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CYRUS COLTON MacDUFFEE

1895–1961

Professor MacDuffee (known to his friends and colleagues as "Colton" or "Mac") died on August 21, 1961 at the age of 66. Professor MacDuffee had been a strong supporter of Pi Mu Epsilon both on the local and at the national level. He served as Director General of Pi Mu Epsilon between 1948 and 1954 and during this period the number of chapters increased from 44 to 55.

MacDuffee did his undergraduate work at Colgate, receiving a B.S. degree in 1917. He then taught at Colgate for two years before starting graduate work at Chicago. (After MacDuffee had become famous as a mathematician, Colgate honored one of its favorite sons by granting him an honorary Sc.D. degree in 1947.)

MacDuffee received his Ph.D. from Chicago in 1921. He then taught three years at Princeton, 10 years at Ohio State, and came to the University of Wisconsin as a Professor in 1935. He left Wisconsin for a Professorship at Hunter College in 1940 but returned to Wisconsin in 1943. He was elected and reelected Chairman of the Department of Mathematics at Wisconsin (1951-57). At the time of his death he was still at Wisconsin and had just completed a very successful eight weeks of Summer School Teaching. In addition to these major positions, he had been a Visiting Assistant Professor at the University of Chicago (summer, 1928), a Fellow of the Institute for Advanced Study (1937-38), and a Visiting Professor at the University of Puerto Rico on two occasions (1947 and 1960-61).

MacDuffee held the following offices with scientific organizations: Pi Mu Epsilon, Director General (1948-54); Mathematical Association of America, President (1945); American Mathematical Society, Vice President (1942) and Editor of Transactions (1937-42); American Association for the Advancement of Science, Secretary of Section A (1957-61).

MacDuffee's mathematical research was in Algebra and centered around the Theory of Matrices. He is well known for his texts and research papers in this area. In addition, he trained many Ph.D. students and was a very successful teacher of undergraduates. His warm approach and broad interests caused students from other fields of learning to sign up for his classes in large numbers. As a soft spoken teacher he communicated an enthusiasm for mathematics and scholarships. He says in his article "An Objective in Education" published in the American Mathematical Monthly in 1945 that: "The pure sciences, taught with the objective of explaining basic principles, have as much aesthetic value as the humanities, and they have a grandeur all their own."

Numerical Studies on Waring's Problem, Modulo p[1]

Monica Gallager, S. J. Lomonaco, Jr. and T. R. Volkman[2]

Let p be a fixed odd prime, n an integer ≥ 2. Let $T(p,n)$ be the least integer such that $\sum_{j=1}^{T(p,n)} x_j^n \equiv c$ modulo p, is solvable in integers x_j, for every c. $T(p,n)$ has been studied in several St. Louis University theses. The theory used in this article is taken from **Torline's** doctoral thesis [1], suitably modified for present purposes. Using this, the authors have for certain **n's**, and for each **n**, many values of **p**, determined $T(p,n)$.

Theory. Unless stated otherwise, all variables are restricted to integral values. It may be assumed $0 < c \leq p-1$. Let g be a fixed primitive root modulo p. Let c have index i, $i = ng + t$, $0 \leq t < n$. Define c to be of n - ic type t. **Torline** has shown that for n an odd prime, $p > n$, $p \neq 2nk + 1$, $T(p,n) = 1$; and if $p = 2n + 1$, $T(p,n) = n$. The primes treated in this article are consequently assumed of the form $2nk + 1$, $k > 1$. n is an odd prime unless the contrary is stated.

The following definitions are introduced. $S = \{1, 2, \ldots, p-1\}$. c is an n-ic residue (or non residue) if it is of n-ic type zero (or non zero). Two elements, m and ℓ, of S are adjacent if and only if $m - \ell = \pm 1$; are doubly adjacent if and only if $m - \ell = \pm 2$, and so on. c is $R(n,p)$ representable if and only if c is congruent, modulo **p**, to the sum of R n-ic residues. Γa is the set of all elements of S which are of n-ic type a. Γa is called $R(n,p)$ representable if and only if an element of Γa is $R(n,p)$ representable; similarly for adjacency and double adjacency. **Torline** [1] has shown that if Γa is $R(n,p)$ representable, then every element of Γa is $R(n,p)$ representable.

Th. 1 For $R \geq 2$, Γa is $R(n,p)$ representable if and only if Γa is is adjacent to Γb, with Γb $(R-1)$ representable.

[1] Received by Editors Jan. 15, 1961. Presented at the National Meeting of Pi Mu Epsilon, East Lansing, Michigan, Aug. 30, 1960.

[2] Work on this paper was done while the authors were participants in a National Science Foundation Undergraduate Research Training Program (NSF - G 12135). All three are members of Missouri Gamma. Miss Gallager and Mr. Volkmann class initiated in 1958, while Mr. Lomonaco's class was 1959.

Proof. Let Γa be $R(n,p)$ representable, c an element of Γa. Hence

$$c \equiv g^{n\ell + a} \equiv \sum_{j=1}^{R(n,p)} g^{n\ell_j} \text{ modulo } p,\ \ell_j \geq 0,\ n\ell_j \leq p-1.$$

Let f be defined by $(\ell_R + f)n = p - 1$. Hence f is integral, ≥ 0, and it follows that $g^{n(\ell + f) + a} \equiv 1 + \sum_{j=1}^{R(n,p)-1} g^{n(\ell_j + f)}$ modulo p.

Now the left hand member is congruent to A, an element of Γa and the sum on the right is congruent to B, an element of $S \cup \{0\}$. Hence $A = 1 + B + \ell p$. But $0 < A < p$, which implies $\ell = 0$ and $A = B + 1$, or that Γa and Γb are adjacent. The remainder of the proof is almost immediate, and no details are given.

The following corollary is immediate.

Corollary 1. If for some a, $0 < a < n$, there exists no R, $0 < R \leq N$ such that Γa is R representable, and if for every a, $0 < a < n$, there exists an R, $0 < R \leq N + 1$, such that Γa is R representable, then $T(p,n) = N + 1$.

Corollary 2. If for every a, $0 < a < n$, Γa is $1(n,p)$ adjacent, then $T(p,n) = 2$.

Proof. Evidently $T(p,n) \leq 2$. But **Torline** has shown that for $p = 2nk + 1$, $k \geq 2$, $T(p,n) \geq 2$, and hence $T(p,n) = 2$.

By a similar argument we get

Corollary 3. If for every a save one, $0 < a < n$, Γa is $1(n,p)$ adjacent, then $T(p,n) = 3$.

Corollary 4. If not all Γa are $1(n,p)$ adjacent, but every Γa is $1(n,p)$ or $2(n,p)$ adjacent, then $T(p,n) = 3$.

It is readily shown that if n is composite, the above theorem and corollaries hold for n odd. For n composite and even, they are valid if the word **"adjacent"** is replaced by the phrase 'adjacent after*.

Procedure:

T (p,n) for certain **n's** and **p's** was evaluated by applying the theory given above. For this evaluation two methods were used, according as $p < 1000$ or not. For $p < 1000$ the table of indices of Jacobi [2] were used, the analysis being made directly from the tables, with a very small amount of hand computation. For odd **n**, the method was as follows:

1) A list was made from Table I, of all numbers of n - ic type O, arranged in order of indices.

2) From Jacobi, Table I, the n - ic type of integers adjacent to the above numbers were observed and listed.

3) If not all n - ic types were observed in the last step, then the n - ic types of numbers adjacent to the n - ic type? just observed were again listed. If all n - ic types appeared, $T(p,n) = 3$. If not, the last step is repeated as many times as needed to have all n - ic types appear, and the number of repetitions obviously determines $T(p,n)$. This method is also applicable to even values of n if 'adjacencies after' are observed.

The method used for $p > 1000$ followed a slightly different routine (although the line of attack was essentially the same). The first step was to find a primitive root of the prime in question. A table of primitive roots of primes less than 5000 was available; primitive roots of larger primes were found by well known devices. Given a primitive root, g, of $p = 2kn + 1$, the process used was as follows:

1) The computer was programmed to compute g^i, reduce this modulo p so as to lie in the range from $-\frac{p-1}{2}$ to $\frac{p-1}{2}$, and print this residue for $i = 1,2,\ldots n$. We term such numbers p - reduced.

2) The computer was programmed to calculate, p-reduced, and g^{nj}, $j = 1,2,\ldots k$.

3) The p-reduced residues of 2) were punched onto I.B.M. cards and sorted in order of magnitude.

4) The computer was programmed to calculate, p-reduce and print g^{nj+a}, $j = 0,1,\ldots k$, for each a, $0 < a < n$. As, for a given a, j ran through its values, the numbers printed were compared with the numbers from 3), until an adjacency was observed, or until $j = k$. If an adjacency was observed, the machine was stopped, a replaced by a + 1, and so on until all values of a had been treated.

If, for every a, an adjacency occured, $T(p,n)$ is 2. If no adjacencies were found for certain values of a, the corresponding integers of 4), for these values of a, were examined for double adjacencies. If these occured for each a in question, $T(p,n)$ is 3. If not, the 'residual' values of a were similarly treated for adjacencies of higher order, until for every a, the minimal order of adjacency was known and hence $T(p,n)$ determined.

The advantage of treating each non zero n - ic type at a time, rather than all such types at once is clear, since for many values of a, an adjacency was observed for very small values of j. Since the time required to calculate, reduce and print each number in 4) was approximately eight seconds, the time saved was substantial.

Results. The results of the numerical investigation for odd prime values of n showed a pattern in every case in which $T(p,n)$ was relatively large for the first few primes (those least in magnitude), p, investigated, and descended rapidly, with increasing p, to a series of 2's and 3's, in varying order, the 3's becoming increasingly rare. For $n = 5,7,11,13,17$ and 19, a series of fifteen or more consecutive primes of the form $p = 2nk + 1$ were observed for which $T(p,n) = 2$, these primes all larger than the largest prime, p, investigated of the same form for which $T(p,n) = 3$. For $n = 23,29,31$, the investigations were not so extensive because of time limitations, and no such uninterrupted series of values of p for which $T(p,n) = 2$ were found. But no case among the primes investigated was found in which $T(p,n) \geq 3$ with $p > 8n^2$.

For the composite values of n considered, $n = 4,6,8,9,10$ and 12, similar sequences of p's for which $T(p,n) = 2$ were found, but in several instances, long series of values of p for which $T(p,n) = 2$ were terminated by the next larger value of p giving $T(p,n) = 3$. For example, with $n = 8$, 14 consecutive primes of the form kn + 1 were formed with $T(p,n) = 2$, but the next prime of this type gave $T(p,n) = 3$. In no case in which $p = kn + 1$, $k > n$, was an instance encountered in which $T(p,n) > 2$.

From the results stated above, it seems reasonable to conjecture that for every n, and for each n, all primes, p, sufficiently large, $T(p,n) \leq 3$. It seems at least possible that 3 can be replaced by 2.

References

[1] Sr. M. F. Torline, C.S.J., Waring's Problem, modulo p Thesis (Ph.D.) St. Louis University, 1955.

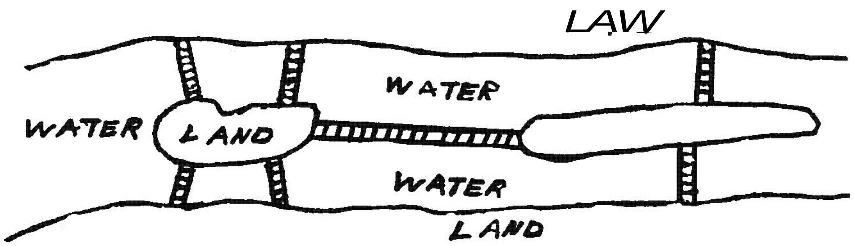
[2] C.G.G. Jacobi, Canon Arithmeticus. Akademic-Verlag Berlin, 1956.

St. Louis University

Konigsberg Bridge Problem[1]

Fred J. Howlett
Nebraska Alpha

Over the Pregel River in Konigsberg, Germany in the 18th century there existed seven bridges situated in the following manner:

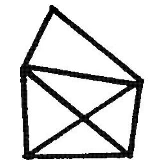


Some people of the time wondered if there was a way that one could walk over all seven bridges, crossing each bridge just once. The famous mathematician, Euler (1707-1783) heard of this problem and solved it. Topology was founded when he presented his solution to the Russian Academy at St. Petersburg in 1735.

The word topology is derived from a Greek word meaning "a place". In mathematics topology was formerly defined to be the study of a situation. An alternative name was "analysis **situs"**. Topology is concerned with intrinsic properties of figures themselves, and not properties concerning their relationship to any surrounding space in which they may be imbedded. Topology is that branch of geometry which deals with properties of figures which are not changed by stretching, shrinking, or blowing up of the surfaces on which they are drawn. For this reason it is sometimes called "Rubber Sheet Geometry". For example, if we take a balloon and draw a closed curve on it, the surface of the balloon will be divided into two parts. If we blow the balloon up or deflate it. we will still have a curve on the surface which divides the surface into two parts. This property has remained unchanged by our distortion or stretching. Although the subject arose as a branch of geometry, in recent years it has been generalized to include manv other branches of mathematics. Nowadavs mathematicians are in fairly general agreement that topology is a study of continuity. Since continuity plays an important part is mathematical analysis, it is not surprising that topology has come under the influence of the **rigorous discipline** of that subiect. Some of the areas in which topology plays a part are "topological equivalence" - congruence of triangles and other figures; "surfaces" such as the Mobius band being topologically different than a cylinder; **"two-sidedness** and **orientability"** - a cylinder with two sides and the mobius band with one side; "classification problems"; **"Euler's** theorem on polyhedra" and the "coloring of maps".

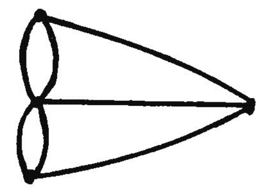
[1]**Received by Editors August 17, 1960. Presented at the National Meeting of Pi Mu Epsilon, East Lansing, Michigan, August 30, 1960.**

Now with a little background history let us get back to the **Konigsberg** Bridge problem. Consider the following figure:



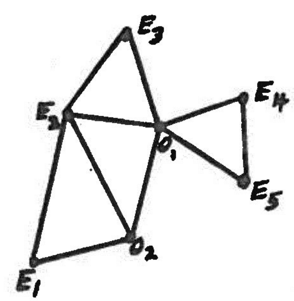
An arrangement of vertices and lines such as this one is known as a ***network.*** If one can draw this network by starting at some point and complete the drawing without lifting the pencil and without using the same line more than once, then the network is said to be traceable. Traceability of a network is another typological property since a network which is traceable will remain traceable even after the surface upon which it is drawn is blown up, stretched, or shrunk.

Now the Konigsberg bridge problem can be reduced to a problem of traceability of networks, if we represent the land by vertices and the bridges by lines, as follows:



Is this network traceable?

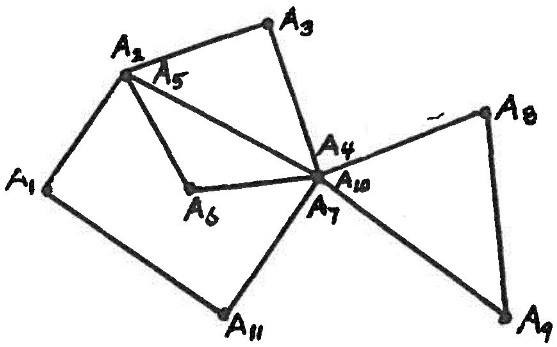
To answer this question and the question concerning the traceability of any network we shall prove a theorem and it's converse. But first the following definition is in order: A vertex is said to be ***even*** if an even number of lines terminate there. **A** vertex is said to be odd if an odd number of lines meet at that point. For example the following figure has five even and two odd vertices.



Theorem: **If** a network is **traceable,** then it must have either zero or two odd vertices.

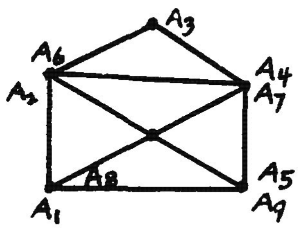
Proof:

Case 1 The initial and terminal points **of** the trace are the same.



We start at some A, and go to A,. If this doesn't complete the trace, we depart from **A,,** leaving A even; we continue to A,. If this doesn't complete the trace we depart from A,, leaving $\mathbf{A_3}$ even. Thus we continue, making every vertex **after** A, even until we return to A,. Having left $\mathbf{A_1}$**.originally,** upon returning to it we make A, even also, hence all vertices are even. q. e. d.
(If a vertex is reached several times, this will not affect the argument since we must leave it as often as we enter it.)

Case II The initial and terminal points are not the same.

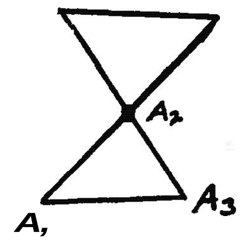


Let the initial point be A,, the terminal point $\mathbf{A_n}$ **(**$\mathbf{A_9}$ in the present case.) As before, every vertex after $\mathbf{A_1}$ and before $\mathbf{A_n}$ must be even. However A, and $\mathbf{A_n}$ must be odd, since we leave A, once more than we enter, and we enter A_n once more than we leave it. q. e. d.
Theorem: (converse). **If** a network has zero or two odd vertices, then it is traceable, in the first case starting anywhere and returning to the same vertex, in the second starting at either of the odd vertices and terminating at the other.
Proof:

Case I. (all vertices even)

Start at A, and go to A,. Since A, is even, we may depart from $\mathbf{A_2}$ and go to A,, eventually returning to A,. This, however, may not trace the whole figure.

Example:

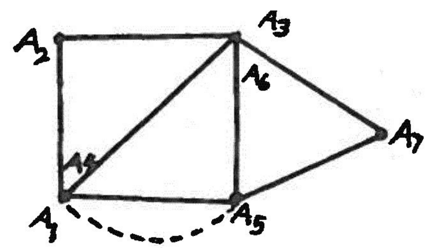


We could go from A, to A and finally to $\mathbf{A_3}$ and the whole figure is not traced. To account for this case we note that there must be more lines emanating from some $\mathbf{A_k}$ **(**$\mathbf{A_2}$ .in our case) which we have used. If the lines we have used are deleted, all vertices will still be even. We can then trace a figure beginning and ending at A but different from the first.

Now we start all over again. From A, we trace to $\mathbf{A_k}$**.** Then we trace all of the second figure, returning to $\mathbf{A_k}$**.** Then we trace from $\mathbf{A_k}$ back to $\mathbf{A_1}$**.**

This may not complete the figure. However, now a repetition of the process will finally complete the figure.

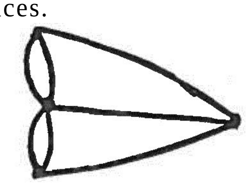
Case II (Two vertices odd.) Set $\mathbf{A_1}$ and $\mathbf{A_n}$ to be the odd vertices. Draw an extra line between A, and A_n. Now all vertices are even.



Hence, **if** we **start at** $\mathbf{A_n}$ and go to $\mathbf{A_1}$**,** we will be able to trace **to** $\mathbf{A_n}$ again as in Case I. Now we leave out the first step and will be able to trace from A, to A . q. e. d.

From the above theorems we are able to conclude that to determine whether a network is traceable or not it is only necessary to determine whether we have zero or two odd vertices or not.

Returning to the problem of the Konigsberg bridges, we note that there are four odd vertices.

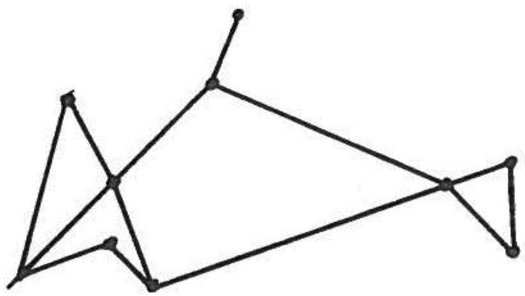


Therefore the network is not traceable and it would be impossible to start at one bridge and complete a journey by crossing every bridge once.

Another Topological property which will not change under distortion is:

In any connected network, vertices minus edges plus polygons is equal to one. i. e. **V-E+P=1**

For example:

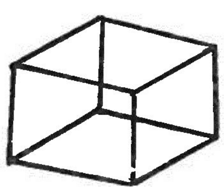


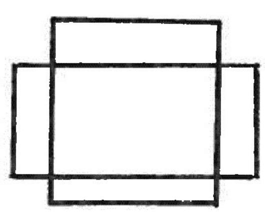
Proof: If any vertex is used just once, erase it and the single edge it lies on. This does not alter the number, V-EtP=1, and leaves the network still connected. Similarly, all such vertices and edges may be removed. Now remove any edges which lie on the boundary of the network. This does not alter **V-E+P** either, because when a boundary edge is removed so is a polygon. This may again leave vertices that are used only once. Repeat the first step and then the second until finally we get **V=1, E=O, P=O;** hence **1-O+O=1.** Therefore since we haven't changed the value of **V-E+P=1** all the way through, the original value must be the same as the end value or as originally **V-E+P=1.** q. e. d.

With this theorem we can easily prove the following Topological **property.**

Theorem: In any simple polyhedron (with no holes) Vertices minus Edges plus Faces is equal to two. i. e. **V-E+F=2.**

Proof: Consider the cube:





Remove one face; then the figure may be cut along the edges and flattened out into a plane to form a connected network as can be seen above. In this figure, or network, by the above theorem, **V-E+P=1,** but the number of polygons is equal to one less than the number of faces on the original cube, or **P=F-1,** by substituting we have:

V-E+**P=1** V-E+**F-1=1** or
V-E+**F=2** q. e. d.

Acknowledgement

I would like to sincerely thank Dr. Walter E. Mientka, Assistant Professor of Mathematics, University of Nebraska, for the use of the material for this presentation. His deep interest in mathematics and teaching has helped immeasurably to develop my own interest in this area.

University of Nebraska

On The Construction of Sequences[1]

by James Collier

1. Introduction. When dealing with bounded sequences one is frequently interested in their limits and (or) limit-points (cluster points). But seldom is any thought given to constructing sequences with specified limits or limit-points. (Recall that a number is a limit-point of a sequence if some sub-sequence has that number as its limit). The problem of finding a sequence whose limit is A is trivial, for surely such a sequence is $\{A + 1/n\}$, $n = 1,2,3,\ldots\ldots\ldots$ If A_1 and A_2 are to be limit-points of a sequence, it is easily verified that $\{\frac{(-1)^n +}{2} A_1 - A_2) + A_2 + 1/n\}$ fulfills those requirements and is, in fact, a closed expression for the nth term of the sequence. It is the purpose of this note to show how to form a closed expression for the general term of a sequence which is to have as its limit-points the finite set of real numbers $\{A_2, A_2, \ldots\ldots, A_k\}$.

2. First, we define a double sequence $\{f_k(n,i)\}$, $(i = 1,2,\ldots\ldots, k-1,k)$, $(n = 1,2,3,\ldots\ldots\ldots)$ for each positive integer k. This sequence is exhibited in the following array.

$$
\begin{array}{llll}
f_k(1,1) = 0 & f_k(1,2) = 0 & \ldots\ldots\ldots\; f_k(1,k-1) = 0 & f_k(1,k) = 1 \\
f_k(2,1) = 0 & f_k(2,2) = 0 & \ldots\ldots\ldots\; f_k(2,k-1) = 1 & f_k(2,k) = 0 \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
f_k(k,1) = 1 & f_k(k,2) = 0 & \ldots\ldots\ldots\; f_k(k,k-1) = 0 & f_k(k,k) = 0 \\
f_k(k+1,1) = 0 & f_k(k+1,2) = 0 & \ldots\ldots\ldots\; f_k(k+1,k-1) = 0 & f_k(k+1,k) = 1 \\
\ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\; f_k(k+2,k-1) = 1 & f_k(k+2,k) = 0 \\
\cdot & & \cdot & \cdot \\
\cdot & & \cdot & \cdot \\
\cdot & & \cdot & \cdot
\end{array}
$$

[1]This work was done by the author during the Summer Science Training Program for Secondary School Students held at St. Louis University, St. Louis, Mo. June 26 to July 21, 1961 supported by the National Science Foundation under Grant NSF G 16612. This program was not one of research participation, but rather a program enriching the students' mathematical background.

Each row has k f's, only one of which is equal to 1, the other k-1 f's in the row being equal to 0. Regarding the first k rows as a k X k square array, the values 1 appear on the diagonal. The same is true for rows k +1 to 2k, rows 2k +1 to 3k, etc. . . In general, $f_k(k-i+1, i) = 1$, $f_k(2k-i+1, i) = 1$, $f_k(3k-i+1, i) = 1$, etc. . . We have then, for each fixed i, a discrete periodic function - that is, each of the k columns is a discrete periodic function in the variable n and with period k.

3. A function which satisfies $f_k(n,1)$ is $1 - \mathrm{sgn}(n/k - [n/k])$ where "sgn" denotes the signum function (which maps all positive reals onto +1, all negative reals onto -1, and 0 onto 0), while the brackets ($[\cdots]$) denote application of the "greatest integer function" (which maps any real number onto the greatest of all the integers which are less than or equal to the number). It is clearly seen that $f_k(n,i)$ is satisfied by $1 - \mathrm{sgn}\left(\frac{n+i-1}{k} - \left[\frac{n+i-1}{k}\right]\right)$, since $n + i = k + 1$ clearly give $f_k(n,i) = 1 - 0 = 1$, and $n + i \neq k + 1$ gives $f_k(n,i) = 1 - 1 = 0$. Consider the sequence $\{F_n\}$ where

$$F_n = A_1(1-\mathrm{sgn}(n/k - [n/k])) + A_2\left(1-\mathrm{sgn}\left(\frac{n+1}{k} - \left[\frac{n+1}{k}\right]\right)\right) + \ldots\ldots + A_k\left(1-\mathrm{sgn}\left(\frac{n+k-1}{k} - \left[\frac{n+k-1}{k}\right]\right)\right) + 1/n.$$

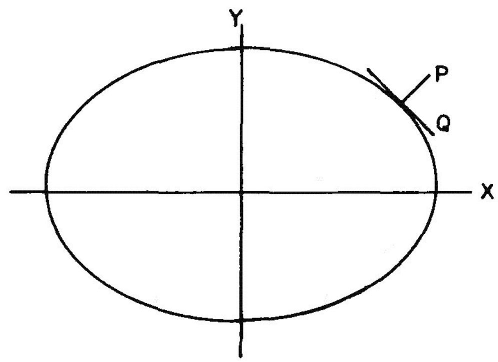
This is a closed expression for a sequence with the limit-points $A_1, A_2, \ldots, A_k$. $F_1 = A_k + 1$, $F_2 = A_{k-1} + 1/2$, $F_3 = A_{k-2} + 1/3, \ldots\ldots$, $F_k = A_1 + 1/k$, $F_{k+1} = A_k + 1/(k+1)$, etc.. .

Hence, it is seen that this sequence consists of k sub-sequences each converging to some A_i.

Webster Groves High School
Webster Groves, Missouri

An Iterative Scheme for Finding A Certain Point on An Ellipse[1]

By Joel Carroll
Land-Air, Inc.



Suppose there is an ellipse given by the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Moreover, let P = (c, d) be a given point in the first quadrant not included in the interior of the ellipse. There exists a point Q = (x, y) on the ellipse in the first quadrant such that the line from P to Q is perpendicular to the tangent line at Q. The problem is to find the point (x, y).

The slope of the line from P to Q is given by:

a) $\frac{y - d}{x - c}$.

The slope of the tangent to the ellipse at Q is given by:

b) $-\frac{b}{a} \frac{x}{\sqrt{a^2 - x^2}}$

[1]Received by the Editors February 16, 1961.

Therefore, expression a) equals the negative inverse of expression b) and

$$y = \frac{a(x-c)}{bx} \sqrt{a^2 - x^2} + d.$$

The x-coordinate of the point Q may be found by solving the following equation:

$$\frac{b}{a} \sqrt{a^2 - x^2} = \frac{a(x - c)}{bx} \sqrt{a^2 - x^2} + d \text{ or}$$

$$\left[\frac{b}{a} - \frac{a(x - c)}{bx} \right] \sqrt{a^2 - x^2} = d$$

The scheme now is to compute x by application of Newton's method. Therefore, let

$$f(x) = \left[\frac{b}{a} - \frac{a(x - c)}{bx} \right] \sqrt{a^2 - x^2} - d.$$

Then by making a particular choice of the initial value of x for the interation, successive applications of Newton's method are used to obtain x. Thus, the problem now becomes the selection of the initial x for the application of Newton's method.

As can be seen, there are four roots of f(x) and we are concerned with only one. It is obvious that two of these roots are real since the line intersects the ellipse in two points. We observe that the point x in which we are concerned is the largest in which the line and ellipse intersect.

Therefore if $0 < c < a$, then the initial x is chosen to be c. If however $c \geq a$, the initial x will be chosen to be $a - \epsilon$, $0 < \epsilon < a$, provided $d > 0$. The trivial solutions are obvious for $d = 0$ or $c = 0$. The choice of ϵ will be such that the point $(a - \epsilon, y)$, where y is evaluated by the question

$$y = \frac{a(x - c)}{bx} \sqrt{a^2 - x^2} + d$$

is not an interior point of the ellipse.

After making a choice of an initial value for x, we proceed with successive applications of Newton's method given by the formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{where } f'(xn) = \left.\frac{df(x)}{dx}\right|_{x_n},$$

observing that as

$$\frac{f(x_n)}{f'(x_n)} \longrightarrow 0, \qquad x_{n+1} \longrightarrow x.$$

The y-coordinate is then computed by substituting the computed value of x into one of the following equations:

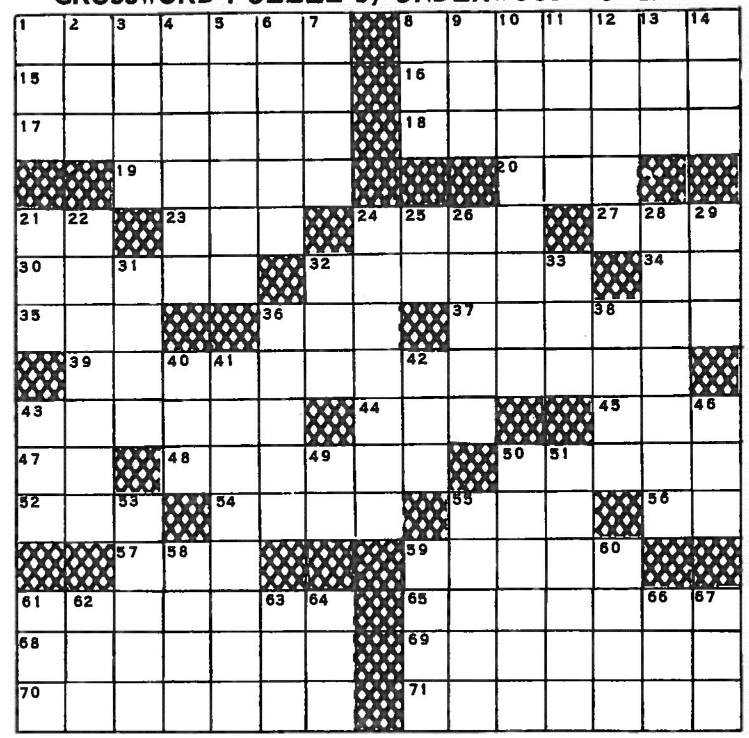
$$1) \quad y = \frac{b}{a}\sqrt{a^2 - x^2}$$

$$2) \quad y = \frac{a\,(x - c)}{bx}\sqrt{a^2 - x^2} + d.$$

By a similar scheme, a point Q may be found in either of the other three quadrants when a point P, not in the interior of the ellipse, is given in that quadrant.

Oxnard, Calif.

CROSSWORD PUZZLE by UNDERWOOD DUDLEY



ACROSS

1. Non-singular
8. $T^{-1} = T^{*\prime}$
15. $\bar{x}$
16. $\lim_{z \to a} (z-a)f(z)$
17. Non-negative. completely additive set function
18. Heads (colloq.)
19. Amusement park attractions
20. —— culpa
21. Cockney leap
23. Born (Fr.)
24. Horn and pitch, for example
27. —— Lupino
30. When $e^x = 0$
32. —— — middling
34. Negative prefix
35. 9(runs)/I.P.
36. Pitch
37. Camera and eye parts
39. $a > b$, $a < b$, or $a = b$
43. Series man
44. He beat 12 down
45. Comparitive ending
46. 6
48. Angular velocities
50. A coverine in the —— of Vitali
52. Collection of anecdotes
54. $x^2 - 2y^2 = 1$
55. Barley beerd
56. One— ——-one mapping
57. Emulsion
59. Encourages
61. Men in the annulus
65. $\sqrt{}$
68. $\iint x^2 dA$
69. Carry on like Picard
70. —— functions, solutions of $x^2y'' + xy' + (x^2 - n^2)y = 0$
71. Abandons

DOWN

1. Aries
2. First woman
3. Can be high or low
4. Bearlike
5. Comedian Sir Harry
6. Reach accord
7. Mathematess Mina
8. "Ode on a Grecian —— "
9. Prefix meaning new
10. T, if $(x, y) = (Tx, Ty)$
11. Spring and low, for example
12. Two-time loser
13. Kegret
14. Is $2^{2281} - 1$ prime?
21. $\int_0^{\pi/2} \cos x \, dx$
22. Concern
24. Type of derivative
25. 2
26. Irreducible
28. Out the neeative x-axis
29. Problem ending (ab.)
31. Have a non-zero derivative
32. Federal Agricultural Organization (ab.)
33. Lubricant
36. Example of 26 down
38. Pitcher Johnny
40. International Labor Organization (ab.)
41. H, if $\lim f_N \notin H$
42. Several 14 downs
43. Government dam-builders (ab.)
46. Former automobile
49. Gill (ab.)
50. Fredholm, Ahlfors, and others
51. No singularities, $|z| < \infty$
53. Chills
55. Let up
58. Says that $(uv)' = u'v'$, e.
59. Dry
60. Blemish
61. Where books are (ab.)
62. Chemical suffix
63. Kind of potent
64. Hobart's island (ab.)
66. Lawyer (ab.)
67, Guitarist —— Paul

SOLUTION ON PAGE 254

PROBLEM DEPARTMENT

Edited by
M. S. KLAMKIN,
AVCO Research and
Advanced Development Division

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication. Address all communications concerning problems to M S. Klamkin, Avco Research and Advanced Development Division, **T-430, Wilmington,** Massachusetts.

PROBLEMS FOR SOLUTION

132. Proposed by L. Carlitz, Duke University.
Let p be an odd prime. Find the number of solutions (x, y, z) of the congruence

$$xyz + a(x+y+z) \equiv 0 \pmod{p},$$

where

$$a \not\equiv 0 \pmod{p}.$$

133. Proposed by Robin Robinson, Dartmouth College.
Prove that for any convex plane area there exists a non-obtuse positive angle θ_1, such that for every angle 6 in the interval, $\theta_1 \leq \theta \leq \pi - \theta_1$, there is a pair of lines meeting at angle θ and dividing the area into four equal parts, while for every angle 6 outside this interval this is impossible. If θ is different from θ_1, $\pi - \theta_1$, or $\pi/2$, the pair of lines is not unique. Also, prove that there exists an area for which θ_1 is any desired non-obtuse positive angle.

134. Proposed by R. F. Wheeling and R. H. G. Mitchell, **Socony** Mobil Oil Company.
If $\phi_1(x)$ and $\phi_2(x)$ are one dimensional probability density functions (**i.e.,** $\int_{-\infty}^{\infty} \phi_r(x)\,dx = 1$) and if there exists a number Xo such that

$$\phi_2(x) \geq \phi_1(x) \quad \text{For } x > X_o,$$
$$\phi_2(x) \leq \phi_1(x) \quad \text{For } x < X_o,$$

then

$$\int_{-\infty}^{\infty} x\,\phi_2(x)\,dx \geq \int_{-\infty}^{\infty} x\,\phi_1(x)\,dx$$

provided both integrals exist.

135. Proposed by T. E. Hull, University of British Columbia.
Suppose that k points are placed uniformly around the circumference of a circle with unit radius. Show that the product of the distances from any one point to the others is equal to k , for any k > 1.

136. Proposed by Michael Goldberg, Washington, D. **C.**
What is the smallest convex area which can be rotated continuously within a regular pentagon while keeping contact with all the sides of the pentagon?

This problem is unsolved but has been solved for the square and equilateral triangle. For the square, it is the regular tri-arc made of circular arcs whose radii are equal to the side of the square. For the triangle, it is the two-arc made of equal **60°** arcs whose radii are equal to the altitude of the triangle.

137. Proposed by Leo Moser, University of Alberta.
Show that the squares of side **1/2, 1/3, ..., 1/n, . . .** can all be placed without overlap inside a unit square.

SOLUTIONS

106. Proposed by M. S. Klamkin, Avco RAD.
An equi-angular point of an oval is defined to be a point such that all chords through the point form equal angles with the oval at both points of intersection (on the same side of the chord). It is a known elementary theorem that if all the interior points of an oval are equi-angular then the oval is a circle.

1. Show that if one boundary point of an oval is equi-angular, the oval is a circle:

2. Determine a class of non-circular ovals containing at least one **equi-angular** point.

3. It is conjectured that a non-circular oval can have, at most, one **equi-angular** point.

Solution by the proposer.

1. Let the x-axis be chosen tangent to the oval at the boundary equi-angular point which is taken as the origin. It then follows that

$$r\frac{d\theta}{dr} = \tan 6.$$

Whence, $r = a \sin 6$ which is a circle tangent to the x-axis at the origin.

2. Let the origin be the equi-angular point. Then we have to find r such that

$$r \frac{d\theta}{dr}\Big]_{\theta} + r \frac{d\theta}{dr}\Big]_{\theta+\pi} = \pi$$

for all θ. One obvious solution is given by

$$\ln r = \int \frac{d\theta}{\pi/2 + F(\sin\theta)}$$

where F is an odd function.

3. The conjecture is still an open problem.

125. Proposed by Leo **Moser,** University of Alberta.
Find the largest number which can be obtained as the product of positive integers whose sum is S.

Solution by L. **Carlitz,** Duke University.

For a given sum S let $P = P(S)$ denote the maximum product; it is of course obvious that the maximum exists. We shall show that if

$$S = 3m, \qquad P = 3^m$$
$$S = 3m+2, \qquad P = 2\cdot 3^m$$
$$S = 3m+1, \qquad P = 4\cdot 3^{m-1},$$

where $S \geq 2$. In each case P is at least equal to the stated value as is evident from the partitions

$$S = 3m = 3+3+\ldots+3,$$
$$(1) \qquad S = 3m+2 = 3+3+\ldots+3+2,$$
$$S = 3m+1 = 3+\ldots+3+4.$$

Now in any partition

$$S = x_1 + x_2 + \ldots + x_k \qquad (x_j \geq 1),$$

we may assume first that $x_j \leq 4$. For if $x_j \geq 5$, the new partition of S with x_j replaced by $3 + (x_j - 3)$ will furnish a larger product since

$$3(x_j - 3) > x_j.$$

In the next place if $x_1 = x_2 = 4$, then the partition in which $x_1 + x_2$ is replaced by

$$3+3+2$$

will give a larger product. Hence there can be at most one 4.

If we assume $x_1 = x_2 = x_3 = 2$ then the partition in which $x_1 + x_2 + x_3$ is replaced by $3 + 3$ will give a larger product. Thus there can be at most two **2's;** however two **2's** can be replaced by a single 4.

Thus the various possibilities leading to a maximum product are described by (1). Moreover the solution is essentially unique, except that in the third case ($S = 3m + 1$), the 4 may be replaced by $2 + 2$.

Also solved by C. Clark, Albert F. Gilman **III,** H. Kaye, P. Myers, Daniel Sankowsky, L. Smith, Ilene Sprung, M Wagner, and the proposer.

126. Proposed by M. S. **Klamkin,** Avco RAD.

Determine an n-digit number (denary system) such that the number formed by reversing the digits is nine times the original number. What other numbers besides nine are possible?

Solution by Daniel Sankowsky, University of Pennsylvania.

It is clear that there are no 2 or 3 digit numbers which satisfy the requirement. If we look for a four digit number with the desired property, we obtain the unique number 1089, (9(1089) = 9801). Similarly to the manner of obtaining the 4 digit number, we also obtain the desired n-digit number 10999. . .989 or $10^n + 9(10^{n-2} + 10^{n-3} + \ldots + 10^3) + 89$.

The reverse of which is 98999. . ,901 or $9\cdot 10^n + 8\cdot 10^{n-1} + 9(10^{n-2} + 10^{n-3} + \ldots + 10^3) + 1$.

The other number besides nine for which the problem is possible is four. For this case the answer is 21999. . .978.

Also solved by C. Clark, L. Smith, J. Thomas and the prosper.

129. Proposed by Leo **Moser,** University of Alberta.
If R be a regular polyhedron and **P** a variable point inside or on R, show that the sum of the perpendicular distances from **P** to the faces of R, extended if necessary, is a constant.

Solution by James M. Horner, University of Alabama.

From point **P** draw lines to all the vertices. This divides the polyhedron into pyramids all having the same area for their bases. The volume of the polyhedron is equal to the sum of the volumes of the pyramids. Consequently the sum of the perpendicular distances from **P** to the faces of R is constant.

Also solved by H. Kaye, L. Smith, M. Wagner and the proposer.

BOOK REVIEWS

Edited by
FRANZ E. HOHN, UNIVERSITY OF ILLINOIS

A Primer of Real **Functions, Carus** Monograph #13, By R.P. Boas, Jr. New York, **Wiley,** 1960. xiii + 189 pp., $4.00.

This little book **moves** lightly and informatively over some of the concepts and methods of "real variables". These include in the first chapter sets in general, sets of real numbers and their properties, and in the second chapter **(which** is also the last) various types of functions (continuous, linear, monotonic, convex, and infinitely differentiable) and their properties such as limits, uniform convergence, and approximations.

Baire's theorem is proved in chapter one and several applications are given. There are a number of very well chosen exercises. Some of them illustrate certain points and others are essential parts of the book. Answers to all questions are given at the end of the book.

No previous knowledge except a course **in** calculus is required of the reader. To quote the author: "This is not a treatise and has not been written like one". It is delightful **in** its style, interesting, and informative and should be recommended to the student of real variables either as supplementary material or for independent study.

University of Maryland **Dagmar** R. Henney

Topology. By. J. G. Hocking and G. **S.** Young. Reading, Mass., **Addison-**Wesley, 1961. ix + 374 pp., $8.75.

This book has been designed as a text for a one-year, first course in topology. The material presented is about equally divided between point set and algebraic topology. A large number of topics are included but usually developed only to a minimal extent, purposely, so as to leave more detailed and advanced study to the choice of the student.

Besides the usual topological notions of separation, connectedness, compactness **(including** the Tychonoff theorem), the local versions of these and uniform structures, the authors go on to discuss decomposition spaces and to prove the **Hahn-Mazurkiewicz** theorem that a compact, connected, locally connected metric space is the continuous image of the unit interval and conversely. Pathological counter-examples show the **limitations** of the various theorems.

A chapter on homotopy theory and a chapter on Euclidean polytopes form the bridge between **point** set and algebraic topology. The fundamental group is applied to a discussion of knots, and then the elementary properties of higher homotopy groups are treated. Polytopes are carried to the simplicial approximation theorem, leading on to abstract simplicial complexes and the imbedding theorem for finite-dimensional complexes.

In the remaining three (of eight) chapters various topics in algebraic topology are taken up, but without most of the machinery of homological algebra. Here we find the universal coefficient theorems, exact sequences, the Eilenberg-Steenrod axioms, cup and cap products, Cech theory, and singular theory. A notable lack is any mention of the **Kunneth** theorem.

This book will be fine for giving a student a broad base in the subject, a good idea of what topology is all about, but none of the depth required for specialization. An extensive bibliography is included.

University of Illinois R.L. Bishop

Sequential Decoding. By J. M. Wozencraft **and B. Reiffen. The Technology** Press of the Massachusetts Institute of Technology and John Wiley & Sons, Inc., New York, 1961. v + **74 pp., $3.75.**

One of a series of Technology Press Research Monographs, this book is a compendium of some significant research done over the past few years at MIT. It is based on the doctoral research of Wozencraft and the subsequent doctoral research of **Reiffen,** and also includes results obtained by others. notably M. Horstein and R. G. Gallagher.

The book represents a self-contained presentation of the concept of sequential decoding for reliable communication. Its basic aim is to provide means for decoding sequentially in such a fashion that the computational complexity grows only algebraically with **n,** the number of code digits **in** each of 2k code words, assuming that the code is binary (for example) and that the rate $R = k/n$ is fixed. This aim is accomplished one information digit at a time: each information digit results from the **elimination** of half of the possible messages, upon receipt of n successive code digits. Just enough of these digits are considered to guarantee that the particular information digit can be decoded correctly, given some desired probabilistic or other criterion of success.

The book is very interesting and well organized. It includes a thorough presentation of introductory fundamentals, a detailed discussion of sequential decoding for the binary symmetric channel, a brief treatment of the corresponding encoding, some experimental results, and extensions to more general channels. Although the extensions and related results are indicated only sketchily, references are given to more detailed expositions. All in all, the authors have made a substantial contribution by circumventing the problem of exponential growth of computational complexity. The short length of the book (which is in itself praiseworthy) is inversely proportional to the content, but makes careful reading essential.

Bell Telephone Laboratories Peter G. Neumann

Transmission of Information, By Robert M. **Fano.** New York, Technology Press and John **Wiley,** 1961. x + 389 pp., $7.50.

This book is the first really comprehensive treatment of information theory which has appeared. The book begins with the introduction of the measure of **information,** and then considers discrete message sources, with special attention to sources of the Markov chain type. The noiseless coding theorem is proved and minimum redundancy codes are developed. Next there is an analysis **of** discrete memoryless channels and continuous channels of the additive Gaussian type. Finally there is a discussion of encoding and de**coding** schemes, with an exhaustive analysis of bounds on the probability of error. Shannon's fundamental theorem appears as a by-product of this analysis.

The book is directed toward communication engineers. but the presentation is precise mathematically and not too difficult to read for one who is well grounded in basic probability theory. Transmission channels and the fundamental coding theorem are not discussed in their full generality, but this is an advantage in the sense that the book becomes accessible to a very wide class of readers. A number of problems are provided, some of which are quite **challenging.**

The one criticism which the reviewer has is that the probabilistic approach to **coding,** i. e., "random coding" analysis, has been overemphasized at the expense of the algebraic approach; parity check codes are given virtually no treatment at all. In general, however, the author has managed to convey the flavor of information theory without lapsing into metaphysics. The book should become a standard reference for some years to come.

Columbia University R. B. Ash

Calculus and Analytic Geometry. By Robert C. Fisher and Allen D. Ziebur. Englewood Cliffs, N. J., Prentice-Hall, 1961. xv + 766 pp., $9 50.

This book is written primarily for adoption in colleges and universities for use in conventional classes of freshmen and sophomores studying analytic geometry and calculus.

While the authors make no great claims to rigor, they have been careful with notation and use of words so as to give more than an intuitive approach to the fundamental concepts which occur in calculus.

The general plan of integration of analytic geometry with calculus is a natural one and by the end of Chapter 7 the basic theory and techniques of the calculus of functions of one variable have been developed. The next two chapters discuss polar coordinates, vectors in the plane, vector equations of curves, parametric equations, derivatives of vectors, arc length, curvature and area of a surface of revolution, the geometry of space of three dimensions, and the applications of the calculus to the third dimension.

One finds in Chapter 10 material which is usually not discussed in such a book. The chapter heading "Linear Systems and Matrices" leads one to expect that topics on linear algebra are discussed. Some of the topics are systems of linear equations, matrices, determinative systems, vectors, inverse of a matrix, the transpose, characteristic values and the diagonalization of a matrix, rotation of axes and graphs of quadratic equations.

In the remaining four chapters partial derivatives, multiple integration, l'Hospital rule, improper integrals, sequences and infinite series are introduced.

In Appendix A a few important trigonometric formulas are given while in Appendix B a rather well-written discussion on limits appears. Earlier in the book some of the basic limit theorems were given without proof. In this appendix the authors show how such theorems may be proved.

Following the appendices a few short tables giving values of trigonometric functions for real numbers 0.0 to 1.6, natural logarithms of numbers 0.0 to 10.0. and exponential and hyperbolic functions of numbers 0.0 to 3.0 are found. Next are the listings of differentiation formulas and the basic integration formulas.

There are answers given to selected problems.

The problem sets throughout the book appear to be ample for the selection of problems fitting the various types of students found in college mathematics.

The content found in the book could be better handled in three semesters than two.

The book will no doubt compete favorably with the flood of analytic geometry and calculus books appearing.

St. Louis University — Francis Regan

Calculus, An Introductory Approach, By Ivan Niven. Princeton, Van Nostrand, 1961, vii + 172 pp.

This text has been designed for a brief course in calculus. This is accomplished by a stringent selection of topics. For example, to avoid a lengthy discussion of the properties of real numbers, the mean value theorem is taken as an axiom, although an intuitive geometrical argument is given for it.

After an introductory chapter on "What is calculus?", more complete discussions of limits, integration, differentiation, and the fundamental theorem are given in that order. The remainder of the book is devoted to a narrow selection of special functions and a few of the more interesting applications. Power series for sine. cosine, inverse tangent, and exponential functions are developed. Logarithm is defined and developed by means of integrals, with the exponential function following as the inverse.

The material is treated throughout with a healthy respect for rigor and motivation. This book deserves wide acceptance for the purpose intended.

University of Illinois — R. L. Bishop

Introduction To Geometry. By H.S.M. Coxeter, New York, Wiley, 1961. xvii + 443 pp., $9.95.

This book consists of many different topics in the various branches of geometry, unified by the idea of symmetry. The first two parts, on Euclidean geometry, are particularly well-done. Only the more interesting, less usual theorems are discussed, and the proofs are selected for their elegance. The nine point circle and Morley's theorem on the trisectors of the angles of a triangle (adjacent ones interesect to form an equilateral triangle) are typical examples. A discussion of transformations leads to a fairly complete account of plane crystallography. The treatment up to Part II is mostly synthetic, while for the remainder of the book coordinates are used freely when appropriate. Conics, complex numbers, and the Platonic solids are among the topics treated in Part II.

Part III is an account of ordered geometry with specialization to the classical affine and-non-Euclidean geometries. It gives a more rigorous and abstract development from a complete set of axioms than do the previous parts.

Part IV gives a brief account of the differential geometry of curves and surfaces, starting with an elementary development of vector algebra. The final two chapters deal with topology of surfaces and four dimensional Euclidean geometry. Particularly interesting is the discussion of the coloring problem for maps on a surface, including Heawood's theorem that an upper bound on the number of colors required to color a map on a non-spherical closed surface having Euler characteristic χ is $(7+\sqrt{49-24\chi})/2$. For a sphere this reduces to the famous conjectured value of 4, although the best the author does is 6, with a reference to the proof that 5 works.

We have here an example of the great vitality which geometry exhibits. A revival of interest in geometry in the undergraduate curriculum could very well start with this book, which contains, as its preface claims, enough material to spread out over the four college years.

University of Illinois — Richard L. Bishop

An Elementary Treatise on Curve Tracing, Fifth Edition (reprint). By P. Frost. New York, Chelsea, 1961. xvi + 210 pp., $3.50.

This is effectively a reprint of the fourth (1918) edition of the book, the difference being simply that all plates are assembled in a single fold-out pamphlet attached to the inside back cover.

The book undertakes the study of the loci of rational equations of low degree, by a variety of almost exclusively algebraic methods. As R.J.T. Bell writes in his preface to the Fourth edition "...there is no other book that contains in such compact form the detailed discussion and the diagrams of so many beautiful curves."

The reading of the book requires a knowledge of college algebra, analytic geometry, the basic theorems of the theory of equations, and a year of calculus. The material, aside from its great intrinsic interest, has applications in the study of differential equations and contributes much to one's knowledge of the numerical behavior of certain algebraic functions.

In view of the current emphasis on precision of statement in modem elementary texts, it is interesting to quote from the pteface of the 1892 edition: "I may mention here that I have used the symbol (a, β) for 'a point whose coordinates are a and β; and instead of speaking of a curve as 'the locus of the equation, $f(x,y) = 0$' or as 'represented by the equation', I have called it simply the curve $f(x,y) = 0$." Evidently the issue of whether to be fully precise or to use simple expressions whose intent is fully clear, was debated even by our grandfathers, who are thus revealed to have had more insight than we usually give them credit for having.

University of Illinois — Franz E. Hohn

The Calculus of Finite Differences, Fourth Edition (reprint). By George Boole. New York, Chelsea, 1961. xii + 336 pp., $1.39 paper, $3.95 cloth.

This is another welcome volume in the Chelsea series of reprints of important mathematical works. This volume treats of interpolation, finite integration, summation of series, Bernoulli's numbers, convergence of series, linear difference equations, mixed and partial difference equations, functional equations, and applications. The material is of value to those concerned with numerical analysis, theoretical or applied, or with actuarial work. As a classic English treatise on the subject, it should be in the library of everyone interested in these fields.

It is inspiring to reflect on the fact that **Boole's** Laws *of Thought* (also a Chelsea reprint) and this book, both dating originally to more than a hundred years ago, presented many of the basic ideas needed for the logical design of computers as well as many of the basic ideas needed for effective numerical use of computers. These two books, by the quality of their exposition and content, have no doubt contributed much to the development of today's computer-centered industrial revolution.

University of Illinois — Franz E. Hohn

Solutions of Equations and Systems of Equations, By A.M. **Ostrowski.** New York, Academic Press, 1960. **ix** + 202 pp., $6.80.

This book is concerned with the theory of numerical solution of equations. Although the prior knowledge required is almost only a knowledge of the mean value theorem, the style is extremely concise and more suitable for graduate study, as is stated in the preface.

The first two chapters are introductory, developing general tools for later use, such as generalizations of Rolle's theorem to higher derivatives, Darboux's theorem on derivatives, and definitions of the concept of interpolating functions and inverse interpolation.

The remaining 12 chapters and 11 appendices are concerned with the theory of particular methods of solution of equations. The results proved are of two sorts: inequalities for the error of an interpolation in terms of values of derivatives and proof of convergence of iterative processes. The method of false position (straight line interpolation) is taken first and subsequent methods are compared by means of a fairly crude computational unit. Various aspects and extensions of the Newton-Raphson method are then studied. Interpolation by linear fractions is introduced as a method of utilizing three interpolation points, apparently a new topic in the literature. The claimed advantage is that the interpolating point is unique, whereas for the more customary quadratic polynomial there are two solutions. Other topics include: a discussion of linear difference equations, finding of a root by using the Taylor series of the inverse function, an explicit formula for the derivatives of the inverse function in terms of the derivatives of the function, theorems on convergence of products of matrices, continuity of the roots of algebraic equations, a discussion of rounding off errors in matrices.

Although there are no problems, illustrative numerical examples are sprinkled throughout.

The selection of topics is rather hit and miss, and certainly the story does not end here; in fact the theory of solutions of equations in several variables is hardly scratched. However, upon reading this elegant book, one gets the impression that this is certainly the right way to develop the subject and to carry the topics further, all that is needed is a facility in the techniques displayed.

University of Illinois — R. L. Bishop

Introduction to Analytic Geometry and Linear Algebra. By Arno Jaeger. New York. Holt, Rinehart and Winston, 1960. **xii** + 305 pp., $6.00.

The author says his book is appropriate for the college freshman who has "mastered" college algebra and trigonometry whereas the reviewer feels more maturity is needed. The techniques of calculus are not required, but the book is about junior level for mathematics majors and early graduate level for non-mathematicians. It has many examples and solid exercises, but is too formal and pedantic for the average student.

It is divided into four major parts, namely: Foundations, Linear Geometry and Algebra, Multilinear Geometry and Algebra, and Quadratic Geometry and Algebra. The connections between geometry and algebra are well-emphasized, and the book is suitable for one or two semesters. It is basically sound but should be supplemented with the remarks of a good teacher.

University of Chicago — James R. Boen

Analytic Geometry and Calculus. By Abraham Schwartz. New York, Holt, Rinehart and Winston, 1960. xi + 864 pp., $9.50.

This text covers the standard material on analytic geometry and calculus in two and three dimensions. In the first six chapters, covering most of the usual differential and integral calculus of functions of one variable, rigorous definitions and proofs are often replaced, quite explicitly, by plausibility arguments, though the Theorem-Proof-Example format is maintained consistently. Chapter 7 contains all the theorems on limits, continuity, existence of integrals, etc., that are normally found in a calculus text, with the usual 'we cannot prove here' replacing proofs that require the compactness of closed bounded sets of real numbers. In the remaining six chapters, formal proofs are almost always given

The order of topics is not exactly usual; e. g., vectors appear early in Chapter 3, but computation of volumes of solids of revolution is postponed until near the end, after three-dimensional geometry. Problems are plentiful, but do not require ingenuity. Alternate answers are in the back of the book. There are tables of trigonometric functions and natural logarithms.

The exposition is very explicit. Not only proofs and commentaries, but even algebraic manipulations are spelled out in more than ordinary detail. Compromises with rigor are apologetically but specifically pointed out, often with a reference to a more accurate version in Chapter 7. On the whole, this is a careful and gentle book, motivated primarily by compassion for the elementary student.

Northwestern University — D. Felinsky

Stochastic Processes, By L. Takacs. Methuen's Monographs on Applied Probability and Statistics. New York, Wiley, 1960. *xi* + 137 pp., $2.75.

This book is primarily a collection of problems and solutions selected from the theory of Markov chains, Markov processes, stationary stochastic processes, recurrent processes, and secondary stochastic processes. A skeleton outline of the relevant theory precedes each of the problem sets, but it is expected that the reader is already familiar with probability theory.

The many books and papers cited in the references provide a convenient guide to the currently available literature on the subjects treated. The book is especially recommended as a supplement to a lecture course on stochastic processes, but should also be useful to anyone who has a good background in probability theory and wishes to pursue his study in any of the several areas covered in this book.

University of Illinois — Donald M Roberts

Boolean Algebra and Its Applications. By J. **Eldon Whitesitt.** Reading, Mass., Addison-Wesley, **1961. x+182** pp., **$6.75.**

This book is designed to serve as a first introduction to Boolean alegbra. No high degree of previous mathematical knowledge is required. Its material falls into two categories. The first is a good presentation of the mathematical foundations, with a definite and helpful emphasis on the equivalence of Boolean algebra to the Algebra of sets, to symbolic logic, and indirectly to discrete probability theory, all of which are covered **in separate** chapters. This method allows a fair degree of mathematical rigor and a broad treatment making the book useful to a large class of students. The material in this section is well-organized and well presented.

The second section deals with applications to Switching Theory, including both combinatorial and sequential circuit synthesis. This section of the book is considerably weaker than the first. None of the systematic reduction methods, such as that due to **Quine** and **McCluskey,** are presented. The actual circuits dealt with are all relay circuits, which do not adapt well to generalization. Not until the final chapter in this section is the notion of a logical element introduced. Sequential circuits are synthesized with the out-of-date sequence chart, rather than, for instance, by the method due to **Huffman.**

Many well-chosen exercises and a good list of references are included.

University of Illinois Digital Computer Laboratory — C. Pottle

Elements of Linear Algebra. **By** J. **Paige, and** J.D. Swift. Boston, **Ginn, 1961.** xvi + **348** pp., **$7.24.**

This book is written for the sophomore or junior mathematics major, physics major, or engineer with the intention of presenting "ideas from linear algebra with sufficient rigor to satisfy mathematical tastes and to nurture an understanding of the meaning and nature of proofs in algebra" (from the Preface). The book is intended to bridge the gap between the computational mathematics of the customary first course in college algebra and the modern highly abstract algebra courses.

After an initial chapter devoted to fundamental definitions, some motivation for the succeeding chapters is given in a consideration of vectors and some fundamental analytic geometry of space, developed in terms of vectors. The remaining chapters discuss vector spaces, Euclidean-n-Space, determinants, linear transformations and matrices, sets of linear transformations and matrices, bilinear and quadratic forms, complex number field, polynomial rings, characteristic values and vectors of linear transformations, and, finally, similarity of matrices.

The development is gradual and well motivated. Not until page **154** is there a definition of a matrix. An equivalence relation is not defined until page **184.** So one should not expect from this book a complete treatise on matrix theory or abstract algebra. However, basic fundamental properties and relations are developed, and the purpose of the book is achieved.

Some special features of the book are the drawings used to explain various types of mappings and geometric properties; the geometrical interpretation given some quite abstract algebraic concepts; numerous illustrative examples to help clarify the theory; a well-selected set of problems; and, something different in a book of this type, a set of answers to problems having unique answers.

The final theorem of the book is the Cayley-Hamilton Theorem, thus providing an indication of how complete a course of linear algebra is therein contained.

Saint Louis University — John F. Daly, S.J.

R. L. Ackoff (Editor): ***Progress in Operations Research, Vol. I.*** New York, Wiley, **1961. $11.50.**
H. W. Alexander: ***Elements of Mathematical Statistics.*** New York, Wiley, **1961.$7.95.**
A. Ambrose and M Lazerowitz: ***Logic: The Theory of Formal Inference.*** New York, Holt, Rinehart and Winston, **1961. $2.00.**
R. Bellman: ***A Brief Introduction to Theta Functions.*** New York, Holt, Rinehart and Winston, **1961 $2.50.**
O. Bolza: ***Calculus of Variations, Second Edition, (reprint).*** New York, Chelsea, **1961. $1.19** (paper).
G.** Boole: ***Finite Differences (reprint). New York, Chelsea, **1961. $1.39** (paper).
G. Cardano: ***The Book on Games of Chance.*** New York, Holt, Rinehart and Winston, **1961. $1.50.**
H.** S. M Coxeter: ***Introduction to Geometry. New York, Wiley, **1961. $9.95.**
G. W. Evans II and **C.** L. Perry: ***Programming and Coding for Automatic Digital Computers.*** New York. **McGraw-Hill, 1961. $9.50.**
R. M **Fano:** ***Transmission of Information.*** New York, The Technology **Press** and John **Wiley, 1961. $7.50.**
F. A. Ficken: ***The Simplex Method of Linear Programming.*** New York, Holt, **Rinehart** and Winston, **1961. $1.50.**
R.** C. Fisher and A. D. **Ziebur:** ***Calculus and Analytic Geometry, Englewood Cliffs, N.J., Prentice-Hall, **1951. $9.50.**
*P. Frost: ***Curve Tracing (reprint).*** New York, Chelsea, **1961. $3.50.**
W. Fulks: ***Advanced Calculus.*** New York, Wiley, **1961. $11.25.**
H. Hochstadt: ***Special Functions of Mathematical Physics.*** New York, Holt, Rinehart and Winston, **1961. $2.50.**
*J. G. Hocking and G. S. Young: ***Topology.*** Reading, Mass., Addison-Wesley, **1961. $8.75.**
C. Jordan: ***Calculus of Finite Differences, Second Edition (reprint).*** New York, Chelsea, **1961, $6.00.**
N. Kazarinoff: ***Analytic Inequalities.*** New York, Holt, Rinehart and Winston, **1961. $2.00.**
A. E. Maxwell: ***Analyzing Qualitative Data.*** New York, **Wiley, 1961. $ ....**
*I. Niven: ***Calculus, an Introductory Approach.*** Princeton, Van Nostrand, **1961. $.....**
A. M **Ostrowski:** ***Solutions of Equations and Systems of Equations: Pure and Applied Mathematics, Vol. 9.*** New York, Academic Press, **1960. $6.80.**
L. J. Paige** and **J. D.** Swift: ***Elements of Linear Algebra. Boston, **Ginn, 1961. $7.25.**
W. W. **Peterson:** ***Error-Correcting Codes.*** New York, The Technology Press and John Wiley, **1961. $7.73.**
J. F. Randolph, ***Calculus and Analytic Geometry.*** San Francisco, Wadsworth, **1961. $8.50.**
H. Sagan: ***Boundary and Eigenvalue Problems in Mathematical Physics.*** New York, **Wiley, 1961. $9.50.**
N. R. Scott; ***Analog and Digital Computer Technology.*** New York, **McGraw-Hill, 1960. $12.75.**
S. Vajda: ***Mathematical Programming.*** Reading, Mass., Addison-Wesley, **1961. $8.50.**
*J. E. Whitesitt: ***Boolean Algebra and its Applications.*** Reading, Mass., Addition—Wesley, **1961. $6.75.**
N. Wiener: ***Cybernetics, Second Edition*** New York, The Technology Press and John Wiley, **1961. $6.50.**
*J. M Wozencraft and B. **Reiffen:** ***Sequential Decoding.*** New York, The Technology Press and John **Wiley, 1961. $3.75.**

*See review, this issue.

NOTE: All correspondence concerning reviews and all books for review should be sent to PROFESSOR FRANZ E. HOHN, **374** ALTGELD HALL, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

OPERATIONS

Science needs you

You need science

UNLIMITED

This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essential as today.

Your election as members of Pi Mu Epsilon Fraternity is an indication of scientific potenital. Can you pursue advanced study in your field of specialization?

To point out the need of advanced study, the self-satisfaction of scientific achievement, the rewards for advanced preparation, the assistance available for qualified students, etc., we are publishing editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

Through these and future editorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

We are most fortunate in this single issue to present four articles from military research agencies. The Army, Navy, and Air Force are constantly engaged in research and applying mathematical techniques to many intriguing problems of design, production, and even combat circumstances. "Operations Research" is a broad heading used for that portion of research designed to assist in decision making by management and others faced with decision problems. Probabilistic techniques are used to evaluate probable outcomes and to estimate their likelihood of occurrence.

Dr. Thomas L. Satty, author of a text on Operations Research, currently Head, Mathematics Branch, of the Office of Naval Research; Dr. Harry Polachek, recipient of the Department of Defense Distinguished Civilian Service Award (1961), Technical Director of the Applied Mathematics Laboratory at David Taylor Model Basin;

Dr. T.E. Harris, past editor of the Annals of Mathematical Statistics, currently at RAND Corporation engaged in research for the Air Force; and Mr. R.E. Girard formerly in Operations Research at Johns Hopkins University, presently at the Research Analysis Corporation, engaged in research for the Army; have contributed articles for this section. Their articles are of extreme interest to mathematics students, as they clearly indicate the need for mathematics in military preparedness.

The following lists contributing corporations with the issue in which their editorials appeared.

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Army Ballistic Missile Agency	Vol. 2, No. 10
AVCO, Research and Advanced Development	Vol. 2, No. 10
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Research Analysis Corporation	Vol. 3, No. 5
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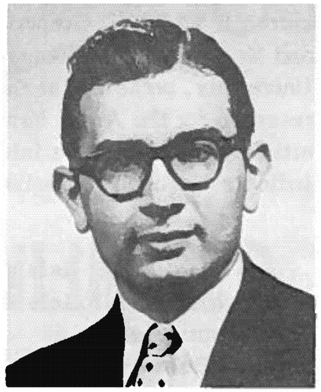
Woodrow Wilson Foundation	Vol. 3, No. 3

OFFICE OF NAVAL RESEARCH

OPERATIONS RESEARCH ORIENTATION AND TRAINING



By THOMAS L. SAATY
Head, Mathematics Branch
Washington, D. C.

Operations research is the scientific approach used to solve problems arising in operations to aid decision making. It has also been defined as the art of giving bad answers to questions to which otherwise worse answers are given. The significance of this definition is best realized by mathematicians who have given a serious look to the world about them. The concept of an operation remains informal and elusive. But we shall have more to say about it later. Roughly speaking, an operation has been conceived as a man-machine relation instituted to achieve given objectives.

Many scientists in parallel with their research in their own specialities practice operations research as it applies to operational phenomena which interest them. Examples of operational problems vary in scope from studying noncongested traffic flow through the Suez Canal or the New York Tunnels, to optimal methods of refining crude oil according to seasonal needs, to sizes of stocks to be kept in inventory to meet fluctuating demands, to the search for lost objects (particularly of people in the high seas) to the allocation and rotation of crops for an entire country, to the analysis and simulation of military strategy and tactics and effectiveness of weapons and so forth.

The pursuit of operations research consists of a) The judgment phase (what are the problems?) b) The research phase (how to solve these problems) and c) The decision phase (how to act on the findings and eliminate the problems). These phases require the analysis of an operation and the collection of evidence, the (mathematical) formulation of problems, the construction of theoretical models and selection of measures of effectiveness to test the models in practice, the making and testing of hypotheses as to how well a model represents the problem, prediction, refinement of the model, and the interpretation of results (usually as possible alternatives) with their respective values (pay off). The decision maker generally combines the finding of the analysis with his experience and tempers it by the dictates of statesmanship to arrive at a more rational decision than he would otherwise make. However, the fact that operations often involve people makes it difficult to follow the dictates of simplified logic. It has neither been easy nor desirable to assign worth to people or to their preferences. Note the important role that the analyst's objectivity plays in this process.

To a considerable extent, operations research as a formal discipline is occupied with the construction of models. This is closely related to the analysis of alternatives for decision making. It is generally assumed that to represent an operation it is preferable to have a model which may be simple and perhaps imperfect than to have none. The structure of a model may be purely logical or it may be a physical analogue. A wind tunnel is an illustration of a physical model. In either case, the model provides a coherent framework for coping with the complexities of a problem.

The literature of science is replete with models. This variety enables one to make some interesting observations. Thus, for example, one rarely regards models as unique or absolute although through the choice of a specific model (e.g., a differential equation) unique solutions to problems may be obtained. A model is formulated to serve a specific purpose. Some models may be suitable for generalization, others may not. These generalizations are more profitably made as extrapolations for scientific purposes and occasionally as useful philosophical observations. A model must be flexible to absorb new information and hence stochastic processes have broader and richer applicability than deterministic models.

Models are usually formed through a process of successive approximation, each time refinements are introduced which are based on how good additional information may be and how satisfactory the model is for prediction purposes. This refinement process may be described as follows:

Data – Model – Prediction – Further Data – Adjustment of the model – Further prediction and so forth.

Economy in the number of parameters which describe the essential properties of an operation, general applicability to a variety of problems and successful prediction are among the first desirable properties of models.

The problems of operations research have stimulated new developments in several mathematical fields, e.g., various aspects of game theory, stochastic processes, the calculus of variations, graph theory, and numerical analysis, etc.

From a theoretical standpoint an interesting and difficult problem is the characterization of the structure of an operation with the view of developing a theory which includes all the elements of the separate theories used so far in the field. This type of coherence is as yet unavailable. The subject of graph theory is receiving considerable attention because of its contribution to the study of flow in networks. Both the concept of flow and the concept of network have immediate bearing on the structure problem.

There are areas in the field of management science where it has not been easy or fruitful to produce theories of use in decision making. The complexity of the concepts and structures involved have so far presented investigators with great difficulties. Some continue to attempt formalizing the structure of the field of management science but there have been no breakthroughs. Often one is limited by the absence of mathematical methods required in this theoretical pursuit.

Areas of advanced planning, policy planning in conflict situations, automation, space problems, psychology and biology require fresh mathematical interest. For example, decision problems would be better understood with a fruitful formal approach to the psychology of values and hence decision criteria. Today the conflicting elements can not be resolved by existing simplified theories. Yet, with theoretical abstraction there is greater promise of achievement. It alone saves one from repetitive use of the same inadequate devices.

So far operations research has utilized the talents of individuals from a wide spectrum of fields ranging from chess players, physical and social scientists to mathematicians pure and applied. The need for imaginativeness and a good background in technical excellence are the basic elements in this choice of individuals. A significant consequence of all this is an opportunity for broad intellectual contacts and cross-fertilization of ideas.

There are, at present, eleven operations research societies in different countries. There is also an International Federation of Operations Research Societies (IFORS). In addition to the journals published by these societies, a journal called International Abstracts in Operations Research, comparable to Mathematical Reviews, has recently made its appearance.

At the end of the article some suggested mathematics courses, which would be helpful in the broad areas of operations research, are given. These are to be taken with courses illustrated as a group at the end of the list.

Computers have come to play a significant role in solving complicated mathematical models describing operations and hence it is desirable to include courses pertaining to computing and computability in the list.

It is best to point out that due to the absence of a formal structure of operations, the courses proposedprovide one only with useful tools. Experience is still required for a better appreciation of the subject. Fortunately there is no monopoly on how to obtain such experience. By examining the world he lives in, the alert individual can see a number of problems which he would like to see solved. The mathematician has the needed intellectual stamina which enables him to enjoy the use of his faculties and his excellence in providing the solution sought. Most of what is required is a little concern about our world and a little daring to change it.

A LIST OF COURSES

I. Optimization

Four Courses
1. Calculus of Variations
2. Linear and Non-Linear Programming with Applications, Econometrics Models
3. Dynamic Programming
4. Game Theory

II. Probability and Statistics

Three Courses
1. Elements of Mathematical Statistics
2. Introduction to Probability and Stochastic Processes
3. Markov Processes and Their Applications

III. Graph Theory

IV. Two Courses
1. Numerical Analysis
2. Computer Programming and Application

V. Application Courses (Teach and Coordinate with other Departments)

Five Courses
1. Queuing Theory
2. Inventory Theory
3. Renewal Processes, Reliability Theory, Testing and Control, etc.
4. Feedback and Automatic Control Theory (Non-linear Differential Equations, etc.)
5. Econometrics (Coordinate with Department of Economics)

Modem Algebra, Matrix Theory and Vector Spaces, Real and Complex Variables, Transform Theory, Statistical Mechanics, Differential Equations, Point set and Combinatorial Topology, Introduction to Algebraic Topology, Advanced Topics, etc.

DAVID TAYLOR MODEL BASIN

MATHEMATICS IN THE NAVY

By HARRY POLACHEK
Technical Director
Applied Mathematics Laboratory

When the present era is evaluated by future historians it is very likely that the development and application of high-speed automatic digital calculators will be ranked first among the many great technical achievements of this period, perhaps even ahead of the development of nuclear energy. This will be the case because the development of nuclear energy has given the human race a great new source of physical power, whereas the development of high-speed automatic devices has made available a new dimension in mental capabilities far beyond that which may be achieved by the human mind alone. Already high-speed calculators are being used to translate from one language to another, to compose symphonies, to design nuclear reactors, to forecast weather, to play chess or to compute fall-out patterns. This is but a small sample of the many complex tasks which are being carried out by these devices. However, we are only in the initial stages of this development. It is only seventeen short years since the first large scale automatic digital calculator has been placed in operation at the Cruft Laboratory of Harvard University. The construction of this calculator was pioneered by Professor Howard H. Aiken, who was at that time a Commander in the United States Navy.

Combined with the use of modem mathematical methods it is possible to apply the techniques of high-speed calculating devices to almost every field of human endeavor. In the field of naval research applications are being made in (1) the design of ship hulls, propulsion equipment, nuclear reactors, propeller systems, radar and sonar devices; (2) the analysis of full-scale or model tests, failure rates of electronic components, material requirements of shipbuilding programs; (3) navigation, fire control and guidance of missiles and satellites; and (4) minesweeping operations, antisubmarine warfare, weapon systems evaluation, simulation of submarine motion, war-gaming, as well as in the solution of many other problems of naval warfare.

In his paper, ***Naval Mathematics at the David Taylor Model Basin****, Admiral *E.* Alvey Wright, the former Commanding Officer and Director of this naval laboratory writes:

"Naval Engineering is acquiring a new dimension. Across the face of our profession, rooted in mathematics and embracing every physical science, is quietly bursting a tremendous new versatility and power that will surely carry naval engineering far beyond present design frontiers. It is the rapid automation of ship calculations in every conceivable and yet unconceived direction. This young dimension might be called naval mathematics."

Mathematics and computer methods are not only finding novel applications in the field of engineering research in general, and the naval sciences in particular; but new mathematical disciplines are continually being developed to accelerate technological progress. Information theory, numerical analysis, theory of games, operations research, linear programming, logic of automata, are but a few of the new mathematical research areas which are undergoing rapid development. The numerical solution of ordinary and partial differential equations is one area in which significant progress has been made. The success in this area alone will have profound influence on future technological development.

In the short span of less than twenty years the efficiency of high-speed digital calculators has increased 100,000-fold; new and unprecedented scientific milestones have been achieved; significant advances have been made in novel and challenging mathematical disciplines. To what extent will we be able to utilize these new advances for the benefit of mankind? - The possibilities are limitless. - The results will depend largely on the foresight and ingenuity of the young scientists in whose hands are entrusted the research efforts of the future.

***Journal of the American Society of Naval Engineers, Vol. 29, May 1957**

THE RAND CORPORATION

MATHEMATICAL TRAINING AND OPERATIONS RESEARCH

By T. E. HARRIS
Head, Mathematics Department

I think of operations research as a field broad enough to include such diverse things as finding better ways of handling fire equipment in a forest fire, improving bomber-tanker refueling tactics, devising an automatic system for indexing reports, or making a mathematical model for some physiological function so as to study improved methods of medical treatment. The list could be longer and no doubt will grow even more in the future as new military, economic, or scientific needs arise.

With such a variety of activities, it seems plain that no single type of training, and indeed no single type of person, will be ideal for all kinds of operations research. I can only make some suggestions for a set of mathematical skills that I think will be helpful in a variety of situations. It is perhaps unnecessary to point out that some of the best operations research is done with no more mathematics than counting, or perhaps not even that. Common sense is truly essential and is sometimes enough.

Today's large electronic computers have fantastic capabilities for handling numerical data. We can expect that future computers will have equally startling abilities to handle language or other non-numerical data. The future operations researcher should begin as early as he can to become familiar with the abilities and limitations of the new machines, large and small. He will be able to formulate and calculate with mathematical models much more complex than his predecessors could use. This will have advantages, but likewise presents dangers. Elaborate computations done with complex models making use of sparse and unreliable data may waste his own time and his clients' money.

Such standard subjects as calculus, matrix algebra, and complex variables are, of course, important. Statistics and probability are old standbys that should not be neglected. The statistics should include statistical inference, **i.e.,** how to make inferences about a population

on the basis of a sample of its members. The probability should include, if possible, something about stochastic processes (random functions), particularly the special variety called Markov processes.

Right after the Second World War, military operations researchers became very enthusiastic about the theory of games of strategy, which was originally developed by von Neumann and Morgenstern with economic applications in mind. The initial enthusiasm waned, possibly because too much was expected. More recently it has become apparent that game-theoretic analysis can be illuminating in certain special kinds of military problems. Today's operations analyst will want to know something about the theory of games. Even if he never applies it, he will find some knowledge of it rewarding because of its intimate mathematical connections with other directions of present-day mathematics.

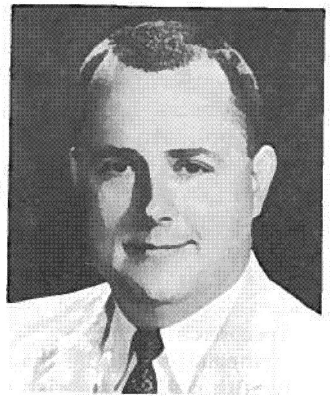
Mathematically, the theory of games has close connections not only with modem theories of statistical inference but with the technique known as linear programming, which has been applied to problems connected with such diverse things as petroleum refining, scheduling of training programs in the Air Force, or (if you allow me to replace "linear" by non-linear") the calculation of chemical equilibria.

Certain problems have a special structure that arises because they require the repeated application of control or decision elements to a system that moves or develops over a period of time. Dynamic programming is a set of techniques that have been developed to handle such problems. These techniques have been applied, for example, to chemical engineering problems, or for determining fuel expenditure rate in missile boosting problems. Here again there are intimate connections with other kinds of mathematics, such as the theory of stochastic processes.

No one should feel that his college curiculum must include all the subjects that I have listed, nor that others might not be more valuable in his particular case. Excellent texts have appeared in the last few years, and the operations researcher can, in some cases, choose among books written at either elementary, intermediate, or advanced mathematical levels, and can supplement his formal education and his on-the-job training by private study. His professional experience will suggest directions in which his study should be concentrated, but I believe he will be better off if, during his student days, he can become acquainted with some of the important current ideas and techniques.

THE RESEARCH ANALYSIS CORPORATION

THE COMBAT SOLDIER NEEDS YOUR HELP



(A Role for the Mathematician in Military Operations Research)

By EDWARD W. GIRARD
Chairman, Tactical Wargaming Group

The role I am about to propose for competent mathematicians in military operations research may come as a surprise. However, it is of great importance to our country at a time when startling advances in science and technology have made the planning, and execution, of military operations, in support of national policy, increasingly complex problems.

When I use the term "Military Operations Research", I mean analytic studies of the operations of highly organized groups of men, who use specialized equipment and techniques to execute the traditional mission of ground forces, "Seek out, and defeat the enemy, occupy his land, and destroy his will to fight".

These remarks on the desirability of advanced study in the field of mathematics are not directed to the man who already loves it. I want to speak to the man who has the capacity to master it as an analytic tool in research, and can get excited over the operational problems of ground warfare in the nuclear age. The woods are full of people in the field of operations research who can spend many a happy hour investigating the pathology of terms in queueing equations where the arrival and service distributions are always Poisson. I am sorry to say that there are not nearly enough people who combine significant mathematical prowess with a deep interest in the central problem of ground warfare in the age of nuclear technology. This is the translation of an investment of men, money, and materiel, and time into maximum combat effectiveness in the hands of the commander at the tactical unit, or building block, level under a variety of conditions, by the development of optimum organization, equipment, and tactical doctrine for employment in military operations.

One must take care to keep in mind that the responsibility for the problem, its solutions, and the decisions relating thereto remain the exclusive domain of the military authorities. The operations analyst tries to give an improved basis for decision or action in the more complex or technical areas, using methodologies which the Army staff has neither the time, nor the qualified scientific personnel, to employ themselves.

Operations Research on the problems of ground combat operations, is of such a nature that even the simplest cases can present awful difficulties, because very little basic work has been done. Wherever one turns in a quantitative analysis, there is the requirement for simplifying assumptions of the most sweeping nature if the work is to be described as a project, and not as a career. Thus, there are no generally accepted models of ground combat corresponding to the role of Newtonian or Relativistic Mechanics in physics. I should hasten to point out here that the well-known classical formulation of combat relationships, the Lanchester Equations, was developed as a model for air warfare, and has not been of significant value in study of ground operations. Furthermore, there is a dearth of good data on which to base a model for the formulation of such generalizing relationships or laws. The reason for the disparity in the state of development in basic research on ground combat, and that of physical science, mathematics or indeed, air/sea warfare, is not hard to find--it is the terrain. Air/sea situations can be well described in terms of the geometry of points in a homogeneous, isotropic continuium, to which the combat potential of the elements of a force may be referenced. On the ground, the tactical dispositions and combat potentials are not defined, except by reference to the proximate features of the terrain. The ground unit commander, up to division level, seeks to take maximum advantage of favorable terrain in the execution of his assigned missions. This requires just as thorough an analysis and appreciation of the terrain, as of the enemy situation, and the capabilities of friendly forces assigned. Thus, any operations analysis of these problems must take account of the terrain factors with at least a corresponding degree of completeness.

At present, at the Research Analysis Corporation, we are conducting intensive investigations of the operations of ground warfare and supporting military functions at all echelons. While this is principally devoted to producing the conclusions and recommendations that the Army has need of in its problems of the present and forseeable future, it also serves to support an effort being made in basic research for the discipline of Military Operations Research. Techniques of nonlinear programming, development of a theory and calculus of values, and basic models of general application are all under active study. In fact, considerable basic research on some aspects of the role of terrain combat operations has already been accomplished.

In all of this, the competent practitioner of mathematics is needed to play an indispensable part in the more mature stages of this work--the formulation of the generalizing relationships, or laws without which operations research will remain an art, or craft, on the fringes of management science, rather than fulfill its promise of attaining the stature of a scientific discipline in its own right.

After all, ground battle is built-up from opposing men with rifles on and in the ground, taking advantage of cover and concealment, each trying to see and shoot his enemy before the reverse situation occurs. To assist them in their task, direct fire from heavier weapons, machine guns, tanks, and close air support, and indirect fire support from artillery and missiles are provided. Not surprisingly, the support systems have proven to be much the more susceptible to analytical description. It now remains to examine the operations of that 200 pound (combat-loaded), fully mobile, all-purpose weapons system that requires a crew of only one lonely man.

Some of the more useful reference works in Military Operations Research include:

Morse, Philip M & Kimball, George, Methods of Operations Research, The Technology Press, MIT and John Wiley & Sons, Inc., New York.

G. Merrill, H. Goldberg, F.R. Helanholz, *Operations* Research, Armament Launching, Vol III of Principles of Guided Missile Design, D. Van Nostaand Co., Inc., New York.

McCloskey and Coppinger, Operations Research for *Management*, Vol II, The Johns Hopkins University Press.

SOLUTION TO CROSSWORD PUZZLE

R	E	G	U	L	A	R		U	N	I	T	A	R	Y
A	V	E	R	A	G	E		R	E	S	I	D	U	E
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UNIVERSITY OF MICHIGAN U. DUDLEY

NEWS AND NOTICES

Edited By
Mary L. Cummings, University of Missouri

Any information that comes under this heading should be mailed directly to me at the University of Missouri, Columbia, Missouri. I do solicit news items, appropriate notices and lists of national fellowships and scholarships won by members of Pi Mu Epsilon at the various chapters throughout the nation. It should be the responsibility of the various chapters to mail them in.at the end of the school year when these are known. In that manner they can be published in the fall issue. One should note the large list sent me for the fall issue of 1960. What happened this year?

We note with interest that Dr. J. Sutherland Frame of Michigan State University has been asked by the Conference Board of the Mathematical Sciences to serve for a year as Director of a project to undertake a study of the design of buildings for mathematics. The project has the support of the Educational Facilities Laboratories.

Murray R. King, representative of the National Life Insurance Company of Vermont, earned attendance at the firm's educational conference held in Honolulu, August 9-13. He qualified for the meeting as one of the top ranking agents in the company's nationwide field force. Mr. King, who maintains an office in Roslyn, Long Island, is a member of the Million Dollar Round Table, an organization of life insurance agents with sales of at least $1,000,000 annually. Mr. King earned a degree with honors in mathematics and astronomy at the University of Kentucky, where he was a member of Pi Mu Epsilon.

An interest story comes from the University of Missouri regarding identical twin graduates, Gerald and John MaGee, members of Missouri Alpha Chapter of Pi Mu Epsilon, who received many honors upon their graduation in June. The brothers were commissioned as ensigns in the Navy, having completed the Naval ROTC program while ranking in the top two per cent of all graduates from the College of Engineering in the past ten years. The past year Gerald won the Missouri Engineers of Chicago Scholarship for high scholastic standing. In May both twins were given Society of American Military Engineers Awards. During the last semester, the grades of the twins were so close that they shared the rank of battalion commander in the ROTC, which goes to the class member with the best grades. Both boys are members of Tau Beta Pi, engineering honor society. Finally, Gerald and John received the additional honor of being among the select few June engineering graduates throughout the nation picked by Vice Admiral Hyman G. Rickover for jobs in the naval nuclear propulsion program headed by the admiral. Both twins had been officers in Missouri Alpha Chapter of Pi Mu Epsilon.

Mr. James M. Bates of Durham, North Carolina, and a member of Pi Mu Epsilon (Ames, Iowa, 1938) has been elected as govenor of district 771 of Rotary International.

William C. Roberts, member of Virginia Alpha Chapter at the University of Richmond, has the distinction of completing his undergraduate career with an academic record of all A grades, the fourth student in the history of the University to achieve this record.

John Michel was graduated with distinction in mathematics from the University of Missouri in June. He presented a paper on the Absolute Value of a Matrix to the Graduate Seminar as part of his requirement. John had held the Continental Oil Company Scholarship of $500.00 during his senior year. He has accepted a fellowship with the Army Mathematics Research Center at the University of Wisconsin.

SOUTH CAROLINA ALPHA TAKES LEAD

South Carolina Alpha is justly proud of its new set of Chapter By-Laws, and feels that this puts them in the front rank among chapters of Pi Mu Epsilon.

The local initiation fee includes a two year subscription to the Journal to commence when the two copies normally obtained have been received. This will provide members with the Journal during that period of graduation and transition to permanent locations.

To further promote interest in mathematics monthly meetings of the chapter are held, at which student presentation of papers is encouraged.

There are three categories of membership: Active voting members are those members of South Carolina Alpha and associate members of South Carolina Alpha whose current dues have been paid and are in regular attendance at meetings. Active non-voting members are those with regular attendance but dues unpaid. They are members or associate members of South Carolina Alpha. Inactive members are those members and associates who have missed two consecutive meetings. They may regain active status by attending two consecutive meetings. Only active-voting members are eligible to hold office. If payment of dues is an undue hardship on a member the Chapter Director has the power to waive this obligation.

These By-Laws were a direct result of the talk made by Dr. Francis Regan, Editor of the Journal, at the national meeting held in East Lansing in August, 1960.

Cther chapters are invited to compete with South Carolina Alpha in furthering love of mathematics and the fraternity.

Martin Duszynski, Jr., Director, South Carolina Alpha, University of South Carolina is to be commended for reporting this initiative taken by his chapter. CONGRATULATIONS.

WINNERS OF PRIZES AND AWARDS AT CHAPTERS

Bucknell University (Pennsylvania Beta): Prize of $40.00 awarded by the chapter to Virginia M. Vidinghoff for outstanding scholarship in mathematics.

Montana State College (Montana Beta):
Outstanding Senior Award, $25.00, to Gary Sackett.
Outstanding Junior Award. $10.00, to Mrs. Kay Seibrand.
These awards were to be used for the purchase of mathematics books.

Montana State University (Montana Alpha):
Book to Mrs. Marleigh Sheaff in physics.
Book to Jack Silver in mathematics.
Mrs. Marleigh Sheaff also was awarded the H.B. Wood Memorial Award of $100.00 for. outstanding junior majoring in mathematics and the N.J. Lennes prize of $1.00 for the highest score on a mathematics examination.

Pennsylvania State University (Pennsylvania Delta): Miss Erika Mares has been awarded a Rotary Foundation Fellowship for advanced study abroad during the year. She is studying mathematics, in preparation for a teaching career at the university level, at the University of Fribourg, Switzerland.

Polytechnic Institute of Brooklyn (New York Iota):
Prizes for freshmen - First to Howard Taub; second and third (tie) to Renny Zarrell and Otto Moller.
Prizes for undergraduates - First to Stewart Nagler; second to Earl Pomerance; third to Paul Feder.
The prizes were books.

St. Louis University (Missouri Gamma): The James W. Garneau Award to Sam Lomonaco, a graduating senior, for excellence in mathematics. The James B. Macelwane Memorial Award to James R. Francoeur. The Engineering Education award and The Electrical Engineering Junior Award to Rudolph F. Trost. The Electrical Engineering Senior Award to Jerome P. Brand. The Pi Mu Epsilon Senior Problem Contest won by Wayne Salus.

University of Arkansas (Arkansas Alpha):
Pi Mu Epsilon Prizes - First to James E. Gauntt, $25.00.
Second to Joseph E. Brownd, $10.00.
Third to William D. Evans, $5.00.

University of Dayton (Ohio Zeta):
Prize for the most outstanding sophomore student to Thomas Grilliot. The award was the two volume set of Courant, Differential and Integral Calculus.

University of Kentucky (Kentucky Alpha):
The Pi Mu Epsilon H. H. Downing Award ($25.00 worth of books) to Roger D. Rosenbaum, who made the highest score on the freshman mathematics examination.
The Pi Mu Epsilon Distinguished Mathematician Book Award for scholastic achievement to Jeanne B. Shaver and Roger D. Rosenbaum. Each received $10.00 worth of books.
The Pi Mu Epsilon Key Award for the best graduate seminar paper (as selected by the faculty) to Richard A. Mulliken. This award consists of a Pi Mu Epsilon key and mathematics books of the student's choice and value not to exceed $20.00.
The Pi Mu Epsilon Key Award for the best mathematical content seminar paper (selected by the faculty) to Clifford Swauger. The prize consists of a Pi Mu Epsilon key and ten dollars worth of mathematics books of the student's choice.

University of Missouri (Missouri Alpha):
Pi Mu Epsilon Prizes in the Calculus - First and second (tie) to Jim D. Campbell and Jerry Jouret. Each received $25.00.
The Continental Oil Company Award for outstanding senior majoring in mathematics to Wilson Brumley and David Martin, $500.00 each.

University of Nebraska (Nebraska Alpha):
Prizes awarded on the basis of examinations. Fall competition
Examination I - First to Mark Teply, second to Patty Edmiston.
Examination II - First to Donald Dermyer, second to Pauline Hill.
The prizes were $10.00 for first and $5.00 for second.
Spring Competition
Examination I - Craig Johnson.
Examination II — Jon Froemke.
The prizes were $10.00.

University of Oklahoma (Oklahoma Alpha):
Prizes to Stephen De Canio and Stanley Burris for having the greatest number of correct solutions to the chapter sponsored Problem Box. The prizes were books.

University of Pennsylvania (Pennsylvania Alpha):
Book Awards to Larry Goldstein, Richard Larson, and Herbert Gintis.

University of Richmond (Virginia Alpha):
Prizes to winners of contests in elementary courses -
Freshman mathematics: First ($10.00) to Joel Gaydos.
Second ($5.00) to William Cale.
Sophomore mathematics: First ($10.00) to Thomas B. Vassar.
Second ($5.00) to Dotti Williams.

The James B. **Crump** Prize for excellence in mathematics to William C. Roberts.
Award for the best paper of an historical or descriptive nature to Joyce Steed for her discussion of the Development of the Calculus.
Award for the best paper of a technical nature to Mary Frances Wright for her presentation of The **Laplace** Transformation. Both awards were copies of **Newman's** four volume The World of Mathematics.

University of Wichita (Kansas Gamma):
The Pi Mu Epsilon Mathematical Scholarship Award of $100.00 to William F. Siegle, Jr.

University of Wisconsin (Wisconsin Beta):
Prizes to Donald R. Ceilesh and Charles H. Giffen for excellence in mathematics. The awards were copies of Bell's Development Of Mathematics.

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Students from St. Louis University having Woodrow Wilson National Fellowships are: Charles Ankenbrandt, physics; Samuel **Lomonaco, Jr.**, mathematics; Roger P. Main, physics; Elizabeth Orange, psychology; Paul A. Westhaus, physics; **Marguerite** Van Flandern, physics.

Those having **National Science** Foundation **Fellowships:** Robert B. Rutledge, mathematics; Charles Ankenbrandt, physics; Samuel Lomonaco, mathematics; Elizabeth Orange, psychology; **Marguerite** Van Flandem, physics; John Cantwell, mathematics; Barry Flachsbart, engineering.

Special Fellowship in nuclear science and engineering, Atomic Energy Commission: Charles **Ankenbrandt,** physics; Kathleen Lips, physics; Marguerite Van Flandern, physics.

New York State Regents College Teaching Fellowship: Patricia Bylebyl, mathematics, who resigned this fellowship with permission to accept a mathematics fellowship at the University of Paris given by the French government. Her travelling expenses will be paid under the Fullbright Fellowship program.

Scholarships in aeronautical engineering at University of Washington: Wayne Salus; Robert Swift.

Fellowship to Air Force Institute of Technology: Joseph **Bullmer.**

Fellowship to University of Southern California (Hughes Aircraft Corp.): John M. Hamm.

Fellowship to Washington University: Thomas Wulfers, chemistry.

Scholarship to University of Kansas: John J. Travalent, engineering.

Fellowship to Massachusetts Institute of Technology: Richard Elder, chemistry.

Fellowships to New York University (Bell Telephone Laboratories): James R. Francoeur; John L. Martin.

Assistantship at **Purdue** University: Thomas B. **Nenninger,** engineering.

All students mentioned above are members of Gamma Chapter of Missouri, St. Louis University. Congratulations!

FALL MEETING AT STILLWATER

All of the meetings of Pi Mu Epsilon were held on Tuesday, August 29, 1961 at Stillwater, Oklahoma. At **8:00** A.M. the National Council had a breakfast meeting, at which they decided to continue the practice of matching local funds up to a maximum of $20.00 per chapter for undergraduate prizes. These prizes are not to be made in cash. The National Council suggests that the prizes be mathematical books and journals of the winner's choice and / or membership in the Mathematical Association of America. To be eligible for funds from the National treasury, the chapter must:

1. File a request on the proper form with R. V. Andree, Dept. of Mathematics, University of Oklahoma, Norman, Oklahoma before Jan. 15, 1962. (Forms available from R.V. Andree, University of Oklahoma)
2. Stipulate that the chapter will at least match the funds supplied from the National treasury, and that the combined amount will be used for undergraduate prizes for mathematical excellence.
3. Agree to send a list of winners and a short description of the basis of selection to Miss Mary L. **Cummings,** Dept. of Mathematics, University of Missouri, Columbia, Missouri.

The national office has plaques with large Pi Mu Epsilon insignia on them available to chapters. If you are interested in purchasing any of them, write to R.V. Andree, University of Oklahoma, Norman, Oklahoma.

At noon a luncheon was held in the Student Union at Oklahoma State University at which members of the council, delegates from the chapters, speakers, and other members of Pi Mu Epsilon in total about thirty were present.

At **1:30** P.M. the following papers were presented:

Conformally Elementary Points, Louis E. DeNoya, Oklahoma Beta, Oklahoma State University.

Transcendence of e, **John Wells,** Ohio Zeta, Ohio University of Dayton.

A Topic in Modern Algebra, Charles **Mullins,** Oklahoma **Beta,** Oklahoma State University.

General Solution to the Linear Diophantine Equation in Two Variables, Thurston Shook, Ohio Alpha, Ohio State University.

It was decided at East Lansing, Michigan in August, 1960 to make two awards for papers presented at this meeting, each award was to be one hundred dollars. One award was for the best paper presented by an undergraduate (as of May **1, 1961),** while the other went to the best paper presented by a beginning graduate student.

R.V. Andree, University of Oklahoma, and Francis **Regan,** St. Louis University were the judges.

Louis **DeNoya,** a graduate student won one of the awards of one hundred dollars, while the undergraduate award of one hundred dollars went to Thurston Shook.

Only one of the four papers presented was given by a beginning graduate student.

The activities of the day were concluded by a tea in the Chinese Lounge of the Student Union.

NEW CHAPTERS INSTALLED

New Mexico Alpha Chapter was installed at New Mexico State University, February 10, 1961. Professor R. H. Bing, Vice-Director General, presented the charter and initiated the candidates. Following the ceremonies, Dr. Bing was guest speaker at a banquet held in honor of the initiates. Other special guests included Dr. Roger Corbett, President of New Mexico State University, and Dr. Earl Walden, Dean of the Graduate School and former Head of the Mathematics Department.

On February 14, 1961, Dr. J. Sutherland Frame, Director General of Pi Mu Epsilon, installed the seventy-ninth chapter, Kappa of New York, at Rensselaer Polytechnic Institute. Members signing the charter were: Dr. Edwin Brown Allen, Jack Hoffman, Eric Joede, Harold Langworthy, and Stephen Meskin. Immediately after the installation, Dr. Frame gave a talk on Continued Fractions to a group of about thirty or thirty-five students and faculty members.

Dr. J. Sutherland Frame again served as installing officer when he launched New York Lambda at Manhattan College, June 7, 1961. Preceding the installation, Dr. Frame gave a lecture on Elementary Concepts in Relativity to an audience of fifty or sixty. He then installed the chapter, initiating the following charter members: Lawrence Chiappetta, John E. Duffy, Frank J. Gordon, Jeremiah Kelly, Stanley S. Leroy, Donald J. McCarthy, Paul F. Mercadante, Harry J. Neylan, Joseph J. Pero, Gabriel Pompilio, John Power, Richard J. Prior, Richard C. Wagner, Francis T. Walsh, Charles F. Weber, Brother C. John. After the installation, the Newton Mathematical Society, which had sponsored the new chapter of Pi Mu Epsilon, was host at a dinner. Over sixty people were present, including the new members of Lambda chapter and former members of the Society living in or near New York. Dr. Frame gave a talk on the history of Pi Mu Epsilon, and several of the students who had presented talks to the Society during the year were given certificates of merit.

June 14, 1961, Dr. Frame was again in New York, this time installing Mu Chapter at Yeshiva College. At 6:00 p.m., he gave a lecture on Power Series for Inverse Functions, following which he initiated these charter members and presented them the charter of the New York Mu chapter: Richard Barth, Herschel Farkes, Aaron Fruchter, Michael Greenebaum, William Kantrowitz, Aaron Lebowitz, Henry Lisman (faculty adviser), Herman Presby, Harry Rauch, Louis Raymon, Joseph Tuchman, Benjamin Weiss, Shlomoh Wohlgemuth. A buffet supper was served at 8:00 p.m. At this informal gathering, Dr. Frame told the group about the history of Pi Mu Epsilon.

CHAPTER ACTIVITIES

Edited By

Houston T. Karnes, Louisiana State University

EDITOR'S NOTE: According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an annual report of the chapter activities including programs, results of elections, etc." The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the Pi Mu Epsilon Journal. Besides the information listed above, we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Chapter Activities Editor Houston T. Kames, Department of Mathematics, Louisiana State University, Baton Rough 3, Louisiana. These reports will be published in the chronological order in which they are received.

REPORTS OF THE CHAPTERS

ALPHA OF MARYLAND, University of Maryland

The Maryland Alpha Chapter held four meetings during the academic year 1960-61. The following papers were presented:

"What We Learned from IGY," by Dr. David Stem.

"Theory of Limits," (film) following lecture by Dr. Leon Cohen.

"New Mathematical Methods as Applied to Quantum Mechanics," by Dr. John Tell.

"Mathematics and General Culture," by Dr. Avron Douglis.

Dr. Elkins, President of the University, gave the annual banquet address. Following the address Dr. Leon Cohen presented the Abramowitz prize on Mathematics, This prize was institutedthis year.

GAMMA OF MISSOURI, St. Louis University

The Missouri Gamma Chapter held four meetings during the academic year 1960.61.

The following papers were presented:

"Weak Rings," by Mr. George Matusek.

"P-adic Numbers," by Mr. Sam Lomonaco.

"Certain Properties of Three Terminal Switching Networks," by Mr. Wade Peterson.

"Some Curious Theorems of Elementary Geometry." by Professor Leonard M. Blumenthal.

At the first meeting of the chapter, Mr. Richard Doyle was elected vice-director to succeed Mr. Daniel D. Cronin, and Miss Marceline C. Gratiaa was elected secretary to succeed Miss Mary Lou Krug.

At the final meeting, Mr. Gerald P. Harshany was elected director for 1961-62.

At the 24th annual banquet, Mr. Daniel D. Cronin acted as toastmaster. The awards were presented by Dr. Waldo Vezeau and were as follows: Pi Mu Epsilon Contest Award to Mr. Wayne L. Salus; Freshman Achievement Award to Miss Virginia Ann Arnoldy; James W. Gameau Mathematics Award to Mr. Sam Lomonaco as outstanding senior.

Dr. Francis Regan has again accepted the position as faculty advisor and permanent secretary-treasurer of the chapter.

The Missouri Gamma News, the chapter bulletin, will be published during the summer of 1961.

ALPHA OF NORTH CAROLINA, Duke University

The North Carolina Alpha Chapter held three meetings during the academic year **1960-61,** of which **onlv** one was a program meeting. At this meeting the following paper was presented:

"**Generalized** Functions;" by Professor F. G **Dressel.**

Twenty eight students were initiated at the final **meetirg** of the year.

Officers for 1961-62 **are:** Director, Carol **Ann** Wilson; Vice-Director, James Richard **Sawers, Jr.;** Secretary, John Otto Meier; and Treasurer, James W. White.

ALPHA OF LOUISIANA, Louisiana State University

The Louisiana Alpha Chapter held seven meetings during the academic year 1960-61. The following papers were presented:

"Continued Fractions," by Dr. J. Sutherland Frame, Director General.

"My Trip to the National Convention," by Mr. Harvey **Carruth.**

"**The** Cantor Set," by Dr. R. J. Koch.

"The Domain of Research Mathematics," by Dr. Richard D. Anderson.

The chapter was indeed fortunate to have a visit **from** the Director General, Dr. J. Sutherland Frame. Dr. Frame was in Baton Rouge to install a chapter at Southern University.

Louisiana Alpha held two initiations, one in the fall and one in the spring. A total of thirty-seven students were initiated.

At the final meeting for the year, the Annual Honors Examination Awards were made to the following persons by Dr. Haskell Cohen: Freshman Award to Allen C. **Milis** and Senior Award to David J. Evans.

Officers for 1960-61 were: Director, Dave Evans; Vice-Director, Kenneth Freeman; Secretary, Elizabeth Sloan; Treasurer, John M. Callaghan; Faculty Advisor, Dr. Haskell Cohen; and Corresponding Secretary, Dr. Houston **T.** Kames.

Officers for 1961-62 are: Director, Kenneth Glenn Freeman; Vice-Director, **DeWitt** Lee Simmers; Secretary, Claire Fasullo; Treasurer, Jeffrey B. **Fariss;** Faculty Advisor, Dr. Haskell Cohen; and Corresponding Secretary, Dr. Houston T. Kames.

GAMMA OF NORTH CAROLINA, North Carolina State University

The North Carolina Gamma Chapter was installed on November 15, 1960 under the direction of the Director General, Professor J. Sutherland Frame. In addition to the installation meeting, the chapter held six meetings during the year. The following papers were presented:

"Analog Computers," by Mr. Lawrence Moore.

"Operations Research," by Dr. Robert M. Thrall.

"**Mathematics** Education in England and Scotland," by Dr. Ian Sneddon, University of Glasgow.

The last talk of the year was given by Dr. Milton **Levine,** Director of the National Register of Scientific **and** Technical Personnel of the National Science Foundation.

Officers for **1960-61** were: Director, Lawrence C. Moore; Vice-Director, Jon R. Howell; and Secretary-Treasurer, Robert D. Davis.

Officers for 1961-62 are: Director, Walter Cummings; Vice-Director, Betty G. Harris; and Secretary-Treasurer, Philip **Nanzetta.**

MU OF NEW YORK, Yeshiva College

The New York Mu Chapter was installed by the Director-General, Professor J. Sutherland Frame, on June 14, 1961. At this meeting Professor Frame gave a talk on "Power Series for Inverse Functions."

Acting officers were: Director, William Kantrowitz; Secretary, Richard **Barth;** and Interim **Director, Benjamin** Weiss.

ALPHA OF GEORGIA, University of Georgia

The Georgia Alpha Chapter held eleven program meetings during 1960-61. The following papers were presented:

"Congruence Relations," by Dr. L. W. Anderson.

"Vector Spaces," by Dr. H. C. Filgo.

"**Proof** of the Axiom of Choice and Other Related Theorems," by Mr. Julio R. P. Bastida.

"**Logic,**" by Mr. John Selden.

"**Problems** of Continuous Functions," by Dr. R. W. Heath.

"Biological Statistics, " by Dr. J. L. Cannon.

"Rational Distance Sets," by Dr. G. B. Huff.

"Mathematics and Astronomy," by Mr. L. H. **Spradley.**

"Ordered Spaces," by Dr. B. J. Ball.

"Modal Logic," by Dr. G. E. Scott.

"The Calculus of Variations," by Dr. M. K. Fort.

Officers for the fall and winter quarters, 1961-62 are: Director, Gordon Howell; Vice-Director, David Lifsey; Secretary, Patsie Bradberry; Treasurer, Rosalie Seymour; and Advisor, Dr. R. P. Hunter.

ALPHA **O**F ARKANSAS, University of Arkansas

The Arkansas Alpha Chapter held nine meetings during the academic year 1961-62. The following papers were presented:

"Astronomy at the Planetarium," by Dr. Richardson.

"The Matrix Representation of Operators and Some Aspects of Spectral Analysis," by Dr. Scroggs.

"A Proof on the Transcendence of **e,**" by Mr. Robert Walls.

"Line Geometry," by Dr. Richardson.

"Magic Squares," by Miss Beth Logan.

"Evaluating Limits," by Miss Marian Brashears.

The service project of Alpha chapter was sponsoring tutoring sessions for all students of mathematics enrolled in courses through Calculus III.

The annual spring banquet was held on April 18, 1961. The guest speaker was Professor John Hoffman from Oklahoma State University.

The winners of the Pi Mu Epsilon prizes to undergraduates, based on a regular examination, were as follows: **First** Prize—Mr. James E. **Gauntt,** winning $25; Second Prize—Mr. Joseph E. **Brownd,** winning $10; and Third Prize—Mr. William D. Evans, winning $5.

New officers for 1961-62 are: Director, **Carroll** Blakemore; Vice-Director, Linda Borden; Secretary, Margaret Tippy Cravens; Treasurer, **Kazuo Oishi;** Tutoring Chairman, Charles Gibson; and Faculty Advisor, Dr. William R. **Orton.**

ALPHA **O**F OKLAHOMA, University of Oklahoma

Oklahoma Alpha Chapter held six meetings during the academic year **1960-61.** The following papers were presented:

"The Theory of Braids," by Dr. Mathews.

"An Arithmetical **Duriosity,**" by Dr. Nicol.

"Number Theory," by Merry Morgan.

"Theory of Heart Potential," by Mr. Paul Berry.

"Calculus of Variations," by Dr. Ewing.

Mr. Stephen **DeCanio** and Mr. Stanley Bums were awarded prizes for presenting the most correct solutions to the problem box.

The annual banquet was held on May 9, 1961. The guest speaker was Dr. Tom Smith who spoke on the history of science.

Officers for **1961-62** are: Director, Paul Berry; Vice-Director, Earl **LaFon;** Secretary-Treasurer, Phillip Briggs; Sponsor, Dr. **W.** N. Huff; and Faculty Correspondent, Dr. Dora **McFarland.**

EPSILON OF NEW YORK, The St. Lawrence University

The New York Epsilon Chapter held five meetings during the 1960-61 year. The following papers were presented:

"What Is an Actuary?" by Lloyd Landau,

"Opportunities for Women in the Field of Mathematics," by Lila Brush.

"Solution of Various Problems by Means of the Pascal Triangle," by Mr. Robert Waterman.

Seven new members were initiated, bringing the total student membership to twenty-one.

The 17th Annual Interscholastic Mathematics Contest was held on the campus of the St. Lawrence University and was open to all secondary schools in St. Lawrence and Franklin counties of Upper New York State. A $400 scholarship was offered to the highest ranking senior. A silver cup was awarded to the winning team, and appropriate medals were given to contestants ranking first, second, and third in the contest.

Pi Mu Epsilon co-sponsored the visit to the campus of Dr. Frank M. Stewart, the MAA-NSF lecturer.

The chapter assisted the Mathematics Department in the orientation of sophomores in their selection of major fields.

Pi Mu Epsilon Awards were made to the outstanding freshman and sophomore in mathematics for 1960-61; these awards will be made in October, 1961.

Officers for 1960-61 were: Director, Margaret Young; Vice-Director, Robert E. Waterman; Recording Secretary, Barbara Zeidler; Treasurer, Richard Smith; Advisor, Dr. O. Kenneth Bates; and Corresponding Secretary, William H. Plows.

Officers for 1961-62 are: Director, Joan A. Herbert; Faculty Advisor, Dr. O. Kenneth Bates; Corresponding Secretary, John J. Kinney. The positions of Vice-Director, Recording Secretary, and Treasurer are to be filled at the first meeting of the academic year 1961-62.

ALPHA OF NEW MEXICO, New Mexico State University

New Mexico Alpha Chapter held fifteen meetings during the 1960-61 academic year. The following papers were presented:

"N-High Subgroups of Abelian-Groups," by Dr. John M. Irwin.

"Generalized Derivatives," by Dr. Edward Gaughan.

"Banach Spaces Isomorphic to a Conjugate Space," by Dr. Seymour Goldberg.

"Simple Continued Fractions" and "Ergodic Estimates in the Theory of Periodic Functions," by Dr. Arthur H. Kruse.

"Projections onto the Subspace of Compact Operators," by Dr. Edward O. Thorp, Massachusetts Institute of Technology.

"Attempts to Solve the Poincare Conjecture," by Dr. R. H. Bing, University of Wisconsin.

"Markov Chains," by Dr. Peter W. Zehna, Colorado State College.

"Representation of Lattice Ordered Rings," by Dr. Donald Johnson, Pennsylvania State University.

"Partition Lattices," by Dr. John Johnston, University of Kansas.

Members of the chapter attended the following lectures, sponsored by the Society of Industrial and Applied Mathematics:

"Constructive Theory of Functions" and "The Monte Carlo Method," by Dr. John Todd, Caliiomia Institute of Technology.

"A Study of Elementary Functions" and "Non-Linear Eigenvalue Problems," by Dr. I. L Kolodner, University of New Mexico.

Officers for 1961-62 are: Director, Carol Lee Peercy; Vice-Director, Edmund J. Peake, Jr.; Secretary-Treasurer, William L. Gaudle.

Officers of 1960-61 were: Director, Roy O. Spruiell; Vice-Director, William L. Gaudle; Secretary-Treasurer, Carol Lee Peercy; and Faculty Advisor and permanent Corresponding Secretary is Dr. Seymour Goldberg.

BETA OF NORTH CAROLINA, University of North Carolina

The North Carolina Beta Chapter held four meetings during the academic year 1960-61. The following papers were presented:

"The Superior Student," by Dr. E. A. Cameron.

"Fibonacci Numbers," by Mr. Clifton Whyburn.

"Lucky Numbers." by Dr. David Hawkins.

"Bernoulli Polynomials," by Dr. J. D. Buckholtz.

Officers for 1961-62 are: Director, Albert Deal III; Vice-Director, Clifton Whyburn; Secretary, Sandra Ness; and Treasurer, Warren Boe.

ALPHA OF MONTANA, University of Montana

The Montana Alpha Chapter held ten meetings during the academic year 1960-61. The following papers were presented:

"Duplicating the Cube," by Mr. Mike Billings.

"Finite Fields," by Mr. Merle Manis.

"Iterated Summability Methods," by Dr. Arthur Livingston.

"Godel Revisited," by Mr. Jack Silver.

"Perfect Sets," by Mr. Keith Yale.

"The Chocolate Cake Problem," by Dr. Howard Reinhardt.

"Lattice Theory," by Mr. George McRae.

"Quasi Groups," by Mr. Don Sward.

Pi Mu Epsilon awards for outstanding achievement in mathematics and physics were presents d to Jack Silver and Marleigh Sheaff, respectively.

Officers for 1960-61 were: Director, Robert D. Engle; Vice-Director, Richard S. Nankervis; and Secretary-Treasurer, Harry Bauer.

ALPHA OF NEBRASKA, University of Nebraska

The Nebraska Alpha Chapter held eight meetings during the academic year 1960-61. The following papers were presented:

"The Concept of Relativity," by Dennis Nelson.

"Permutations, Combinations, and Probability," by Jim Glathar.

"The Sentimental Criminal," by Professor Reinhardt.

"Estimation and Elementary Probability Problems," by Professor Heuer.

"Systems Development," by Professor Henzeker.

"Euler," by Dennis Nelson.

"Tour of the University Computing Center," by Professor Christopher.

During the fall, 1960, the Prize Examination winners were Mark Teply, first; Patty Edmiston, second—in Group I. And Donald Dermyer, first; and Pauline Hill, second—in Group II. The spring, 1961, winners were Craig Jonson, first—in Group I; and Jon Froemke, second—in Group II.

During the second semester, a "Problem of the Week" was printed in the campus newspaper.

Eleven new members were initiated in the fall and twenty-nine in the spring.

Officers for 1961-62 are: Director, Larry Dornhoff; Vice-Director, David Bliss; Secretary, William White; Treasurer, Richard Altrock; and Faculty Advisor, Professor H. L. Hunzeker.

IOTA OF NEW YORK, Polytechnic Institute of Brooklyn

The Iota Chapter of New York held four meetings during the academic year 1960-61. On May 12, 1961, the annual initiation was held at which time eight new members were received into membership. Following the initiation Professor George Bachman presented a paper on "Geometric Algebra."

Freshman and undergraduate contests in mathematics were held. The winners were: freshman contest, Howard Taub, Otto Moller, and Renny Zarrell; undergraduate contest, Stuart Nagler, Errol Pomerance, and Paul Fefer. Prizes were Dover Books on Mathematics.

Officers for 1961-62 are: Director, Errol Pomerance; Vice Director, Marc Plotkin; and Secretary, Lester Rubenfield.

ZETA OF PENNSYLVANIA, Temple University

The Pennsylvania Zeta Chapter held eighteen meetings during the academic year **1960-61.** The following papers were presented:

"Homogeneous Coordinates," By Dr. Albert Schild.

"The Transcendence of **e,"** by Mr. **Eli** Mandelbaum.

"The Job of an Actuary," by Mr. Robert Ward, Associate Actuary of Provident Mutual Life Insurance Company.

"Measures of Simplicity," by Dr. Sidney **Axinn.**

"Zeno's Paradoxes and Their Resolutions," by Mr. David Drasin.

"Calculus of Finite Differences," by Irwin Goodman.

"Mathematics and Economics," by Mr. David Ostroff.

"Group Theory in Physics," by Dr. Henri Amar.

"**Problems** in Graduate School and in Industry," by Mr. Thomas Slook.

"The History of the Mathematics Department at Temple University," by Dr. Walter S. **Lawton.**

On March **11, 1961,** a mathematics conference was held. The students were divided into four groups and in each group there were two speakers. The speakers and their subjects were as follows:

Group I	Subject
Dr. F. Gavalos General Electric Company Missiles and Space **Vehicles** Department Philadelphia, Pennsylvania	"What is Applied Mathematics?"
Mr. Thomas Slook Assistant Professor of Mathematics Temple University Philadelphia, Pennsylvania	'The Difference between Problems in College and Problems in Industry."
Group II	
Mr. Malcolm Smith **Remington** Rand Univac Division of Sperry Rand Corporation Blue Bell, Pennsylvania	"Programming **Computers--** A Profession for You."
Dr. Margaret Lehr Chairman, Mathematics Department Bryn Mawr College Bryn Mawr, Pennsylvania	"The New Mathematics " or "The New Look in Mathematics."
Group 111	
Dr. Martin Zeffert Fidelity Mutual Life Insurance Co. Philadelphia, Pennsylvania	"General Duties of an Actuary."
Dr. Henri **Amar** Associate Professor of Physics Temple University Philadelphia, **Pennsylvania**	"Mathematics in College Physics."
Group IV	
Mr. Maurice **Slud** General Electric Company Missiles and Space Vehicles Dept. Philadelphia, Pennsylvania	"Problems in an Industrial Laboratory."
Brother Damian Chairman, Mathematics Department Lasalle College Philadelphia, Pennsylvania	"Theoretical Mathematics."

ZETA OF PENNSYLVANIA. Temple University

Officers for the fall semester, **1960-61** were: Director, Nicholas Macri; Vice-Director, Leonard Braitman; Secretary, Barbara G. Hoffman; and Treasurer, **Lilyan** Fireman.

Officers for the spring semester, **1960-61** were: Director, Leonard Braitman; **Vice-Director,.Carl** Benner; Secretary, Barbara G. Hoffman; Assistant Secretary, David Drasin; and Treasurer, **Lilyan** Fireman.

New officers for the fall semester, **1961-62** are: Director, David Drasin; Vice-Director, Carl Benner; Secretary, Sheila Machinton; Assistant Secretary, Eleanor Griff; and Treasurer, Betty Rabinowitch.

ALPHA OF NEVADA, University of Nevada

The Nevada Alpha Chapter held four business meetings and several program meetings during the academic year **1960-61.**

A new annual award was presented to Miss Gail Chadwick as the most outstanding freshman student in mathematics. Alpha Chapter co-sponsored with the Department of Mathematics the annual Nevada State Prize Examination in High School Mathematics as well as several lecturers. The lecturers were: Professor Leo Moser of the University of Alberta and Dr. Reaves of the American Astronomical Society.

Officers for **1960-61** were: Director, Eugene Isaeff; Vice-Director, Jerry **Morrison;** and Secretary-Treasurer, Margot Mills.

Officers for **1961-62** are: Director, Albert King; Vice-Director, John Miller; and Secretary-Treasurer, Daniel **Kinkel.**

ALPHA OF MICHIGAN, Michigan State University

The Michigan Alpha Chapter held seven meetings during **1960-61.** The following papers were presented:

"Constructions with a Compass Alone," by Dr. Leroy Kelly.

"Lattice Point Problems," by Dr. John Kelly.

"The Definition of the Natural Numbers," by **Dr.** Paul Axt.

"Some Problems in Topology," by Dr. Patrick Doyle.

The guest speaker for the annual banquet was Dr. R. H. Ring from the University of Wisconsin.

Officers elected during the winter term were: Director, Fred **Gilman; Vice-**Director, Stanley Steinburg; Secretary, Carolyn Hams; and Treasurer, Merlin Wheeler.

BETA OF MONTANA, Montana State College

Montana Beta Chapter held six meetings during the academic year **1960-61.** The following papers were presented:

"The Brachisfochrone," by Mrs. **Vinnie** Miller.

"The Meaning of Number," by Dr. Hans R. Fischer.

"*A* Construction in Semi-Groups," by Mrs. Ellen **Missall.**

"Testing the Truth of Astronomical Hypotheses," by Dr. Robert **Kraft,** visiting lecturer.

"Maximum Likelihood Function," by Mr. **Willis** Alberda.

"The Application of Determinants and Matrices to the Euclidean Algorithm," by Mr. Donald Kastella.

Dr. **Leo** Moser, visiting lecturer from the University of Alberta, was the speaker at the annual initiation, where twenty members were initiated.

Montana Beta instituted a prize in mathematics to be given to the outstanding junior and senior student in mathematics at the annual honors day assembly. The junior prize, **$25** worth of mathematics books, was presented to Mrs. Kay Leibrand. The senior prize, likewise **$25** worth of mathematics books, was given to Mr. Gary Sackett. These prizes were made possible by a contribution from the national organization made on a matching basis.

Dr. William Swartz became faculty advisor of the Montana Beta Chapter replacing Professor J. **Eldon** Whitesitt who was becoming head of the Mathematics Department.

BETA OF ALABAMA, Auburn University

Mr. James N. Issos has been nominated as Beta Chapter's delegate to attend the summer meeting of Pi Mu Epsilon at Stillwater, Oklahoma.

Officers for 1961-62 are: Director, Philip F. Pollacia; Vice-Director, Edgar Rose; Secretary, Peggy Jo Smith; Treasurer, Alfred Lasaine; Corresponding Secretary, S. L. Thompson; and Faculty Director, Dr. R. K. Butz.

GAMMA OF NEW YORK, Brooklyn College

The New York Gamma Chapter held thirteen meetings during the academic year 1960-61. The following papers were presented:

"Turing Machines, Their Significance to Mathematics and Philosophy," by Professor William Miller.

"What Moves and What Doesn't: Some Fixed Point Theorems," by Mr. Jay Goldman.

"Odd Perfect Numbers," by Professor Forman.

Officers for the spring semester, 1960-61 were: Director, Jay Goldman; Vice-Director, Eugene Spiegel; Secretary, Ann Fassler; and Treasurer, Nathaniel Reisenberg.

Officers for the fall term, 1961-62 are: Director, Nathaniel Reisenberg; Vice-Director, Lawrence Smith; Secretary, Leon Gerber; Treasurer, Irving Kessler; and Faculty Advisor, Professor Woodbridge.

ALPHA OF VIRGINIA, University of Richmond

Virginia Alpha Chapter held nine meetings during the academic year 1960-61. The following papers were presented:

"Elementary Point-Set Topology," by Professor Marion Stokes.

"Linear Programming," by Professor James B. Worsham.

"The Atwood Machine," by Paul Smith.

"An Introduction to Game Theory." by Betty Pritchett.

"Solving Problems," by Raoul Weinstein.

"Graduate Training and Careers in Mathematics," —a panel discussion led by members of the mathematics staff.

Winners of prize examinations sponsored by Alpha Chapter were: Freshman Mathematics, First Prize—Joel Gaydos ($10); Second Prize—William Cale ($5). Sophomore Mathematics, First Prize—Thomas B. Vassar ($10); Second Prize—Dotti Williams ($5).

Awards presented were: Best paper of historical or descriptive nature (The Development of the Calculus by Joyce Steed); best technical or research paper (The Laplace Transformation by Mary Frances Wright).

William C. Roberts, a member of the chapter and a mathematics major, completed his undergraduate career with an academic record of all "A" grades, the fourth student in the history of the University to achieve this record. He was named the winner of the James B. Crump Prize, a University award for excellence in mathematics.

The guest sneaker for the annual banquet was Professor Robert N. Thrall of the University of Michigan. Professor Thrall was a Visiting Lecturer of the Mathematics Association of America. Both Professor and Mrs. Thrall were honored guests at the banquet.

Officers for 1960-61 were: Director, Joyce Steed; Vice-Director, Raoul Weinstein; Secretary, Ann Jones; and Treasurer, Betty Pritchett.

Officers for 1961-62 are: Director, Paul Cohen; Vice-Director, Alice Hall; Secretary, Charlotte Adams; and Treasurer, Harold Smith.

ZETA OF OHIO, University of Dayton

The Ohio Zeta Chapter held meetings once a month during the academic year 1960-61. The following papers were presented:

"Elements of Quantum Theory," by Mr. Brian Brady.

"A Comparison of Proofs of Desargue's Triangle Theorem," by Mr. Ralph C. Steinlage.

"Transcendence of e," by Mr. John M Wells.

Two lectures by Professor Leon Henkin from the University of California were given during the year.

Six new members were initiated this year. Faculty members who were previously associated with other chapters were invited to affiliate with this chapter.

An Annual Award was made to the sophomore who was outstanding in mathematics; Mr. Thomas J. Grilliot was the recipient of this year's award.

Officers for 1960-61 were: Director, Lawrence Raiff; Vice-Director, John M Wells; and Secretary-Treasurer, Ralph C. Steinlage.

Officers for 1961-62 are: Director, Ralph C. Steinlage; Vice-Director, Walter Fuchs; and Secretary-Treasurer, Thomas J. Grilliot.

ALPHA OF KENTUCKY, University of Kentucky

The Kentucky Alpha Chapter held five meetings during the academic year 1960-61. The following paper was presented at the program meeting:

"On Combinational Relations and Chromatic Graphs," by Lael Kinch.

Winner of the freshman mathematics examination of the Pi Mu Epsilon H. H. Downing book award for scholastic achievement was Roger D. Rosenbaum. Winners of the Pi Mu Epsilon Distinguished Mathematician Book Award for scholastic achievement were Jeanne B. Shaver and Roger D. Rosenbaum. Pi Mu Epsilon Key Award for the best mathematical content seminar paper was presented to Clifford Swauger and the Pi Mu Epsilon Key Award for the best graduate seminar paper was presented to Richard A. Mulliken. Miss Martha Watson and Mr. Lael Kinch were recipients of National Science Foundation Cooperative Fellowships for the academic year 1961-62. Mr. Jerry King received a National Science Foundation Cooperative Fellowship for summer, 1961.

Officers for 1960-61 were: Director, William Sledd; Vice-Director, John Pfaltzgraff; Treasurer, Clifford Swauger; Secretary, Barbour Lee Perry; Librarian, Thomas Bagby; and Faculty Advisor, Dr. T. J. Pignani.

Officers for 1961-62 are: Director, John Pfaltzgraff; Vice-Director, Clifford Swauger; Treasurer, Tom Bagby; Secretary, Evelyn Rupart; Librarian, Jim Caveny; and Faculty Advisor, Dr. T. J. Pignani.

The guest speaker at the annual banquet was Professor Lewis Wellington Cochran of the Department of Physics who gave a speach on "Recent Research in Atomic and Nuclear Physics at the University of Kentucky."

BETA OF FLORIDA, Florida State University

The Florida Beta Chapter held seven meetings during the academic year 1960-61. The following papers were presented:

"The Calculus of Variations," by Forrest Dristy.

"The Pseudo Arc," by Dr. Morton L. Curtis.

"Combinatorial Topics," by Dr. Marion F. Tinsley.

"UICSM Program," by Dr. Max Beberman, University of Illinois.

"Game Theory," by Dr. Robert Thrall, University of Michigan.

During the year twenty-three new members were initiated. At the annual banquet in May winners of the Pi Mu Epsilon sponsored mathematics contest were announced. They were as follows: Emile B. Roth, first place; and John C. Wells, second place. The banquet speaker was Dr. Thomas R. Lewis, Assistant Dean of the Graduate School.

Officers for 1961-62 are: Director, Donald VanderJagt; Vice-Director, Donald Kiser; Secretary-Treasurer, Naomi Cheeley; and Faculty Director. Dr. Ralph McWilliams.

ALABAMA ALPHA, University of Alabama (December 13, 1961)

Janet H. S. Allsbrook
Margaret E. Hall
Gerhard P. Beduerftig
Glen R. Bethune
Elias Issa Bishara
Clarence A. Blackwell
James A. Blackwell, Jr.
Robert E. Boone, Jr.
William Gary Briscoe
Bobby Franklin Bryan
James Lawrence Byers
Jelks H. Cabaniss, Jr.
Ronald William Case
Charles M. Chambers, Jr.
George David Cole
William B. Compton, III
George Larry Copeland
Manning J. Correia
Bryan Leonard Coulter
Phillip Earl Deering
Felix F. Delgado
Ray Michael Dowe, Jr.
Robert B. Driver
Gary Neil Drummond
Sarah Frances Duke
Owen James Felis
Thomas W. Franklin
John R. Gilbert, Jr.
Stanley Morris Guthrie
Don Ward Haarbauer
Emmet Lavern Hadding
William Hartselle, HI
Charles F. Hendrix
Harry H. Holliman
James Melvin Horner
Paul Lewis Jernigan
Gordon Lee Kelly
Frank H. Kendall
George David Lacy
William Hugh Lawler
Ernest Bryan Longmier
AM Bullock Martin
Clayton A. McAdams
Bernard D. McElroy
Samuel Rex McWaters
Eldon Lynn Miller
Alvin D. Moore
Truitt Moore
Edward P. Moravec, Jr.
John Max Nelson
Ghassan B. Osman
James Lester Pate
Chris Pane
Jerry K. Patterson
Albert E. Peirce, IV
Bryan Wayne Pilgrim
Felix Logan Pitts
James Eldridge Pope
Bonnie Carl Riddle
Charles Louis Seebeck, III
Julio L. Serra-Jones
Frances Anita Sikes
Robert Silber
Jan Harald Sodergren
William Ray Strahan
Virgil H. Strahan
Howard Franklin Trucks
William S. VanDevender
George Eliot Weeks

ALABAMA BETA, Auburn University (June 1, 1960)

Leon Hartwell Allen, Jr.
Henry A. Bailey
John Hone Bartol, Jr.
William Dalton Breakfield
George E. Clausen, Jr.
George Robert Connolly
Kenneth Harold Cranford
Robert Todd Downey
Mary Fondle
Ronald Lee Funderburg, Jr.
Robert A. Hale
Faye C. Harris
Julia Frances Harris
Harold Harrison
Melvin V. Hood, Jr.
Gordon W. Hutt
Charles Lauffer
Ragan Bun Madden
Brenda E. Minus
George Miller
Linda Pollard
Edgar B. Rose
Edward J. Seiffert
William Larry Smith
Forrest E. Steber
Walter Gary Suttle
Barbara Joyce Thomas
Carol Thomas
AM Martina Walker
Margaret Wilcox

ARKANSAS ALPHA, University of Arkansas (March 1961)

Linda Lee Borden
Jim Dee Brazil
Mike D. Bums
Sherwood J. Charlton
Charles C. Chesley
William C. Coe
Margaret T. Cravens
William D. Evans
Raymond D. Fitzgibbon
Charles Henry Gibson
John James Harton
Phillip Maxwell Jolley
Stephen Lowell Kooker
Harry R. Lirtrel
Michael L. Meistrell
Edward Newton Mosley
James Robert Norwood
Martha Ella Moore
Jerry Rowland Rogers
David Paul Ross
David John Sloop
Troy Lamar Wilson
Joe Robert Dyer
John Herman Ivester

CALIFORNIA ALPHA, U.C.L.A., (May 1961)

James A. Adams
George G. Barnes
George Beckwith
Richard Binder
Joy A. Burkhart
Dolores Cangiano
Daniel Etter
Joel Friedman
Derek J. Fuller
Robert Gordon
Marilyn Heller
Sheldon A. Herman
David L. Hilliker
Arthur Z. Kitzler
Adam M. Kliszewski
Walter H. Kroy, Jr.
Jerome S. Lipman
Henry L. Loeb
Thomas A. McCullough
Michael Minovich
Giovan Natale
Weston I. Nathanson
K.V. R. Rao
Evelyn M. Read
Lewis C. Robertson
Stephanie Rosen
John H. Sandoz
George Senge
Richard G. Shook
Stoddart Smith, Jr.
Michael E. Tarter
Arnold L. Villone
Robert Wengler
Joel J. Westman
Roger Williams
Pritchard White
William R. Yucker

CALIFORNIA ALPHA (continued)

(February 1960)

George L. Johnston
Betty Y. T. Lou
Carl R. Webb

(March 1961)

William Hall
Richard Tafel

CALIFORNIA GAMMA, Sacramento State College (March 22, 1961)

David Glen
Philip A. Van Veldhwzen
Robert J. Wahl

DISTRICT OF COLUMBIA ALPHA, Howard University (May 19, 1961)

Jerome Antnony Atkins
Orell Oliver Desnoes
Dean Lewis King Downing
Christopher Benedictus Gadegbeku
Salome Ursula Hanley
Vincent I. Henry
Giri S. Lingappaiah
Herbert J. Miller
Dr. Albert L. Scipio, II
Victor Brentnal Smith
James Daniel Steele

FLORIDA BETA, Florida State University (May 1961)

Howard C. Brown
Paul A. Brown
Jewell Naomi Cheely
John P. Crews
Eleanor B. Dean
Richard Joseph Fleming
Edward P. Guettler
Elizabeth AM Magarian
Samuel Leverte McCall, Jr.
Eugene C. Padgett
Peter Milton Rice
Albert Thurman Siegrist

GEORGIA ALPHA, University of Georgia (March 1, 1961)

Carl Warren Franklin
Bradbury P. Foss, Jr.

(May 17, 1961)

Merle A. Barlow
William Horace Morgan

ILLINOIS ALPHA, University of Illinois (May 25, 1961)

William Russell Abel
Walter Edward Beck
Merlyn James Behr
John Grant Bergman
Dorothy Ann Bollman
John Dahlmand Bramsen
Richard Lyle Brewer
Judith Hoffman Brock
O. Robert Brown, Jr.
William F. Cutlip
Sr. M. Drufenbrock
David Ray Dyroff
Sen Fan
Kathy K. Forbes
Richard Phillip Giles
Richard Dixon Gillis
Henry Richard Hammes
Sr. Raymond M. Harty
Kad Cornell Kelley
Mona Dee Klebe
Robert Paul Kopp
Anatol Kuczura
Myrna Pike Lee
Charles Shi Tung Liang
Robert Dean Macek
Sister John McGrath
Ewen Austin McLennan
Sister Mary G. Meyer
Roy Alden Meyerholtz
John Peter Miller
John Nazarian
Wayne Aaron Norby
John Hoyt Painter
Leonard Leon Palmer
Louise D. Petermann
Robert Joseph Pfeifer
Bernice Vonhorn Pollyea
Robert Earl Rector
Joel William Robbin
Harold G. Robertson
Harl Edgar Ryder, Jr.
Steven Browning Sample
David George Schaeffer
Clare AM Seitz
Richard Eugene Shermoen
Michael Samuel Skaff
Mandyam Dhati Srinath
Constance M. Swanson
Douglas Henley Taylor
Frank Yungfoong Tse
Mary Ellen Walsh
Ann Regina Wertz
Walter Raymond Westphal
Russell Charles White
Lyman John Wilmot
Mary Katherine Yntema
Lester Jay Zimmerman

ILLINOIS GAMMA, DePaul University (February 8, 1961)

Robert V. Anderson
Arthur J. Becker
Kay Carroll
Harriet Coleman
Christine Cwiertniak
Donald A. Hausser
Don Lencioni
Robert Marchini
Carmin J. Scotti
Peter Stang
Michael Sullivan
Joseph Z. Yao, Ph.D.
Bernard Zak
Vincent Zippard
Philip Zipse

ILLINOIS DELTA, Southern Illinois University (May 15, 1961)

Frank Alex Asta
John Samuel Bmwn
Fred H. Bunnell
Jerry J. Cummins
Allen Loy Davidson
Bruce Davis
William A. Ettling
Donald Dean Funkhouser
Ronald David Hall
Rita Jane Henderson
Neil D. Jones
Glenn Richard King
Ivan J. Lach
Mary Theresa Lackey
Larry Lynch
Joseph M. Mabry
Sara Millspaugh
Hans Tymen Schreuder
Evelyn H. Seyer
Lawrence E. Wagy

INDIANA ALPHA, Purdue University (May 7, 1961)

Joseph G. Brown
Rex Comerford
David A. Edelman
Susan Krause
Marjorie M. Marlin
John Pearson
Ray P. Schrameyer
David Sherer
Thomas Smith
William Spell
Powell Sprunger
Tom Stafford
Peggy J. Stein
Robert A. Wagner
Steven A. Wallace
Katherine L. Yang

IOWA ALPHA, Iowa State University (May 27, 1961)

Robert J. Adolphson
James J. Allen
Billy D. Arendt
John E. Bailey
Thomas Arthur Beshaw
Cmmwell C. Bowen
Samuel Philip Bowen
Gary Franklin Braley
Dennis Keith Branstad
Irvin A. Christenson
Robert L. Cmft
David Risdon Crosley
J. Ann Davison
Charles David DeBoer
Gerald L. Dutcher
Kenneth James Dykstra
Zoe A. Ekquist
Virgil Bruce Elings
June E. Eyerly
Eldon M. Fisher
William S. French
John L. Glossinger
Roger M. Goetz
James E. Halligan
Roxanne Rae Harper
Richard Vernon Helgason
A. Douglas Herrmann
Craig D. Hollowell
Robert Ronald Kinsey
Howard Wilson Lambert
Roland Eugene Lentz
loan Urban Loeb
David O. Lomen
Sharon E. McDonald
Francis J. McGrath
Wayne T. Murphy
Hamld H. Niebel
Margaret Ohlson
Vilnis Ozols
Jerry Dean Pearson
Leonard Dean Peterson
Anita Pmfitt
Roger James Radloff
William C. Randall
Mary Patricia Read
Thomas Charles Reihman
Jean Louise Royer
James S. Rue
Charles E. Runge
Donovan F. Sanderson
Nancy Ann Wilbanks Sorensen
LeRoy Richard Thompson
Malcolm David Tobey
James R. VanDoren
Arthur W. Warrick
Sandra Wiggins Short
George Roland Wilde

KANSAS ALPHA, University of Kansas (April 28. 1961)

Richard C. Basinger
Harry E. Bean
Richard A. Bell
William E. Daeschner
Robert L. Ferguson
Barbara K. Foley
Frederick A. Foos
Sandra S. Janousek
Nancy J. Layle
Boo-Sang Lee
Patsy R. McCurley
Richard E. Phillips
Billy J. Power
Desmond J. Powers
Robert D. Rati
Erwin A. Schroeder
Frank R. Spitznogle
Mary L. Wheat
John W. White

KANSAS BETA, Kansas State University (May 8, 1961)

Marshall D. Anderson
Jason C. Annis
David H. Baehr
Edward C. Bertnolli
William B. Bickford
Jon B. Bryan
Lin-Chuan Cha
Roshan L. Chaddha
Sidney Hsin Huai Chow
William Critser
Bernard B. Evans
R. Richard Gobel
Duane G. Harder
Duane Arthur Huber
Daniel C. Jones
Paul E. Justus
Lucinda Lou Keller
Craig W. Lawson
Ronald P. Leinius
Donald H. Mabry
Harry W. McLaughlin
Michael H. Miller
Larry D. Nelson
William S. J. Pi
Charles T. Prevo
James A. Smith
Mary Jeane Starkey
Donna Sue Stratton
John Stephen Tripp

KANSAS GAMMA, University of Wichita (April 21, 1961)

Eric F. Bachert
Gary D. Crown
Roger A. Fields
Justus Fugate
John E. Fuller
Cecil Kurtz Gordon
George Dewey Harshberger
Newton Kenneth Hawthorne
Robert H. Hodge
Glenn L. Koester
Donald C. Martinson
Louis M. Michel
Richard N. Schuh
William F. Seigle, Jr.
Kenneth F. Seipel
Peter C. Van Der Voorn
Larry H. Werner

KENTUCKY ALPHA, University of Kentucky (May 11, 1961)

Bartlett Graves Dickinson
Martha Dickinson Frasier Hartono
Hughes B. Jenkins. Jr.
Richard Andrew Mullikin
James A. Pearson
Ikie Neal Presson
Adelbert Lee Roark
Irene Louise Rose
William Hayden Smith
James C. VanMeter
Charles M. Walker
James Bernard Willett

LOUISIANA ALPHA, Louisiana State University (May 17, 1961)

John Joseph Arnold
Claire Fasullo
Donald Wayne Hammons
J. M. Jose' Jullienne
Edwin E. Klingman
James L. Mullenex
Jon M. Plachy
Florence D. Sanchez
Jane F. Savely
Arthur T. Shankle
DeWitt L. Sumners
Jesus Urrechaga-Altuna

LOUISIANA BETA, Southern University (May 11, 1961)

Noah Booker, Jr.
James Carter
Gloria Mae Jordan
Walter Earl Lamorta
Charles H. Pugh
Ineatha W. Ruffin
Barbara Ann Vidrine

MARYLAND ALPHA, University of Maryland (April 28, 1961)

Joel A. Echison
Stanley Edlavitch
Elinor R. Evenchick
Theodore L. Felsentreger
Roberta L. Haase
George Kalish
Simon R. Kraft
John E. Lagnese
William Miller
Robert J. Pirih
Marvin C. Sachs
Evelyn R. Woolley
Dennis Wortman

MINNESOTA ALPHA, Carleton College (Charter Members)

Benet R. Brabson
James M. Bruce
Boyce Burdick
Stephen H. Burch
Linda M. Buswell
Martin Engert
Alison Krotter
Kenneth O. May
June L. Matthews
Charles L. Odoroff
Robert Oelhaf
Judith K. Roth
Seymour Schuster
Frederick W. Stevenson
Donald H. Taranto
Frank L. Wolf
Charles R. Wyman

MISSOURI ALPHA, University of Missouri (May 8, 1961)

Jesse Charles Alexander
Gerald Baker
George E. Bement
Kathryn Lee Boehm
Stanley Raymond Bull
Jim D. Campbell
John George Erhart
Richard Earl Fairley
Alice Lee Ferris
Laurel Elaine Ford
Stanley Nelson Garber
Yim Fun Gee
Lawrence D. Gore
George Allen Grimm
Jerry Robert Henry
Mary Alice Hudson
Marvin Joseph Jostes
Vladimir J. Kedrovsky
Thomas Gordon Kurtz
Leo Jemme Lange
Paul Larry Leath
Kenneth Rodger Limstrom
David Paul Lohman
Eldon Earl Miller
Robert Lemy Moffitt
Robert Arnold Montgomery
Jimmie Lee Nelson
William Francis Obermark
Charles Vinton Ohrenschall
Gary M. Redeker
Bob G. Rees
Vernon David Reynolds
Roger John Rohlfs
Anthony Frank Rolli
Francis Michael Sharp
Kathryn Made Shiveley
James Houston Whitt

MISSOURI GAMMA, St. Louis University (April 20, 1961)

William Abkemeier
Sr. M Magdalene Baca
Dale C. Barnum
Thomas Berry
Marianne Bueltmann
Suthep Chanthrasomsak
Algimantas John Ciplickas
Ann Cunningham
Leo R. Dohogne
Stephen Dubas, S.J.
Edward J. Dunne
Thomas Ebner
Richard Elder
Margaret Mary Esker
Denis John Feld
Henry M Fendrich
Lawrence E. Follis
Dorothy Ford
Mary Ann Gagen
Edwin Heckman
Francis E. Judge, S.J.
Peter J. Kenny
George Kertz
Albert A. Klein, Jr.
M. Jane Krekemeyer
Mary Ann Kroupa
Shirley Leber
Richard Leitten, S.J.
William P. Lonc, S.J.
Elaine Louis
F. H. Luebbert
Mrs. James McBrayer
James B. McCormick
Mercedes McGlynn
Frank Thomas Magiera
Michael J. Mahon
J. J. Mansell
John Albert Mellen
Carla Montani
John J. Mossinghoff
Jane Mueller
Thomas B. Nenninger
Elizabeth C. Orange
Wade Peterson
Joseph Potts
Mary Ann Pozsgay
Gordon Francis Quinn
William Reddy
Edwin Riemer
Sharon Ryan
Wayne Leory Salus
Glenn A Sartori
Marilyn Scheer
Carl Scheibal
Margaret Anne Schmitt
Kathleen Ann Schoen
Francis A. Schiermeier
Vincent A. Scognamiglio
Jerome Shen
Michael Dennis Shovlin
Richard Francis Smith
Richard Sontag
Joan Made Stepzinski
Loretta Stock
Robert Paul Swift
Robert Van Haaren
Norval Dale Wallace
AM Beryl Wells
John Charles Wine

MONTANA ALPHA, Montana State University (May 10, 1961)

Bert Campbel Clinkingbeard
Frederick Matthew DeMatinis
Hilda Christine Holtz
Keith Melford
Roland J. Keefer
Allen Duane Luedecke
Diane Elizabeth Mossey
Richard Samuel Nankervis
Marleigh Chandler Sheaff
John Matt Ulvila
Robert H. I. Yang

NEBRASKA ALPHA, University of Nebraska (May 14, 1961)

Cad Joseph Bern
Dan Roger Blazek
Howard Eugene Bosley
James Denton Church
Donald Francis Costello
Floyd A. Erickson, Jr.
Jon Froemke
Kenneth Elmer Gobbet
Rodney Stuart Goering
Arvis Grindulis
David Blair Gustavson
Kenneth Anthony Heimes
Earl Lynn Kemper
Raymond Ralph Lange
Donald John McGurk
Ronald Melvin Mathesen
Lonnie Lee Miles
Robert Eugene Mitchell
Jack Lee Mundell
Michel Jon Rediger
Robert Lorn Scheffler
Lawrence Charles Smith
Olaf Martin Stokke
Charles William Stuber
Dwight V. Sukup
Dean Leroy Ulrichson
James Thomas Wallen
Michael Lee White
Bruce James Williams

NEVADA ALPHA, University of Nevada (May 18, 1961)

Roger Kent Ferguson
Gus Maddux Griffin
Michael J. Hebett
Mary Susan Hickman
Donald H. Hutcheson
Howard A. Hyatt
Albert James King
Daniel Robert Kinkel
John Campbell Miller
Richard Ward Morris
Merlin Lee Puck
Louis C. Smith
Robert David Smith

NEW HAMPSHIRE ALPHA, University of New Hampshire (April 11, 1961)

Mary E. Capron
Robert H. Colburn
Paul T. Cronshaw
Nguyen Duc Cuong
Gregory J. Eskijian
Faramarz Fardshisheh
John David Hourigan
Marilyn LaCava
Robert S. Lunt
Richard O. Norman
Richard W. Story
Judith A. Treamer
Ingrid Willeke
Klaus Willeke

(June 1961)

Douglas Earl Krebs

NEW JERSEY ALPHA, Rutgers University (April 4, 1961)

John Aklonis
Robert Boysen
Joseph Baumgarden
Anthony Cerami
Norman Father
George Fausel
Jay Feln
Arthur Galya
Michael Hagler
Daumants Hazners
Dennis Herzo
John Horvath
Allan Jenks
Kenneth Jungblut
Nicholas King
John Kjellgren
Walter Korowjow
Arthur Krumins
George Kwaitkowski
Donald Lehman
John MacDonald
Thomas Maniscalco
Nicholas Miskovsky
Edward Page
Alan Stevens
Robert Tucker
Peter Unterweger
David Van Derveer
Michael Vasile
Dennis Viechnicki
Richard Vnenchak
Robert Vojack
Carl Zanoni
George Zezenski
Anatol Zinchenko

NEW JERSEY BETA, Douglass College (May 2, 1961)

Allyn Jean Barnett
Elizabeth Ann Burd
Elaine R. Glickman
Elizabeth C. Horwitz
Deborah Kuryk
Lois J. Malon
Martha P. Medoff
Rosemarie G. Moreken
Catherine E. Reiser
Sarah W. Toms

NEW YORK ALPHA, Syracuse University (May 3, 1961)

Richard Adelman
Jane L. Alexander
Paul Francis Bala
Mira Bhargava
Lois H. Brady
Henderson D. Booth
Richard A. Brualdi
Polly A. Burkhead
Barbara L. Charles
Ronald H. Curran
James H. Carney
Michael M. Dach
Donald V. Davies
Andrew H. Davy
Paul G. Duchow
Earle D. Eason
Morton I. Fox
Lawrence Friedman
Lloyd A. Fulk
Bradley K. Fullager
Herbert Gish
Michael Henry Goldbaum
Katharine J. Kharas
Richard H. Klein
Seymour Laster
John La Veile
Diana Leitner
John S. Ludwich
Harry C. Marcus
Ross Martin
Marilyn S. Meinhardt
Charles K. Mesztenyi
Barbara Missert
Elizabeth L. Munro
James A. Paige
William L. Reddy
Edward S. Rose
Letty G. Shapiro
Beverly H. Sheeran
Linda A. Sullivan

NEW YORK BETA, Hunter College (March 28, 1961)

Doris Grossman
Ann Hahne
James Jones
Martin Kotler
Joanne Palmer
Norman Weiss
Joan Yares

NEW YORK GAMMA, Brooklyn College (April 17, 1961)

Michael H. Berkowitz
Isaac Chavel
Leon E. Gerber
Robert Kaufman
Irving Kessler
Michael Krosky
Stanley Osher
Bennett S. Pallant
Harriet Rovner
Harold Sackrowitz
Amy Shuster
Lawrence Smith
Jerome H. Spruck
Joel Stemple
William Vitale

NEW YORK EPSILON, St. Lawrence University (March 28, 1961)

John Herbert Reule

NEW YORK IOTA, Polytechnic Institute of Brooklyn (May 12, 1961)

Clifford Meyer Gerstenhaber
Paul Gilbarg
Fred Grossman
Dennis D. McGuire
Hugh S. Seidman
Michael Silverman
Howard I. Teich
Dieter Rolf Tuerpe

NEW YORK LAMBDA, Manhattan College (Charter Members) (April 1961)

Lawrence P. Chiappetta
John E. Duffy
Frank J. Gordon
Jeremiah Kelly
Stanley S. Leroy
Donald J. McCarthy
Paul F. Mercadante
Harry S. Neylan
Joseph J. Pero
Gabriel J. Pompilio
John T. Power
Richard J. Prior
Richard C. Wagner
Francis T. Walsh
Charles F. Weber

NEW YORK MU, Yeshiva College (Charter Members) (June 1961)

Richard S. Barth
Stanley L. Boylan
Hershel M. Fatkas
Aamn S. Fmchter
Michael Greenebaum
William Kantmwitz
Aaron Lebowitz
Pmf. Henry Lisman
Herman M. Presby
Prof. Harry E. Rauch
Louis Raymon
Joseph G. Tuchman
Sam Vogel
Benjamin Weiss
Shlomoh Wohlgemuth

NORTH CAROLINA ALPHA, Duke University (April 1961)

Frank Maurice Armhrecht, Jr.
Travis Charles Bmesche
Barbara Thompson Dale
Stanley David Echols
Louis E. Grenzer
James Page Hamilton
Scott Hampton Hendrix
Caroline Clark Hilton
Joan Hoffman
Annie Lewis Johnston
George Wesley Lockwood
James N. MacNaughton
John McHugh
Thomas O. McIntire
John Otto Meier
Robert Chapman Newman
Sidney Joseph Nurkin
Jesse Q. Ozholt
Bertie Kent Paylor
Carl Frederick Rolle
Robert E. Smith
Eugene H. Thompson, Jr.
Robert Earl Thornberry
Dabney W. Townsend, Jr.
Lynne Margaret Vogel
James M. White
Lyle K. Williams, II
Carol Ann Woodard

NORTH CAROLINA GAMMA, North Carolina State College (May 25, 1961)

Harvey Thomas Banks
Willard Lee Garrison

OHIO ALPHA, University of Ohio (May 19, 1961)

John F. Bridge
Mary AM Bright
C. Keith Cretcher
Mary Louise Davis
Kian Seng Dy
Henry Gates
Sally McConnell Ginet
Richard M. Gordon
Ronald Dane Graft
Edward Emerson Hagenlocker
Robert Ihnot
Stephen Kimbleton
Donald Allen Lawver
Charles R. MacCluer
John Olson
James F. Ramaley
Gerald J. Rankin
James Gordon Root
Dietrich Schmeer
Charles T. Shirkey
David C. Stickler
Price E. Stiffler
Duane Arthur Warner

OHIO GAMMA, University of Toledo (March 1961)

Karen L. Ackland
Randy G. Bohn
Gilbert S. Fair
Terrill K. Staley
Bmno A. Rosa

OHIO DELTA, Miami University (April 9, 1960)

Steven Hirst
Ned C. Hoelzer
Arthur L. Houston
Janet L. Johnson
Joel Lynn Morrison
Mary Mosser
David B. Phillips
John M. Puckett
Gerald L. Strategier
Hamld E. Zealley

(November 16, 1960)

James Paul Henderson
Marian Howe
Helen Louise Malott
Jane Pycraft
Constance M. Welsh
Madelon F. Whitenack

(April 5, 1961)

Douglas E. Camemn
William W. Ford
David Michael Henderson
Brady Duane Holcomb
Marian E. Holloway
Norman S. Humphrey
Richard S. Keister
Karl Chapman Kumpf

OHIO EPSILON, Kent State University (March 1961)

Vera Linda Chapman
Nicholas J. Cortese
Georganne Dutka
Alfred Joseph Ganse
Raymond Eugene Gmss
Robert O. Hall, Jr.
Ivan Glen Hargis
Vera Maria Hrabec
Manfred Benedict Kieser
Mary Lou Kinsey
Joan Debora Mikluschak
Eugene LeRoy Miller
Judith Naona Miller
Frank Bernard Molls
Joan Rees
William Dean Sell
Gerald D. Smith
Ronald Joseph Sovie
Richard E. Stonemetz
Clifford Earl Thompson
Yvonne Marie Torma
David R. Work
Laverne D. Yut

OHIO ETA, Fenn College (Fall 1961)

Alfred Gene Brown
Alfred Joseph Bmey
Yi Chang
Mabel Beeman Greber
Lawrence Walter Kreps
Shirley Ann Lilge
Frank Laverne Schweingmber
David George Shields
Marguerite Joanne Jost Hrabak
William Roger Geiger
Robert Martin Stark

OHIO ZETA, University of Dayton (May 11, 1961)

Edward William Albers
Walter Paul Fuchs
Thomas Edward Gantner
Howard S. George
Thomas Joseph Grilliot
Ronald Walter Thomas

OKLAHOMA ALPHA, University of Oklahoma (May 9, 1961)

Stanley N. Burris
Curtis L. Greer
Dorothy Ann Key
Earl P. LaFon
Leland M. Nicolai
Robert F. Rossa
Jose A. Villalba

OKLAHOMA BETA, Oklahoma State University (April 26, 1961)

Bill Anderson
Nancy Kay Beach
David Elliot Bee
Bmoks W. Bell
John H. Blankenship
Corrine Kay Carpenter
Piyasak Chu-kes
Louis E. De Noya
Robert Allen Ewing
Bob Fincher
Earl Donald Folk
Meredith R. Goss
Herman Patrick Hinrichs
Don Larry Huffman
Carmi Humes
Barbara Kimmell
Frank Koons
Larry N. Kysar
Jerry Lamb
Bmce Miller
Robert E. Miller
Hamld Gil Nickel
Robert E. Oliver
Dwayne A. Rollier
Edward L. Skidmore
Phyllis Ann Smith
Charles Stewart
Howard Theilen
Mary Lou Thunnan
Burge Tmxel

OREGON BETA, Oregon State University (May 24, 1961)

Ralph C. Applebee
Robert L. Baker
Barbara Jean Bauer
Edward T. Bayliss
Jose Bedregal
Ralph H. Bergstad
Richard S. Brabmok
Karen J. Butler
Arthur Calderon, Jr.
James Robert Carlon
Joel A. Carlson
James A. Carr
Dwight P. Clark
Thomas T. Claudson
Michael C. Clock
Dale R. Comstock
John Hariand Darrah
G. Fielden Dickerson
David W. Digby
Oren W. Dixon
Aeimuddin A. Faruqui
Dale E. Fisk
Gary James Ford
Daniel Richard Gmschkowsky
Peter Harvey
Victor Hauser
Stephen M. Henning
John Hutcheon
Robert E. Kvarda
Hamld G. Lawrence
Arthur D. Libeman, Jr.
Frederick Thomas Lindstmm
Dan W. Lozier
Joseph A. Mansfield
Jack Meredith
John S. Murray
K. Narayanan
Howard 9. Noonchester
Steve P. Ogard
Robert Stanley Ottinger
David Allen Pearson
Paddy Prenter
Wallace H. Reed
Wayne V. Roberts
Michael J. Schaer
Tom Alvin Stone
James W. Walker
Leonard Dale Webb
Clarence Robert Young

PENNSYLVANIA ALPHA, University of Pennsylvania (May 12. 1961)

Haig Alexander
James G. Beitchman
Alan J. Bennett
Rochelle Caplan
George L. Cassiday
Howard M. Darmstadter
Harvey Feldman
Hugo J. Finarelli
Stanley Friedman
Herbert M. Gintis
Phyllis E. Goss
J. Elaine Howe
Ziyad I. Husami
Judy A. Korman
Elizabeth M. Lamond
Leon Liebman
David S. Perloff
Wayne A. Rebhorn
Leonard E. Rennick
Marion Rita Sokal
Michael L. Swerdlow
Robert Wang
Barbara Ann Wisniewski

PENNSYLVANIA BETA, Bucknell University (April 26. 1961)

Bernard J. Johns
Joseph E. Koletar
Donald M. Olssen
J. Stephen Strobeck
Marlin A. Thomas
Virginia Margaret Vidinghoff

PENNSYLVANIA GAMMA, Lehigh University (December 7, 1960)

Grant S. Anderson
William R. Buehler
George E. Clark
John D. Cutnell
Thomas L. Gabriele
Ronald P. Groff
David H. Hibner
Charles G. Hodge
Richard B. Hoffman
Richard A. Inciardi
Howard B. Lange
Raymond E. Noonan
Paul J. Peters
James R. Rice
Ivan J. Sacks
Peter E. Solender
William E. Toikka
Walter B. Wagner
Richard E. Weiser
Lawrence H. Whitney
Douglas H. Yano

PENNSYLVANIA DELTA, Pennsylvania State University (May 10, 1961)

John Tressler Baskin
Edward Bolger
Charles Lawrence Booth
John Cheroki, Jr.
Paul A. Chiavacci
D. F. Clemson, Jr.
John E. Diem
Bernard E. Droney
Jere E. Fidler
Charles A. Gaston
Harvey Gerber
Donald L. Harnett
Robert W. Hawkes
William W. Hicks, Jr.
Janet M. Hughes
Barent Charles Johnson
Dorothy L. Jones
Marine Louise King
Richard G. Luke
David Marker
William E. Mastrocola
Harvey Meyer
William B. Millhiser
Thomas H. Savits
Chalmers F. Sechrist
Bachchan Singh
William Randle Smith
Robert A. Wojtkowski
Gail Frances Whitfield
Richard Wilbur Zdarko
Patsy A. Zitelli

PENNSYLVANIA ZETA, Temple University (June 7, 1961)

Sandra Greenfield
Eleanor Gdff
Joan L. Hillwerth
Lawrence S. Jackman
Murray Kirch
Barry Litvack
Estelle Ludwig
Elaine Simonson

SOUTH CAROLINA ALPHA, University of South Carolina (May 17, 1960)

Edgar A. Altman, Jr.
William H. Castine
Eleanor Hazel Cmwn
Anthony C. DeLoach
Charles Harold Hall, Jr.
William Edward Headley
Donald Leroy Husa
John J. Jackson
Philip W. Koetsch
Wai-Kit Keung
Jesse W. Moore
Joseph Peter Nicoletti
John H. Overton, Jr.
Roderick Ead Smith
Cad Vassy Stone
David Stuckey Watson

(May 16, 1961)

Lemy Li-gong Chang
Forrest James Crayford, Jr.
Aubrey Shealy DesPortes
Eugene Lee Dominy
Marvin Hue Edgar Eargle
Elnathan Haskell
John Malcolm Johnstone, Jr.
Stephen Alton McMillan, Jr.
Joseph Emil Matt
Patricia Irene Mobley
Howard Edward Moore, Jr.
John William Shirley
Robert E. Spivack
William Cleveland Smith

(June 10, 1961)

Kamal M. Bitar
Jeffrey Z. Brooker
Judy Lee Hagood
Billy Hoyte Maddox
Joseph Frederick Neely
Thomas Charles Sayetta
Joseph Leslie Williams, Jr.

SOUTH DAKOTA ALPHA, State University of South Dakota (April 6, 1961)

Allen Fred Anderson
Arthur Paul Alberding
Susan Belle Colvin
Beverly Anne Cooper
Thomas Lee Eichman
John Cad Gertendch
William John Gray
Sandra Ann Haisch
John W Handsuch
Russell Floyd Hanson
Louis Charles Knoll
Bruce Calvin Linder
John Allen Lushbough
Jane Eliot Nickolisen
Cliff Duane Pomeroy
Robert Paul Schenk
Marilyn Margaret Sen
Leander Adolph Stroschein
Edward Earl Wutsch

TEXAS ALPHA, Texas Christian University (April 27, 1961)

Temple Baker
Lillian Made Bales
Rodene T. Capalongan
H. Margaret DuBose
Billy S. Goodson
Hadey Ead Hicks
Charles Walter Kight III
Doris Suzanne Luton
Richard Lysiak
Mary Jo McCrorey
Mary E. Miller
Laura Jane Read
Hugo John Selinger
William E. Simon
Paula Thompson
Bobby R. Uzzell

VIRGINIA ALPHA, University of Richmond (April 24, 1961)

Reubena Catherine Connaway
Mary Louise Grayson Foy
Barbara Sue Oglesby

VIRGINIA BETA, Virginia Polytechnic Institute, Charter Members (May 1961)

John Jerry Bartko
Paul Francis Clemens
George Edward Culbertson
George Ernest Gautney, Jr.
William Alexander Glenn
Svend Theodore Gormsen
Boyd Harshbarger
Thomas Watkins Hatcher
Flora Elizabeth Hickerson
Vymn Martin Klassen
Clyde Young Kramer
Leonard McFadden
Carl Allan Persinger
Eldon Eugene Posey
Leo Arthur Robinson

WASHINGTON GAMMA, Seattle University (May 12, 1961)

Raymond Louis Bernadelli, Jr.
Jack N. Fox
Lawrence Charles Hebner
Mary Ann Hoare
Leo Landkammer
Sr. Judith Ann McPhee
Chung-Jen Tan
Lawrence Michael Wagner

WISCONSIN ALPHA, Marquette University (April 16, 1961)

Douglas James Bohn
Jill Louise Clausen
Robert John Ennis
Michael Lee Gauer
Susan Alice Hahn
John Anthony Immekus
Raymond Michael Kramer
Gerald G. Madson
Donald Francis Madura
Judith Francine Marks
Gerald Lawrence May
Anthony Nickolaus Michel
Marie Ann Puetz
Patricia Ann Quade
Ronald Peter Reak
Daniel Richard Revall
Damian Earl Schumacher
Thomas Raymond Siffeman
Richard John Talsky
Channaine Camille Tucker
James Tylicki

WISCONSIN BETA, University of Wisconsin (April 1961)

Carl N. Brooks
Donald R. Ceilesh
Ronald F. Chapuran
Hildur A. Davey
Dirk A. Dahlgren
Ronald G. Faich
Louis D. Friedman
Charles H. Giffen
Elaine M. Good
Jeanette A. Hinkins
Clinton J. Kolaski
Warren M. Krueger
Palma A. Mark
Roxanne Naeseth
David M. Ozonoff
Barbara Simon
Jonathan D. Sondow

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