays in business.

That requires a lot of meters, one at least for every customer — over 650,000 for a metropolitan area the size of St. Louis. It is a tribute to the meter manufacturing industry that such instruments can be made for a very few dollars and record energy consumption year after year with an error not over a few tenths of one percent.

Periodically, however, these meters must be given a test for accuracy of registration and, if necessary, recalibrated.

But that costs money, in a big way, and is time-consuming, and that is where in the last few years application of the principles of statistical analysis, sample testing, has put utility company meter testing on a more economical basis, and as an added dividend vastly improved the overall accuracy of electric meters in service.

It is common practice for the electric utility company in a community and the local telephone company to share use of poles belonging to one, with the other, to carry their respective power and telephone circuits. Either company may own an agreed upon proportion of the "joint-use" poles and receive compensation from the other in the form of a rental fee.

When it is considered that several hundred thousand poles are involved, it is not unusual that inventory statistics may eventually become inaccurate.

In several instances the problem of "joint-use" ownership has been solved by application of a statistical sampling technique based on determining the ownership ratio within sample clusters throughout the area involved.

Very few electric power companies operate on the "lone wolf" basis today. Usually they are part of a much larger integrated system. For instance, our company here in St. Louis has power interchange agreements, through high voltage interconnections, with power companies in the states of Missouri, Illinois and Iowa, representing a total generating capacity of 4,500,000 kilowatts. Our neighboring utilities in turn are interconnected with other companies forming a vast network comprising approximately 40% of the total generating capability in the United States. At the present time this total is close to 175,000,000 kilowatts.

To operate such a system efficiently and economically is a problem – complex and of utmost importance.

The cost of generating electricity at any particular plant within the interconnected system is directly related to the age of the generating units. Operating efficiencies steadily improve through the years, hence the most recently installed units are the most economical of operation. Selection of additional units to carry the total load depends upon such factors as; cost of transmitting power to the numerous load centers; incremental cost of generation; cost of buying power from other members of the interconnected system, and where hydro-electric power is a part of the system, a consideration of water availablilty in the reservoir and expected rainfall is necessary.

All of these factors must be evaluated and put into mathematical formulation to reveal the most economical operating arrangement, usually involving the application of automatic computers; to provide answers quickly and frequently during the daily load cycle to which the system is subjected.

Optimum economy of system operation requires that available generating capability be held to a minimum, commensurate with adequate reliability of service, taking into account equipment and circuit outages, due to failure or maintenance of equipment, or the elements themselves. First, second, and possibly higher order contingencies concerning the occurance of events that may lead to a disruption of the service continuity of specific parts of the electric system, is the real problem here, and is the object of some considerable research today. The evaluation of these questions in the past may have been quite vague and difficult, but application of statistical and probability methods promises results that can, and in some few instances has, effected real economies in system operating procedures.

These are but just a few examples of the application of mathematical procedures directed to the improvement, and greater ease of operation, of the electric power industry. Many more could be cited and certainly many more are just over the horizon – projects that we are not aware of today, or are in the embryonic stage that will come to the forefront because modern forms of mathematics and computational equipment will permit study on a truly economic basis.

MATHEMATICAL ASSOCIATION OF AMERICA

MATHEMATICS TODAY AND TOMORROW

Mathematical ability and mathematical training are commodities in greatest demand today. Science is the new American frontier, and mathematics is the language of science. New pioneers in all fields of science, engineering, and technology will need to be experts in this language.

At the same time, the nature of mathematics has changed drastically. A broader conception of the subject today has stimulated amazing new theoretical developments, and in turn has led to new possibilities of application in the physical, biological, and social sciences. There are more research mathematicians alive today than the total number in the several thousand-year history of the subject.

Our colleges are being called upon to fill an endless need for professional mathematicians, for mathematically trained scientists, and for a variety of mathematically skilled personnel in hundreds of activities. Our business schools often demand the very newest techniques developed by the mathematician. Medical research may soon require mathematical training comparable to that required of the nuclear physicist. Our engineers must be prepared to meet the needs of the rapidly changing American technology. The new industrial revolution – automation – ***each year*** demands many thousands of mathematically trained men and women to command our "giant brains".

Such topics as operations research, linear programming, theory of games, stochastic processes, and machine-simulation were unheard of a generation ago. Today government, industry, and our universities are clamoring for more experts in these fields.

Vast sums of money are being spent by the United States government – through the National Science Foundation, the Department of Health, Education, and Welfare, and the research arms of the various military services - to increase the amount of new mathematics produced and to interest more students in a scientific career.

It is fair to say that mathematics will play a central role in the American culture of tomorrow. We must train our young men and women to be able to attack and solve problems that did not exist when they attended school: problems which require the ability to think mathematically. This requires an educational system that teaches not only fundamental mathematical techniques, but stresses understanding and originality of thought in its mathematics courses. It is this new emphasis on the role of mathematics, the new demands made upon mathematical education. and the broader view of the nature of mathematics itself which motivates these recommendations.

The rate of development in mathematics since 1900 has been truly amazing. There are hundreds of journals all over the world reporting on the most recent discoveries in both pure mathematics and in its ever increasing applications. Our educational system has until recently not responded to these developments. We find thousands of people who now regret their failure to appreciate earlier the significance of mathematics in the modern world and who must return to college later in life to learn techniques demanded by their professions.

Our teachers on all levels, in primary and secondary schools as well as in colleges, must be competent to teach mathematics with an understanding of traditional mathematics and an appreciation of the modern point of view; and they must be able to convey to our students a new insight into the nature of mathematical thought and of its role in our culture. The training of these teachers should be one of the primary concerns of our civilization.

NEWS AND NOTICES

Edited by
Mary L. Cummings, University of Missouri

INSTALLATION OF NEW CHAPTERS

The seventy-sixth chapter of Pi Mu Epsilon, Louisiana Beta, was installed at Southern University on October 14, 1960, with Director-General J. Sutherland Frame present as installing officer. The campus of Southern University is about fifteen miles north of Baton Rouge, on the Mississippi River.

Professor Frame presented the charter to the following charter members: Matthew Crawford, William Fun, John Gipson, Carolyn Hines Harris, Emma Dee Jenkins, Percy Lee Milligan, Rogers Newman, Delores Spikes, Harry Washington, Lloyd K. Williams. Additional members initiated were: Trenton Cooper, Clyde L. Duncan, Carolyn Jacobs, Frances Kraft, Frankie B. Patterson, Leroy Roquemore, Washington Taylor, Llewellyn S. Whitlow, David Williams, Winifred W. Williams.

Professor Frame had a busy afternoon the day of the installation ceremonies. At 1:30, he gave a talk on "Space Drawings with the Trimetric Ruler" to a class of twenty-five. At 4:00 p.m., he gave a lecture on "A Bridge to Relativity Theory". The installation took place at 5:15. This was followed by a banquet at 7:00 p.m., attended by about thirty people, including President Felton Clark and Dean Harrison of Southern University. After several short informal speeches by members present. Professor Frame concluded the program with a talk on the history of Pi Mu Epsilon

Professor Frame says, "I feel that Southern will accept the challenge and develop an active chapter".

On November 15, 1960, the seventy-seventh chapter of Pi Mu Epsilon, Gamma of North Carolina, was launched, at North Carolina State College in Raleigh.

Professor Frame was again installing officer. At 4:00 p.m., he gave a lecture on "Continued Fractions" to an audience of sixty people, including Professor W. T. Whyburn from the University of North Carolina in Chapel Hill, representing the North Carolina Beta Chapter, and Professor F. G. Dressel of Duke University, representing the Alpha Chapter of North Carolina. Immediately after the talk, fourteen students were installed as charter members of the new Gamma Chapter.

The banquet at 6:30 p.m. was attended by the charter members, some faculty members, and a few guests. Professor Frame talked briefly on the history of Pi Mu Epsilon

Speaking of the new chapter, Professor Frame says, "This group of students appears to have been carefully selected out of a much larger number of eligible students, and I believe that a strong chapter will be active in Raleigh".

While on his two installing trips, Professor Frame had the opportunity to visit the Pi Mu Epsilon chapters at Louisiana State University and the University of North Carolina. At the former university he lectured to a meeting of the chapter, while at the latter university, he was entertained at luncheon with student officers.

DEPARTMENT DEVOTED TO CHAPTER ACTIVITIES

Edited by
Houston T. Karnes, Louisiana State University

EDITOR'S NOTE: According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an annual report of the chapter activities including programs, results of elections, etc." The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the Pi Mu Epsilon Journal. Besides the information listed above, we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Chapter Activities Editor Houston T. Karnes, Department of Mathematics Louisiana State University, Baton Rouge 3, Louisiana. These reports will be published in the chronological order in which they are received.

REPORTS OF THE CHAPTERS

ALPHA OF VIRGINIA, University of Richmond.

The Virginia Alpha Chapter held six meetings during the academic year of 1959-60. The following papers were presented:

"Four-Dimensional Graphs of Complex Functions" by Mr. Malcolm Murrill
"Logical Systems" by Professor H. P. Atkins
"The Four-Color Problem" by Paula Williams
"The Development of a Formula for the Normal Age Distribution of a Population" by Marie Grasty
"The Development of Logarithms" by Edith Jones
"A Mathematical Treatment of Error" by William J. Bugg

Activities during the year included the annual banquet at which Professor Tibor Rado of Ohio State University gave a speech on "Mechanical Brains." Winners of the prize examinations for students in elementary courses were: Freshman Mathematics; tie between Patricia Long, Maurice Novick and Robert Powers. Sophomore; first prize, Joyce Chang, second prize, C. Y. Man.

Officers for 1959-60 were: Director. Paula Williams; Vice-Director, William E. Seward; Secretary, Anne Loving; Treasurer, William J. Bugg.

Officers for 1960-61 are: Director, Joyce Steed; Vice-Director, Raoul L. Weinstein; Secretary, Ann Jones; Treasurer, Betty Pritchett.

INITIATES

ALABAMA BETA, Auburn University, (January 17, 1961)

Edward B. Anders
Arthur D. Carpenter
David Livingston Colbert
Eldridge R. Collins, Jr.
Ralph S. Cunningham
Robert E. Hammett
John Taylor Hannon, Jr.
Royce D. Harbor
Cordelia Houston
John David Irwin
Alfred LaSaine
Janie Lomax
Lewis C. Martin, Jr.
John B. McManus
Joel W. Muehlhauser
Thomas L. Osborne
William Steve Pesto
Philip Frank Pollacia
Thomas Al Saunders
Peggy Jo Smith
Bruce L. Spencer, Jr.
Nathan Wayne Stark
Dwight L. Wiggins, Jr.
Edward L. Wills

ARIZONA ALPHA, University of Arizona, (May 5. 1961)

Robert W. Blum
James Bunch
Michael Calderon
Hal W. Crawford
Tomas C. Darr
Wayne Dawson
Robert H. DeVore
Herbert Goullabian
Larry J. Hall
Howard Jelinek
Vuryl J. Klassen
Robert V. Lee, Jr.
James W. Miller, Jr.
Peter G. Nelson
Eugene V. Sherman
Gerald N. Soma
John B. Terry

ARKANSAS ALPHA, University of Arkansas, (November 18, 1960)

James B. Barksdale
Ronald Duain Davis
Freida Carol Davis
William Brian Disney
David W. Dubbell
Nora Lee Ford
Travis J. Galloway
Justin J. Garrett
J. D. Hansard, Jr.
Holly Louise Hartrick
Ernest Loyd Hazlewood
Charles Everett Head
William Higgenbottom
Lillie Lee Johns
William King Johnston
Robert M. Merrifield
Allan Wayne Parse
Stephen H. Rowland
William S. Shipp
Phillip Lance Smith
George Wallace Sorrells
Charles Larry Thompson
Michael Allen Weaver

CALIFORNIA ALPHA, U.C.L.A. (January 18, 1961)

William E. Adams
Barry Boehm
Stephen Bornstein
Wilbur Edwin Bosarge
Doneld Briglia
Wayne S. Carpenter
Cheryl Clark
John Alexander Copeland, III
Gary D. Darby
Caleb Davis
Frank Falch
R. Don Freeman
Michael D. Fried
Robert E. Greene
Paul F. Gruber
Theodore Guinn
Lon Day Hadden
Llewella L. Hall
Carolyn L. Harris
Linore W. Hobbs
Mary Ellen Jones
Edward Landesman
Joseph Carby Mendez
Ronald Lee Olive
Eugene Robkin
Robert Rodman
Edward Scher
Donald Lamed Smith
Judith G. Soren- in
George Stem
Keith F. Taylor
John Varady
Donald Wilken
Lynda L. Wolfinger
Ronald Wyllys

FLORIDA BETA, Florida State University, (November 9, 1960)

Ann Carol Brennan
George William Crofts
Linda Ruth Eshleman
Edna Elizabeth Hawes
Julia Kathrine Hobbs
Mary Patricia Kelley
Donald Joe Kiser
Jim Wade Miller. Jr.
James W. Shiver
Lawrence A. Smith
Richard Clinton Wadle
Ruth Wright Wing

GEORGIA ALPHA, University of Georgia. (December 15, 1960)

James H. Anderson
John W. Boyd, Jr.
Patricia A. Bradberry
John Lane Dolvin
Allen Lee Echols
Mack Earnest Elder, Jr.
Linda Hope Evans
Joseph Faber
Lillian Louise Greene
James M. Hartly, Jr.
Laura Augusta Hay
Gordon C. Howell, Jr.
Mildred Louise Hyde
Dewitt Earl Lavender
Eileen Little
George Lane Maness
James Virgil Peavy
John Richard Ray
Rosalie Seymour
Robert Glenn Stockton
Vesta Lois Stovall
Lucy Carol Watrous
Sara Ellen Weaver
Mary Frances Wellborn
Ramona E. Westbrook
Nancy Claire Wright

ILLINOIS BETA, Northwestern University, (November 13, 1960)

Sara Aslanian
Robert Beck
Robert Bissell
John Bryson
John L. Cooper
David A. Dixon
Alvin W. Filstrup
Louis Goldberg
Charles F. Hepner
Gerald W. Iseler
R. T. Malmgren
Winfred L. Morris
Arthur Palmer
John Roberson
Charles Rulon
Joel W. Russell
Elmer Schaefer
Philip Schaefer
Thomas Tourville
Annie Marie Watkins

IOWA ALPHA, (January 31, 1961)

Glen Albers, Jr.
Kendall E. Atkinson
Ruben B. Babayan
Gerald R. Baumgartner
Robert F. Berry
Payl E. Chase
Norman B. Dillman
Susan Jane Dobson
Jerry G. Doige
Richard C. Eden
William F. Egleston
Dennis Lee Fear
Lyle D. Feisel
Charles A. Goben
Raymond R. Guenther
William M Hartman
Dan Andrew Hayes
Eugene W. Holden
Kempton L. Huehn
Bong Taick Kown
Kenneth C. Kruempel
Marvin M. Lentner
Orval G. Lorimor
Ronald E. McClellan
Joyce Elaine McGee
Richard Keith Miller
Sidney Dean Nolte
John Robert Peterson
Paul C. Phillips
Robert Nelson Sackett
Carol J. Schultz
Jan D. Schwitters
Wayne H. Specker
Jerry R. Tennant
William J. Wahn
Leon N. Wordle

KANSAS ALPHA, University of Kansas, (November 30, 1960)

Rebecca Ann Brown
Player E. Cook
Harold W. Fearing
Frank Feiock
Emilie Louise Hopkins
Joseph L. McNickols
Donald A. Morris
Andrew Page
Robert Keith Remple
F. Edward Spencer, Jr.
Howard L. Taylor
Marvin E. Turner
Neal Wagner
B. Hobson Wildenthal
John Wesley Wyman
Charles W. S. Ziegenfus

KANSAS GAMMA, University of Wichita, (December 8, 1960)

Mufid Abla
John R. Burchfiel
Bruce K. E. Donaldson
Lawrence Camden Garlow
Michael J. Mailhiot
Maganbhai P. Patel
John Bart Sevart
Alan Leon Shore
Verl K. Speer
Lawrence Taylor

KENTUCKY ALPHA, University of Kentucky (December 15, 1960)

John Crawford Adkins
Mary Edna Logan
Charles W Plummer
James H. Rolf
Jacoab Regina Smits

LOUISIANA ALPHA, Louisiana State University, (November 7, 1960)

Patricia Ann Aedsole
Thomas Loris Boullion
Alton Aubrey Braddock
George M. Chandlee, Jr.
William H. Chatoney
Robert V. Courtney
Gaston M. Dubrock, Jr.
Marlin Dutt
Jeffrey Bert Fariss
Eugene Lance Forse
Rex Elwyn Fox
Ted Thomas Gradolf
Dudley W. Griffith
Patricia Rae Hedblom
Anuar Emir Mahomar
Carey Sisemore Mathis
Betty Jo Neal
Valgene Otto Peters
John M Rucker
Sarah Abernethy Ruse
Walter E. Schlemmer
Robert Carl Smith
Henry Howard Thoyre
Ann T. Tinsley
John Franklin Wheeler

LOUISIANA BETA, Southern University, (September 15, 1960)

Matthew W. Crawford
William Franklin Furr
John Edward Gipson
Carolyn Hines
Emma D. Jenkins
Percy Lee Milligan
Rogers E. Newman
Dolores R. Spikes
Harry K. Washington
Lloyd K. Williams

LOUISIANA BETA, (September 27, 1960)

Trenton Cooper
Clyde L. Duncan
Carolyn W. Jacobs
Frances S. Kraft
Frankie B. Patterson
Leroy C. Roquemore
Washington T. Taylor
Llewellyn Whitlow
David Williams
Winfred W. Williams

MISSOURI ALPHA, University of Missouri, (December 5, 1960)

John Howard Anderson
William E. Beckner
James Robert Bickel
Robert Rush Bigger
Walter Robert Bowles
George Russell Brower
John Courtney Cartland, Jr.
James Chih Chen
Charles Claude Cox
Ronald Gregg Craven
Jerrold Ira England
Paul Willard Garvis
William Alfred Gates
Larry Laverne Gilworth
Gerald Lee Goe
Eldon Eugene Heaton
J. V. Hood
Willis Gall Jones, Jr.
Jerry Alan Jouret
Marshall Kai Lee
Joseph William Loeffelman
Warren Alan Mosby
Collin Daryl Nipper
David Virgil Porchey
James T. Sacamano
George Howard Schnetzer
Lake Romero Stith
Allen Borden Webb
W. Todd Wipke
Gary Paul Zeller

NEBRASKA ALPHA, University of Nebraska, (December 14, 1960)

James Arnold Anderson
John Allen Anderson
Roger D. Bengston
David Harold Bliss
Fred William Forss
Gary Gene Gilbert
Francis Marvin Green
Louis Earlyon Lamberty
David Lee Sorensen
Charles Thomas Spooner
Sidney Stastny
Leon Eugene Wallwey

NEW JERSEY ALPHA, Rutgers University (April 4, 1961)

John Joseph Aklonis
Joseph F. Baumgarden
Robert Lorenz Boysen
Anthony Cerami
Norman Farber
George William Feusel
Jay Shelden Fein
Arthur T. Galya
Michael H. Hagler
Daumants Hazners
Dennis Paul Herzo
John Richard Horvath
Allan Jenks
Kenneth R. Jungblut
Nicholas Joseph King
John Kjellgren
Arturs Krumins
George T. Kwiatkowski
Walter S. Koroljow
Donald R. Lehman
John A. MacDonald
Nickolas M. Miskovsky
Edward A. Page
Allan J. Stevens
Robert J. Tucher
Peter J. Unterweger
David F. VanDerveer
Michael J. Vasile
Dennis J. Viechnicki
Richard J. Vnenchak
Robert J. Vojack
Carl A. Zanoni
George M Zezinski
Anatol Zinchenko

NEW JERSEY BETA, Douglass College, Rutgers University, (October 17, 1960)

Jean Shropshire Harris

NEW MEXICO ALPHA, New Mexico State University, (December 15, 1960)

Elias M Armijo
Eileen M. Arnold
Benjamin A. Benn
Dennie R. Cartlidge
Alfred C. Carver
William L. Caudle
Walter B. Miller
Edmund J. Peake, Jr.
Carol L. Peercy
Anthony J. Perrotto
Paul L. Sperry
Roy O. Spruiell
Kenneth J. Thomas
Everett L. Walter
Frand J. Williams

NEW MEXICO ALPHA, New Mexico State University (February 10, 1961)

J. Mack Adams
John E. Crane
Richard H. Dale
Harry S. Davis
Allan B. Gray, Jr.
Kenneth E. Guthrie
Darel W Hardy
Ernest G. Holeman
Jimmie L. Johnson
Richard E. Johnson
James W. Moore
Lloyd G. Palmer
Nancy A. Parsons
Harry T. Prescott
D. Michael Tucker

NEW YORK BETA, Hunter College, (November 2, 1960)

Karin Alff
Elizabeth Bradbury
Samuel Chapman
Judi Cohl
Patricia Joyco
Charles Masielle
Ann Nelson
Pearl Sokal
Illene Sprung
Andrew Tescher

NEW YORK GAMMA, Brooklyn College, (November 28, 1960)

Lawrence Chimerine
Robert Cohan
Sandra Klein
Robert S. Mankin
Elaine R. Morritt
Thomas A. Mormino
Eric H. Ostrov
Isaac R. Pfeiffer
Nechemiah Reiss
Nathaniel Riesenberg
Dianna Roth
John K. Young

NEW YORK EPSILON, St. Lawrence University, (October 15, 1959)

Lila J. Brush
Herman Hagedom
Fred Hecklinger
Anita Hills
Antonia Hoffmann
Carol Lachenbach
Lloyd Landau
Patricia Linderoth
Richard Palmer
Margaret Potter
Bruce Roberts
Robert J. Robinson
Richard C. Smith
Robert Waterman
Margaret Young
Barbara Zeidler

(October 18, 1960)

Bruce D. Boss
Joan A. Herbert
Jacquelyn Kaufman
Arthur B. Kessner
Julia E. Neuse
Andrew Steinmetz

NEW YORK ETA, University of Buffalo, (Fall, 1960)

Jerry Ehman
Robert Kinzly
Jerold McClure
Robert Pompi
Mrs. Cynthia Ritvo
Robert Smith
Robert Stalder
Donald Trasher
Richard Uschold

NEW YORK IOTA, Polytechnic Institute of Brooklyn, (June 15, 1960)

Jerome Dancis
Marc Plotkin
Errol Pomerance
Janett Rosenberg
Lester A. Rubenfeld
Sheldon Trubatch

NEW YORK KAPPA, Rensselaer Polytechnic Institute, (June 6, 1960)

Dr. Edwin Brown Allen
Stuart Antman
Jerry Bank
Spero Criezis
Jack Friedman
Thomas Giammo
Robert Griswold
Jack Hoffman
Eric Jaede
Martha Jochnowitz
Theodore Jungreis
Harold Langworthy
Stephen Meskin
Richard Roth
Dr. Robert Talham

NORTH CAROLINA ALPHA, Duke University, (December 15, 1960)

Perry Rutledge Grace
Gary W. Husa
Margery Ann Katz
Janice Edna Murphy
John Bradbury Reed
John S. Thaeler
John Randolph Tinnell

NORTH CAROLINA BETA, University of North Carolina, (November 4, 1960)

Edwin John Blythe, Jr.
Warren J. Boe
Wilber Ray Boykin
Larry Wesley Brown
John R. Dowdle
Julia Dunning
Ian Morgan Happer
Shirley Ann Harris
John Charles Hellard
Hughes Bayne Hoyle, III
James Lee Kerney
George G. Killough
Sigrid B. Lund
Kay Nichols Lynn
David Franklin McAllister
David Eugene Price
Ralph Connor Reid, Jr.
Clyde Gordon Roberts
Robert Charles Rohlfs
Lewis Odie Rush, Jr.
Richard Lenoir Sanders
Francis Gerald Smith
Kosmo Davis Tatalias
Wilfred Turner
Emanuel Vegh
John Bason Wagoner
Grayson Howard Walker

NORTH CAROLINA BETA (cont'd.) (December 16, 1960)

Anil Kumar Bose
Joan Brooks
Henry Lee Butler
James L. Comer
John David Harrell, Jr.
Camilla Joseph
George J. Michaelides
Sandra G. Ness
Stewart B. Priddy
James A. Reneke
Miriam G. Shoffner
Nathan Simms, Jr.
William H. Turner
Albert R. VanCleave
Richard P. Yantis

NORTH CAROLINA GAMMA, North Carolina State College, (October 20, 1960)

Joel Vincent Brawley, Jr.
Walter Bradley Commings
Robert Dabney Davis
Peter Murray Gibson
Reid Kent Gryder
Betty Gall Harris
Arthur Bruce Hoadley
Jon Russell Hosell
William Patrick Kolodny
Douglas Eugene Lingle
Marvin Samuel Margolis
Lawrence Carlton Moore, Jr.
John Tunstall Welch
James Blake Wilson

(January 13, 1961)

Henry Foust Atwater
Mitchell Detroy Brackett
James Philo Caldwell, Jr.
Charlie Harrison Cooke
Gus Perry Couchell
Robert Edward Dalton
James Frederick Daughtry
Charles Alfred Davis
David Richard Decker
John Walter Dulin
Roscoe Edison Elkins
David Lloyd Flanagan
John Edward Fletcher
Henry Horace Gatewood
John Henry Heinbockel
Jafar Hoomani
Clinton Jefferies, Jr.
George Edward Levings
Paul Froneberger Long
Linda Catherine Maddry
Gary Alan Massel
Samuel Calvin Matthews
William Baxter Michael
Robert A. Morrow, Jr.
Philip Newcomb Nanzetta
Edward Samuel Oberhofer
Ronald Lee Olive
Richard Wayne Philbeck
Thomas Gilmer Proctor, HI
Jerry Allen Roberta
Robert Jones Smith
James Thornwell Spence
Timothy Nugent Taylor
David Boyce Teague
Fred Toney, Jr.
Keppel Duane Wait
Robert M Woodside
Grover Karl Warmbrod
Stevie Mike Yionoulis

OHIO BETA, Ohio Wesleyan University, (April 13, 1961)

John Findley Berglund
Suellen Bowden
Ernest I. Glickman
Paul C. Hart
Lewis H. Jones IV
Constandy Khalil Koury
Leslie H. Leighner
Patricia S. Martin
Ronald Moulder
Carol Lynette Robinson
Leslie Shelia Smith
Richard Chase Windecker

OKLAHOMA ALPHA, University of Oklahoma, (December 8, 1960)

Paul M Berry
Holland Ford
Robert L. Kinzer
Thomas A. Lewis
Katie Richards
Thad B. Welch, Jr.

PENNSYLVANIA ALPHA, University of Pennsylvania, (January 10, 1961)

Leon H. Assadourian
Stephen M Belikoff
Kenneth Brait
John E. Burroughs
Frank C. Caizzi
Raymond W. Carlson
Deborah F. Chemock
Peter J. Clelland
Peter M Corritorri
John W. Docktor
Lanny Edelsohn
Larry J. Godlstein
Janet Hesse
Barbara E. Hoffman
Richard B. Hyman
Harvey B. Keynes
Dietrich Kindl
Jennie Lavin
Gretta Leopold
David Liberman
Theodore Live
Judith A. Nelson
Kenneth R. Paton
Peter L. Podol
Michael L. Rosenzweig
Andrejs Rube
Daniel A. Sankowsky
Howard Silberman
Myrna Skobel
Barry M Trost
Richard P. Whittaker

PENNSYLVANIA BETA, Bucknell University, (November 18, 1960)

Rosemary J. Berhslter
William M Brelsford
Alice Diane **Budde**
Meredith Ann Conger
Diana **Deichmann**
Charlotte C. **deShields**
Kenneth Roy **Erickson**
Judith **Ayres Fisk**
Judith Claire Halter
Carolyn Fay **Hocker**
Elizabeth **O. McLeister**
Ray Lyman Ott
Harry Elwood Ritter
Hilda Ann **Schwartz**
Alan Glen Stromberg
Louise Sara **Switkes**
Janet Mae **Symons**
Edmund James Vallecorse
Katherine G. Vanderbeck
George Walter Van Dine
Stanley George Wheeler
Carolyn Grace **Wilcox**

PENNSYLVANIA ZETA, Temple University (January 6, 1961)

Mary R. Adsms
Eileen Applebsum
Carl Benner
Martin Cohen
Ruth Cohen
Robert E. Craig
Ellis A. Monash
Betty Rabinowitch

VIRGINIA ALPHA, University of Richmond, (October 13, 1960)

Charlotte Desper Adams
Sharon Margaret **Alderson**
Barbara Dale Boatwright
Charles Ellery Clough
Paul Edward Cohen
Robin Lewis **Cramme**
Preston Williams Forbes
Jack Wilson Fretwell, Jr.
Robert Thomas **Grissom**
Alice Verlander Hall
Elizabeth **Beaman** Hesch
Dilek Asiye Kahyaoglu
Gall Marie **Matthews**
Clarence Monk
Martha **Renick O'Kennon**
James Hall Revere
Frederick Richard **Shull**
Harold Lee Smith
Margaret Lee Strawhand
Kenichi Sugahara
Jane Bryant Thompson

WASHINGTON BETA, University of Washington, (November 15, 1960)

Iwao Abe
Norris Dean Adams
Willie C. **Aikens**
Mary **Atkinson**
John E. Brown
George **S.** Campbell
Chih-hsing **Chien**
Stella **Chang Chien**
John **S.** Y. Chiu
Donald Con-igan
Walter A. **Danley**
Daniele Durand
Robert C. Echerlin
J. W. Gelzer
George **S. Gunderson**
Ernest Y. Hu
Carl Harvey **Jackins**
Kazuo Kanda
Allen **S.** Kennedy
Clarice **MacDonald**
Delbert E. **McClure**
Charles P. Moshier
John M Musser, Jr.
Lloyd A. Osbom
N. Tenney Peck
G. James Ronsh
Carter V. Smith
Eileen Ting
Ross Vicksell
Wayne N. Walker
Theodore W. **Wilcox**
Claude Prior **Wrathall**

WASHINGTON GAMMA, Seattle University, (December 16, 1960)

Lewis **S.** Coleman
Alvin A. Cook, Jr.
Frank R. De Meyer
James D. Estes
Gary **M.** Haggard
John E. **Hopcroft**
Bonnie Ann Lawrence
Lawrence J. **McHugh**
Michael R. Ogilvie
Harold **J.** Shakerley
Elmar Zemgalis

WISCONSIN ALPHA, Marquetle University, (July 25, 1960)

Lynn Louise Asp
Phillip J. Blank
Lillian Catherine Bozak
Patricia A. **Brennan**
Mary Lynne Coon
Marlene E. **Derr**
James Stuart Fitch
James R. Gebert
Nancy Rose Harter
A. Kathleen **Hartl**
Victor E. **Henningsen**
Gerald Joseph Janusz
Charles John **Kircher**
Ronald Anthony **Kojis**
Nicholas J. **Kormanik**
John H. Linehan
Thomas Anthony Lipo
Judith C. **Matteson**
Brenda M **McCarthy**
Darrell R. Meulemans
John **Coll** Morhfeld
Elizabeth M Neumann
Richard John **O'Farrell**
Theodore Frank Raske
James **Allan** Robinson
Robert James Robinson
Larry T. Roth
Doris J. **Sikorski**
Joseph Leo Skibba
Gunars Charles **Svikis**
Emery **J. Szmrecsanyi**
Agnes Mary Talacko
Alan James Toepfer
Chester E. **Tsai**
William R. Vitacco
Gerald W. **Wadina**
Thomas P. Ward
Friedrich Weber
Henry J. Wellenzohn
Robert J. Wenzel
Kenneth P. Woelfel
Edward A. **Zanoni**
Kathleen A. **Zierden**

WISCONSIN APLHA, Marquette University (November 20, 1960)

Roseann **Carroll**
George Anthony Dempsey, Jr.
Edward **Allan Gillis**
Rev. Francis X. Hudson
Mary Margaret **Lickly**
Phillip G. **Maresca**
Winston **Ashton** Richards
David Walter **Zdan**

WISCONSIN BETA, University of Wisconsin, (September 28, 1960)

Alex Bacopoules
Laurie Barrett
Lois Jean Fiedler
Sister Mary **McKillop**
Suzanne **C.** Newton
William E. Schilling
Stanley **S.** Schmidt
Indranand Sinha
Deetje **J. Wildes**

(November 15, 1960)

Phyllis August
Garret **J.** Etgen
Marvin C. Gaer
David B. Halmatad
Dorothy R. **Hatton**
Ronald J. **Loring**
Gerald W. Marlette
Louis D. **Melnick**
Frank J. Mestecky, Jr.
David F. Monk
John N. Moschovakis
James **A. Murtha**
Dolores B. Peck
Margery L. Rinehart
Stephen P. Slack
Joseph M Weinstein
Theodore **O. Wiese**

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