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A GEOMETRIC INTERPRETATION OF THE SOLUTIONS OF 3x3 RECTANGULAR GAMES[1]

VIRGINIA S. TRASHER

INTRODUCTION

The rectangular game, which we will consider, is a game of strategy between two players, P_1 and P_2, consisting of one move, in which the players simultaneously make a choice from a set of alternatives available to them. Corresponding to each of the possible pairs of choices available, a number is assigned which represents the amount paid to P_1 by P_2. Any rectangular game can be represented by an mxn game matrix where P_1 has m alternatives and P_2 has n alternatives. The entry a_{ij} corresponds to the amount paid to P_1 when he makes the choice "i" and P_2 makes the choice "j".

A solution of the rectangular game includes the value of the game, which is the maximum of the minimum expectations to P_1, plus an optimal strategy for each player, that is, a method of playing for P_1 such that he can expect to gain at least v, the value of the game, and a method of playing for P_2 such that he can expect to pay P_1 not more than v.

The value of the game is unique, but the optimal strategies for each player may be unique or may be infinite in number. In the latter case, the set of all optimal strategies is determined by a finite number of optimal strategies.

A well-known geometrical interpretation of the 2xn (or nx2) game can be used to completely determine the solution of the game for both players. If P_1 has two choices, any optimal strategy $\| x, y \|$ must satisfy the conditions $x \geq 0$, $y > 0$ and $y = 1 - x$. Linear equations which represent the expectation, E, to P_1 for each possible choice of P_2 are plotted with E as the vertical axis and x as the horizontal axis. If we consider the infinite strip bounded by $x = 0$, $x = 1$ and with a least upper bound determined by portions of the lines which determine the expectation to P_1, we find that the solution is given by those points of the upper bound where the E-coordinate is a maximum. See Figure 1.

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[1] Received by Editors Aug. 25, 1960. Resented at the National Meeting of Pi Mu Epsilon, East Lansing, Michigan, Aug. 30, 1960.

$$A = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$$

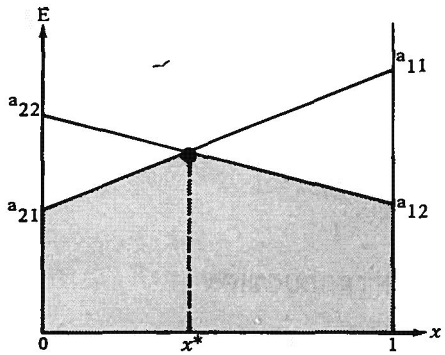


Figure 1: Solution Point

This same method can be generalized to 3-dimensional games, but instead of working with lines intersecting in a plane, we are concerned with planes intersecting in a prism. The solution points will then be maximum points in a volume determined in a similar manner to the area described above.

Since the points in this volume which determine the solution are those points which maximize the minimum expectation, or minimize the maximum expectation, depending on the player being considered, these points will be determined by the lines which are intersections of **the expectation** planes. By studying these lines of intersection as they are projected into the plane $E = 0$, we can learn much about the nature of the solution by considering a 2-dimensional graph.

GEOMETRIC INTERPRETATION OF 3x3 GAME

Given the matrix A, of the 3 x 3 rectangular game, where

$$A = \begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix},$$

we will denote a strategy, X, for P_1 by $\|x, y, z\| = \|x, y, 1-(x+y)\|$, since $x + y + z = 1$ for any strategy.

We write the following equations making use of each choice available to P_2.

(1) $E = (a_{11} - a_{31})x + (a_{21} - a_{31})y + a_{31}$

(2) $E = (a_{12} - a_{32})x + (a_{22} - a_{32})y + a_{32}$

(3) $E = (a_{13} - a_{33})x + (a_{23} - a_{33})y + a_{33}$

These three equations represent the expectation to P_1 for any strategy he may choose, in terms of each corresponding choice available to P . Geometrically, these equations represent three planes in 3-dimensional space. See Figure 2.

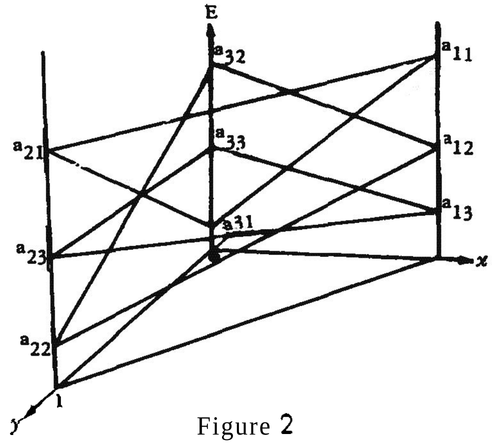


Figure 2

We shall refer to these planes as Ex-planes. Since every strategy must satisfy the condition that $x + y + z = 1$ and $x \geqq 0$, $y \geqq 0$, $z \geqq 0$, the solutions of the game are contained within the infinite prism bounded by the planes $x = 0$, $y = 0$ and $x + y - 1 = 0$.

DEFINITION: The Ex-prism is an infinite triangular prism bounded by the planes $x = 0$, $y = 0$ and $x + y - 1 = 0$.

DEFINITION: The Ex-volume is the portion of the Ex-prism without a lower bound and with an upper bound determined by portions of the Ex-planes such that no Ex-plane cuts the interior of the Ex-volume.

P_1 wishes to choose a scheme of making a choice at each play such that his expectation will be a maximum, regardless of the choice P_2 makes. P_2 wants to play in such a way that he must pay the minimum amount to P_1; therefore, P_1 should play in such a way that his expectation will be the maximum of the minimum expectations.

In order to maximize his expectation, P_1 must play a strategy which corresponds to the maximum E-coordinates of the Ex-volume. Therefore, the x and y coordinate corresponding to these maximum points of the Ex-volume determine optimal strategies for P_1. We shall call these maximum points solution points.

The problem then is to find all solution points.

Since most graphing is done in 2-dimensions, the 3-dimensional interpretation does not provide a good method for investigating the solution points. If the Ex-planes intersect within the Ex-prism, the lines of intersection of the Ex-planes with each other and with the sides of the Ex-prism contain all possible solution points, and therefore, we can project these lines into the plane $E = 0$ and examine them for solution points. If the planes do not intersect within the Ex-prism, it is necessary to investigate only the intersection of the Ex-planes with the sides of the Ex-prism.

All solution points will lie within the triangle bounded by the lines $x = 0$, $y = 0$ and $x + y - 1 = 0$. We shall call this triangle the Ex-triangle and the projected lines of intersection Ex-lines. Points of intersection of the Ex-lines with each other, points of intersection of the Ex-lines with the sides of the Ex-triangle and the vertices of the Ex-triangle represent possible solution points and will be called ***critical points.*** If only one of these is a solution point, P_1 has a unique optimal strategy. If two of these points are solution points, then all points on the line segment determined by these two points are solution points and P_1 has an infinite number of optimal strategies. If three or more points are solution points, then all points in the convex area determined by these points are solution points.

The procedure for finding all solution points is as follows:

1. Using equations (1), (2), and (3) obtained from the game matrix, eliminate the variable E between the three equations taken in pairs. This yields the equations of 3 vertical planes passing through the intersection of the planes taken two at a time. We are interested in the intersection of these planes with the plane $E = 0$. Denote these lines of intersection by L_{12}, L_{13}, and L_{23}, where the subscripts denote the planes which intersect.

2. Plot L_{12}, L_{13}, and L_{23} in the xy-plane by finding the x and y intercepts and/or the intersection of these lines with the line $x + y - 1 = 0$.

3. Using the x and y coordinates of each critical point, determine the strategy $\| x, y, 1-(x+y) \|$. Then calculate the three expectations corresponding to the three pure strategies available to P_2, as given by (1), (2) and (3). Associate with each critical point the minimum expectation of P_1. The maximum of these minimum expectations is the value of the game. All critical points which yield this maximum value are solution points.

We justify the above remarks by referring to the Fundamental Theorem for Arbitrary Rectangular Games which states that if S_m represents the set of all possible strategies for P_1 and S_n represents the set of all possible strategies for P_2, then

$$\max_{X \in S_m} \min_{Y \in S_n} E(X, Y) = \min_{Y \in S_n} \max_{X \in S_m} E(X, Y) = v$$

Since the lines we are considering represent the intersection of planes which may possibly form the boundary of the Ex-volume, points on these lines determine possible solutions to the game. By considering the expectations associated with each critical point, and considering the maximum of the minimum expectations at those points, we see by the above theorem that we have found the point or points whose corresponding strategies yield the value of the game.

There is a theorem of the theory of games which states that if $E(X^*, Y^*) = v$ and $E(X^*, Y) \geq v$, where X^* and Y^* denote a particular strategy for P_1 and P_2 respectively, and Y is any other strategy for P_2, then X^* and Y^* are optimal strategies for the respective players. Therefore, the point whose coordinates correspond to the value of the game yields the optimal strategy $\| x, y, 1-(x+y) \|$ for P_1.

If a solution point corresponds to one of the vertices of the Ex-triangle, the game has a saddle point. In this case v, the value of the game, is an entry of the game matrix which is at the same time the minimum of its row and maximum of its column. Thus, there is one choice that can always be made by each player which is an optimal strategy for him.

If a solution point is on one of the edges of the Ex-triangle, P_1 need never use more than two of his choices. If a solution point is interior to the Ex-triangle, P_1 may include all three of his choices in an optimal strategy.

If the solution point is unique and in the interior of the Ex-triangle, then P_1 must use all three of his choices in a mixed strategy. The same is true for P_2. In this ease the three planes intersect at a unique point which gives the maximum expectation for P_1. The game then has a unique solution for both players, since P_2 must use all three of his choices to insure that P_1 cannot gain more than v.

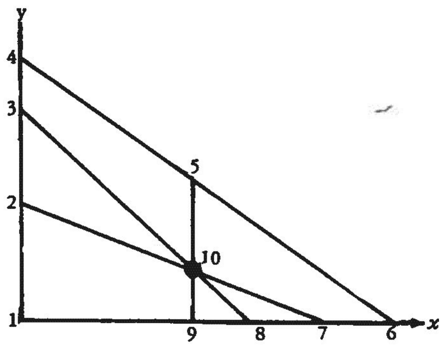
We illustrate the method just described by the consideration of the following example.

$$A = \begin{Vmatrix} 3 & 5 & 4 \\ 1 & 6 & 7 \\ 8 & 2 & 3 \end{Vmatrix}$$

$E = -5x - 7y + 8$ $\qquad$ L_{12}: $-8x - 11y + 6 = 0$

$E = 3x + 4y + 2$ $\qquad$ L_{13}: $-6x - 11y + 5 = 0$

$E = x + 4y + 3$ $\qquad$ L_{23}: $2x - 1 = 0$



Minimum Expectation at each Critical Point:

(1) (0, 0, 1): 2	= 2.00	(6) (1, 0, 0): 3	= 3.00
(2) (0, 5/11, 6/11): 42/11	= 3.82	(7) (5/6, 0, 1/6): 23/6	= 3.83
(3) (0, 6/11, 5/11): 46/11	= 4.18	(8) (3/4, 0, 1/4): 15/4	= 3.75
(4) (0, 1, 0): 1	= 1.00	(9) (1/2, 0, 1/2): 7/2	= 3.50
(5) (1/2, 1/2, 0): 2	= 2.00	(10) (1/2, 4/22, 7/22: 93/22	= 4.23

Conclusion:

$X^* = \| 1/2,\ 4/22,\ 7/22 \|$ $\qquad$ $Y^* = \| 4/11,\ 13/22,\ 1/22 \|$

$v = 93/22 = 4.23$

If the three Ex-planes intersect, we have at most ten critical points since the three lines of intersection either meet at a point, are parallel, or are co-incident. This gives us the three vertices of the triangle, the six intersections of the lines with the sides of the triangle and the point of intersection of the three planes to consider. However, the point of intersection of the three planes may be outside the Ex-triangle, or some pair of lines may be parallel, thus reducing the number of critical points. There are at least three critical for every game, since each vertex corresponds to a possible saddle point.

The reader will recall that a common method of solution of the rectangular game involves considering all square submatrices contained in the game matrix. In the case of the 3x3 game this would involve considering ten matrices which corresponds to the consideration of the ten critical points.

The above method gives the optimal strategies for P_1 and the value of the game, but does not directly give the optimal strategies for P_2.

A similar interpretation would yield the strategies for P_2 by changing the definition of Ex-volume such that it is the infinite part of the Ex-prism with no upper bound and with a lower bound determined by portions of the Ex-planes, such that no Ex-plane cuts the interior of the volume. Then at each critical point, the maximum of the expectation to P_1 is considered. The points which represent the minimum of these maximum values are the solution points and determine all optimal strategies for p,. However, the above game can be examined for P 's optimal strategies by the solution of three equations in three unknowns, since we know from the optimal strategy for P_1 that P_2 has a unique optimal strategy which makes use of all three choices.

The following equations yield the optimal strategy for P_2 in the preceding example.

$$3x + 5y + 4z = 93/22$$
$$x + 6y + 7z = 93/22$$
$$8x + 2y + 3z = 93/22$$

REFERENCES

1. Kuhn, H. W. and Tucker, A. W. (editors), "Contributions to the Theory of Games," *Annals of Mathematics* Study No. 24, Princeton University Press, Princeton, 1950
2. Kuhn, H. W. and Tucker, A. W. (editors), "Contributions to the Theory of Games," *Annals of Mathematics* Study No. 28, Princeton University Press, Princeton, 1953.
3. McKinsey, J. C. C., *Introduction* to *the Theory of Games,* McGraw Hill Book Company, New York, 1952
4. Vajda, S., *The Theory of Games and Linear Programming,* John Wiley and Sons, New York, 1957.

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ON THE NUMBER OF REPRESENTATIONS BY CUBIC POLYNOMIALS MODULO P.*[1]

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Let p be a prime number and let f(x) be a polynomial modulo p whose degree, m, is a divisor of $p - 1$. The following conjecture has been proposed:

Conjecture. If f(x) assumes as few distinct values as possible (that is, $\frac{p-1}{m} + 1$), then f(x) is essentially an m^{th} power. In other words, $f(x) \equiv a(x + c)^m \pmod{p}$.

The conjecture clearly holds for $m = 2$. In this paper the conjecture is proved for $m = 3$. Throughout the paper p will stand for a prime number greater than 2.

Theorem 1. If f(x) is a polynomial of degree m and if m is a factor of $p - 1$, then f(x) assumes at least $\frac{p-1}{m} + 1$ distinct values modulo p.

Proof. Suppose that f(x) assumes at most $\frac{p-1}{m}$ distinct values as x runs through the p values in a complete residue system modulo p. Since p is prime and since f(x) is of degree m, each such value can be assumed by f(x) at most m times. But this accounts for no more than $\frac{p-1}{m} \cdot m$ or $p - 1$ values of the variable x. Hence, f(x) must assume at least $\frac{p-1}{m} + 1$ distinct values.

Theorem 2. If f(x) is a polynomial, modulo p, whose degree, m, is a factor of $p - 1$, and if f(x) assumes $\frac{p-1}{m} + 1$ distinct values, one of which is assumed exactly once, then every other value of f(x) is assumed exactly m times.

Proof. Since f(x) assumes exactly $\frac{p-1}{m} + 1$ values and some value, say v_1, is assumed only once, the remaining values that are assumed, $v_2, v_3, \ldots, v_k$, where $k + 1 = \frac{p-1}{m} + 1$, must each be taken on exactly m times. For suppose one of them were to be taken on fewer

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than m times. Then at least one other value would have to be taken on more than m times. But this is impossible since no congruence of degree m can have more than m distinct roots modulo p.

Theorem 3. Any integer b, not congruent to 0 modulo p, $p \neq 3$, may be represented in at least two ways as the difference between a quadratic residue and a non-residue; that is, there exist residues w^2 and z^2, and non-residues u and v such that $b \equiv u - z^2$ and $b \equiv w^2 - v$.

Proof. Let g be a primitive root modulo p. We first show that there must exist integers α, β, γ, and δ such that

(1.) $g^{2\gamma+1} - g^{2\delta} \equiv 1$ and (2.) $g^{2\alpha} - g^{2\beta+1} \equiv 1 \pmod{p}$.

Since not every element in the sequence $1, 2, 3, \ldots, p - 1$ is a quadratic residue, we choose the first non-residue appearing in the sequence and express it in the form $g^{2\gamma+1}$. Since the element immediately preceding it in the sequence is a residue, we may express that element in the form $g^{2\delta}$ and write $g^{2\gamma+1} - g^{2\delta} = 1 \pmod{p}$. Thus (1.) certainly holds.

Suppose $\left(\frac{-1}{p}\right) = 1$. Then, in the sequence $(p - 1), (p - 2), \ldots, 3, 2, 1$ we may choose the first non-residue following $p - 1$ and express it in the form $g^{2\beta+1}$. The element immediately preceding it in the sequence is a residue and, consequently, can be expressed in the form $g^{2\alpha}$. Thus we have $g^{2\alpha} - g^{2\beta+1} \equiv 1 \pmod{p}$ and (2.) holds in this case.

If $\left(\frac{-1}{p}\right) = -1$ and $p \neq 3$, it cannot be true that every element of the set $1, 2, 3, \ldots, \frac{p-1}{2}$ is a quadratic residue. Indeed, $2^{\frac{p-1}{2}} \equiv -1 \pmod{p}$; so that 2 and $\frac{p-1}{2}$ must have opposite quadratic characters. Consequently, in the sequence $1, 2, 3, \ldots, p - 1$ there must be at least one non-residue which has a residue as its immediate successor. Expressing these elements in the form $g^{2\beta+1}$ and $g^{2\alpha}$ we have $g^{2\alpha} - g^{2\beta+1} \equiv 1 \pmod{p}$.

From (1.) and (2.) we obtain by repeated multiplication by g:

$$\begin{array}{llll}
(1.) & g^{2\gamma+1} - g^{2\delta} \equiv 1 & (2.) & g^{2\alpha} - g^{2\beta+1} \equiv 1 \\
 & g^{2\gamma+2} - g^{2\delta+1} \equiv g & & g^{2\alpha+1} - g^{2\beta+2} \equiv g \\
 & g^{2\alpha+3} - g^{2\delta+2} \equiv g^2 & & g^{2\alpha+2} - g^{2\beta+3} \equiv g^2
\end{array}$$

and, in general,

$$g^{2\gamma+1+h} - g^{2\delta+h} \equiv g^h \qquad g^{2\alpha+h} - g^{2\beta+1+h} \equiv g^h$$

Recalling that even powers of g represent quadratic residues, while odd powers yield non-residues, and that the set $1, g, g^2, g^3, \ldots, g^{p-2}$ is a complete residue system modulo p, we see that the theorem is proved.

Theorem 4. If $f(x)$ is a polynomial of degree 3 and if $p \not\equiv 3$, then there exist integers a, b, c, and d such that $f(x) \equiv a(x+b)^3 + ex + d \pmod{p}$.

Proof. Let $f(x) \equiv a_0x^3 + a_1x^2 + a_2x + a_3$. Since $f(x)$ is of degree 3, if follows that $a_0 \not\equiv 0 \pmod{p}$. Thus,

$$f(x) \equiv a_0\left(x + \frac{a_1}{3a_0}\right)^3 + \left(a_2 - \frac{a^2}{3a_0}\right)x + a_3 - \frac{a_1^3}{27a_0^2}.$$

The theorem, therefore, holds with $a \equiv a_0$; $b \equiv \frac{a_1}{3a_0}$ c $a_2 - \frac{a_1^2}{3a_0}$, and $d \equiv a_3 - \frac{a_1^3}{27a_0^2} \pmod{p}$.

In view of theorem 4 it follows that we need consider only polynomials of the form $f(x) \equiv a(x+b)^3 + cx + d$ where $a \not\equiv 0 \pmod{p}$. Furthermore, there is no loss of generality in assuming that $a \equiv 1 \pmod{p}$. Indeed, the polynomial obtained by the substitution $X = x + b$ that is, $X^3 + cX + D$ where $D = d - cb$, assumes exactly the same number of distinct values as does $f(x)$. Thus, in the remainder of the paper we can, without loss of generality, consider only polynomials of this convenient form.

Theorem 5. if $p \equiv 1 \pmod{3}$, there exists an integer which is represented exactly once by the polynomial $x^3 + cx + d$.

Proof. If $c \equiv 0 \pmod{p}$, the theorem clearly holds since d will then be represented only once.

If $c \not\equiv 0 \pmod{p}$, we first show that there exist integers r, v, and t such that $x^3 + cx - rt \equiv (x - r)(x^2 + vx + t) \pmod{p}$ where $x^2 + vx + t$ is irreducible modulo p. Since the coefficients of corresponding powers of x must be congruent, we have $0 \equiv v - r$ or $v \equiv r$, and $c \equiv t - rv \pmod{p}$.

Therefore, $c \equiv t - v^2$ or $t \equiv c + v^2 \pmod{p}$. Thus, we seek to show that v can be determined so that the polynomial $x^2 + vx + c + v^2$ is irreducible modulo p. But, this polynomial will be irreducible, if its discriminant $v^2 - 4(c + v^2) = -3v^2 - 4c$ is a quadratic non-residue modulo p.

Since $p \equiv 1 \pmod{3}$, we know that $\left(\frac{-3}{p}\right) = 1$. Thus, $-3v^2 - 4c$ can be written in the form $w^2 - 4c$. By theorem 3 we may determine w^2 so that $w^2 - 4c$ is a non-residue. Hence, we may choose v so that $x^3 + cx - v(c + v^2) \equiv (x - v)(x^2 + vx + c + v^2)$ where $x^2 + vx + c + v^2$ is irreducible. It follows that $x^3 + cx$ represents $v(c + v^2)$ exactly once, and, therefore, that $x^3 + cx + d$ represents $d + v(c + v^2)$ exactly once.

Theorem 6. If 3 is a divisor of $p - 1$ and if $f(x)$ is a polynomial of degree 3 which assumes exactly $\frac{p-1}{3} + 1$ distinct values, then $f(x)$ is essentially a cube.

Proof. In view of the comments preceding theorem 5 we may suppose that $f(x) \equiv x^3 + cx + d$. The theorem will follow, if we show that with $c \not\equiv 0 \pmod{p}$, $f(x)$ has to assume more than $\frac{p-1}{3} + 1$ values. By theorem 5 we know there exists an integer v such that $F(x)$ assumes the value $f(v)$ exactly once. Hence, by theorem 2, every other value will be assumed 3 times.

Let $p - 1 = 3k$, and let $f_1, f_2, \ldots, f_k$ be the values of $f(x)$ that are each assumed 3 times. By Fermat's theorem,

$$x^p - x \equiv 0 \pmod{p} \text{ or } x^p - x \equiv x(x-1)(x-2)(x-3)\ldots(x-(p-1)),$$

it follows that

$$\prod_{i=1}^{k}(x^3 + cx + d - f_i) \equiv \frac{x^p - x}{x - v} \pmod{p}.$$

Thus,

$$x^{3k} + kcx^{3k-2} + \cdots \equiv x^{3k} + vx^{3k-1} + v^2x^{3k-2} + \cdots$$

If $v \equiv 0 \pmod{p}$, the above congruence is clearly impossible since $kc \not\equiv 0$, because $k \not\equiv 0$ and $c \not\equiv 0 \pmod{p}$. If $v \not\equiv \pmod{p}$, the congruence still cannot hold. Thus, it cannot be true that $f(x)$ represents no more than + 1 distinct values, and, at the same time, $c \not\equiv 0 \pmod{p}$. Thus, we have $c \equiv 0 \pmod{p}$, so that $f(x)$ is essentially a cube.

SOME PARADOXES IN PLANE GEOMETRY

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Can you imagine a geometrical figure with the paradoxical property that it can be rigidly moved a certain distance, say **one inch,** and become smaller because of the motion? An easy example of this will be given shortly, but not until something is said about the meaning of the words in the question. By "rigidly moved" I mean, as usual, moved so that all points in the figure stay the same distance apart as they were before the motion. By "smaller" I understand that one geometrical figure is smaller than another if it is a part of the other, for example a side of a triangle is smaller than the triangle.

An example of a figure which can be made smaller by a rigid motion is given in Appendix I, below,

The example given in Appendix I is a figure that "goes to infinity", **i.e.** goes off the paper, no matter how large a sheet you have.

This is not essential. Can you imagine a figure which can be rigidly moved and made smaller, as in the previous example, but which is also bounded, **i.e.** lies entirely within some square, say a 10 inch by 10 inch square?

An example of such a figure is given in Appendix **II:**

The example of Appendix I is one-dimensional, **i.e.** requires only a line to contain it. The example of Appendix **II** is two-dimensional, **i.e.** is not contained in any straight line. It is within the reader's ability to prove that no answer to the second question could be **one-dimensional.** An indication of the proof will be given in Appendix **III.**

For a few pleasant moments, thumb through W. Sierpinski: "On the congruence of sets and their equivalence by finite decomposition" **Lucknow** 1954, parts of which are easy - parts are deeper.

Finally we may ask: can we make up these diagrams out of pieces of wire so that they will weigh something? Clearly not since we would have to do something an infinite number of times. But we may ask, in a general way, can we associate a (finite) weight with each geometrical figure so that congruent figures have the same weight, and a figure weighs more than any of its parts? The answer is no, as these examples show. These ideas are treated in measure theory.

Appendix I. From a point A draw a horizontal line L to the right, L is to extend indefinitely to the right; it has A as its left end point. A rigid motion of L which will make it smaller is a motion to the right, say one inch. After this motion it is a part of its former self. The motion has the same effect as chopping off the left hand inch of L.

Appendix **II.** On the circumference of a circle of radius one inch, mark a point. Call it point 1, or P_1 for short. Proceed clockwise one inch along the circumference **(i.e.** one inch of arc, not chord) and mark P2. Proceed clockwise one inch and mark P_3. Continue. The figure we are interested in consists of the points $P_1, P_2, P_3, \ldots$

A rigid motion of this figure which will make it smaller is a clockwise rotation so that P_1 moves to where P_2 was, P_2 moves to where P_3 was, etc. The effect of the motion is to chop off P_1.

For those who are acquainted with mathematical notation (= short hand) the two examples can be given in very brief space: the first is the positive X axis with translation to the right as the motion; the **second** is the set $P_n = (\cos n, -\sin n)$ or e^{-in}, $n = 1, 2, 3, \ldots$ with $P_n \to P_{n+1}$ as the motion.

The sucess of the example depends on the fact that no two of the P's coincide. This can be proved starting from the fact that π is irrational.

Appendix **III:** Let F be a bounded one dimensional figure. There is a smallest interval I which contains F. A motion of F to the right or left will move I along with it, while a reflection of F can't make F smaller since a reflection of the reflection yields F again.

DO YOU KNOW

If two points are taken at random within a sphere of radius **r,** that the probability that their distance apart is less than a given value c is;

$$\frac{c^3}{r^3} - \frac{9}{16}\frac{c^4}{r^4} + \frac{1}{32}\frac{c^6}{r^6}$$

PROBLEM DEPARTMENT

Edited by
M. S. KLAMKIN,
AVCO Research and
Advanced Development Division

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication. Address all communications concerning problems to M. S. Klamkin, Avco Research and Advanced Development Division, T-430, Wilmington, Massachusetts.

PROBLEMS FOR SOLUTION

138. Proposed by David L. Silberman, Beverly Hills, California.
The points of the plane are divided into two sets. Prove at least one set contains the vertices of a rectange.

139. Proposed by Leo **Moser,** University of Alberta.
Show that there exists a unique sequence of non-negative integers, $\{a_i\}$, such that every non-negative η can be expressed uniquely in the form $\eta = a_i + 2a_j$.

140. Proposed by Michael Goldberg, Washington, D. C.
What is the smallest area within which an equilateral triangle can be turned continuously through all orientations in the plane. This problem is unsolved and similar unsolved ones exist for the square and other regular polygons.

141. Proposed by D. J. **Newman,** Yeshiva University.
Determine conditions on the sides a and b of a rectangle in order that it can be imbedded in a unit square.

142. Proposed by Pedro A. **Piza,** San Juan, Puerto **Rico.**
Show that unity can be expressed as the sum of four squares less the sum of four squares (all squares distinct) in an infinitude of ways.

143. Proposed by M. S. Klamkin, AVCO Corporation.
If τ is a rational approximation to $\sqrt{N}$, find an always better rational approximation.

SOLUTIONS

118. Proposed by Leo **Moser,** University of Alberta.
Split the integers 1, 2, 3, ..., 16 into two classes such that the $\binom{8}{2} = 28$ sums formed by taking the sum of pairs is the same for both classes.

Editorial note: In the solution of this problem in the Spring, 1961 issue, a simple numerical solution was given. The following is an analytic solution of the problem generalized.

Solution by D. J. **Newman,** Yeshiva University.

To split the integers 1 through n into two sets A and B, such that the sum sets of each are the same (counting multiplicities).

Let

$$A = \{a_1, a_2, \dots\}, \quad (1 \in A),$$
$$B = \{b_1, b_2, \dots\},$$

and

$$A(x) = \sum x^{a_i}$$
$$B(x) = \sum x^{b_i}$$

Then,

$$A(x) + B(x) = \frac{x - x^{n+1}}{1-x}.$$

The requirement that the sum sets be identical can be expressed by

$A^2(x) - A(x^2) = B^2(x) - B(x^2)$. Letting $A(x) - B(x) = C(x)$, it follows that

$$[A(x) + B(x)]\, C(x) = C(x^2) \text{ or } C(x^2) = \frac{x - x^{n+1}}{1-x}\, C(x).$$

Whence

$$\frac{C(x^{2^k})}{x^{2^k}} = \frac{1-x^n}{1-x} \cdot \frac{1-x^{2n}}{1-x^2} \cdots\cdots \frac{1-x^{2^{k-1}n}}{1-x^{2^{k-1}}} \cdot \frac{C(x)}{x}$$

Letting $k \to \infty$ gives (since $C(x^{2^k})/x^{2^k} \to 1$)

$$C(x) = x \cdot \frac{1-x}{1-x^n} \cdot \frac{1-x^2}{1-x^{2n}} \cdot \frac{1-x^4}{1-x^{4n}} \cdots\cdots,$$

and since $C(x)$ = polynomial, we must have n = a power of $2 = 2^m$ and

$C(x) = x(1-x)(1-x^2)\ldots(1-x^{2^{m-1}})$. The coefficient of x^k in $C(x)$, then, is 1 if $k-1$ has an even number of ones in its binary expansion. Otherwise, the coefficient is -1.

Finally,
A = set of all $k(\leq 2^m)$ with an even number of ones in the binary expansion of $k-1$.
B = the complimentary set or the set of all $k(\leq 2^m)$ with an odd number of ones in the binary expansion of $k-1$.

Also solved in the same way by the proposer and J. Lambek in their joint paper, "On Some Two Way Classifications of Integers," Canadian Math. Bull., Vol. 2, No. 2, 1959, pp. 85-89. Also, other related problems are considered.

127. Proposed by Harry Furstenberg, University of Minnesota.
Show that

$$\{\text{Rank } \|A_{rs}\|\}^2 \geq \text{Rank } \|A_{rs}^2\|.$$

Solution by L. Carlitz, Duke University.

Put $A = \|A_{rs}\|$, $B = \|A_{rs}^2\|$, and $C = \|A_{is}A_{js}\|$, where in C the row index is ij. Clearly

$$\text{Rank } B \leq \text{Rank } C$$

Let rank A = t and assume the first t rows are linearly independent. Then for given i we can find constants $C_1, C_2, \ldots, C_t$ (independent of S) such that

$$A_{is} = \sum_{p=1}^{t} C_p A_{ps}.$$

It follows that

$$A_{is}A_{js} = \sum_{p,q=1}^{t} C_p C_q A_{ps} A_{qs},$$ so that the ij row of C is linearly independent on a certain set of $\frac{t(t+1)}{2}$ rows. Therefore,

$$\text{Rank } C \leq t(t+1)/2,$$
and also
$$\text{Rank } B \leq t(t+1)/2 \leq t^2.$$

Also solved by James M Horner, H. Kaye, Paul Meyers and the proposer.

130. Proposed by H. Kaye, Brooklyn, N. Y.
If P is a variable point on the circular arc $\overset{\frown}{AB}$, show that $\overset{\frown}{AP} + \overset{\frown}{PB}$ is a maximum when P is the mid-point of the arc $\overset{\frown}{AB}$.

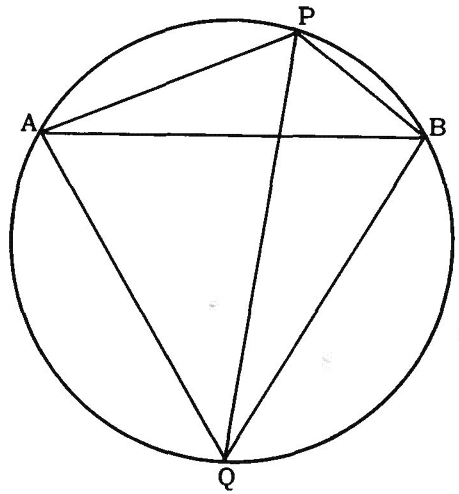
Solution by Theodore Jungreis, New York University.

Let angles subtended by arcs $\overset{\frown}{AP}$ and $\overset{\frown}{PB}$ be 6, and ϕ, respectively (6 + T = constant). Then

$$\overset{\frown}{AP} + \overset{\frown}{PB} = 2r\,[\sin\theta/2 + \sin\phi/2];$$
$$= \text{const.} \cdot \cos\frac{\theta-\phi}{2}.$$

Whence $6 = \psi$, and P is the mid-point.

Solution I by Leon Gerber, Brooklyn College.



Q is such that AQ = BQ = s. By Ptolemy's theorem,

$$AB\cdot PQ = AP\cdot BQ + PB\cdot AQ \quad \text{or}$$
$$AP + PB = \frac{AB}{s}\,PQ.$$

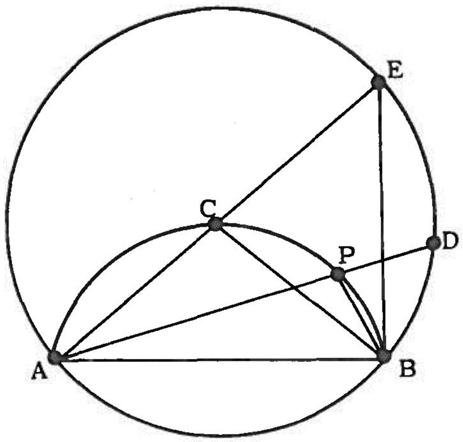
In order for PQ to be amaximum it must be a diameter and thus P is the mid-point of AB.

Solution II by Leon Gerber.

By considering the largest ellipse which has A and B as foci and which intersects the arc AB it follows that P is the mid-point of arc AB.

Also solved by John Doctor, Leon Gerber (Solution No. III), James M. Horner, Paul Meyers, F. Zetto and the proposer.

Editorial Note: A more general problem with solution has been given by C. M. Ingleby (Mathematical Questions from the "Educational Times", Vol. VII, 1867, p. 21) **i.e.,** "To find a point P in a given circular arc ABC, so that the chords AP, BP drawn therefrom to the ends of the chord AB may be given or a maximum."



"With C the middle point of the given arc as centre, and CA or CB as radius, draw a circle; and from either end of AB, draw in this circle a chord AD equal to the given line and cutting the given arc in P; join BP, then the chords AP, PB are together equal to AD. For join AC and produce it to meet the circle in E; and join EB, DB; then the triangles BCE, BPD, are evidently isosceles. The sum of AP and PB is a minimum when the given line is equal to AB; and a maximum when the given line is equal to AC + BC, that is, when P coincides with C."

It is of interest to note that the given property characterizes circles, i. e., if an arc has the property that for any sub-arc AB, AP + PB (P is a variable point on AB) is a maximum when P is the mid-point of AB, then the given arc is circular.

131. Proposed by M. S. Klamkin, AVCO RAD.

Solve the following system of equations:

$$\begin{aligned} ax_1 + bx_2 + bx_3 + \dots + bx_n &= c_1, \\ bx_1 + ax_2 + bx_3 + \dots + bx_n &= c_2, \\ &\vdots \\ bx_1 + bx_2 + bx_3 + \dots + ax_n &= c_n. \end{aligned}$$

Solution by H. Kaye, Brooklyn, N. Y.

It follows immediately that

$$(a-b)(x_1 - x_r) = c_1 - c_r \quad (r = 2, 3, \dots, n),$$
$$[a + (n-1)b]\,(x_1 + x_2 + \dots + x_n) = \Sigma c_r.$$

The trivial case $a = b$ is easily solved by inspection. So we assume $a \neq b$. Then

$$x_r = x_1 - \frac{c_1 - c_r}{a - b},$$

where

$$[a + (n-1)b]\,\left\{nx_1 - \Sigma \frac{c_1 - c_r}{a - b}\right\} = \Sigma c_r.$$

In solving the latter equation for x_1, we consider the two cases $a + (n-1)b \neq 0, = 0$. The first case leads to no difficulties. The second case implies $\Sigma c_r = 0$ and x_1 is arbitrary.

Also solved by L. Carlitz, Leon Gerber, James M. Horner, Theodore Jungreis, Paul Meyers, David L. Silberman, J. Thomas, M. Wagner, F. Zetto and the proposer.

QUOTE . . .

Mathematics possesses **not only** truth but supreme beauty – a beauty cold and austere, like that of sculpture without appeal to any part of our weaker nature, sublimely pure, and capable of a stern perfection such as only the greatest art can show.

Bertrand Russell: ***The Study** of **Mathematics***

BOOK REVIEWS

Edited by

FRANZ E. HOHN, UNIVERSITY OF ILLINOIS

Progress In Operations Research. Edited by Russell L. Ackoff. New York, Wiley, 1961. xii + 505 pp., $11.50.

This book has ten chapters by different authors, each devoted to a different aspect of operations research, with emphasis on work since 1955. It is remarkable for the extensiveness of its coverage. In addition to some general discussion, there are many brief summaries of individual projects and expositions of modern techniques of problem solving.

To touch on some of the chapters: ***Dynamic Programming,*** by Stuart Dreyfus is technical writing at its best. Anyone who has conquered his fear of "Σ" and a matrix can follow the mathematics. The theme is that "... the essential idea of dynamic programming is the characterization of the problem by means of a functional equation or its discrete analogue, the recurrence relation." As this is developed, it becomes an introduction to operations research from a mathematician's viewpoint. It will be greatly enjoyed as good reading as well as good instruction. The chapters on Inventory, Dynamics of Systems. Sequencing, and Replacement are excellent catalogues of recent projects, with brief descriptions of each. They fulfill the purpose of the book admirably. Unfortunately, Military Gaming is more history than progress. The censor is still with us.

The book is uneven in quality. In parts it is longwinded, in parts concise. There are bibliographies at the end of each chapter. The book will be most useful in a reference library.

Jane I. Robertson

Calculus of Finite Differences. By C. Jordan. New York, Chelsea Publishing Company, 1961. xxi + 654 pages, $6.00.

This is Chelsea's second reprint of Professor Jordan's classic treatise which was originally published in Budapest in 1939. It remains a great storehouse of information and reference for the professional .mathematician. However, it should not be considered suitable for use as a text for an advanced undergraduate course in the Calculus of Finite Differences. The book contains no exercises.

The standard topics in the Calculus of Finite Differences are discussed. In addition, such items as generating functions, Bernoulli and Euler polynomials, and Boole polynomials are presented. These topics are of interest to professional mathematicians, but may be of little concern to a computer-oriented student whose interest in Finite Differences stems from interpolation, quadrature, and other problems in numerical analysis. The presence of tabular values of Stirling Numbers of the First and Second Kind, Bernoulli Numbers, and others, increases the value of the book as a reference volume.

The photographic method of printing is not entirely adequate. Subscripts are sometimes unreadable, while fraction lines and periods have occasionally disappeared. Some typographical errors do occur. An important early example appears on page 16. The formula should read:

$$\Delta^{2m+1} = E^m (\mu \delta^{2m+1} + 1/2\, \delta^{2m+2})$$

The limitations which have been noted in no way detract from the wealth of information appearing in this volume and it should be on the shelves of all mathematicians' libraries. Its value to students with little interest in formal mathematical developments may be slight.

United States Air Force — Russell E. Welker

Elements of Mathematical Statistics. By Howard W. Alexander. New York, Wiley, 1961, xi + 367 pp., $7.95.

The subjects covered in this text are: discrete and continuous distributions; sampling distributions; statistical inference; regression and correlation; and analysis of variance, one and two way classification. Also, in addition to tables, answers to odd-numbered problems and bibliography, the text contains an appendix on matrices.

The material is clearly presented with many examples to help the reader understand the subject matter. There are many interesting exercises, some of which are intended for the better students.

Some of the more interesting items of this text are: the definition of the sample variance as the sum of the squared deviations from the sample mean divided by m-1 instead of m; the proof of the central limit theorem without the use of the moment-generating function; the development of the distribution of the sum of two chi-squared variables without the use of the moment generating function; the use of orthogonal transformations in obtaining distribution functions arising in the analysis of variance test; the use of the F-distribution instead of the normal distribution to obtain the significance or nonsignificance of the sample correlation coefficient.

This reviewer feels that the ordering of the material is not entirely satisfactory. On page 161 of the text the author mentions the ratio $t = [\bar{x} - m] / [s/\sqrt{n}]$ but does not prove that this ratio has t distribution until page 263. Confidence intervals appear in two places, page 161 and page 199. This reviewer feels that these two discussions should have been combined.

On page 160 the author, no doubt, wanted to write $U = [x - \mu_x]/\sigma_x$ instead of $(x - \mu_x)\sigma_x$. The hypothesis of theorem 21.2 deals with random variables whereas the hypothesis of theorem 21.3 deals with independent random variables. This reviewer feels that the word "independent" is redundant.

This reviewer recommends use of this text.

San Diego State College — Joseph M. Moser

Introductory Topology. By Stewart Scott Carirns. New York, The Ronald Press Company, 1961. ix + 244 pages, $8.75.

This is a textbook intended for an introductory course in topology. It begins with illustrative geometric examples and an intuitive treatment of the topological classification of surfaces. These two chapters appeal to the intuition and suggest the general nature of the subject matter. Then a chapter on fundamental concepts in general topology follows. The central core of the book consists of the remaining five chapters (146 pages): complexes and polyhedra, simplicia1 homology and cohomology groups, their topological invariance proved via singular theory, manifolds and the Poincaré-Lefschetz duality theorem, fundamental group and covering surfaces. The book closes with an appendix on group-theoretic background.

Throughout the book, there is a constant emphasis on geometric content, while the algebraic language is kept to a minimum (e. g., there is no mention of functors or diagrams). For an introductory course, this approach has the merit that it enables the beginning student to grasp the main ideas quickly. It is also due to this approach that the author was able to cover the fundamentals of classical algebraic topology in 146 pages. The material is well organized and the text is amply supported by illustrative figures and exercises. It is a fine text, well suited for the intended purpose.

Northwestern University — Ky Fan

Logic: The Theory of Formal Inference. By Alice Ambrose and Morris Lazerowitz. New York; Holt, **Reinhart** and Winston, 1961. vi + 78 pp., $2.00

This book is an introduction to those subjects considered in classical logic. Few mathematical methods or illustrations from the field of mathematics are used. The text is a good starting point for the beginner and could be pursued by a college freshman or sophomore without great difficulty.

The book is divided into three chapters: **Truth-Functions,** Quantification, and Classes. The first chapter develops the propositional calculus first by means of functions and then by the axiomatic method. The second chapter is concerned with the categorical syllogism and its modem interpretation plus an introduction to the functional calculus. The last chapter discusses classes, primarily in their relation to the categorical syllogism. Each chapter is followed by adequate exercises.

For the student interested in classical logic or for the student beginning a study of mathematical logic, this text is a fine starting point.

University of Illinois — Gordon E. Cash

Lectures on the Calculus of Variations. Second Edition, by O. Bolza, New York, Chelsea, 1961. ix + 271 pp., paperback, $1.19

This is a reprint of the 1904 edition with minor changes: some of the addenda have been incorporated **into** the text and a number of corrections and improvements in notation have been made.

The discussion is confined tc "the case in which the function under the integral sign depends upon a plane curve and involves no higher derivatives than the first."

The book was rated "excellent" by Professor Earle Raymond 1edrick who aptly described it (Bulletin of the American Mathematical Society, 1905) as "the first accurate and critical presentation of all the modern methods which is thoroughly readable."

University of Illinois — **Franz E.** Hohn

Elementary Differential Equations, Second Edition. By W. T. Martin and E. Reissner. Reading, Massachusetts, Addison-Wesley, 1961. xiii +331 pp., $6.75.

This book was written as an **introduction** to the techniques of solving differential equations. The authors have omitted rigorous proofs. The justification of each technique is made plausible by its application to simple problems. Each new method is clearly explained, has many examples, and is followed by a large number of exercises. The material is well written and organized. It is an excellant textbook for a course immediately following calculus where the main emphasis is on the technique of solving differential equations.

The translation of scientific and engineering problems into mathematical equations is covered in the first chapter. The second chapter covers differential equations of the first order while the next two chapters cover second-order and higher-order differential equations. Chapter five **covers** systems of first-order differential equations and chapter six covers approximate solutions of first-order differential **equations** and **Picard's** theorem. Chapter seven covers finite difference equations and the last chapter covers partial difference equations.

The second edition contains twice as many exercises as the first, additional work on linear and plane motion with variable mass, a new section on differential operator methods, a revision of the idea of general solutions, additional work on Fourier series, and various minor improvements over the first edition.

University of Illinois — George L. Kvitek

Statistical Independence in Probability, Analysis and Number Theory. The Cams Mathematical Monographs, No. 12. By Mark Kac. New York, **Wiley,** 1959. xiv + 93 pp., $3.00.

Starting with simple trigonometric formulae, the author presents an elegant and mathematical approach to the statistical independence of tosses of a coin. His intuitive power and his mathematical preciseness lead the reader in a natural and elementary way to the "Law of Large Numbers" and to the "Normal Law". The manner of presentation of these problems allows the author easily to generalize the notion of statistical independence. Special kinds of such independence create an heuristic approach to new and deep results in number theory and in some topics of analysis. These new methods can be made precise.

In the content of this book and **in** the sets of noneasy problems given at the end of each chapter, the author certainly shows what he intended to show, that is: "(a) extremely simple observations are often the starting point of rich and fruitful theories and (b) many seemingly unrelated developments are in reality variations on the same simple theme."

University of Illinois — **Z.** Ciesielski

Probability and **Experimental** Errors in Science (**an** elementary survey). By L. G. Parratt. New York, **Wiley,** 1961. xv + **255** pp., $7.25.

Many books which are written as texts must be judged and reviewed on two levels. On the one hand, they should be evaluated in terms of their main purpose, and on the other, they should be examined for their potential utility as self-teachers or reference works. Anyone who is contemplating teaching undergraduate science majors a combination of the philosophy of science. scientific measurement, **probability,** and statistics should unquestionably read this book **Whether** or **not they** choose it as a base for such a course, they will surely get good ideas from it.

It is harder to decide on the merit of ***Probability*** and ***Experimental Errors in Science*** as a general addition to the personal library of a scientist. The book has the **adventage** of being concise, in some sections, and therefore useful either as an introduction or a rapid review. This advantage necessarily implies the complementary disadvantage; there are not enough examples to make the motivation and development clear.

The portion of the book which is most self sufficient is the clear, interesting introduction it gives to a **priori** probability and elementary game theory. Anyone who has not been exposed to these ideas would enjoy the forty relevant pages. In a similar way, the thirty pages used to introduce frequency distributions and precision indices are quite readable and informative. However, the corresponding introductory sections on measurements and errors do not stand so well alone; they need **the** supplementary material which would be provided by a teacher **and** a course. The novice needs additional examples and explanations; an experienced scientist will find little new.

The second half of the **book** deals with subjects (such as statistical analysis, the normal probability distribution, **and** the **Poission** distribution) **to** which relatively few scientists have been exposed formally. It is unfortunate that the requirements of a text and a general book have become more divergent by this stage. The treatment given the subject is appropriate for students who have been progressing slowly, and learning formalism and approach continuously. The general reader therefore is at a disadvantage; he must either invent his own exa**mples or cons**ult other books.

In summary. ***Probability and Experimental Errors in Science*** is designed as a text for the training of scientists. Parts of it would be useful, in a more general way, for scientists who want to know about probability and the statistical analysis of data. The same author could have written a similar book for this general audience. Until he or someone else does, this existing book might provide a worthwhile compromise.

University of Illinois — P. Axel

The Book on Games of Chance. By Gerolamo Cardano. Translated by Sydney Henry Gould. New York; Holt, Rinehart, and Winston, 1961. v +57 pp., $1.50

The main portion of this book is devoted to calculating odds for certain card and dice games. Unfortunately, Cardano assumes a prior knowledge of the games he discusses, and since the book was written around 1520, most of his games are unknown to the modern reader. However, there are amusing sidelights; he spends a chapter on an attempt to justify'gambling as a pastime, and two other chapters are concerned with cheating and fraud. As a work of mathematics, this book is significant as the beginning of the mathematical theory of probability, predating the Pascal-Format correspondence, which historians have identified as the beginning, by about 130 years. Cardano shows a very clear understanding of the then unknown laws of probability, pointing out several errors his contemporaries had made in calculating odds.

University of Illinois — Marc E. Low

Programming and Coding for Automatic Digital Computers. By G. W. Evens II and C. L. Perry. New York, McGraw-Hill, 1961. xii + 249 pp., $9.50.

Too many books, pamphlets and papers on programming which one sees are lacking in depth and read like "popular science" in a Sunday supplement. Happily enough, this book is not one of these; it is seriously written for the serious student. The authors suggest that it is suitable as a text for college juniors, and this reviewer agrees.

The contents of the book as indicated by the chapter headings and page numbers are as follows:

This book emphasizes machine language programming instead of automatic programming. Flow diagrams are used extensively to describe program logic. This reviewer is pleased to see this emphasis and use of flow diagrams; however, the subject of automatic programming deserves a more extensive treatment than it receives here. (The chapter on automatic programming was written by R. E. Keirstead, Jr.)

Sample machine programs are based on a code for a fictitious computer. Most books on programming do this. However, as any beginner who has just waded through the manuals supplied by the computer manufacturer can tell you, what he needs badly is a good introductory programming text based on the *real* computer. In the absence of such a text, the present text, supplemented by the manufacturer's manuals, is a good guide to learning about how to use a computer.

One chapter in the book is somewhat unusual; chapter 9, "Mathematical Aid for Programming and Computer Design". This chapter contains a description of the application of Boolean algebra to programming and, as far as this reviewer knows, it is the first time that this has been done in a textbook on pro'gramming and coding.

University of Illinois — Lloyd D. Fosdick
Digital Computer Laboratory

Advanced Calculus. By Watson Fulks. New York, Wiley, 1961. xv + 521 pp., $11.25

This book is meant to serve as an introduction to analysis rather than as a text for a standard course in advanced calculus. Such a book would be ideal for students who are majoring in mathematics, and who have just completed a course in elementary calculus. The contained material could be reasonably covered in two semesters, or one year, and such a course could well replace the standard courses in advanced calculus and introduction to real variables. In other words, the topics normally covered in advanced calculus courses are in this book, but the approach here is from a rigorous point of view. The book would hardly be appropriate as a text for students in engineering or the applied sciences who seek an advanced calculus course which only emphasizes techniques.

The book Is divided into three parts. Part I is an introduction to the theory of functions of one real variable. Part II is an introduction to vector analysis, and to functions of several real variables using vector methods. Part III is a study of the convergence of infinite series and improper integrals. Certain special functions such as the gamma and beta functions are dealt with in this section, and the book concludes with a chapter on Fourier series. There is an excellent selection of exercises, especially from the viewpoint of variety of levels of difficulty.

What impresses the reviewer most about this book is its readability, and the fact that the author is able to motivate any new idea which he introduces. It is unusual to find a book on this subject where the material is rigorously presented and at the same time so easy to follow. It appears that this book accomplishes both of these goals, and hence it is highly recommended either as a text or reference book to any student who is taking an introductory course in real analysis.

McMaster University — Howard L. Jackson

Special Functions of Mathematical Physics. By H. Hochstadt. viii + 81 pp., $2.50.

Analytic Inequalities. By N. Kazarinoff. vii + 89 pp., $2.00.

A Brief Introduction to Theta Functions. By R. Bellman. x + 78 pp., $2.50.
All published by Holt, Rinehart and Winston, New York, 1961.

These three little books are most welcome additions to the Athena Series of Holt, Rinehart and Winston. They share a common appearance (attractive), typography (large and clear), and purpose, which is to introduce undergraduates to some advanced fields very early in their careers.

Hochstadt's book discusses orthogonal polynomials. Legendre functions, Bessel functions and Mathieu functions. The treatment includes general theory and applications. A small complaint is that in several places asymptotic formulas are written down, with no definition of what an asymptotic formula is, or of the mysterious "$\approx$" symbol which appears in the formulas.

Analytic Inequalities is a much-needed discussion of the classical inequalities of analysis at an elementary level. After some introductory remarks about the algebra of inequalities, the author proves the inequalities of arithmetic and geometric means, of power means, and those of Cauchy, Holder and Minkowski. The excercises are numerous and interesting.

Finally. Bellman's book on theta functions is a very readable account of these functions and their applications. Many of the remarkable identities involving theta functions are proved here, some in several ways. Among the applications discussed are the functional equation of the Riemann zeta function and the evaluation of Gaussian sums. This book should certainly fulfill its aim of generating interest in analytic function and number theory among undergraduates.

University of Illinois — Herbert S. Wilf

Error-Correcting Codes, By W. W. Peterson Cambridge, Mass., M.I.T. Press and New York, Wiley, 1961. x + 285 pp., $7.75.

This book is indispensable to anyone interested in learning about or working with error-correcting codes. It introduces the subject in a thorough and self-contained fashion, and presents most of what is currently known about such codes.

The first chapter contains some preliminary remarks on the nature of error-correcting codes, the second chapter contains a brief discussion of groups, fields, and matrix algebra, and the third chapter shows the application of matrix algebra to linear **error-correcting** codes (sometimes called group codes). In Chapter 4, some of the theoretical **limitations** on the **error-**correcting capabilities of codes are discussed; and in Chapter 5, specific examples of important kinds of linear codes are given. Chapter 6 contains a discussion of rings and fields, particularly Galois fields, which provides the necessary mathematical background for the remainder of the book. In Chapter **7,** we are shown how to combine linear switching elements into circuits which will accomplish the polvnomial and matrix operations discussed in Chapter 6. Chapters 8, 9, and 10, discuss cyclic codes, **Bose-Chaudhuri** codes, and burst-error-correcting codes, respectively. Finally, in Chapters 11, 12, and 13, the author takes up some of the more advanced topics, such as special decoding methods, recurrent codes, and codes which check arithmetic operations.

The book contains four appendices, the most interesting being a table of irreducible polynomials over **GF(2)** which is complete for degrees 2 through 16 and representative for degrees 17 through 34. Finally, there is a list of 129 references, listing everything of importance published on the subject of error-correcting codes through 1960, and an index.

The only disappointing feature of this book is that, despite its hard covers, the pages are not stitched but merely glued in. The material itself is well organized, well presented, and carefully motivated. There are a half-dozen or so exercises at the end of each chapter, a large number of examples scattered throughout the text, and guides to further reading on each topic discussed which make the book, if treated with care, a valuable reference work.

University of Illinois — Clinton R. Foulk

Calculus and Analytic Geometry. **By** John **F.** Randolph. San Francisco, Wadsworth, 1961. xi + 618 pp.,

This book is intended for classes of engineering and mathematics students. As such, its development is midway between an intuitive approach and a rigorous exposition. Professor Randolph introduces the standard notation for a set of elements each satisfying a given condition, and then uses this notation to develop the concepts of functions, limits, derivatives, etc. Following the first chapter, the book covers the standard topics and in addition includes chapters on vector analysis and differential equations. There is an excellent appendix which contains most of the rigorous development.

As the author states, Chapter 1 does seem to start rather abruptly. It contains the notions of sets, functions, and most of the basic topics of plane analytic geometry, including slope and equations of a line, translation of coordinates, and conic sections. Because of this rather brief treatment, this reviewer thinks that the text would best suit the "average" student who has already been exposed to analytic geometry and thus could spend more time on the unfamiliar concept of sets and its applications, using the material on analytic geometry as a thorough review.

The book is well illustrated with figures which accompany the explanatory material of the text, and answers are provided for approximately **one-**half of the problems. Many examples showing applications of the theory are included in the book. and are of sufficient difficulty to be a real aid to the student. Both the teacher and the student should enjoy using this text.

University of Illinois — John S. Cross

Cybernetics, Second Edition. By Norbert Wiener. Cambridge, Mass., M.I.T. Press and New York, Wiley, 1961. xvii + 212 pp., $6.50.

The first edition of this book appeared in 1948; this second edition contains the full text of the first, revised and corrected where necessary, plus a new introduction and two new chapters. The subtitle of "Cybernetics" is "Control and Communication in the Animal and the Machine". **A** more concise description of this science would be hard to find.

Professor Wiener's intent in this book is twofold. First, to discuss the broad concept of control mechanisms, both electromechanical and **neurol-**logical, and second, to put forth the various mathematical disciplines which are, in his opinion, most suitable for the formulation and also the solution of the many and facinating problems which arise in control mechanisms. The first of these intents is addressed to the layman; the style is extremely clear, and the range of subject matter enormous. The author's greatest interest is in relating various behaviorisms of the human nervous system to those of electro-mechanical systems, with the eventual hope of understanding the basic principles which govern the nervous system. Perhaps an example is in order. Most electro-mechanical control systems are so designed that they generate, in one form or another, a signal which indicates the deviation, at every instant of time, of their acutal behavior from their intended behavior. This signal is then used to redirect the system in such a way as to decrease this deviation. This is known as the use of negative feedback for control and correction. In an improperly designed feedback system it is possible for the machine to overcorrect, and then, in an attempt to recorrect, to overcorrect in the opposite direction, on and on without limit. The result is that it may oscillate about its intended behavior, ceaselessly overcorrecting, or its oscillations may even increase in magnitude until the system destroys itself. Examples of negative feedback, at least on the conscious level in human being, are the actions of a blocker attempting to intercept the runner on a football field, or the maneuvering of a fielder going for a fly ball. What about more basic and reflexive actions? It turns out that there is a pathological condition, associated with a certain kind of brain damage, in which a person attempting, for example, to bring a glass of water to his lips will instead involuntarily swing his arm back and forth in oscillations of increasing amplitude until the glass is entirely emptied. This is analogous to previously mentioned behavior of an improperly designed negative feedback system, and suggests that negative feedback may be a valid principle in the study of the nervous system.

The mathematical methods which the author proposes as the appropriate tools for the further study of control systems are mainly probabilistic in nature. The idea is that most systems are not intended to operate in the face of one possible contingency, but in a whole variety of contingencies. Generally, then, one attempts to design a system which will perform acceptably in all, or at least a majority of cases. But if we cannot predict with certainty just when, or how frequently the various cases can occur, we are already in the realm of **probability** theory and statistical inference. We cannot go further into details; suffice it to say that the mathematical methods are of a high order of complexity, many of them representing precisely those researches which are responsible for the author's reputation in the field of mathematics.

Summing up, this is a book which is not, in its entirety, easy reading. **But** it is interesting, even exciting reading, which offers rewards to any reader who will read it slowly and carefully, and not give up when the mathematical level rises above his head.

University of Illinois — Amich **Feinstein**

An Introduction to the Theory of Numbers. By I. Niven and H. S. Zuckerman New York, Wiley, 1960. viii +250 pp., $6.25.

This book is an excellent introduction to the theory of numbers. It has a reasonable chance of becoming the most popular textbook for introductory courses in number theory in American colleges and universities. The writing is concise and to the point, but relatively clear. Undergraduates without very much mathematical experience might find the book-a little difficult for self-study, but most readers are sure to like its direct approach to each topic and the authors' unwillingness to beat around the bush. The authors seem to feel that the beauty of number theory can speak for itself.

A very significant feature of the book is the excellent collection of problems. Classes using the book and individual readers should try to spend a lot of time on the problems, for the text probably needs this kind of amplification.

Although the book is only 250 pages lone, there is plenty in it for a one-year course. Because of the conciseness of the presentation and the desirability of spending considerable time on the problems, the usual five-pages-per-day pace needs to be cut down by a factor of at least two. The book is also readily adaptable to a one-semester course.

A rather wide variety of topics are taken up, but the authors do not push any of them to the point of exhausting the reader. The titles of the chapters are: (1) Divisibility, (2) Congruences, (3) Quadratic Reciprocity, (4) Some Functions of Number Theory, (5) Some Diophantine Equations, (6) Farey Fractions, (7) Simple Continued Fractions, (8) Elementary Remarks on the Distribution of Primes, (9) Algebraic Numbers, (10) The Partition Function, (11) Density of Sequences of Integers.

Typographically the book is attractive, except that complicated exponents are rather poorly done. The standard of workmanship in this respect might be acceptable in a mathematical journal with the customary meager budget, but seems out of place in a textbook which is certain to be a profitable item for its publishers.

The only real howler in the book is the following starting sentence at the top of page 3: "In general we shall use roman letters a, b, c, ..., m, n, ..., x, y, z to designate integers unless otherwise specified". Fortunately the printer ignored this sentence and followed the hallowed tradition of using italic letters for mathematical symbols.

Here are a number of minor criticisms which readers may find helpful. On lines 5 to 10 from below on page 91 the authors invoke L'Hospital's rule, even though the definition of the derivative is all that is required. In section 5.3 the formulas would become slightly neater if, instead of considering the solutions of $ax + by = c$ in positive integers x, y, we were to consider the solution in non-negative integers. (The two problems are equivalent, since we can pass from one to the other by adding $a + b$ to c or subtracting $a + b$ from c). In sections 5.10 and sections 5.11 it seems unnecessary (and a little confusing) to introduce the function $P(n)$. In section 5.14 it would have been well to mention explicitly that if we restrict ourselves to transformations of determinant $+1$, then we must enlarge the set of reduced binary quadratic forms to include forms $ax^2 + bxy + cy^2$ with $0 < -b < a < c$ in addition to those with $0 \leqq b \leqq a \leqq c$. In chapter 8 the letters a, b, c, d, e are not intended to be integers, although this is not specified. (The letter e, incidentally, is used to mean the base of natural logarithms on page 166 and to mean something else on page 168.) The proof of Theorem 8.5 (which asserts that the product of the primes not exceeding x is less than 4^x) is made rather tortuous by the authors' unwillingness to use mathematical induction. (It would have been helpful if this theorem had been put earlier in Chapter 8 than it is). In Chapter 9 it might have been desirable to give the definition of divisibility and related concepts for any integral domain and to use the customary notation for polynomial rings. In the treatment (in section 9.9) of the splitting laws for the rational primes in quadratic fields $R(\sqrt{m})$ with unique factorization (where m is a squarefree rational integer), the behavior of the prime 2 and of the odd primes dividing m is not discussed, even though this would have

been very easy to do. Also it would have been well to give some indication of how frequently unique factorization arises (nine or ten times for $m < 0$, probably infinitely often for $m > 0$). In Chapter 10, the authors are to be congratulated on their careful treatment of convergence questions, but it must be admitted that it might have been clearer just to introduce the concept of a formal power series. In section 11.2 the proof given for the formula

$$\sum_{n=1}^{\infty} \frac{\mu(n)}{n^2} \cdot \sum_{n=1}^{\infty} \frac{1}{n^2} = 1$$

is more complicated than necessary, even if we accept the authors' wish to avoid using the theorem on multiplication of two absolutely convergent series. Finally the proof of formula (11.10) is based on a counting argument in which it is never made clear just what sets are being counted.

Needless to say, these minor objections do not seriously interfere with the overall value of this first-rate book.

P. T. Bateman

University of Illinois
and
University of Pennsylvania

Boundary and Eigenvalue Problems in Mathematical Physics. By Hans Sagan. New York, Wiley, 1961. xviii + 381 pp., $9.50.

The well known volumes on "Methods of Mathematical Physics" by Courant-Hilbert have had a great deal of influence in arousing interest in the field of classical analysis. Written in an elegant, but terse style, they require a background and mathematical maturity that many students of physics and engineering lack. The volume under review should fill the need of those who require a rigorous, but a less condensed exposition of many of the topics treated in Volume I of Courant and Hilbert's, "Methods of Mathematical Physics".

The author writes clearly on subject matter which is becoming more and more necessary for students of physics and applied mathematics. These are: calculus of variations, partial differential equations, Fourier series, self-adjoint boundary value problems, non-homogeneous boundary value problems, orthogonal functions, eigenvalue problems, spherical harmonics and Green's functions.

The various sections are supplied with problems some of which test the understanding of the text and others of which round off the treatment of the material presented. The student with adequate courses in advanced calculus and differential equations should have sufficient background for the comprehension of the contents.

In summary, this is a well written and readable book that should prove of value to those seeking a knowledge of mathematical methods and unifying concepts which can be used to solve many important physical problems.

University of Illinois E. J. Scott

BOOKS RECEIVED FOR REVIEW

*S. S. Calms: *Introductory Topology.* New York, Ronald Press, 1961. ix + 244 pp., $8.75.

E. M Grabbe, S. Ramo, and D. E. Wooldridge, editors: *Handbook of Automation, Computation, and Control. Volume 3: Systems and Components.* New York, Wiley, 1961. xxi + 1158 pp., $19.75.

P.P. Korovkin: *Inequalities.* New York, Blaisdell, 1961. vii + 60 pp., $0.95.

A.N. Kostovskii: *Geometrical Constructions Using Compasses Only.* New York, Blaisdell, 1961. xii + 79 pp., $0.95.

H. Levi: *Algebra, Fourth Edition.* New York, Chelsea, 1961. 189 pp., $3.25.

*W. T. Martin and E. Reissner: *Differential Equations, Second Edition.* Reading, Mass., Addison-Wesley, 1961. xiii + 331 pp., $6.75.

G. B. Mathews: *Theory of Numbers, Second Edition (Reprint).* New York, Chelsea, 1961. xii + 323 pp., $3.50.

*L. G. Parratt: *Probability and Experimental Errors in Science.* New York, Wiley, 1961. xv + 255 pp., $7.25.

C. A. Scott: *Projective Methods in Plane Analytical Geometry, Third Edition (Reprint),* New York, Chelsea, 1961. xii + 290 pp., $3.50.

A. S. Smogorzhevskii: *The Ruler in Gwmetrical Constructions.* New York, Blaisdell, 1961. viii + 86 pp., $0.95.

I. S. Sominskii: *The Method of Mathematical Induction.* New York, Blaisdell, 1961. vii + 57 pp., $0.95.

V. A. Uspenskii: *Some Applications of Mechanics to Mathematics.* New York, Blaisdell, 1961. vii + 58 pp., $0.95.

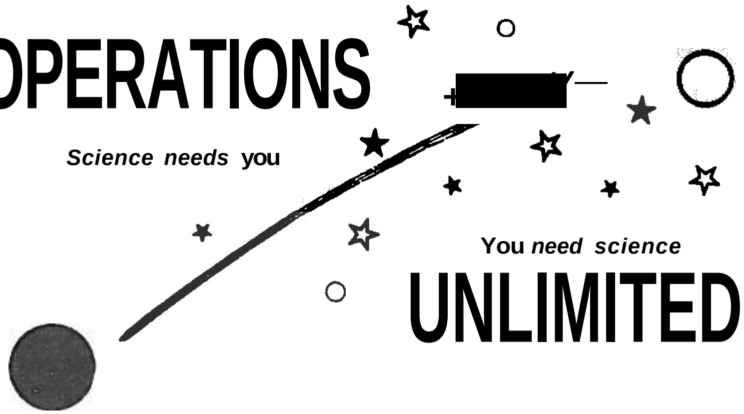
N. N. Vorob'ev: *Fibonacci Numbers.* New York, Blaisdell, 1961. viii + 66 pp., $0.95.

*See review, this issue.

NOTE: All correspondence concerning reviews and all books for review should be sent to PROFESSOR FRANZ E. HOHN, 374 ALTGELD HALL, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

OPERATIONS

Science needs you



You need science

UNLIMITED

This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essential as today.

Your election as members of Pi Mu Epsilon Fraternity is an indication of scientific potential. Can you pursue advanced study in your field of specialization?

To point out the need of advanced study, the self-satisfaction of scientific achievement, the rewards for advanced preparation, the assistance available for qualified students, etc., we are publishing editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

Through these and future editorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

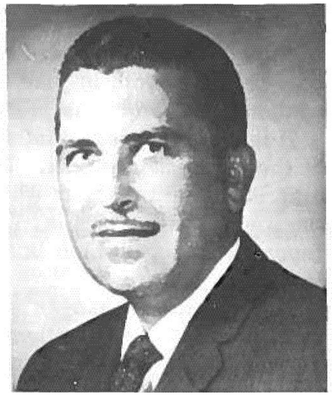
In this issue we are pleased to present an article from Remington Rand Univac Division of Sperry Rand Corporation, one of our country's leading manufacturer of electronic computer equipment. From the ENIAC (Electronic Numerical Intergrator and Calculator) first issued at the Aberdeen Proving Ground to today's UNIVAC LARC and UNIVAC III solid-state computers capable of calculating and processing complex data at speeds that almost defy the imagination, the development has progressed through several "generations" of computers. The first generation employed mercury line memory and vacuum tubes; the second, magnetic drum memory -and vacuum tubes; the third, magnetic core memory and solid-state components. Solid-state components such as diodes, transistors, and amplifiers depend on the controlled flow of electrons in solid substances. With these advances came greater speed, more memory capacity and smaller size, allowing a tremendous increase in applicability.

Remington Rand's contributions in this field together with the tireless effort of other producers of electronic computers have provided an ever increasing demand for the skilled mathematician as emphasized in Dr. Leutert's article. Almost every phase of scientific development is now coupled with computer analysis and techniques. Don't underestimate the advantages of advanced training in this field.

The following lists contributing corporations with the issue in which their editorials appeared.

Aeronutronics	Vol. 3, No. 2
Army Ballistic Missile Agency	Vol. 2, No. 10
AVCO, Research and Advanced Development	Vol. 2, No. 10
Bell Telephone Laboratories	Vol. 2, No. 10
Bendix Aviation Corporation	Vol. 2, No. 8
David Taylor Model Basin	Vol. 3, No. 5
E. I. du Pont de Nemours and Company	Vol. 3, No. 2
Emerson Electric Company	Vol. 2, No. 7
General American Life Insurance Company	Vol. 2, No. 9
Hughes Aircraft Corporation	Vol. 2, No. 9
International Business Machines Corporation	Vol. 2, No. 8
Office of Naval Research	Vol. 3, No. 5
Eli Lilly and Company	Vol. 3, No. 2
Mathematics Teachers College, Columbia U.	Vol. 3, No. 3
McDonnell Aircraft Corporation	Vol. 2, No. 7
Monsanto Chemical Company	Vol. 2, No. 7
National Science Foundation	Vol. 3, No. 3
North American Aviation, Inc.	Vol. 2, No. 9
Olin Mathieson Corporation	Vol. 2, No. 7
RAND Corporation	Vol. 3, No. 5
Research Analysis Corporation	Vol. 3, No. 5
Shell Development Company	Vol. 3, No. 1
Sperry Rand Corporation	Vol. 3, No. 6
Union Electric Company	Vol. 3, No. 4
Woodrow Wilson Foundation	Vol. 3, No. 3

THE SPERRY RAND CORPORATION



MATHEMATICIANS IN THE COMPUTER INDUSTRY

DR. W. W. LEUTERT
Director, Systems Programming,
Remington Rand Univac Division,
Sperry Rand Corp.

A critical shortage of competent mathematicians is developing rapidly throughout the computer industry.

This is an industry which as little as five years ago, was dominated completely by engineers. Today, Systems Programming or "software" is as important as the hardware built by engineers. Tomorrow, software is likely to exceed hardware in the effect it has on the success of any company in this field. As a consequence, almost without exception, the Systems Programming department in a company is at the same level as the engineering or the marketing department, both economically and organizationally.

It is easier today to find capable computer engineers than to recruit competent computer mathematicians. As in every other lucrative new area of endeavor there is no shortage of charlatans in the computer field.

There are some simple economic facts behind this development. During the past 10 years the computer population in the U.S. has increased by a factor of about 1000. Estimates indicate that over 5000 computers were in operation by the end of 1961. Computers are now an integral part of our economy. Programming costs are high, running between $5 and $10 per instruction or line of coding. Some companies have literally spent millions of dollars on programming alone. The population of computers has grown far more rapidly than has the population of competent programmers. Consequently the level of proficiency of the average programmer is dropping at an accelerated rate. Less than 10 years ago one of my colleagues at another location believed that a programmer was useless unless he had a Ph.D in mathematics. He succeeded at that time in staffing his installation according to his principles. Today this would no longer be possible.

Partly because of necessity and partly because of interest and healthy competition among the computer manufacturers a computer user of today expects his supplier to carry a growing share of his programming load. It is in this area that mathematicians are in increasing demand. As an extreme example, I have seen a case where two sets of mathematicians programmed the same problem correctly for the same computer. The solution times differed by a factor of over 100. This is why competence is so important.

There are several ways to reduce programming and operating costs of a computer both in time and money. Many interesting and unsolved mathematical problems present themselves in this area.

It is quite obvious that a computer can be used to take over much of the programming task. This means, that the manufacturer will furnish computer programs which in turn prepare the computer programs the user is interested in. These assemblers and compilers accept as input a language which is different from the machine language built into the hardware.

Such source languages are close to the language preferred by the user. For example, mathematical compilers use a language close to standard mathematical formalism. Data processing languages approximate the English language.

Any computer manufacturer would like to see compilers produced at low cost. The manufacturer would also like the compiler itself to use minimum computer time and minimum memory space to prepare computer programs which solve the user's problems in minimum time occupying minimum memory space.

Obviously, a compromise must be made between these contradictory objectives. In addition, the user would like a compiler to detect input errors and to correct them. Very little serious mathematical study has been given to the resulting mathematical problems.

The next step is to consider hardware and software as two aspects of one system and to design both simultaneously. Again, competent mathematicians are needed for this purpose. Executive routines sequence computer programs and may permit concurrent solution of several problems. Computer scheduling problems have barely been formulated in mathematical terms.

There is a growing demand to have computer manufacturers supply programming packages for specific classes of problems such as linear programming. Finally, a user expects some direct support on his own premises.

To succeed in this environment a mathematician should have thorough training in fields such as logic, linear algebra and matrix algebra. Numerical analysis and applied mathematics are helpful. English is essential for advancement into managerial positions. A certain amount of standardization must be accepted in the technical area. Flow charting conventions and quality control procedures in checking out programs are two such examples.

Fortunately more and more computer manufacturers are using sound management techniaues in their Systems Programming Departments. This includes not only a definition of group and individual objectives but also a tangible recognition of accomplishments as well as a weeding out of unsuitable talent. It is not unusual that up to 50% of inexperienced trainees do not succeed in this environment and leave within the first year of employment. I do not know of any mathematician, however, who worked hard and passed this hurdle, who has switched permanently into another field.

NEWS AND NOTICES

Edited By
Mary L. Cummings, University of Missouri

Dr. Alan J. Goldman (New York Gamma) has been appointed Chief of the Operations Research Section of the Applied Mathematics Division at the National Bureau of Standards, U. S. Department of Commerce, Washington, D. a.

As head of this newly organized section, Dr. Goldman will direct research in mathematical and computational techniques for the analysis and improvement of complex systems of activity-patterns. He will also be concerned with the application of these techniques to select problems, of general methodological significance, arising in the Bureau or other government agencies.

A mathematician who has done research in linear programming, the mathematical theory of games, operations research, and topology, Dr. Goldman received his B. A. from Brooklyn College in 1952, and his M. A. and Ph. D. from Princeton University in 1954 and 1956 respectively.

Army 2d Lt. William B. Nethery Jr., recently was assigned to the U. S. Army Chemical Center in Maryland.

The 25-year-old officer is a 1960 graduate of Louisiana State University, Baton Rouge. He is a member of Tau Beta Pi, Pi Mu Epsilon, Pi Epsilon Tau and Alpha Chi Sigma fraternities.

FELLOWSHIP OPPORTUNITIES

Each December for the past three years, the American Mathematical Society has published a special issue of the ***Notices*** of the society listing the various assistantships and fellowships in mathematics which are available at colleges and universities throughout the country. The 1961 issue also contains a list of foundations and organizations other than colleges and universities which offer such aid for graduate and undergraduate study in mathematics. Students who are interested in obtaining financial assistance, especially for graduate-level study, should find this issue of the ***Notices*** most helpful.

CHAPTER ACTIVITIES

Edited By

Houston T. Karnes, Louisiana State University

EDITOR's NOTE: According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an annual report of the chapter activities including programs, results of elections, *etc.*" The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the ***Pi Mu Epsilon Journal.*** Besides the information listed above, we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Chapter Activities Editor Houston T. **Karnes,** Department of Mathematics, Louisiana State University, Baton Rouge 3, Louisiana. These reports will be published in the chronological order in which they are received.

REPORTS OF THE CHAPTERS

ALPHA OF KANSAS, University of Kansas

The Kansas Alpha Chapter held seven meetings during the academic year 1960-61. The following papers were presented:

"Logic and Mathematics," by Professor J. M. Bochenski, University of Freiberg.

"Mathematical Proofs," by Robert K. Remple, Honor Initiate of 1960-61.

Officers for 1960-61 were: Director, **DeWayne** S. **Nymann;** Vice-Director, Charles Stuth; Recording Secretary, Thomas Kezlan; Treasurer, Katheleen **O'Donnell;** Corresponding Secretary, Wealthy Babcock; Librarian, Gilbert Ulmer.

Officers for 1961-62 are: Director, Charles J. Stuth; Vice-Director, Raymond E. **Pippert;** Recording Secretary, Janice A. Wenger; Treasurer, Kathleen **O'Donnell;** Corresponding Secretary, Wealthy **Babcock;** Librarian, Gilbert Ulmer.

BETA OF KANSAS, Kansas State University

The Kansas Beta Chapter held four meetings during the academic year 1960-61. The following papers were presented:

"Digital Approximation of the Solutions of Differential Equations Using Trapezoidal Convolution," by Dr. Charles A. Halijak.

"A Generalization of **Waring's** Problem," by Dr. Richard Yates.

"Some Aspects of Projective Geometry," by Dr. Beatrice **Hagen.**

"India's Education System and Some Recent **Developments,"** by Dr. Roshan Chaddha.

Initiation was held May 8, 1961, at which time Dr. Chaddha spoke on the above listed topic and twenty-seven new members were initiated.

Officers for **1960-61** were: Director, F. J. **McCormick;** Vice-Director, Grace **Woldt;** Secretary, Evelyn **Kinney;** and Treasurer, S. T. Parker.

Officers for 1961-62 are: Director, John Meux; Vice-Director, Charles Hulijak; Secretary, Evelyn Kinney; and Treasurer, William Stamey

ALPHA OF FLORIDA, University of Miami

The Florida Alpha Chapter held seven meetings during the 1960-61 academic year. The following papers were presented:

"The Seven Bridges of **Konigsberg,"** by Robert Kelley.

"Continuous Fractions," by Harvey Davis.

"The **Banach-Tarski** Theorem," by Dr. P. R. **Halmos.**

"Why Was Junior Disturbed or Testing an Analog Computer," by Dr. B. Howard.

"The Banach Theorem on Contraction Mappings," by Dr. H. Meyer.

Social activities included the Initiation Banquet at which the guest

speaker, Dr. Tomlinson Fort. spoke on "The History of Pi Mu Epsilon." A scroll was presented to Dr. Herman Meyer in recognition of his untiring efforts to promote interest in mathematics.

Officers for 1960-61 were Director, Robert L. **Kelley; Vice-Director,** Harvey Davis; Secretary-Treasurer, Gloria **Cashin.**

Officers for 1961-62 are: Director, **Al** Sadaka; Vice-Director, Larry **Hawkins;** Secretary-Treasurer, Gloria **Cashin;** Faculty Advisor, Miss **P.** Elliott.

NEW CHAPTERS INSTALLED

On October 3, 1961, Director General J. Sutherland Frame installed the eighty-third chapter of Pi Mu Epsilon, to be known as Virginia Beta, at Virginia Polytechnic Institute.

The ceremony **for** installing the new chapter was held in the YMCA chapel at five-thirty. Eleven of the thirteen chapter members and **twenty-**nine others were initiated into Pi Mu Epsilon at that time.

At the seven o'clock banquet attended by about sixty people, Dean Johnson, Dean Dean, and Vice President **Pardue** made brief remarks, as did also Professor Svend **Gormsen,** faculty adviser, Mr. George Gautney, Mathematics Club President, Professor T. W. Hatcher, chairman of the mathematics **department, and** Professor Boyd Harshbarger, of the statistics department.

Commenting on the **new chapter,** Professor Frame said, "I feel that the chapter at VPI promises to be strong and active."

Minnesota Alpha became the eighty-fourth chapter of Pi Mu Epsilon, installed at **Carleton** College, October 5, 1961, by Director General Grame.

On arriving at the **Carleton** campus. Professor Frame had lunch with members of the mathematics department. In the afternoon, he gave a talk on "Continued Fractions" to an audience of about seventy-five. The **installation** ceremony was held at five-thirty, and was followed by a banquet, at which Professor Frame talked about the history of Pi Mu Epsilon. Later, everybody was invited to the farm home of Professor Kenneth May for a social evening with cider and cookies.

Director General Frame installed the eighty-fifth chapter, Ohio Eta. at Fenn College, on November 17, 1961.

At four o'clock, he addressed the prospective chapter on "Professional Opportunities in Mathematics." At the installation, the following became charter members: W. R. Geiger, Mabel B. Greber, Marguerite Hrabak, Shirley Ann Lilge, D. G. Shields, Professor Alien W. **Brunsom (faculty** advisor), and Professor W. R. Van **Voorhis** (faculty corresponding secretary). Professor Frame briefly summarized the history of Pi Mu Epsilon at the dinner **which** followed the installation ceremony.

On November 27, 1961, Director General Frame presided at the installation of the eighty-sixth chapter, Louisiana Gamma, at Tulane University. Professor Frame's schedule of the **day included** an advanced talk on "Group Representations" at **2:00** p.m., an elementary talk on "Continued Fractions" at **4:30,** and the installation ceremony at **5:30,** where twelve people became charter members. Later, at the banquet, he talked on the history of Pi Mu Epsilon.

INITIATES

ALABAMA ALPHA, University of Alabama (December 11, 1961)

William L. Albritten
Gustavo A. Aramayo
Oren W Babb
Bessie L. Barno
Charles F. Bearden, Jr.
John C. Bostick
Richard H. Bouldin
Linda M Campbell
William H. Campbell
Charles H. Copeland
Robert M Cox, Jr.
Henry J. Crowder
Paul R. Davis
Robert C. Davis
David S. Dillard, Jr.
James T. Dixon
Dianne Dobbs
Hugh E. Dodgen
J. T. Doke
William E. Dunn
William G. Esslinger
Joseph C. Franklin
Barbara M Fromm
Sylvia L. Fugate
David L. Gabardi
Richard M Gibbs
Richard G. Gibson
Sammy F. Gilbert
Jay W Gray
William J. Gray
Gary P. Herring
Geoffrey S. Hicks
John Z. Higgs
Gerald J. Hogg
Linda A. Hogue
Charles G. Hooks
Christopher H. Horsfield
Thomas A. Hosty
Wyllie W Hudson
William E. Hughes
G. B. Johnson
Nancy C. Jones
Thurman D. Kitchin, III
David A. Kulpe
Christopher E. Laird
William M Lampkin
Joyce M Lawson
John W Long
Pearson M Lowell
Patricia A. Lucas
Elaine M. Meyers
Walter T. Meyers
Mohammed Kheir Mikdashi
Charles R. Mitchell
Cherry R. Mixon
Robert E. Moyer
Virginia L. McCahan
Melvin C. McIlwain
Hubert T. Nagle, Jr.
Walter G. Northcutt, Jr.
Ben B. Peete, Jr.
Dieter H. Pukatzke
Rodolfo C. Quijano
James D. Ramsey
Lewis E. Rayfield, Jr.
Don P. Reid
James H. Riddle
William E. Roberts
Ward D. Robertson, Jr.
Samuel D. Rosenberg
George J. Sakellarides
William E. Salter
Sondra J. Sanderson
Blinn D. Sheffield
Philip N. Smith
Farley M Snow
William S. Spivey
John Stark
Claudia J. Stewart
Mary B. Stone
James R. Storey
William M Terrill, Jr.
Ray F. Tipsword
James M Tucker
L. D. Wallace
Charles L. Watkins
Gaines L. Watts
Jerry D. Weiler
Joseph F. West
Karen A. White
Peggy R. Wright
Roy B. York, Jr.

ALABAMA BETA, Auburn University (November 13, 1961)

Robert A. Agrusti
Thomas W Athey
William J. Barksdale
Michael D. Bentley
Harold E. Bumgarner
E. B. Carson
Harry E. Chan
Janjai Chayavadhanangkur
Joseph R. Copeland
Gail C. Fitzgerald
George E. Frizzell
James A. Hutchby
William Ingram
Faye Jenkins
James J. Johnson
James H. Johnston
S. B. Khleif
Jess C. Laughlin, Jr.
William Locke
Sarah J. Marsh
William O. Millet, Jr.
David Mischke
Mary E. Newman
Eugene Omasta
Marilyn Pinson
Dorothy Porterfield
W. D. Ray
Marvin Reed
Sam Roberts
Frank Salzman
Diane Scarborough
James R. Smith
Joel J. Speights
Paul W Spikes
David B. Stewart
John B. Switzer
Charles P. Thomas
Russell L. Weaver

ARKANSAS ALPHA, University of Arkansas (November, 1961)

Charles E. Blackstock
James G. Blaylock
Darrell M Bolding
Ralph G. Brodie
Rex L. Clark
George R. Essig
James E. Gauntt
Edward K. Gordon
Mary Jo Hammond
Ruth C. Hill
Tom H. Magness
Martha E. Moore
Phillip R. Meinert
William F. Ogden
Lynda F. Stair
David W Terry
Douglas W Tiberus
Gus W Winfield

ARIZONA ALPHA, University of Arizona (January 4, 1962)

Charles E. Backus
Robert W. Clarke
Donald R. Kerr, Jr.
Kalman J. Miller
John D. Patterson
James G. Seal
Isaac R. Steinberg
Robert Taylor
Daniel L. Thomas
Garth W. Warner
Philip M Weber

FLORIDA ALPHA, University of Miami (May 27, 1961)

Robert Adt, Jr.
Denys O. Akhurst
Henri Bader
Luis Baez
Steveh A. Bernstein
James C. Bidwell
Leonard S. Bobrow
Michael R. Botwin
Paul L. Brown
Albert Caputo
hfelvyn Ciment
Hallie J. Cohen
Albert Comparini
Irene Cooper
John H. Curtiss
Basil Dimitriades
Jacquelin E. H. Elliott
Howard Feinberg
Madeline Fiduccia
Joyce Fortgang
Howard Frank
Robert B. Frary
Neil J. Freeman
Patricia E. Gaynor
Shlomo M Gerchakov
Joan Vera Gerhard
Denny W Glenn
Larry Hawkins
Jules B. Kaplan
Stephen Klein
Emily G. Kozakoff
James M Kronick
Joel Kutnick
Roland J. Lavelle, Jr.
Darlene Macik
Amer W. Nelson
Al R. Sadaka
Robert S. Tarzon [illegible]
Wanda L. Taylor
Penny Zinn

FLORIDA BETA, Florida State University (December. 1961)

William E. Bird
W. Thomas Blackshear
Thomas R. Caplinger
Marjorie A. Carlson
Jane L. Finchum
Robert M. Holley, Jr.
Stinson H. Lenkerd
Sharon L. Moses
Bernard L. Palmer
Edward G. Platt, Jr.
L. J. Rhodes
William H. Ruckle
Harry G. Sharp, III
Daniel L. Solomon
John W. Ulrich
Robert H. VanDam

GEORGIA ALPHA, University of Georgia (January 10, 1962)

Nancy A. Fox
Juanice Forte
Dorothy A. Paschal
Saralyn Souter
Robert J. Simpson

GEORGIA BETA, Georgia Institute of Technology (January 14, 1962)

Neal C. Aaron
Daniel L. Albritton
John F. Cooke
Joseph D. Cornwell, III
Charles P. Frahm
William E. Heierman
Roland R. Kneece, Jr.
William H. Miller
Anne S. Minkin
Ferdinand E. Schlaepfer
Benjamin M. Smith
William A. Steed, Jr.
Lester N. Tharp
Nguyen Duc Tuong
Jerry L. Whitten

ILLINOIS GAMMA, DePaul University (Fall. 1961)

William G. Valence

KANSAS ALPHA, University of Kansas (January 10, 1962)

Martha L. Anderson
Thomas D. Beisecker
Bruce A. Burns
Robert H. Bussard
Larry L. Dike
Thomas J. Fitzgerald
Elizabeth A. Fly
Wilfred M. Greenlee
Larry F. Heath
Stanley Kranzler
Phillip R. Long
Kenneth C. Matson
Marc N. Murdock
Charles R. Nicolaysen
Carol F. On
Phillip H. Roberts, Jr.
Derald D. Rothmann
Joanne K. Stover
Darrel R. Thoman

KANSAS GAMMA, University of Wichita (December 6, 1961)

Robert G. Blaisdell
Herbert H. Coin
Don C. Cox
L. Darlene Fearey
Thomas C. French
David W Johnson
Jerry G. Mrazek
Earl R. Norman
Vernal Piantanida
Herbert E. Smith, Jr.
Gordon D. Stevenson
Jimmy R. Wallis

KENTUCKY ALPHA, University of Kentucky (January **18, 1962)**

August D. **Brackett**
Bobbye J. Connell
Louis E. Furlong
Thomas L. **Hayden**
Lael F. Kinch
Rodney **J.** Roth

LOUISIANA ALPHA, Louisiana State University (December **13,1961)**

Hector A. Arredondo
Elizabeth **Bollinger**
Tommy B. Borne
Henry E. **Corkern**
Larry D. **Crandall**
Merrill A. Espigh
Charles B. Holloway
Ann M. Johnson
Bob **J. Kellenbeck**
Howard B. **Lambert**
Ann H. **LeLaurin**
Austin **J.** Lemoine
Eugene D. Lischewski
Elton B. Martin
Alfred L. Mason
Wallace R. **Mixon**
Joseph P. Patti
Anna Louis B. Rauer
Harold B. Reirer
Roy M. Roberts
John F. Rogers
Hugh H. Walker
Henry H. Wall, **III**

LOUISIANA GAMMA, Tulane University (November **27, 1961)**

George P. Barker
Charles R. Blackburn, **II**
Robert E. Bonini
Harry R. Carson
Norman **S.** Fertel
John H. Fielder
Sylvia **A.** Ibele
Robert C. Kagy
Stanley A. Kurzban
Dallas **Mallerich** Jr.
Patricia S. **Merkle**
Samuel **Merrill, III**
John P. Riley, Jr.
George W. Tiller
Alexander D. Wallace
Fred B. Wright

MINNESOTA ALPHA, Carleton College (October 5, **1961)**

Theodore Bergstrom
Barbara **Bragman**
Sidney Came
Margo Cleavenger
Donald Cooper
Barbara Dreyer
John Dyer-Bennet
David Hildebrand
William B. Houston, Jr.
Paul S. Jorgensen
Thomas Kieren
Bertha **Lauritzen**
Donald Olivier
Robert Peterson
Ellen Rosen
Mary **Stelma**
Kenneth W. Wegner
Richard Wolff
George Woodworth

MISSOURI ALPHA, University of Missouri (December **5, 1961)**

Francis E. Burk
Jerry L. Daniel
Ira D. Dodge, **III**
Roger W. Gardner
David L. Gill
Gleen D. Harbison
Robert N. **Healy**
Wilma T. Heeren
Warren C. Jackson
Donald F. James
Samuel E. King
Roger L. Kyllonen
James C. Lynch
James E. Lyon
Patricia **McCarty**
Samuel L. hlagruder
Emma **D.** Martins
Roland P. Meyer
Jack C. **Mixon**
Robert A. Mollenkamp
Harry P. **Planchon,** Jr.
Roger **L.** Proctor, Jr.
Herman N. Ramakers, Jr.
Hannah L. **Rickman**
Ronald Roy Salmons
Eddie L. Simmons
David R. Smith
Ronald E. Sonderegger
Gene M. Sweeney
Dennis L. Tebbe
George E. **Tillman**
Lester M. Waganer
Philip M **Wolfe**

MONTANA BETA, Montana State College (May **19, 1961)**

Jere **J.** Alien
Clarissa E. Bliss
Jon R. **Carlson**
David **Chamberlin**
John M Clark
Arthur E. Clements
Barney M. Considine
Clinton **J.** Cooney
Robert **J.** Day
Dr. Hans R. Fischer
Dennis **Garoutte**
Robert H. Golder
Dr. Herbert Gross
Donald W. Haase
Lindsay Hess
Kenneth M Lochner
Stewart C. Keeton
Dennis **Osgard**
Margaret **J.** Palmer
Marcia Peterson
Fr. John F. **Redman**
Gary Rogers
Roger R. Schell
Dorothea **Striebel**

NEBRASKA ALPHA, University of Nebraska (December-14, **1961)**

Dale E. Anderson
Joe Charles Anderson
Kenneth B. Chatfield
Orville D. Dodd
Gary R. Fleischmann
Robert E. Healey
Robert G. Ladd
Steven C. **Lange**
Ronald L. Morse
Gary **J. Policky**
James D. Reierson
David **J. Sandfort**
Donita **J.** Schmidt
David A. **Scholz**
Charles **J. Sherfey**
Robert M. **Stearley**
Robert Stevenson

NEW JERSEY ALPHA, Rurgers University (Fall, **1961)**

James N. Alexander
Ernest **J. Bastian,** Jr.
Alien B. Borodin
Steven A. **Brawer**
John M. Cannel
Albert H. Clark, Jr.
John M Clarke
William Culverhouse
Robert H. Dawson
Donald B. Dinsmore
Philip L. **Feidelseit**
Leonard H. **Goldman**
David A. Greenberg
Frederick A. Grimm
J. Michael Hartstein
Robert P. Hill
William G. Hill, Jr.
John **J.** Jarka
Lewis **Joris**
Kalju Keero
Marc W. Konvisser
Allen M Krieger
Stanley K. **Kulpinski**
Peter Kusulas, Jr.
Kenneth R. Lee
Richard B. Mansfield
William R. **Matthes**
Henry C. Mazzoni
William F. **McCord**
Cyril **J. Mumber**
Peter R. Member
Frank Niver
Walter V. Paliga
Ralph E. **Portmore**
Norman **Primost**
Stanley **J. Radomski**
Donald L. **Reisler**
Louis **J. Rivela**
Gilbert Sandier
Constantine P. Sarkos
William F. **Selders**
Richard **J.** Sieredzki
Charles **J. Szyszko**
Michael L. Tropp
Richard **J. Wasowski**
David G. **Whiteman**
Walter **Woshakiwskj**

NEW YORK BETA, Hunter College (November **6, 1961)**

James Henderson
Nancy L. Knaff
Lois B. **Mazor**
Veda F. Osborne
Lorraine S. **Paladino**
Diana **J. Polley**
Gabriel Rosenberg
Marie **J.** Tucci

NEW YORK GAMMA, Brooklyn College (December 4, **1961)**

Harvey Abramson
Phyllis Barrel
Robert Blumenthal
Alien Cohen
Sheldon Ross
Stephen Seide
Shaul Stahl
Robert Taylor
Michael Tinkler
Martha **Wallach**

NEW YORK EPSILON, St. Lawrence University (October **11, 1961)**

Joan M. Castro
David S. Daniels
Claire S. Durham
Leslie **C. Higbie**
Charles Hyams
Herbert W. **LaVere**
Robert L. **Lockwood**
Barbara L. **McKeon**
Barbara A. Raine
Michael F. Sullivan
Judith M **Witzig**

NEW YORK ETA, University of Buffalo (May **17, 1961)**

Claudia R. Britt
Naomi R. Certner
Robert O. Green
Hugh **C.** Joudry
Gordon E. **Keller**
Donald W. **Reinfurt**
Steven **J. Rojek,** Jr.
James P. Stevens
Margaret L. Vitanza
Carol R. **Wendel**

NORTH CAROLINA GAMMA, North Carolina State College (December 5, **1961)**

David Anderson
Franklin Benson
Jimmie H. **Caldwell**
Carbon B. Carver
Stephen B. Denny
William S. **Guion**
James C. Halsey
Leland K. **McDowell**
Hugh B. Noah
Van B. Noah
Albert K. **Pearson**
William Hi Roberson
Richard H. Shachtman
Samuel D. Scott, **III**
William M. Spence
Henry F. **Tisdale**
Robert E. Williams
Raymond S. **Winton**
James W. York, Jr.
Richard H. Williamson

OHIO DELTA, Miami University (November 20, **1961)**

Phillip R. Brown
Saralyn **Brugh**
Emily E. Carson
Dennis **A. Costarakis**
Leslie Jane **Galle**
Mark E. Humphrey
James L. Hussey
David A. James
Janet D. Jenkins
James R. Krabill
James H. Morrison

OREGON ALPHA, University of Oregon (May 17, 1961)

Larry L. Archambeau
Kirk W. Battleson
Wendell T. Beyer
Ted L. Calouri
Frank Cater
Julia N. Chambliss
Henry H. Chau
George A. Chrones
Wing F. Chung
Leslie L. Edwards
Bryan C. Ellickson
Janet H. Fesq
Diana R. Glover
Janice H. Hinton
Bryan T. Hodges
H. William Houghton
Richard C. Janzig
Preston L. Joiner
Jeanne Kullberg
Julie C. T. Liu
Ernest C. McGoran
William McGuire
Carole H. McQuarrie
Douglas Mathews
Scott Niven
Thomas B. Paine
John R. Pond
Harvey Richmond
Richard L. Robinson
Virindra M. Sehgal
Timothy Thomas
Steven K. Thomason
Lée Trippett
I. Fei Tsai
Virginia Walsh
Peter C.C. Wang
William H. Wright

PENNSYLVANIA ALPHA, University of Pennsylvania (Fall, 1961)

Steven Kingan
Brian Kritt
Bruce Kuklick
Edward Miller
Ogden G. Nackoney
Longine Prusinowski
Arnold Rosen
David Rosenglick
Wayne Saslon
Arlene Tunick

PENNSYLVANIA BETA, Bucknell University (November 29, 1961)

Gail K. Anderson
James R. Bensinger
Thomas A. Benton
William D. Besselievere
Gay M. Brook
Edward R. Chertkof
Jeanne E. Corson
Ints Delgalvis
Forbes L. DeRusha
Jay W. Faberman
Richard H. Fidler
Elaine L. Fiorentino
David N. Goss
Riccardo J. Illingsworth
Louis W Johnson
Michael B. Jones
Marlene A. Kresge
Allen L. Metzler
Arthur F. Michaelis
Ronald A. Petticoffer
Terrence M. Quick
Mary L. Schmidt
Virginia E. Speer
Mary J. Taylor
Ellen M. Thomas

TEXAS ALPHA, Texas Christian University (December 12, 1961)

James C. Bolen
Virginia I. Coblentz
James A. Gardner
Denwood F. Ross
Kenneth G. Sloan
Aleksandar Svager
John H.M. Whitfield

VIRGINIA ALPHA, University of Richmond (October 30, 1961)

Walter H. Carter, Jr.
Robert L. Davis
Pauline F. Fones
Wallace E. Garthwright Jr.
Stuart V. Grandis
Claude C. Gravatt, Jr.
Eleanor K. Koontz
Betty C. McMullin
Jean W Morris
Thomas J. O'Connor
Ronald N. Orr
William S. Ryan, Jr.
Thomas C. Smith
Margaret E. Stafford
Cecelia A. Stiff
William R. Tolbert
Thomas B. Vassar
James F. Watts

VIRGINIA BETA, Virginia Polytechnic Institute (September 29, 1961)

Orville W Addington
Bruce C. Allnutt
Gene H. Anguil
Robert H. Appleby
Rolf E. Bargmann
William H. Beyer
Kimiko P. Bowman
Hugh G. Campbell
Harvey J. Charlton
Poo-Sen Chu
Howard E. Clark
Jane K. Cullum
Roger E. Flora
James P. Grimes
Waldemar E. Heinzelmann
William W Hokman
Burt C. Home
David C. Hurst
James E. Jackson
James R. Kent
Peter C. Kim
Richard L. Kimball
John W. Layman
Robert S. Lecky
Lydia R. Lohr
Anna C. Miller
Jessie W Moore
Aneurin V. Morris
Raymond H. Myers
Wesley E. Pace
Frederick W. Patterson
Alice M. Pletta
Charles. P. Quesenberry
Louis B. Rail
James P. Ray
Gerald D. Repass
Daniel M. Sheehan
Thomas P. Sternberg
Robert J. Taylor
Charles C. Thigpen
Benon J. Trawinski
William A. Turner
Donald R. Vaught
Larry J. Walker
Shin-Hon Wong

WASHINGTON BETA, University of Washington (May 17, 1961)

Brian R. Alspach
Jesse W Armstrong
Robert Atkins
Patricia Bevan
Robert G. Bliesner
Joel Bloomor
Robert L. Brandon
Leslie A. Burnell
Rose L. Burnell
Chester M Burrows
Esko Cate
Jeremy F. Caughlan
Tony Y. Chinn
John Churchill
Robert Crenshaw
James D. Danberg
Donald B. Daniels
Robert J. Douglas
Morris L. Eaton
Melvin Eide
Lauren Defaccio
Gordon Fall
Roger L. Farrell
David A. Gaudio
Erick Gidlund
Malcolm Gray
David J. Griffiths
Leonard D. Gross
Joseph Hall
Norman Hamilton
Ramona Hammerly
Noel Hardy
William B. Hemphill, Jr.
Kenneth D. Hill
Victor Huang
Duane Hudson
Kichio Ishimitsu
Sam Key
Richard Koyama
George Kutter
Jack N. Lahti
Francis J. Lawlor
R. Leon Leonard
Robert Li
Duaine Linstorm
Everts C. Lyle
Kenneth H. Lynch
Thomas A. McCoy
Norchiko Mihara
Richard Moores
Herbert L. Munson
James F. Nale
William L. Newcomb
John Nylander
Richard Oettel
Marjorie Omori
R. Jerome Peterson
H. D. Pitzler
Ronald R. Roulet
Marvin C. Schiffman
Werner Schimmelbusch
Jerry Schwarz
Charles Sienkiewicz
R. H. Stoltman
Donald Taylor
Okey Townsend, Jr.
John P. Turneaure
Don R. VanderStoepe
Norman Vincent
Ronald L. Wadsack
Kirke Wolfe
William J. Wilson

WASHINGTON GAMMA, Seattle University (November 16, 1961)

Nickolas V. Arvanitidis
Joseph A. Bossi, Jr.
David R. Ferguson
Robert H Kuhner
James J. Merkel
Houng Yu Yang

WISCONSIN ALPHA, Marquette University (November 19, 1961)

Edward J. Blanc
William E. Bodden
Joseph A. Haertle
Gerald T. Hunt
Michael E. Kelly
Hannelore Klaus
John C. Lemanski
Charles R. Lemke
Edward H. Majewski
Thomas Joseph Meskel
Robert J. Norbutas
Pamela L. Parks
Vincent A. Rowe
Gerald W. Scholand
Henry E. Thompson
Richard J. Yezek

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