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PI MU EPSILON JOURNAL  
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## WHEN LINES ARE POINTS<sup>1</sup>

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by Harry Levy

The beginning student derives a great deal of pleasure from working in mathematics. In part his sense of satisfaction stems from his realization that mathematical problems have a definiteness that problems in other areas often seem to lack, and that the correctness, or lack of it, of his solutions is not a matter of opinion but a hard mathematical fact. It may therefore be surprising to some of you when I observe that mathematics resembles the fine arts and indeed may be called one of them; it does so because the mathematician like the poet, the sculptor, and the composer, creates something out of nothing using only his talent and his knowledge, the resources of his intellect. This may have been the frame of mind that led the youthful geometer **Janos Bolyai** to write to his father, in 1823, "I have created a new universe out of nothing." Like all great artists, Bolyai was passionately proud of his creation, and like all great art, **Bolyai's** work is immortal. It lives on in today's nuclear age to which it has been bound by the work of Lobachevski and Gauss, of Riemann and Ricci, of Einstein, and of many others.

But the artistry of mathematics is only one of its aspects. Mathematics is a science, because mathematicians observe and study existing mathematical structures, and discover and formulate the basic principles that govern them. Perhaps one characteristic that distinguishes mathematics from other areas of human endeavor is that the scientific side of mathematics and its artistic side are united in one coherent whole; I intend this evening to suggest how Riemann, a great German mathematician of the middle nineteenth century, might have been led to the creation of elliptic **geometry** (one of the two classical non-Euclidean geometries) by a "**scientific**" study of a particular Euclidean structure.

Before going on to my main objective, I should like to note that when a scientist is confronted with something apparently new, he frequently uses the method of analogy. He compares the new with the familiar, and, drawing on his knowledge of the latter, he is often able to direct his studies in a meaningful way. In using this method in mathematics, proper vocabulary and suitable choice of symbols can serve to illuminate the structure being studied and to clarify the problems whose solution is being sought. The primitive elements of geometry are usually called points, and when a mathematician is investigating any kind of mathematical structure by geometrical methods, he is very likely to call the constituent elements of his structure points, even though, in some other context, he may refer to them as lines, or functions, or matrices, or spheres, or transformations.

The structure that we propose to investigate is a bundle of lines in ordinary, that is, Euclidean (**three-dimensional**) space. For our purposes, a bundle of lines means the set of all lines through a given point. Some

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<sup>1</sup> These remarks were presented to the Undergraduate Mathematics Club, University of Illinois, Urbana, Illinois.

of you may wonder whether such a commonplace structure can possess any exciting properties. Since we wish to examine relationships between lines of the bundle, since our "space", that is, the bundle, is a set of lines, it is clear that we are to be concerned with a geometrical analysis in which the primitive elements are lines. However strange it may seem, we cannot hesitate. From this moment on, whenever we speak of a *point* we shall mean a *line* of the given bundle. We formalize this first step of our analysis thus:

1. There exists a set of objects called points; their totality, together with the relationships between them, is called an elliptic plane.

We shall denote the elliptic plane by  $E_2$ , and the notation  $X \in E_2$  is read " $X$  is an element of  $E_2$ ", that is,  $X$  is a point of the elliptic plane. To prove a theorem in elliptic geometry, it suffices for our purpose to select a suitable theorem about lines of a bundle and then translate the relationships involving lines of the bundle into geometrical language appropriate to the elliptic plane. In the bundle of lines, each two lines determine a real number, their angle, which we can, for convenience, measure in right-angle units. We commented earlier on the importance of vocabulary; what shall we call this number when we are concerned with elliptic geometry? It would seem that the word "distance" might prove useful; using this terminology enables us to put our second observation of elliptic geometry as follows:

2. Each two points of  $E_2$  determine a real number; if  $P, Q \in E_2$ , we call the number they determine the (elliptic) distance between them, and we denote it by  $e(P, Q)$ . We call  $e$  the distance function or the metric of  $E_2$ ; it satisfies the following conditions:

- (a)  $0 \leq e(P, Q) = e(Q, P) \leq 1$ ;
- (b)  $e(P, Q) = 0$  if and only if  $P = Q$ ;
- (c)  $e(P, R) \leq e(P, Q) + e(Q, R)$ .

The proof of (a) and (b) is indeed trivial; (c) can be established by elementary solid analytic geometry if we introduce rectangular coordinates chosen so that the direction cosines of the lines representing  $Q, P, R$  are  $(1, 0, 0)$ ,  $(\cos \theta, \sin \theta, 0)$ ,  $(\cos \alpha, \cos \beta, \cos \gamma)$ , respectively, where  $\cos \theta \geq 0$ ,  $\cos \alpha \geq 0$ .

The Euclidean distance function satisfies these three conditions with the exception of the second inequality in (a), and indeed these conditions play an important role in defining metric geometries that are more general than either Euclidean or elliptic. The last inequality in (a) tells us that in elliptic geometry, as in spherical geometry, distance is bounded. Condition (c) is called the triangle inequality; it can be used to define the curves that play the role of Euclidean straight lines. But even in elliptic geometry there are difficulties that we need to take care of. Returning to our bundle of lines, we see that  $\angle PQ + \angle QR = \angle PR$ , where  $P, Q, R$  are lines, if and only if  $Q$  is coplanar with  $P$  and  $R$  and lies in the acute angle sector formed by them. But we would like to prolong an elliptic line beyond any one of its points, and we can be satisfied with

the following formulation:

3. If  $P, Q$  are distinct points of  $E_2$ , there exists one (and just one) point  $P_Q^*$  such that

$$e(P, Q) + e(Q, P_Q^*) = e(P, P_Q^*) = 1.$$

We define the elliptic line through  $P$  and  $Q$  to consist of all points  $X$  such that

$$e(P, X) + e(X, P_Q^*) = e(P, P_Q^*).$$

Circles arise naturally in any two-dimensional metric geometry.

4. If  $P \in E_2$ , and  $r$  is a positive real number, the (elliptic) circle with center  $P$  and radius  $r$  is the set of points  $X$  distant  $r$  from  $P$ ,  $e(P, X) = r$ .

Thus an elliptic line corresponds to a pencil in the Euclidean bundle, and an elliptic circle to a circular cone (with one exception)

We now have points, lines, circles, and distance in our emerging geometry. Do these concepts differ significantly from their Euclidean counterparts? If we turn to elementary intersection and incidence relations, we find some results that are ordinary, and others that deviate sharply from the Euclidean pattern.

(A) Two distinct points in  $E_2$  are incident to (that is, lie on) exactly one line.

(B) Two distinct lines in  $E_2$  have exactly one point of intersection. Thus the phenomenon of parallel lines does not exist in  $E_2$ .

(C) Every line in  $E_2$  is a circle of radius one.

Here we have a property that suggests spherical geometry, but the parallelism is not complete since every circle in spherical geometry has two centers.

(D) Concurrent lines in  $E_2$  have collinear centers, and lines with collinear centers are concurrent:

A somewhat less elementary property of elliptic lines is the following:

(E) Every line in  $E_2$  is a simple closed curve of length two.

A Euclidean line is cut when one of its points is deleted, and the two portions form the two rays emanating from the point. Removal of one point of an elliptic line leaves the line connected; to cut the line two points must be removed, and of the two segments then formed, one has length equal to the distance between the deleted points, and the other, two decreased by that distance. But a more interesting situation arises when we examine (E) and (B) together. We are doomed to fail if we attempt to portray these two properties on a blackboard. Have we found a contradiction? No, the validity of (E) and (B) is irrefutable. We may have gone astray in tacitly supposing that in every two-dimensional geometry a simple closed curve determines an exterior and an interior. We need only to consider on a torus either a generating circle or a parallel circle to see the error in such an assumption.

Let us return to our model, the bundle of lines. Let  $p$  be a pencil of lines in the bundle, and let  $P$  and  $Q$  be any two distinct lines not in  $p$ . Then clearly the pencil determined by  $P$  and  $Q$  is separated by  $P$  and  $Q$  into two sectors of lines, one of which has a line in common with  $p$ ,

and the other does not. Thus, in  $E_2$ , any two points not on a given line of  $E_2$  can be joined by a segment which does not intersect the given line. If  $E_2$  were cut along the given line, it would remain connected. We thus have the following:

(F) *An elliptic line is a nonbounding curve.*

There is one farther very fundamental difference between the elliptic plane and the Euclidean plane. The latter is orientable, and it possesses two orientations. A line reflection interchanges the two orientations, whereas rotations and translations preserve both orientations.

Thus an orientation-preserving motion in the Euclidean plane can never be the product of an odd number of line reflections. We cannot here prove that  $E_2$  (like the Mobius band) is nonorientable, but we can prove that the Euclidean distinction between products of an odd and of an even number of line reflections has no significance in  $E_2$ .

A point reflection in Euclidean space leaves fixed each line of the bundle through the point; it therefore acts as the identity transformation when interpreted in  $E_2$ . On the other hand, it is the product of successive reflections in three mutually orthogonal planes. A reflection in a plane when interpreted in  $E_2$  becomes a reflection in a line (the elliptic line corresponding to the pencil in the plane). Thus in  $E_2$  the product of three line reflections can be the identity, so that the product of two line reflections can be a single line reflection.

Those of you who might wish to follow up these preliminary remarks on elliptic geometry could investigate one or more of the following problems:

(a) The problem of coordinatization. One way to establish coordinates in  $E_2$  is to represent a point by direction numbers of the corresponding line of the bundle.

(b) The problem of axioms. It is possible to establish a system of axioms which give  $E_2$  a logical existence independent of Euclidean space.

(c) The development of relationships between an elliptic and a Euclidean plane. Such a development can create for the Euclidean plane a unity which would otherwise be lacking. For example, all "conics" in  $E_2$  are closed curves, and by means of them we can find common properties of conics in the Euclidean plane.

University of Illinois

## CONFORMALLY ELEMENTARY POINTS<sup>1</sup> 329

By Louis E. De Noya  
Oklahoma Beta

### INTRODUCTION

In 1953 Professors Peter Scherk and N. D. Lane published a paper (3) which employed unique methods for the study of conformal differential geometry; methods which are rather synthetic in character. If we consider the conformal plane as the one point compactification of the Euclidean plane it is apparent that methods from complex variables might be used in its study. However, certain desired properties are lost when extensions to conformal spaces of higher dimensions are attempted. The methods of Scherk and Lane are directly applicable to higher spaces (2, 5, 7). They are related to methods developed by Scherk in his studies of projective spaces (6).

The results presented in this paper are due primarily to these two men. Some of their work has been modified and new proofs have been offered for particular theorems by both the present writer in his master's report (1) and by F. A. Scherk in his master's thesis (7). The introduction of conformally elementary points and an indication of their use are our goals.

### PRELIMINARIES

The points studied here are considered to be interior points of real arcs embedded in the conformal plane. This restriction to interior points is one of convenience for the theory has also been developed to **emcompass** end points of arcs.

A proper circle  $C$  will decompose the plane into two regions, the interior  $C_*$  and the exterior  $C^*$  of  $C$ . The interior lies to the left of the orientation of  $C$ . The infinite sequence of circles  $C_1, C_2, \dots$  will converge to the circle  $C$  if there exists to every pair  $C' \subset C$  and  $C'' \subset C$  an integer  $k = k(C', C'')$  such that  $C' \subset C_{n+k}$  and  $C'' \subset C_n^*$  for every integer  $n > k$ . Convergence of a sequence of points to a point and convergence of a sequence of circles to a point are defined similarly. It will be convenient to denote by  $C(P, Q, R)$  the circle  $C$  drawn through the three mutually distinct points  $P, Q$  and  $R$ .  $C(\pi P)$  will denote the circle  $C$  of the linear pencil  $\pi$  of circles which passes through the point  $P$ . Only pencils of the second kind with fundamental point  $s$  will be considered (cf. Fig. 1).

Finally, suppose that  $s$  is an interior point of an arc  $A$ . Then  $s$  is called a point of support (intersection) with respect to a given circle  $C$  if there

<sup>1</sup> Received by Editors September 30, 1961. Presented at the National Meeting of Pi Mu Epsilon, Stillwater, Oklahoma, August 29, 1961. This paper was judged best for graduate students. Mr. De Noya won a hundred dollar prize.

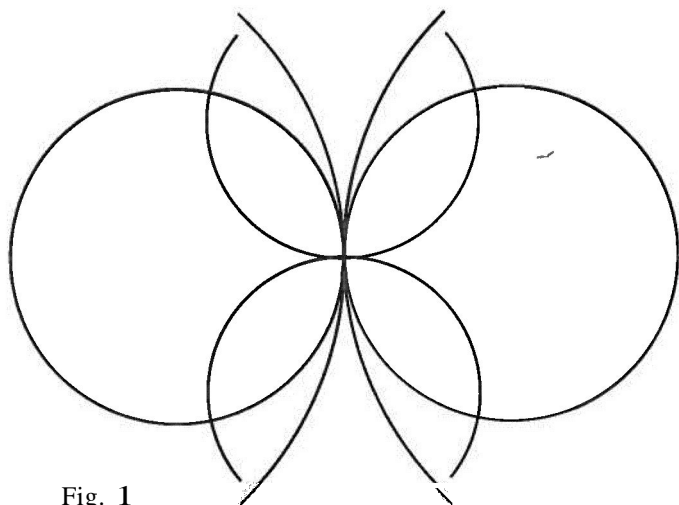


Fig. 1

is some neighborhood of  $\mathbf{s}$  decomposed by  $\mathbf{s}$  into two open neighborhoods which lie in the same (different) regions bounded by  $C$ .  $C$  is termed a supporting (intersecting) circle of  $A$  at  $\mathbf{s}$ . By definition the point circle  $\mathbf{s}$  supports  $A$  at  $\mathbf{s}$ . It may be that for a given arc there are points of the arc which are neither points of support nor of intersection, e.g. the arc

$$t \sin \frac{1}{t} \text{ at } t = 0.$$

#### DIFFERENTIABILITY

We shall say that an arc  $A$  is differentiable at an interior point  $\mathbf{s}$  (or that  $\mathbf{s}$  is a differentiable point) if two conditions are satisfied. The first of which is

$$t \sin \frac{1}{t} \text{ at } t = 0.$$

##### Condition I.

Let  $\mathbf{s} \in A$ . Then for every point  $P \neq \mathbf{s}$  and for every sequence of distinct points  $\{\bar{\mathbf{s}}\} \rightarrow \mathbf{s}$ , such that each  $\bar{\mathbf{s}} \in A$ , there exists a circle  $C_0$  such that  $C(\bar{\mathbf{s}}, \mathbf{s}, P) = C_0$ .

The circle  $C_0$  is called the tangent circle of  $A$  at  $\mathbf{s}$  and is clearly independent of how  $\{\bar{\mathbf{s}}\}$  converges. However,  $C_0$  does depend continuously upon  $P$ . The point  $\mathbf{s}$  is itself the tangent point-circle of  $A$  at  $\mathbf{s}$ .

##### Theorem 1

The set  $\tau = \tau(\mathbf{s})$  of all tangent circles of  $A$  at  $\mathbf{s}$  is a pencil of the second kind with fundamental point  $\mathbf{s}$ .

Proof. Let  $P, Q$  and  $R$  be three mutually distinct points. If the sequence of points  $\{\bar{R}\} \rightarrow R$ ,  $\bar{R} \neq R$ , then the angle between the circles of the sequence of circles  $C(\bar{R}, R, P)$  and  $C(\bar{R}, R, Q)$  converges to zero. Thus, the circles have (in the limit) the one point  $R$  in common and are therefore tangent. Suppose now that  $R = \mathbf{s}$  and  $\bar{R} = \bar{\mathbf{s}}$ , where  $\bar{\mathbf{s}} \in A$  and  $\mathbf{s} \in A$ , then

$$\lim_{\bar{\mathbf{s}} \rightarrow \mathbf{s}} \angle [C(\bar{\mathbf{s}}, \mathbf{s}, P); C(\bar{\mathbf{s}}, \mathbf{s}, Q)] = 0$$

Since the only restriction placed upon the points  $P$  and  $Q$  was that they be distinct it follows that for every other distinct point  $M$  there is one and only one circle through  $M$  and tangent to  $A$  at  $\mathbf{s}$ . Hence, if two tangent circles have another point in common they must be identical. Moreover, let some circle  $C$  touch the tangent circle  $C_0$  at  $\mathbf{s}$  and let  $P \in C$ ,  $P \neq \mathbf{s}$ . Then  $C_0$  and  $C$  are identical, i.e.  $C \subset \tau$ . Therefore the totality of tangent circles of  $A$  at  $\mathbf{s}$  constitutes a pencil  $\tau = \tau(\mathbf{s})$  of the second kind with fundamental point  $\mathbf{s}$ .

##### Corollary

If  $C(\tau, P)$  and  $C(\tau, Q)$  have another point in common they are identical. Thus, there is one and only one circle of  $\tau$  through each point  $P \neq \mathbf{s}$ .

The above then yields

##### Theorem 2.

Suppose that the point  $\mathbf{s} \in A$  satisfies Condition I. Let  $\pi$  be a pencil of the second kind with fundamental point  $\mathbf{s}$ ,  $\pi \neq \tau$ . If  $\{\bar{\mathbf{s}}\} \rightarrow \mathbf{s}$  ( $\bar{\mathbf{s}} \in A$ ,  $\bar{\mathbf{s}} \neq \mathbf{s}$ ), then

$$\lim_{\bar{\mathbf{s}} \rightarrow \mathbf{s}} C(\pi, \bar{\mathbf{s}}) = \mathbf{s}.$$

##### Theorem 3.

Let  $\mathbf{s}$  be an interior point of an arc  $A$  and assume that  $\mathbf{s}$  satisfies Condition I. Then every non-tangent circle either supports or intersects  $A$  at  $\mathbf{s}$ .

Proof. Suppose that  $C$  neither supports nor intersects  $A$  at  $\mathbf{s}$ . Then the non-support supposition implies that there is a sequence of distinct points  $\{\bar{\mathbf{s}}\} \rightarrow \mathbf{s}$  such that  $\bar{\mathbf{s}} \in (A \cap C)$  and  $\bar{\mathbf{s}} \neq \mathbf{s}$ . Now let  $P \in C$ ,  $P \neq \mathbf{s}$ . Then  $C = C(\bar{\mathbf{s}}, \mathbf{s}, P)$  for every  $\bar{\mathbf{s}} \neq \mathbf{s}$ .

But Condition I implies that  $C = C(\tau, P)$  which contradicts the hypothesis  $C$  was a non-tangent circle. Therefore our supposition was false and the theorem is proved.

##### Condition II.

If  $\{\bar{\mathbf{s}}\} \rightarrow \mathbf{s}$ ,  $\bar{\mathbf{s}} \neq \mathbf{s}$ , then  $C(\tau, \bar{\mathbf{s}})$  converges to some unique circle  $C(\mathbf{s})$ .  $C(\mathbf{s})$  is the osculating circle at  $\mathbf{s}$ .

The arc  $A$  is differentiable at  $\mathbf{s}$  if both conditions I and II are satisfied. It can be shown that the two conditions are independent (1, 3, 7).

The following three theorems deal with non-osculating circles. It will be noted that they, together with the preceding theorem, provide us with a complete classification of distinct types of differentiable points.

##### Theorem 4.

Let  $\mathbf{s}$  be a differentiable interior point of an arc  $A$ . Then every non-osculating circle either supports or intersects  $A$  at  $\mathbf{s}$ .

The proof of this theorem is similar to that of Theorem 3 and will be omitted.

**Theorem 5.**

If the osculating circle  $C(s)$  is different from the point circle  $s$ , then every non-osculating tangent circle supports  $A$  at  $s$ .

Proof. Let  $C$  be any proper non-osculating tangent circle of  $A$  at  $s$ , i.e.  $C \subset \tau$ . Then by Theorem 4  $C$  either supports or intersects  $A$  at  $s$ . Suppose that  $C$  intersects. If a sequence of points  $\{\bar{s}\}$  exists such that  $\bar{s} \subset (A \cap C^*)$ ,  $\bar{s} \neq s$ ,  $\{\bar{s}\} \rightarrow s$ , then each  $C(\tau, \bar{s})$  lies in the closure of  $C^*$ . Hence, by Theorem 1 we have  $C(\tau, \bar{s}) \subset C^* \cup s$  and  $\{\bar{s}\} \rightarrow s$  gives us

$$C(s) \subset (C^* \cup C)$$

Now similar consideration a sequence  $\{\bar{s}_1\} \subset (A \cap C^*)$  with  $\bar{s}_1 \neq s$ ,  $\{\bar{s}_1\} \rightarrow s$  gives us that

$$C(s) \subset (C^* \cup C)$$

By combining  $C(s) \subset (C^* \cup C)$  and  $C(s) \subset (C^* \cup C)$  we have that  $C(s) = C$ , which contradicts the hypothesis. Thus,  $C$  supports  $A$  at  $s$ .

We now consider the final case, when the osculating circle  $C(s)$  is the point circle  $s$ . The proof follows from the above.

**Theorem 6.**

If  $C(s) = s$ , then the non-osculating tangent circles at  $s$  either all support or all intersect  $A$  at  $s$ .

## CHARACTERISTIC

We now introduce the important concept of the characteristic of a differentiable point. It is not our purpose to exhibit the reasoning behind the definition but to indicate how by using it we may classify differentiable points.

The characteristic  $(a_0, a_1, a_2, i)$  has the following properties

$$i = 1 \text{ or } 2$$

$$a_0 = 1 \text{ or } 2$$

$$a_1 = 1 \text{ or } 2$$

$$a_2 = 1, 2 \text{ or } \infty$$

$i = 1$  if  $C(s) \neq s$ ;  $i = 2$  if  $C(s) = s$ . The number  $a_0$  is even or odd as the non-tangent circles through  $s$  intersect or support;  $a_0 + a_1$  is even or odd as the non-osculating tangent circles support or intersect;  $a_0 + a_1 + a_2$  is even if  $C(s)$  supports, odd if  $C(s)$  intersects, while  $a_2 = \infty$  if  $C(s)$  neither supports nor intersects.

Theorem 5 places a restriction upon the characteristic in that if  $C(2) = s$ ,  $i = 1$  and  $a_0 + a_1$  is even. Moreover, the agreement that the point-circle always supports implies that  $a_0 + a_1 + a_2$  is even. With these restrictions in mind it is seen that there exists ten distinct types of differentiable points in the conformal plane (1, 3, 7).

## ORDER

An arc  $A$  in the conformal plane is said to be of finite cyclic order if it has only a finite number of points in common with any circle. If some circle  $C$  meets  $A$   $n$  times and no circle meets  $A$  more than  $n$  times where  $n$  is a specific integer, then  $A$  is said to be of bounded cyclic order and  $n$  is called the (cyclic) order of  $A$ . If  $s$  is any point on  $A$ , the order of  $s$  is the minimum of the orders of all neighborhoods of  $s$  on  $A$ . In this note we shall deal only with arcs of order three. However, the next theorem, which will in part justify our choice for the characteristic, does not specify the order of the arc and yet does concern the order of a point on this arc.

**Theorem 6.**

Let  $s$  be a differentiable interior point of the arc  $A$ . Suppose that  $s$  has the characteristic  $(a_0, a_1, a_2; i)$ . Then the order of  $s$  is not less than  $a_0 + a_1 + a_2$ .

The proof of this theorem is too long to include here. We have listed it because we shall indicate how it can be sharpened when  $s$  is conformally elementary.

Suppose now that we have three mutually distinct points  $u, v, q$  such that  $u$  and  $v$  converge on an arc  $A$  to a point  $s$  and that

$$\lim C(u, v, q) = C.$$

Then  $C$  is termed a general tangent circle of  $A$  at  $s$ . Moreover if  $q$  also converges to  $s$ , then  $C$  is a general osculating circle of  $A$  at  $s$ . We say that  $A$  is strongly differentiable at  $s$  if the following two conditions are met.

Condition I'. Let  $R \neq s$ ,  $q \rightarrow R$ . If two points  $u$  and  $v$  converge on  $A$  to  $s$ , then  $C(u, v, q)$  always converges.

Condition II'.  $C(t, u, v)$  converges if the three mutually distinct points  $t, u, v$  converge on  $A$  to  $s$ .

It can be shown that strong differentiability implies ordinary differentiability and that  $C(s)$  is the one and only osculating circle, for if in I' we let  $Q = R$  and  $u = s$  we have that I' implies I and

$$\lim C(u, v, q) = C(\tau, R).$$

Condition II' and II are similarly related.

Elementary Points. We now have sufficient background for the introduction of conformally elementary points. We say that a point  $s$  of an arc  $A$  is conformally elementary if there exists a neighborhood of  $s$  decomposed by  $s$  into two neighborhoods of order three. It has been shown (4, 6, 7) that their closures are strongly differentiable at  $s$ . Theorem 6 can now be made more exact.

**Theorem 6'**

Let  $s$  be a differentiable conformally elementary point with characteristic  $(a_0, a_1, a_2; i)$ , then the cyclic order of  $s$  is precisely  $a_0 + a_1 + a_2$ .

It is, of course, much more satisfactory to deal with an equality rather than an inequality as in Theorem 6.

We have indicated that strong differentiability implies ordinary differentiability. The converse is not true. However, if the differentiable point is conformally elementary we have

#### Theorem 7

Suppose that  $s$  is a conformally elementary **point-on** the arc  $A$ . Then

i)  $s$  satisfies Condition **I'** if and only if it satisfies Condition **I** and  $a_0 = 1$  (i.e. if the non tangent circles through  $s$  support),

ii)  $A$  is strongly differentiable at  $s$  if and only if it is differentiable at  $s$  and  $a_0 = a_1 = 1$ .

Besides providing a useful link between ordinary and strong differentiability conformally elementary points are used with much success in the study of vertices of closed curves in the conformal plane. Finally, they are instrumental in supplying a conformal proof of the Four Vertex Theorem. The study of conformally elementary points does not belong in an introduction to conformal differential geometry. These points are used to unify certain preliminary results and to give order and meaning to more extensive and more profitable studies. It has been this author's expressed purpose to provide the material necessary for their introduction and to indicate how some of this material is further related.

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Oklahoma State University

## GENERAL SOLUTION TO THE LINEAR DIOPHANTINE EQUATION IN TWO VARIABLES<sup>1</sup>

By T. W. Shook  
Ohio Alpha

#### INTRODUCTION

It is well known that the Diophantine equation

$$ax + by = c \quad \text{-----} \quad (1)$$

has integer solutions if and only if  $(a,b)/c$ , in which case the general solution is given by:

$$\begin{aligned} x &= x_0 + \left(\frac{b}{(a,b)}\right)t \\ y &= y_0 - \left(\frac{a}{(a,b)}\right)t \\ (t &= 0, \pm 1, \pm 2, \dots) \quad \text{-----} \quad (2) \end{aligned}$$

where  $x_0$  and  $y_0$  are a solution to equation 1.

Normally, the initial solution is found by an algorithm which is often lengthy and cumbersome. This paper expresses  $x$  and  $y$  directly as a function of  $a$ ,  $b$ , and  $c$ , eliminating the algorithm altogether.

#### THE GENERAL SOLUTION

Suppose  $a$  and  $b$  are two integers with  $0 < a \leq b$ . We define a sequence of remainders recursively as follows:

$$\begin{aligned} \langle ab \rangle_0 &= b \\ \langle ab \rangle_1 &= a \\ \langle ab \rangle_{k+1} &= \langle ab \rangle_{k-1} - \langle ab \rangle_k \cdot \left[ \frac{\langle ab \rangle_{k-1}}{\langle ab \rangle_k} \right] \\ (k &= 1, \dots, \lfloor \frac{b}{a} \rfloor - 1) \quad \text{-----} \quad (3) \end{aligned}$$

#### WHERE:

$$\lfloor \frac{b}{a} \rfloor = \text{MAX} \{ k \mid \langle ab \rangle_k > 0 \} \quad \text{-----} \quad (4)$$

#### THEN DEFINE:

$$\begin{aligned} \langle ab \rangle &= b \sum_{j=0}^{\lfloor \frac{b}{a} \rfloor - 1} \frac{(-1)^j}{\langle ab \rangle_j \langle ab \rangle_{j+1}} \\ \langle ab \rangle &= -a \sum_{j=1}^{\lfloor \frac{b}{a} \rfloor - 1} \frac{(-1)^j}{\langle ab \rangle_j \langle ab \rangle_{j+1}} \quad \text{-----} \quad (5) \end{aligned}$$

The following theorem then gives the general solution:

$$\begin{aligned} \text{THEOREM} - \text{Given the Diophantine Equation} \\ ax + by = c \quad \text{-----} \quad (1) \end{aligned}$$

<sup>1</sup> Received by Editors October 11, 1961. Presented at the National Meeting of Pi Mu Epsilon, Stillwater, Oklahoma, August 29, 1961. This paper was judged best for undergraduates. Mr. Shook won a hundred dollars prize.

One may assume without loss of generality that  $0 < a \leq b$ . Then (1) is solvable if and only if  $\lfloor a, b \rfloor / c$ , in which case the general solution is:

$$x = \frac{c \lfloor ab \rfloor}{(a, b)} + \frac{bt}{(a, b)} \quad y = \frac{c \lfloor ab \rfloor}{(a, b)} - \frac{at}{(a, b)}$$

$$t = 0, \pm 1, \pm 2, \dots \quad \text{----- (6)}$$

**PROOF** – We need only show that  $\lfloor ab \rfloor$  and  $\lfloor ab \rfloor$  are integers satisfying

$$a \lfloor ab \rfloor + b \lfloor ab \rfloor = (a, b) \quad \text{----- (7)}$$

If one substitutes the values given in (5) for  $\lfloor ab \rfloor$  and  $\lfloor ab \rfloor$ , equation (7) becomes a trivial identity.

We prove that  $\lfloor ab \rfloor$  and  $\lfloor ab \rfloor$  are integers by induction on  $\lfloor ab \rfloor$ .

$\alpha$ ) When  $\lfloor ab \rfloor = 1$ ,  $a \mid b$  and  $(a, b) = a$ . The Formula (5) yields  $\lfloor ab \rfloor = 1$  and  $\lfloor ab \rfloor = 0$ .

$\beta$ ) Suppose  $\lfloor ab \rfloor = k > 1$  and let  $r = \lfloor ab \rfloor_2$ . Then  $\lfloor ra \rfloor = k - 1$

so  $\lfloor ra \rfloor$  and  $\lfloor ra \rfloor$  will be integers by the induction hypothesis.

Also note that  $\lfloor ab \rfloor_j = \lfloor ra \rfloor_{j+1}$  and  $(r, a) = (a, b)$ . Then

$$\begin{aligned} \lfloor ab \rfloor &= -a(a, b) \sum_{j=1}^{k-1} \frac{(-1)^j}{\lfloor ab \rfloor_j \lfloor ab \rfloor_{j+1}} \\ &= a(r, a) \sum_{j=1}^{(k-1)-1} \frac{(-1)^j}{\lfloor ra \rfloor_j \lfloor ra \rfloor_{j+1}} \lfloor ra \rfloor, \text{ an integer.} \end{aligned}$$

Solving Equation (7) for  $\lfloor ab \rfloor$ ,

$$\lfloor ab \rfloor = \frac{(a, b) - b \lfloor ab \rfloor}{a} = - \left\lfloor \frac{b}{a} \right\rfloor \lfloor ab \rfloor + \frac{(a, b) - r \lfloor ab \rfloor}{a}$$

The right-hand term of the above expression is

$$\frac{(a, b) - r \lfloor ab \rfloor}{a} = \frac{(r, a) - r \lfloor ra \rfloor}{a} = \lfloor ra \rfloor, \text{ which is an integer.}$$

Hence  $\lfloor ab \rfloor$  and  $\lfloor ab \rfloor$  are integers. This completes the proof.

Ohio State University

Edited by  
M. S. KLAMKIN  
State University of New York  
at Buffalo

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication. Address all communications concerning problems to M. S. Klamkin, Division of Interdisciplinary Studies and Research, State University of New York at Buffalo, Buffalo 14, New York.

#### PROBLEMS FOR SOLUTION

144. Proposed by **Hüseyin Demir, Kandilli, Eregli, Kdz., Turkey**.  
Find the shape of a curve of length  $L$  lying in a vertical plane and having its end points fixed in the plane, such that when it revolves about a fixed vertical line in the plane, generates a volume which when filled with water shall be emptied in a minimum of time through an orifice of given area  $A$  at the bottom.  
(Note: The proposer has only obtained the differential equation of the curve.)
145. Proposed by David L. **Silverman, Beverly Hills, California**.  
For what integers  $a$  and  $b$  ( $0 < a < b$ ) are the roots of  $x^4 + (a+b)x^3 + (a+b+ab)x^2 + (a^2+b^2)x + ab = 0$ , integers?
146. Proposed by C. W. Trigg, **Los Angeles City College**.  
Find a set of three-digit numbers, each of which is a permutation of the same three digits, which when divided by the sum of the digits yields two pairs of alternate integers.
147. Proposed by Leo **Moser, University of Alberta**.  
Show that the maximum number of terms of different form in a polynomial of degree  $n$  in  $k$  variables is the same as the maximum number of terms of different form in a polynomial of degree  $k$  in  $n$  variables.
148. Proposed by M. S. Klamkin, State University of New York at Buffalo, Buffalo.  
If a convex polygon has three angles of  $60^\circ$ , show that it must be an equilateral triangle.



## SOLUTIONS

124. Proposed by M. S. Klamkin, State University of New York at Buffalo.  
Prove the impossibility of constructing the center of a circle with a straightedge only, given a chord and its midpoint. (For the original version of this problem see the spring, 1961 issue, p 183).

The proof is a slight variation of the one given to prove the impossibility of constructing the center of a given circle with straightedge only (see H. Rademacher and O. Toeplitz, The Enjoyment of Mathematics, Princeton University Press, Princeton, 1957, pp. 177-180). It is established there, that there exists a conical projection transforming the given circle into another circle such that the centers do not correspond to each other. Now rotate the given circle about an axis through its center and perpendicular to its plane such that the given chord becomes parallel to the line of intersection formed by the planes of the two circles. It now follows that the midpoint of the chord transforms into the midpoint of the transformed chord (since they are both parallel). The impossibility of the construction now follows by a "Reductio ad Absurdum" argument. We first assume that the center of the given circle with a given chord and its midpoint can be constructed by straightedge alone. The previous construction immediately leads to a contradiction. For, whatever the construction might be, it would consist in drawing a certain number of straight lines and finding their intersection with one another and with the given circle with its given chord and midpoint. Now if the whole figure, consisting of the given circle, chord and midpoint, together with all the points and lines of construction, is conically-projected by the above transformation, the transformed figure will satisfy all the requirements of the construction, but will produce a point other than the center of the transformed circle. Consequently, such a construction is impossible.

While almost all of the previous proof is given in Rademacher and Toeplitz (ibid) and also R. Courant and H. Robbins, What is Mathematics, Oxford University Press, New York, 1943, P. 152, it was repeated here for the sake of completeness.

132. Proposed by L. Carlitz, Duke University.  
Let  $p$  be an odd prime. Find the number of solutions  $(x, y, z)$  of the congruence  
(1)  $xyz + a(x+y+z) \equiv 0 \pmod{p}$ ,  
where  $a \not\equiv 0 \pmod{p}$ .  
Solution by the proposer.  
The congruence can be written as  
(2)  $(xy + a)z \equiv -a(x+y) \pmod{p}$ .

If  $xy + a \not\equiv 0$ , then  $z$  is uniquely determined by (2); thus there are  $p^2 - p + 1$  solutions satisfying this condition. On the other hand, if  $xy + a \equiv 0$ , it is necessary that  $x + y \equiv 0$ , and  $z$  is arbitrary. Now the system

$$\begin{aligned} xy + a &\equiv 0, \\ x + y &\equiv 0, \end{aligned} \pmod{p},$$

reduces to  $x^2 \equiv a$ , which has  $1 + \left(\frac{a}{p}\right)$  solutions, where  $\left(\frac{a}{p}\right)$  is the Legendre symbol. Hence, the total number of solutions is  $p^2 - p + 1 + p \left\{ 1 + \left(\frac{a}{p}\right) \right\}$ .

134. Proposed by R. F. Wheeling and R. H. G. Mitchell, Socony Mobil Oil Company.

If  $\phi_1(x)$  and  $\phi_2(x)$  are one-dimensional probability density functions (i.e.,  $\int_{-\infty}^{\infty} \phi_i(x) dx = 1$ ) and if there exists a number  $X_0$  such that

$$\begin{aligned} \phi_2(x) &\geq \phi_1(x) & \text{for } x > X_0, \\ \phi_2(x) &\leq \phi_1(x) & \text{for } x < X_0, \end{aligned}$$

then

$$\int_{-\infty}^{\infty} x \phi_2(x) dx \geq \int_{-\infty}^{\infty} x \phi_1(x) dx,$$

provided both integrals exist.

Solution by F. Zetto, Chicago, Illinois.

Since  $[x - X_0] [\phi_2(x) - \phi_1(x)] \geq 0$ , for  $-\infty < x < \infty$  it follows that

$$\int_{-\infty}^{\infty} [x - X_0] [\phi_2(x) - \phi_1(x)] dx \geq 0,$$

or that

$$\int_{-\infty}^{\infty} x [\phi_2(x) - \phi_1(x)] dx \geq 0.$$

Also solved by Paul Meyers, K. Smith, J. Thomas, M. Wagner and the proposers.

135. Proposed by T. E. Hull, University of British Columbia.  
Suppose that  $k$  points are placed uniformly around the circumference of a circle with unit radius. Show that the product of the distances from any one point to the others is equal to  $k$ , for any  $k > 1$ .

Solution by David L. Silverman, Beverly Hills, California.  
Place the circle in the complex plane with center at  $(0, 0)$  and

one vertex at (1, 0). The other vertices will then coincide with the  $k$ th roots of unity. If we denote them by  $\Gamma_1 = 1, \Gamma_2, \Gamma_3, \dots, \Gamma_k$  it follows that

$$z^k - 1 = \prod_{i=1}^k (z - \Gamma_i).$$

Consequently,

$$\prod_{i=2}^k (z - \Gamma_i) = 1 + z + z^2 + \dots + z^{k-1}$$

The desired result is obtained by letting  $z = 1$ . Also solved by John T. **Bagwell**, Jr., H. **Kave**, Paul **Meyers**, Ted Newton, K. Smith, G. Tarns, M. Wagner, F. **Zetto** and the proposer.

Editorial Note: The same problem was proposed by Robert P. **Goldberg** in the November, 1961, Mathematics Magazine and solved in the May-June, 1962 issue.

#### ERRATA

106. Proposed by M. S. Klamkin, State University of New York at Buffalo.

An equi-angular point of an oval is defined to be a point such that all chords through the point form equal angles with the oval at both points of intersection (on the same side of the chord). It is a known elementary theorem that if all the interior points of an oval are equi-angular, then the oval is a circle.

1. Show that if one boundary point of an oval is equi-angular, the oval is a circle.
2. Determine a class of non-circular ovals containing at least one **equi-angular** point
3. It is conjectured that a non-circular oval can have, at most, one equi-angular point.

It has been pointed out by Michael Goldberg, Washington, D. C., that there is an error in the proposer's solution to part 2 (Fall, 1961 issue). He also gives a geometrical solution for this part.

The corrected version of part 2 should read:

2. Let the origin be the equi-angular point. Then we have to find  $r$  such that

$$r \frac{d\theta}{dr} \Big|_{\theta} + r \frac{d\theta}{dr} \Big|_{\theta+\pi} = 0, \text{ for all } \theta$$

One obvious solution is

$$\ln r = \int_0^{\infty} F(\sin t) dt$$

where  $F$  is odd, i.e.,

$$r = ae^{b \cos \theta}$$

Edited by

FRANZ E. HOHN, UNIVERSITY OF ILLINOIS

Undergraduate Research in Mathematics, Report of a conference held at **Carleton** College, Northfield, Minnesota, June 19 to 23, 1961, with support from the National Science Foundation, edited by Kenneth O. May and Seymour Schuster. **Carleton** Duplicating Service, **Northfield**, Minnesota, 1961.

Despite the fact that "undergraduate research in mathematics" is not well defined, this report should be of great interest to anyone concerned with the development of mathematical maturity in undergraduate students. Although some of the conferees wished to restrict the term "research" to research leading to original results in mathematics worthy of publication in at least the Monthly, and others wished to extend the word to cover routine term papers of the nature found in undergraduate courses in, say, English, this reviewer feels that despite the semantic disagreement two important contributions are made in this report.

First, there is a discussion of the various techniques used to stimulate undergraduate study. These techniques include, among others, the Moore Method, undergraduate theses, undergraduate seminars, term papers, honors programs and honors sections, competitions, clubs, and local undergraduate publications. There are several detailed reports describing the utilization of these methods.

Second, there is a discussion of the support available for undergraduate research and independent study in mathematics under the NSF's Undergraduate Science Education Program. Here, the discussion ranged from the availability of NSF funds to a summary of the programs now being held under the NSF grants. Mathematicians, it appeared, have not been participating as actively in this program as other scientists.

For the very few hours needed to read this report, the time will be well spent.

University of Illinois

Hiram **Paley**

Introduction to Probability and Statistics, Second Edition. By H. L. Alder and E. B. **Roessler**. San Francisco, W. H. Freeman, 1962. xii + 289 pp., \$5.50.

This second edition remains virtually unchanged from the first except for the addition of two new chapters on the **F-distribution** and analysis of variance. As before, the book is intended for use in a one-semester introductory course in statistics where calculus is not a prerequisite.

With the exception of brief discussions on index numbers and time series, the topics covered are the usual ones and, as in other books written on this level, the quality of mathematics suffers. Of course, this is partly due to the severe limitations of writing about statistics on this level. However, at least some of the **definitions** could be improved. For example, the first **definition** given is: "A measurable characteristic is called a variable." Later on, we find: "A measurable characteristic of a population, such as its mean or standard deviation, is called a population parameter or simply a parameter."

A distinguishing positive feature of the book is the authors' easy, fluid style of writing. Also, there are hundreds of exercises requiring only a small amount of computation, and the answers are given in the back of the book.

University of Illinois

Gus Haggstrom

**A Second Course in Number Theory.** By Harvey Cohn. New York, Wiley, 1962. xiii + 276 pp., \$8.00.

As the title of this book suggests, a reader needs as background a first course in number theory. But titles cannot reveal everything, and so it should be pointed out that a reader also needs some understanding of abstract algebra, particularly a little group theory. The author does give a review of elementary number theory and group theory, but this might not be easy reading without some prior familiarity with these topics.

Following this review material, Professor Cohn launches forth on his main topic, ideal theory in quadratic fields, including discussions of units, the algebra of ideals, unique factorization with ideals, norms, and class numbers. The third and final part of the book comprises the application of this theory of ideals to three topics: a proof of the **Dirichlet** theorem on the infinitude of primes in an arithmetic progression, quadratic **reciprocity**, and quadratic forms. There is a good bibliography and several numerical tables.

The writing has a historical orientation which is rare outside of books on the history and culture of mathematics. To illustrate this we cite two examples out of many. In the preface it is remarked that "A student completing this course should acquire an appreciation for the historical origins of linear algebra, for the zeta-function tradition, for ideal class structure, and for genus theory." On page 241 it is observed that "At this juncture number theory was strongly influenced by **Riemann's** theory of functions. . . .

This book fills a gap in the literature very neatly. There is, in fact, no book (certainly not in English) covering these particular topics, although of course there are a few that overlap in minor ways. The author is to be congratulated on opening up a large area of mathematics to a wider class of readers. Graduate students will find that this book is one of those helpful works that serve as bridges between the more elementary books and the articles in the journals.

University of Oregon

Ivan Niven

**Discrete Variable Methods in Ordinary Differential Equations.** By Peter Henrici. New York, Wiley, 1962. xi + 407 pp., \$11.50.

Professor Henrici has written a book which will be of vital interest to all students of numerical analysis. Although this book is concerned with the numerical solution of ordinary differential equations, the underlying concepts are of importance in all areas of numerical analysis. This text is the first book on the numerical solution of **o.d.e.** which was written with high speed digital computers in mind.

The book is divided into three parts which are as follows:

Part 1. One-step methods for initial value problems.

Part 2. Multistep methods for initial value problems.

Part 3. Boundary value problems.

In each part, there is a buildup of the theoretical apparatus with precise definitions followed by applications. There is a considerable discussion of **roundoff** error in terms of random variables, and the numerical examples show the appropriateness of the stochastic model. A good set of problems is given at the end of each chapter.

This book represents a standard of rigor and clarity which all future text books in numerical analysis should aim for.

Stanford University

Gene H. Golub

**Theory of Numbers, Second Edition.** By G. B. Mathews. New York, Chelsea, 1961. xii + 323 pp., \$3.50 clothbound.

**Conic Sections.** By G. Salmon. New York, Chelsea, 1962. xv + 399 pp., \$3.50 clothbound, \$1.95 paperbound.

**Projective Methods in Plane Analytical Geometry.** By C. A. Scott. New York, Chelsea, 1961. xii + 290 pp., \$3.50 clothbound.

**The Logic of Chance.** By J. Venn. New York, Chelsea, 1962. xxix + 508 pp., \$4.95 clothbound, \$2.25 paperbound.

These four books are reprints of classical works, originally published in the last century, that were outstanding in their day. They are famous for clarity, completeness, and charm and belong in every reference library.

**University Calculus with Analytic Geometry.** By C. B. Morrey, Jr. Reading, Mass., Addison-Wesley, 1962. xiv + 754 pp., \$12.50.

Like many others of the modern texts published recently by Addison Wesley, this is another excellent book.

It is designed for a sequence of courses, totalling 12 semester hours, for students who are well prepared in algebra and trigonometry but who do not have any background in analytic geometry. The approach and the language of the book are definitely modern. Besides the usual topics discussed in most calculus books, the author devotes a substantial amount of space to analytic geometry and to the theory of vectors in the plane and in space. Theory is emphasized from the beginning, even to the point of requiring the student to prove many of the theorems as exercises.

An early chapter gives an introduction to the fundamental concepts of limit, differentiation, and integration. The discussion is informal at the start, but rigorous proofs are given of all essential theorems. Union, intersection, and difference of two sets and also interior points as well as limit points are defined and made use of in Chapter 8 on the definite integral. The differential is discussed carefully and extensively.

The reviewer recommends this book as a text where a combination geometry-calculus book course is in order.

University of Maryland

Dagmar Henney

**Stochastic Service Systems.** By John Riordan. New York, Wiley, 1962. + 139 pp., \$6.75.

This book, a member of the **SIAM** Series in Applied Mathematics, is a well written introduction to the probability theory of queueing and traffic systems, and of certain variants of these which arise especially in telephone problems. The mathematical level is comparable to that of Feller's book, and a student who has had an introduction to probability theory from such a book should find little difficulty in reading Riordan's book. Many interesting topics which are not treated in standard references on queueing theory are contained in the book; for example, nonstandard queueing disciplines such as "last-come, first-served", systems with defections or balking, busy periods, etc. There are also historical remarks and references which are quite up-to-date.

Cornell University

J. Kiefer

Numerical Analysis, with Emphasis on the Application of Numerical Techniques to Problems of Infinitesimal Calculus in Single Variable, Second Edition. By Zdenek Kopal. New York, Wiley, 1962. xvi + 594 pp., \$12.00.

The author has written a highly personal book which strongly reflects his own philosophy and interests. The numerical techniques described are mainly finite difference methods for approximating continuous problems. The first chapter gives a short history of number systems and **numerical** analysis. Chapters II through VII develop the classical subjects of numerical analysis: polynomial interpolation, numerical differentiation, **integration of o.d.e.**, boundary value problems, and mechanical quadrature. Chapter VIII describes methods for solving integral and **integro-differential** equations and Chapter IX is devoted to operational methods in numerical analysis which include polynomial and rational approximations. The book is concluded by several appendices which contain several useful tables.

Each chapter begins with motivational material. There are extensive **bibliographical** notes and problems at the conclusion of each chapter. Throughout the text, illustrative numerical examples are given.

Although this is a very readable and interesting book, it will find its greatest use as a reference for the practicing numerical analyst.

Stanford University

Gene H. Golub

Infinite Series. By I. I. Hirschman, Jr. New York, Holt, Rinehart, and Winston, 1962. x + 173 pp., \$4.00.

This book is another welcome addition to the **Athena** Series of Holt, Rinehart and Winston. As other members of the series, it has an attractive format.

As an indication of the coverage, a listing of the chapter headings and the corresponding lengths in pages is as follows:

- Chapter 1. Tests for Convergence and Divergence (35 pages).
- 2. Taylor Series (20 pages).
- 3. Fourier Series (25 pages).
- 4. Uniform Convergence (21 pages).
- 5. Rearrangements, Double Series, Summability (28 pages).
- 6. Power Series and Real Analytic Functions (18 pages).
- 7. Additional Topics in Fourier Series (14 pages).

#### Appendix

Set and Sequence Operations, Continuous Functions (6 pages)

Although the treatment is rigorous, emphasis has been placed upon the applications of the theory developed rather than upon the theory itself. Consequently, there is more material than usual on problems and applications, which, according to this reviewer, definitely enhances the book. The pace set by the author is rather leisurely, which should make the book attractive to those students studying on their own.

It should be noted that in problem 9, p. 15, it has been tacitly assumed that  $\lambda$  is a constant.

Otherwise,  $\sum_{n=1}^{\infty} n^{-\lambda}$  can diverge even if  $\lambda > 1$  (i.e.  $\lambda = 1 + 1/n$ ).

Similarly for problem 11.  
AVCO Corp.

M. S. Klamkin

Selected Topics in the Classical **Theory** of Functions of a Complex Variable. By M. H. Heins. New York, Holt, Rinehart and Winston, 1962. xi + 160 pp., \$3.50.

The advent of the present monograph comes as a rare pleasure to the reviewer, as it must to other mathematicians who enjoy classical analysis and its recent developments. For many reasons this is a most interesting and stimulating book.

To begin with, the selection of topics has been done with considerable care. Among the contents are to be found the Cauchy-Goursat theorem, the **Mittag-Leffler** and Weierstrass theorems, the Riemann mapping theorem, the **Bloch** and Schottky theorems, the big **Picard** theorem, the theory of harmonic and subharmonic functions, the Dirichlet problem, the classification of regions (as hyperbolic and parabolic), the theorems of Iverson and Milloux-Schmidt, the **Phragmén-Lindlöf** theorem, **Wiman's** theorem, **Carleman's** method, results on the boundary behavior of conformal mappings, and the strong form of the Cauchy integral theorem and formula. In a number of these areas the author has made significant research contributions, and **this** is not without influence on the development of the text.

A few of the topics discussed may already have been encountered by the student in a first course in function theory. If so, he will find it fruitful to renew his acquaintance with them here; the treatment is almost certain to be different in some respect from that previously seen. This difference is not merely for its own sake, however, since there is always a point to the author's departure from the traditional. For example, his use of Fourier series methods in deriving basic properties of analytic functions is highly illuminating, as is his presentation of the Caratheodory-Koebe proof of the Riemann mapping theorem.

The many topics that are new to the student will greatly enrich his knowledge of function theory and provide a base for attacking the current literature. Indeed, the latter facet of the book is perhaps its strongest. With due acknowledgement of **Pólya** and **Szegő's** celebrated *Aufgaben und Lehrsätze*, the author adopts a style in which the problems often form an integral part of the development. The student should therefore work all of them, as well as the informal problems which arise as details in the text itself. This poses a serious challenge, but the student who accepts it and works his way through the book's 155 pages will be amply rewarded in ability to cope with the research literature.

Alternatively, the book lends itself well to use as a source of advanced topics for inclusion toward the end of a first course in complex function theory. An attempt has been made to keep it reasonably self contained, even to the extent of inserting material from real function theory in the appendices, and many of the topics are treated independently of one another.

To carry out a program of the above sort is no mean task. The author has done it **skillfully**, particularly with regard to the organization of material and the difficult choice of what should be made explicit and what left to the reader. Not the least of the author's **skills**, however, is his command of a prose style which virtually puts the work in a class by itself. Pleasantly informal, the writing is filled with interesting observations, side remarks, and references, so that one almost **has** the feeling of being present at a well polished lecture. In all likelihood this book is destined to become a classic in its field.

University of Washington

Maynard G. Arsove

Geometrical Constructions Using Compasses Only. By A. N. **Kostovskii**. New York, **Blaisdell**, 1961. xi + 79 pp., \$0.95. (Translated from the Russian by **Halina Moss**).

The Ruler in Geometrical Constructions. By A. S. Smogorzhevskii. New York, **Blaisdell**, 1961, viii + 86 pp., \$0.95. (Translated from the Russian by **Halina Moss**).

These two booklets, presupposing hardly any more **geometry** than that to which an American high school student is generally exposed, form a readable introduction to the problems implied by the titles. The authors restrict themselves to the synthetic approach almost exclusively. They develop with a minimum of axiomatic formalization the additional ideas of inversive and projective geometry needed and which are not generally part of the beginning students' background.

These books should make excellent sources of additional material for high school students and will be of interest to anyone who has ever toyed with ruler and compass.

A highlight of the "Ruler" book is a proof of the impossibility of constructing with the ruler alone the centers of two given nonconcentric circles unless they intersect. However, beyond mentioning them, the three classical impossible construction problems are not discussed.

The translation is well done. Only two misprints were detected; these are too minor to point out.

University of Nebraska

Edwin **Halfar**

Handbook of Automation and Control, Volume 3. Edited by M. **Grabbe**, S. Ramo, and D. E. **Wooldridge**. New York, **Wiley**, 1961, xxi + 1153 pp., \$19.75.

Preceding volumes of this Handbook were devoted to the fundamental theories of Logical Design and Information Theory (Vol. 1) and to Programming and Design of Computers (Vol. 2). The latest (and final) volume is called "Systems and Components"; a better title might have been "Process Control". The reviewer feels, as for the first two volumes, that overlap is considerable—compare Ch. 27 of Vol. 3 to Ch. 16 of Vol. 2—and that the depth of thought is highly variable—as witnessed by the complete irrelevance of Ch. 2 ("The Human Component") and the very fine treatment of "Transmission Systems" in Ch. 15.

It seems, however, that in overall quality this new volume is superior to the predecessors: The sections on "Manufacturing Process Control," "Chemical Process Control" and "Industrial Control Systems" (including Nuclear Reactors) are, in general, quite usable as a first introduction to the fields: This contrasts strongly with the approach of Vol. 2 which presupposes often a good deal of preliminary knowledge. Incidentally, Chapter 26 on "Semiconductor Devices" and Chapter 27 on "Transistor Circuits" (both of these are in the Component Section) are examples of outstanding clarity and show handbook writing at its best.

In summary: The reviewer wished that about 50% of the chapters were rewritten to reach the high level of competence of the other half. This criticism, however, can be levelled at almost any compendium. One should congratulate the editors for having achieved an entirely useful encyclopedia of the automation and control field. The density of information on the more than 3,000 pages is definitely high and, this is a very positive point, there seem to be practically no errors.

University of Illinois

W. J. Poppelbaum

Handbook of Statistical Tables. By D. B. Owen. Reading, Mass., **Addison-Wesley**, 1962. xii + 580 pp., \$12.50

This is an unusually complete collection of 113 tables of functions used in statistics, many of them more extensively tabulated than ever before. The volume will be much appreciated by both the advanced student and the practicing statistician.

An Introduction to Probability and Statistics. By Howard G. Tucker. New York, Academic Press, 1962. xii + 228 pp., \$5.75.

Here is a bold new textbook on probability and statistics for undergraduate mathematics majors that differs radically in scope, depth, and presentation from others now on the market. For an introductory text, the author has given us a sound mathematics book which is concise, well written, and relatively free of errors.

In the preface the author draws attention to these features of the book:

- "1. Random variables are treated as measurable functions.
2. Sampling is treated in terms of product spaces.
3. Distributions are derived by the transformation method.
4. Probability is given an axiomatic treatment.
5. A chapter on the matrix theory needed is inserted in the middle of the book.
6. The **Neyman** theory of confidence intervals is given a systematic treatment.
7. A more natural definition of the multivariate normal distribution is given.
8. Expectation is given a unified treatment; the expectation of a random variable  $X$  is defined to be

$$\int_0^{\infty} P[X > x] dx - \int_{-\infty}^0 P[X \leq x] dx,$$

provided that both of these improper **Riemann integrals** are finite. Formulas and properties in the discrete and absolutely continuous case are then derived from this definition."

A further indication of the content of the book is that among the theorems proved are a version of the Law of Large Numbers, the **Neyman-Pearson** Fundamental Lemma, the Cramer-Rao Inequality, and **Cochran's** Theorem.

Compared to other books aimed at the same level, relatively little space is devoted to motivational material and examples. The exercises are commonly used for filling in steps in proofs and for proving corollaries, but there are still many good exercises on applications. The answers are not given.

It is unfortunate that this excellent text may be at too high a level to be very useful in the classroom. Many teachers will find that a course based on this text will overshoot the maturity of their students, particularly if many of them come directly from calculus. More motivational material and examples would have been helpful without compromising the author's aim to write a sound introductory statistics book.

University of Illinois

Gus Haggstrom

**Stability** by Liapunov's Direct Method, with Applications. By Joseph LaSalle and Solomon Lefschetz. New York, Academic Press, 1961. vii + 134 pp., \$5.50.

This monograph, the fourth in a series edited by Richard Bellman, represents (according to the cover) the first "detailed and elementary account in English of Liapunov's direct (second) method." The book presents no new material but does give a concise account of Liapunov stability theory as it appears in the literature of the past two years. Indeed, in the short space of 134 pages the reader is at least introduced to some of the modern concepts of stability analysis in a rather painless and pleasing way.

In Chapter One, essential material is given for application in later chapters. Ideas are rapidly and lucidly presented in a manner which gives the student an opportunity to learn yet does not insult the more mature reader. By inclusion of this chapter, the book is rendered essentially self contained.

Chapter Two deals with differential equations, immediately and conclusively put in the normal form. The reader is guided quickly through the ideas of trajectories in phase space, critical points and eigenvalues, etc. A great deal of notation is brought out in this chapter and it appears that one entirely unfamiliar with the terminology might easily lose his way among the definitions. However, the main purpose of the chapter is to lay a solid mathematical foundation for the stability theory which is the subject of the book. Having previously given definitions and elementary properties of vector spaces and coordinate transformations, the authors now begin a careful, clear development of stability notions applied to the vector set of differential equations in normal form. Beginning with autonomous systems, Liapunov functions are defined and Liapunov's stability theorems are given with proofs either stated or outlined. Several types of stability are described and examples are given to explain the more subtle aspects of the theory. More general and practically important topics complete this chapter: Stability and the Theorems of Liapunov for Nonautonomous Systems, Converse of the Theorems of Liapunov, the Extent of Asymptotic Stability, Stability under Persistent Disturbances. Chapter Two contains the bulk of the theoretical development of Liapunov's work, condensed into fifty pages. This is, indeed, the main contribution of the book.

Chapter Three is devoted to the study of stability theory applied to control mechanisms. Several typical control problems are worked out in detail and and comments are made concerning more desirable ways to approach control problems. It might be considered that this chapter introduces a philosophy of control from the viewpoint of the mathematician. Such a discussion is important to the student learning to apply notions of stability to practical control problems and is welcomed by the systems analyst as an analytic approach to the quantitative study of non-linear control systems. The technical content of Chapter Three is not new to the scholar working the control field; however, it is a good concise source of pertinent information for reference.

Chapter Four is a collection of assorted topics in stability and is of interest primarily to the specialist.

In total, the book leaves a good impression. It might be criticized in that there is little discussion of the philosophy underlying the selection of Liapunov functions. Many of the coordinate transformations seem to have insufficient motivation at the outset. The logic behind their selection is apparent only at later stages of the solution. Still, it must be admitted that most people attack stability problems this way and little is really known about the selection of Liapunov functions. This reviewer must emphasize, however, that several significant contributions have been made along these lines since publication of the book. Thus the text does not contain the current advances concerning selection of Liapunov functions. The unconventional

notation used in numbering certain equations was confusing at first to this reviewer. In Chapter Three some simple block diagrams of the systems under discussion would have added to the clarity of the presentation. These minor criticisms are, however, offset by the clear style of discourse and appropriate geometrical diagrams which help give an insight into some of the subtle differences between the several types of stability described.

The book is well suited for use as a reference text in a course on stability or for self study. The authors have written a monograph which should be well received as a clear, concise introduction to an intensely important subject. University of Illinois. Stephen J. Kahne

**Lie Algebras.** By Nathan Jacobson. New York, Wiley-Interscience, 1962. ix + 331 pp., \$10.50.

This book gives a very concise, closely knit presentation of the theory of Lie algebras. The objective is to provide a thorough background in the most general context in recognition of the growing relevance of the theory in a number of different fields, such as Lie groups, algebraic groups, and free groups. In particular, the restrictions on the base field are minimal and are introduced only when necessary. For example, the classical classification of semi-simple Lie algebras is extended to split semi-simple Lie algebras, namely, those which have a Cartan subalgebra  $H$  such that for every  $h \in H$ ,  $\text{ad } h$  has its characteristic roots in the base field. This is a fairly minor point, but it illustrates the spirit in which the author approaches the subject.

The prerequisites, as stated by the author, are a thorough foundation in linear algebra for the first nine chapters, plus some knowledge of Galois theory and the structure theory of associative algebras for the last chapter. This already puts the book out of reach of the average first year graduate student, to say nothing of that intangible, mathematical maturity, clearly necessary for negotiating such a compact work.

Briefly, the first four chapters are concerned with the structure of Lie algebras and the classification of the semi-simple ones. Levi's decomposition theorem and an introduction to the cohomology theory of Lie algebras are included, and Dynkin's method is followed in the classification. The next chapters are concerned with representations of Lie algebras. The universal enveloping algebra is introduced and used to reduce the problem to that of representations of associative algebras, the theorem of Ado-Iwasawa is proved, Cartan's classification of irreducible representations of a semi-simple Lie algebra is given, via modules, and Weyl's formula for the character of such a representation is derived. The ninth chapter studies the automorphism group of a Lie algebra, while in the last chapter the classification of simple Lie algebras over arbitrary fields of characteristic 0 is investigated. At the end of each chapter there is a section of stimulating, non-trivial problems.

Much of this material is obtainable elsewhere, ultimately in the works of E. Cartan, more readably in the volumes of Chevalley on Lie Groups, the Paris "Sophus Lie" seminar notes, Bourbaki, and the notes of H. Freudenthal on Lie groups. However, the completeness and generality of the present exposition make it a valuable addition to the literature both as a reference and as a text for an advanced graduate course. Its appearance is sure to be welcomed by many experts in related fields.

Northwestern University

R. J. Crittenden

**The USSR Olympiad Problem Book.** By D. O. Shklarsky, N. M. Chentzov, and I. M. Yaglom. San Francisco, W. H. Freeman, 1962. xvi + 452 pp., \$9.00. (Revised, edited, and translated by I. Sussman and J. Maykovick.)

**The Contest Problem Book.** By Charles T. Salkind. New York, Random House, 1961. vi + 154 pp., \$1.95.

The first of these two books is a collection of unconventional problems of varying levels of difficulty, many new, some familiar, and a few very well-known. The problems come from arithmetic, elementary number theory, analytical trigonometry, and algebra. Very little knowledge beyond that taught in college algebra and trigonometry is required for their solution. (When unusual knowledge is required, it is provided in the text.) On the other hand, many of them require a high degree of computational skill and insight.

The problems are 320 in number and are arranged according to subject matter. Their statement occupies the first 79 pages of the book. Then follow 343 pages of solutions and, for those who may give up less easily, 30 pages of hints and answers.

These problems are taken from the Olympiad contest examinations given to Russian students from the seventh to tenth grades, which correspond in academic attainment to our ninth to twelfth grades. However, the problems are interesting enough and difficult enough to be useful and challenging to college students in this country.

This volume is an attractive and valuable contribution to the mathematical literature.

The second of these collections is a compilation of the problems from the annual high school contests sponsored by the Mathematical Association of America. Solutions to the problems are given, as is an index which classifies the problems according to subject. The examinations are those for the years 1950 to 1960 inclusive.

These problems are easier than their Russian counterparts. They are also much more varied in nature and scope. (Geometry, for example, is well represented here.) As a result, this book will be more useful in high schools in this country than the first volume will. Indeed, it should be made available to every able mathematics student as a source of diversion, inspiration, and training in original thinking.

University of Illinois

Franz E. Hohn

**Concepts of Tensor Analysis and Differential Geometry.** By Tracy Y. Thomas. New York, Academic Press, 1961. vii + 119 pp., \$5.00.

This is the first volume in a series entitled "Mathematics in Science and Engineering" which is designed to present the theory and application of recent scientific and mathematical developments. The author of this volume gives an introductory account of the subject described in his title. He succeeds in including a great deal of the standard material in his twenty-five chapters. The brevity of his treatment will appeal to those students of applied mathematics who would like to become familiar with the formal aspects of the tensor calculus, and to many others who want only a bird's-eye view of the subject.

University of Illinois

Harry Levy

**Lectures in Projective Geometry.** By A. Seidenberg. Princeton, Van Nostrand, 1962. x + 230 pp., \$6.50.

This is an extremely well-written introduction to the subject, intended as a text for a two-semester course at the junior or senior level in college. The author introduces the main topics of projective geometry on an intuitive basis "as an extension of high-school geometry", then proceeds axiomatically. The book is completely self-contained, even to the extent of introducing such topics as determinants — the stated prerequisites are two years of high-school mathematics. This greatly increases the accessibility of the material treated: the book could be recommended to the graduate student who has had no time and/or opportunity to take a course in geometry, or equally well to the high-school teacher wishing to enrich his background by independent study. Its value would seem greatest as a course text for the advanced undergraduate; even if one foresees no extensive study of geometry in his future, this is an excellent place to acquire familiarity both with the axiomatic method and with many of the basic concepts of abstract algebra — but a person using the text for the latter purpose should be cautioned that Professor Seidenberg's treatment might well entice him into a full-time study of geometry.

University of Chicago

D. A. Moran

**Russian Reader in Pure and Applied Mathematics.** By P. H. Nidditch. New York, Wiley, 1962. x + 166 pp., \$2.25.

This little book contains one hundred brief readings from many different areas of pure and applied mathematics. The variety is excellent. Interlinear translations of all passages are provided. Extensive notes explaining all grammatical peculiarities are given. The book will be useful to all who are learning to read mathematical Russian. Indeed, it fills a long-felt need for a collection of specifically mathematical readings. Previous collections of scientific passages have only rarely included mathematical material.

University of Illinois

Franz E. Hohn

**The Method of Mathematical Induction.** By I. S. Sominski. New York, Blaisdell, 1961. vii + 57 pp., \$0.95.

This little booklet begins by outlining the method of mathematical induction and pointing out the equivalence of this principle and the fact that any set of positive integers contains a smallest number. Examples are given in which induction fails because one of two essential components of a proof by induction is omitted or bungled.

There follow 52 theorems appropriate for proof by induction. Some are proved. These are chosen to illustrate a variety of techniques. Others are left as exercises. A brief chapter then gives proofs of some elementary theorems from algebra, and the final chapter gives solutions to the exercises some of which are particularly simple and some of which are challenging. Most of the examples and exercises deal with algebraic identities. Some familiar inequalities are also included. Very few geometrical theorems appear. No determinantal identities are given.

The book is easy to read. Since so little background is presumed, it is suitable for good high school students. They should find in it much that is of value but they may wish the style were a little livelier.

University of Illinois

Franz E. Hohn

Mathematical Statistics. By S. S. Wilks, New York, Wiley, 1962. xvi + 644 pp., \$15.00.

Here is a book that may be fairly said to represent the solid core of modern statistical theory, carefully and systematically presented. The student who has **mastered** this volume will have a very thorough and balanced grasp of the present state of the subject, and will be in a good **position** to begin to work at the growing **edge** of this large area of mathematics.

The task of preparing an up-to-date account of any active field of mathematics is like trying to board a moving train. This book had its beginning in a **lithoprinted** text under the same title published by Princeton University Press in 1943. In the ensuing 19 years the subject has advanced so rapidly that one may say that the character of the book has radically changed since the **lithoprinted** version. A few topics have receded into the background, or even **disappeared**. For example, in the original version there was a section devoted to the **Pearson** system of distribution functions, and another to the chi-square test of goodness of fit; in the present text the first of these topics is not mentioned, and the second appears only in two exercises. Chapter **IX** of the early version was devoted to analysis of variance, treated from the regression standpoint; in the present version the term "analysis of variance" does not appear in the table of contents, although it is adequately treated in the text.

These changes are, of course, symptomatic of the extensive changes in mathematical statistics over the past several decades. Even more interesting from this standpoint are the topics appearing in Professor Wilks' new book that did not appear in the earlier one. Chapter **I**, entitled "Preliminaries", contains a rather detailed account of the necessary set theory (including sequences of sets), **Borel** fields, probability measures, and probability spaces. These matters, if mentioned at all, received only passing mention in the earlier book. Chapter **16**, on statistical decision functions, deals with a topic which, in 1943, was little more than a gleam in the eye of John von Neumann. The chapter presents a rather brief but entirely adequate account of certain central ideas of decision theory. Sequential statistical analysis, dealt with in Chapter **13**, has likewise made its debut since 1943; in fact, it may be regarded as a forerunner of decision theory. Another new topic is that of time series, to which Chapter **17** is devoted.

According to the preface, "the purpose of this book is to introduce mathematical statistics to readers with good undergraduate backgrounds in mathematics." Only a very few undergraduates, however, would be sufficiently grounded in mathematics and in statistics to undertake to read this book at the beginning of their graduate work. It presumes a sophistication in set theory and **related** matters which is likely to be gained only in a rather solid course in real variables. The reader without considerable background in matrix theory would be in difficulties at some points, but this is more easily remedied. Finally, it **must be** said that this book needs to be supplemented by a good deal of experience with actual data. In practical terms, for almost all students this means that one or two preliminary courses in statistics should precede the study of this book. In this connection, a rather minor criticism is that although the term "experimental design" is used a number of times, it is never defined, as far as the reviewer could determine.

In summary, this is a book which will retain its usefulness for many years, and which is likely to become one of the permanent classics in the field of mathematical statistics.

Earlham College

Howard W. Alexander

Mathematical Programming. By S. Vajda. Reading, Mass., Addison-Wesley, 1961. ix + 310 pp., \$8.50.

Suppose that  $N$  horses **run** in a race and that a gambler will receive a dollar for each dollar bet if the  $i^{\text{th}}$  horse wins and will lose the amount of the bet otherwise. How should the gambler divide his stake among the horses so that **his** smallest possible stake after the race is as large as possible?

To see the nature of this problem we may assume **that** the gambler's stake is unity and that he allocates the amount  $x_i$  to the  $i^{\text{th}}$  horse. Then these variables must satisfy the relations

$$x_1 + x_2 + \dots + x_N = 1$$

$$x_i \geq 0, \quad i = 1, 2, \dots, N.$$

Furthermore, if the  $i^{\text{th}}$  horse wins, the gambler's stake will become

$$a_i x_i \cdot (1 - x_i) = (a_i + 1) x_i - 1.$$

If we let

$$\text{Min}_i [(a_i + 1) x_i - 1] = v,$$

then our task is to choose the variables  $x_i$  **so as to** maximize the value of  $v$ . We may restate the relations and problem **to** read:

Choose the **non-negative** variables  $x_i$  subject to the conditions

$$(a_i + 1)x_i - 1 \geq v, \quad i = 1, 2, \dots, N,$$

$$\sum_{i=1}^N x_i = 1,$$

so as to permit the largest possible choice of the value  $v$ .

We have just stated a linear **programming** problem, the solution of which may be found in the book under review. The essential ingredients are that we consider a physical situation demanding an optimization, try to formulate the **problem mathematically** as the maximization of a linear form whose variables are subject to linear inequality constraints, and then solve the resulting equations computationally. It has been found in recent years that many problems in operations research, game theory, and engineering are of this type. The computational problem is enormous, since we may have to deal with hundreds of inequalities involving thousands of constraints.

The first four chapters of this book are preparatory in nature. Chapter five through eight form a satisfactory introduction to this field and are readily accessible to upper division and graduate students. Interesting **sets** of problems and answers are available so that the book is suitable for independent study. The **remaining** chapters, devoted to quadratic **programming**, stochastic linear programming, and dynamic programming, are not written in the lucid expository style for which the author is well known and may safely be avoided. A useful bibliography is included.

This is a useful introduction to a field that is rapidly growing in importance. RAND Corporation R. Kalaba



*Higher Algebra for the Undergraduate, Second Edition.* By M. Weiss. Revised by R. Dubisch. New York, Wiley, 1962. ix + 177 pp., \$4.95.

This book may well be the answer to that difficult problem of changing from "cookbook" problem solving algebra to the axiomatic modern algebra of the present day. Usually, the transition is a very painful one for students and teachers alike. The students simply don't see the need for axioms, and for proving things which seem obviously true. They find modern algebra dull, and very troublesome, and even if they don't give up, they frequently end up with far from a love for the subject. This text should alleviate many of these difficulties.

The original edition was well received, and the present one, retaining the same spirit, will be widely used. Important material on linear algebra has been added, additional exercises included, and the format generally improved.

After introducing the real and complex number systems, the authors discuss the elementary theory of groups, rings, integral domains, and fields, pleasantly guiding the student through these abstract concepts. Next follow the subjects of polynomials, matrices, systems of equations and determinants. The last chapter is devoted to the important topic of homomorphisms. Notice how the best (and most difficult) is saved to the last.

The reviewer feels that a careful study of this text should provide most of the material and much of the maturity needed for a serious graduate course in modern abstract algebra.

San Jose State College

T. J. Cullen

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- R. Stoll: *Sets, Logic, and Axiomatic Theories*. San Francisco, Freeman, 1961. x + 206 pp., \$2.25, paperbound.
- \*T. Y. Thomas: *Concepts from Tensor Analysis and Differential Geometry*. New York, Academic Press, 1961. vii + 119 pp., \$5.00.
- T. Y. Thomas: *Plastic Flow and Fracture in Solids*. New York, Academic Press, 1961. ix + 267 pp., \$8.50.
- J. Todd: *A Survey of Numerical Analysis*. New York, McGraw-Hill, 1962. xvi + 589 pp., \$12.50.
- \*H. G. Tucker: *An Introduction to Probability and Mathematical Statistics*. New York, Academic Press, 1962. ix + 228 pp., \$5.75.
- \*J. Venn: *Logic of Chance (Reprint)*. New York, Chelsea, 1962. xxix + 508 pp., \$4.95 clothbound, \$2.25 paperbound.
- \*M. Weiss and R. Dubisch: *Higher Algebra for the Undergraduate, Second Edition*. New York, Wiley, 1962. ix + 171 pp., \$4.95.
- H. S. Wilf: *Mathematics for the Physical Sciences*. New York, Wiley, 1962. xii + 284 pp., \$7.95.
- \*S. Wilks: *Mathematical Statistics*. New York, Wiley, 1962. xvi + 644 pp., \$15.00.

\*See review, this issue.

NOTE: All correspondence concerning reviews and all books for review should be sent to PROFESSOR FRANZ E. HOHN, 374 ALTGELD HALL, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

#### NOTICE TO INITIATES

On initiation into Pi Mu Epsilon Fraternity, you are entitled to two copies of the Journal. It is your responsibility to keep the business office informed of your correct address, at which delivery will be assured. When you change address, please advise the business office of the Journal.



This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essential as today.

Your election as members of Pi Mu Epsilon Fraternity is an indication of scientific potential. Can you pursue advanced study in your field of specialization?

To point out the need of advanced study, the self-satisfaction of scientific achievement, the rewards for advanced preparation, the assistance available for qualified students, etc., we are publishing editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

Through these and future editorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

The National Aeronautics and Space Administration was established in 1958 to conduct research into space problems of flight and vehicles, conduct activities for space exploration, and to provide the widest appropriate dissemination of information on these activities. We are fortunate to have an article by Dr. Mattison L. Story from the Educational Services Branch of NASA in this issue. Certainly mathematics in this respect helps make the satellites go round.

The Aeronautical Charting and Information Center is responsible for providing the Air Force with aeronautical charts, flight information, terrain models, maps, intelligence on air facilities, and related cartographic services as well as the research necessary to carry out these objectives. Mr. J. Donald Define, mathematician in the Geophysical Studies Section of the Geo-Sciences Branch of ACIC, has participated in this program to increase interest in mathematics, science, and research with a very interesting article, published in this issue.

The following lists contributing corporations with the issue in which their editorials appeared.

Aeronautical Chart and Information Center	Vol. 3, No. 7
Aeronutronics	Vol. 3, No. 2
Army Ballistic Missile Agency	Vol. 2, No. 10
AVCO, Research and Advanced Development	Vol. 2, No. 10
Bell Telephone Laboratories	Vol. 2, No. 10
Bendix Aviation Corporation	Vol. 2, No. 8
David Taylor Model Basin	Vol. 3, No. 5
<b>E. I. du Pont de Nemours</b> and Company	Vol. 3, No. 2
Emerson Electric Company	Vol. 2, No. 7
General American Life Insurance Company	Vol. 2, No. 9
Hughes Aircraft Corporation	Vol. 2, No. 9
International Business Machines Corporation	Vol. 2, No. 8
Office of Naval Research	Vol. 3, No. 5
Eli Lilly and Company	Vol. 3, No. 2
Mathematics Teachers College, Columbia U.	Vol. 3, No. 3
<b>McDonnell</b> Aircraft Corporation	Vol. 2, No. 7
Monsanto Chemical Company	Vol. 2, No. 7
National Aeronautics and Space Administration	Vol. 3, No. 7
National Science Foundation	<b>Vol. 3</b> , No. 3
North American Aviation, Inc.	Vol. 2, No. 9
Olin Mathieson Corporation	Vol. 2, No. 7
RAND Corporation	Vol. 3, No. 5
Research Analysis Corporation'	Vol. 3, No. 5
Shell Development Company	Vol. 3, No. 1
Sperry Rand Corporation	Vol. 3, No. 6
Union Electric Company	Vol. 3, No. 4
Woodrow Wilson Foundation	Vol. 3, No. 3

## NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

### MATHEMATICS AND NATIONAL PURPOSE

MATTISON L. STORY  
Education Specialist  
National Aeronautics & Space Adm.  
Washington, D. C.



Quantification processes are becoming increasingly vital to every aspect of our national life. Today's complex systems, which are nowhere more evident than in our prodigious space program, daily reflect the endless miracles of modern mathematics. Our so-called "leaping technology" is, in the fullest sense, uniquely dependent upon the amazingly rapid calculations and precise exactitudes now possible in this all-important realm.

The immense challenge which the "organized complexity" of modern life constantly poses could never be met without the systematic formulations and concise organizing principles which mathematics indispensably furnishes. Each Gargantuan accomplishment in modern space technology can be attributed to some specific new implementation of theorem or formula -- the actual physical structuring of complex instruments from symbolic predesigns.

It is thus entirely safe to say that mathematics has become a basic and essential tool of national purpose. The vast space enterprise undoubtedly furnishes the single, most dramatic evidence of its strategic new role as a kind of bellwether of national progress. Its increasing essentiality in all areas, stemming again from inevitable conditions of growth and complexity, can be equally demonstrated in the many diverse facets of our daily life as we cope with such commonplaces as gross national product, standard-of-living indices, opinion poll techniques, and population-explosion predictions.

The status of mathematics in relation to other educational disciplines has perhaps undergone a comparable change. While never actually **subordinated**, it has often been regarded as occupying the position of a prerequisite or intermediate discipline in the curricular scale. Today it is additionally claiming a highly independent stature as a kind of interface between man's scientific concepts and the vehicles and artifacts which enable him to bring them to realization.

Evidence of the dramatically changed status of mathematics can be

**demonstrated** in a variety of ways. A highly pragmatic aspect of its enhanced recognition lies in the simple fact that mathematical know-how is now worth a great deal of money. Happily, the professional skills of the trained mathematician are not readily marketable but continue to be in the highest possible demand.

Far more significant, as a factor of change, **is the** growth of a new attitude toward the mathematician. He has, in a real sense, become the new heroic image of our society. Nothing is of greater social significance, perhaps, than our changing ethic of heroism which promises to supplant the warrior with the thinker as a symbol of greatness. The archetypal hero, traditionally celebrated in epic and fable, has invariably been the man of action, the warrior-statesman whose deeds of physical prowess marked him out for leadership and for worshipful adulation of the masses. The emergence of such intellectual giants as Albert Einstein to positions of widespread public recognition and heroic esteem augurs well for a future in which a new and civilizing recognition of intellectual greatness will perhaps become the keystone of our value system.

The gratifying progress now being made in the improvement of mathematics education is perhaps the most felicitous trend of all. Not only in our advanced curricula for specialized mathematics students but in the common core of education for all citizens, mathematics programs are being **strengthened** and refurbished in school systems throughout the land. This is our surest guarantee of national prestige. It is the bedrock of our future space program. Nothing is more heartening than this recognition that a widespread study of mathematics is, in effect, the best means of strengthening and undergirding our national potential for future greatness.

## AERONAUTICAL CHART AND INFORMATION CENTER

# ACIC AND ITS AEROSPACE MISSION

By

J. Donald Define

Geophysical Studies Section  
Geo-Sciences Branch of ACIC



Mathematics, the Queen of the Sciences, is inherent in many fields, often where least expected. At the Aeronautical Chart and Information Center (ACIC), you might expect to find a large cartographic operation producing many navigational charts and aids for normal use as well as special support activities such as the Mercury Orbit Chart which has been and will be used by the Astronauts. Some glimmer of the Aerospace Mission might have come to your attention when reading of the Center's work on Lunar Charts and the production of a Lunar Atlas as part of the preparation for a manned lunar landing. At this point however, you might consider this the sum total of **ACIC's** work and, as a mathematician, consider that the cartographic field, even when allied with astronomy, is not a field worthy of your attention.

However, this impression, as with many impressions, only partially reveals the situation. Specifically, there are a number of fields with which the Center is concerned: Cartography, certainly, but also photogrammetry, geodesy, and astronomy along with the requisite computer support.

Cartography, although basically concerned with accurate geographic portrayal, must preserve certain physical relationships of the earth's surface as precisely as possible. One of the more challenging aspects of cartography is the separate field of map projections where the principal concern is the representation of a three dimensional space. Here a good mathematical background is essential with emphasis on such areas of study as Theory of Functions of a Complex Variable, Differential Geometry of Curves and Surfaces and Advanced Calculus.

Photogrammetry is involved principally with obtaining reliable measurements through photography. At ACIC, aerial photographs are used to provide improved cartographic coverage. What sort of background is needed for a photogrammetrist? Error Theory, Matrix

Algebra, Vector Analysis and Least Squares Adjustments are certainly essential.

Geodesy - the science involving study of the exact size and shape of the earth and its internal structure - is a branch of applied mathematics. In the future, geodesy will surely embrace a similar study of the moon (selenodesy) and most probably the planets - the term planetodesy has already been proposed. A variety of subjects are required background for this science. Geodesists are keenly interested in positioning points on a non-mathematical surface, the geoid, which is an equipotential mean sea level surface; and the transfer of the points to a mathematical surface, the ellipsoid, for distance and azimuth computations. Some essential mathematical tools include: Numerical Analysis (including the Numerical Solution of Differential Equations, Numerical Integration, Fourier Analysis, **Legendre** Polynomials and Least Squares), Vector Analysis, Matrix Algebra, Projective and Differential Geometry, Bessel Functions, Spherical Harmonics, Theory of the Potential, Error Theory and Probability and a smattering of Matrix and Tensor Calculus.

Astronomy may be considered as another branch of applied mathematics. Positional astronomy is used to determine point locations on the earth's surface and, together with geodesy, contributes to determination of the earth's size and shape. Solar eclipses, star occultations, precise observations of the moon and of the motion of artificial earth satellites are of vital interest to ACIC astronomers. Background mathematics should include: Bessel Functions, **Legendre** Polynomials, Least Squares Adjustments, Calculus of Finite Differences, Differential Correction Methods, Celestial Mechanics, Potential Theory, and Numerical Integration of Orbits.

The academic background of an applied mathematician at ACIC must be varied and, at the same time, complete. To provide some of the necessary knowledge, ACIC has offered individuals with appropriate mathematical foundation the opportunity of attending one year of advanced training in geodesy and photogrammetry at the Ohio State University's Institute of Cartography, **Photogrammetry**, and Geodesy. Similarly, a number of qualified ACIC employees have received graduate instruction in astronomy at the University of Cincinnati and the Yale University Observatory. Advanced mathematics and computer courses are included in both programs. The Center will no doubt continue to provide advanced training related to current and future projects.

Mathematicians are an integral part of the technical and scientific work force at ACIC. A variety of opportunity exists for the application of mathematical knowledge to the cartographic, photogrammetric, geodetic, and astronomic projects currently in work at the Center or anticipated for the future to support the **USAF** aerospace effort.

### Edited By

Mary L. Cummings, University of Missouri

Amy Major Robert J. Weeks, a 1949 graduate of the University of Illinois, was one of a class of 665 U. S. officers and 85 officers from 48 allied nations, who completed the regular course at the U. S. Command and General Staff College, Fort Leavenworth, Kansas, June, 1962.

Amy Second Lieutenant **Eldon J. Nosari** recently completed the eight week officer orientation course at the Artillery and Missile Center, Fort Sill, Oklahoma. He received bachelor's and master's degrees from the University of Arkansas, and was formerly an instructor at the University.

Amy Second Lieutenant Bruce C. Tyson, Jr. has been assigned to the Chemical Research and Development Laboratories at the U. S. Army Chemical Center, Maryland. He is a 1958 graduate of Duke University and a 1960 graduate of Princeton University.

Amy Second Lieutenant Alfred E. Bruns, recently completed the eight week field artillery officer orientation course at the Artillery and Missile School, Fort Sill, Oklahoma. The school emphasizes leadership as well as the practical application of artillery tactics and techniques. Lt. Bruns is a graduate of the University of Missouri.

Paul Henley, a 1962 graduate of the College of Engineering, University of Missouri, has received a Chemical and Engineering News merit award as one of the country's outstanding scholars. He is one of twelve students from American universities to receive the 1962 award, which is given for scholarship and outstanding achievement in campus extra-curricular activities. In March, 1961, **Henley** was presented the American Institute of Chemical Engineers Scholarship Award as the member of the student chapter of the Institute at the University with the highest scholastic rating during his freshman and sophomore years.

We note with interest that **Henley** was the 1961 captain of the University of Missouri football team. As left guard, he made the All Conference first team during his junior year, while during his senior year he was named to the second team in both the Academic All American and All Conference selections.

A lecture by the famous French mathematician, Professor Jean Dieudonne, was the high point of a series of lectures presented by Pi Mu Epsilon at the University of Maryland. Seventy-five midshipment of the Naval Academy were present at the lecture in addition to many others. Dr. Dieudonne outlined the aims and methods of that famous body of French mathematicians (the **Bourbaki** group) that tries to put order into large chunks of mathematics. Though they try to classify mathematics, they do not attempt to write an encyclopedia. The books published by the society are painstakingly prepared, the completion of a single book taking perhaps ten to twenty years. One person is elected to write a first draft of the book, which is read during a general session and **discussed** at great length. A second man writes a second draft, and it is read during the following year, with further examination and rewriting. Finally, after ten, twelve, fourteen years, if agreement has been reached, the work is turned over to the editor.

Professor Dieudonne treated his audience to some astounding remarks, namely, that trigonometry and analytic geometry are silly subjects taught in a silly way, and that lattice theory and non-associative algebra are weeds in the rapidly growing theory of mathematics. What do **you think**?

At the University of Kentucky, the Pi Mu Epsilon Distinguished Mathematician Award for scholastic achievement went to Thomas **Steadman Bagby**. He received mathematics books valued at ten dollars and ten more dollars to be spent on books of his choice.

The Pi Mu Epsilon Key Award for the best graduate seminar paper-content

and presentation was presented to Jim Caveny. The award consists of a Pi Mu Epsilon key and a book award equivalent to that of Mr. **Bagby**.

The Pi Mu Epsilon chapter at the University of Buffalo awarded Richard M. Meyer and Albert D. **Polimeni** memberships in the Mathematical Association of America, the former for having achieved the highest average in mathematics, and the latter for having obtained the highest grade on the comprehensive examinations.

We wish to point out that **Carleton** College, which has one of our newest chapters, publishes a journal of undergraduate mathematics called **Delta Epsilon**. It is published by students and faculty of the college and serves as "a medium for immediate publication for private circulation and discussion of undergraduate work." Donald C. **Olivier** is student editor, while Professor Kenneth O. May is faculty editor. The paper first appeared in 1960. A four year grant from the National Science Foundation enables the staff to continue with the publication.

Students and faculty at Carleton, alumni of the college, mathematicians at other institutions interested in undergraduate research, and libraries, may receive Delta Epsilon by requesting it.

Two members of Pi Mu Epsilon at the University of Missouri, Wilson E. **Brumley**, and David Norman Martin, were graduated with honors in mathematics. Both **Brumley** and Martin held Continental Oil Company scholarships.

Other members at the University of Missouri receiving awards and scholarships were:

David O. Lambeth, a Henry N. Ess Scholarship for scholastic excellence.

Patrick D. Harris, Bendix Aviation Corporation Honors Scholarship in Science and Engineering. Harris also received the Weinbach Prize in Electrical Engineering for being the highest ranking member of the graduating class in electrical engineering.

Kathryn Lee Boehm, the Sigma Delta Epsilon Award of \$25 for excellence in science.

Billy G. Kay, Hamilton Watch Award to the person who has most successfully combined a proficiency in his major field of science with achievements in the social sciences and the humanities. Kay also received the Missouri Engineers of Chicago Scholarship.

Herbert Black, the Frederick O. Norton Memorial Prize for high scholastic standing and promise of leadership in social service appropriate to the engineering profession.

Edward K. Bower, first prize in the annual Calculus Competition.

**Mathukumalli** Vidyasagar, second prize in the Calculus Competition.

#### DIGITAL COMPUTING FOR PROMISING UNDERGRADUATES

Beginning June 17, 1963, the Digital Computer Laboratory of the University of Illinois will conduct an eight-week undergraduate training and working program, concerned with the use and construction of computers, for a limited number of advanced undergraduates.

Students in residence at any college or university in the United States or Canada who will be juniors or seniors in the fall of 1963 and who are interested in learning about and working with stored-program digital computers are invited to apply for admission.

Successful applicants will receive a stipend of \$400.00 and travel expenses to and from Urbana, Illinois.

No academic credit will be given to students engaged in this program.

Application forms may be obtained by writing to Professor C. W. Gear, Digital Computer Laboratory, University of Illinois, Urbana, Illinois.

The closing date for receiving applications is February 15, 1963.

#### A STATEMENT ON THE PEACE CORPS FROM

R. SARGENT SHRIVER, JR., DIRECTOR

The United States is sending some of its most outstanding young men and women as Peace Corps Volunteers to the developing nations. As teachers, engineers, nurses, coaches and surveyors, and in community development work, these Volunteers are providing leadership and knowledge to people throughout the world.

Fraternalities and sororities have prided themselves on their ability to attract and develop leadership. Responsibility, too, has come with this leadership.

Let me suggest that an even greater responsibility and challenge awaits you now. The chance to serve overseas, and thus to continue the work of more than 4,000 Peace Corps Volunteers now in the field, offers a rare fulfillment and experience. Inform yourself about the Peace Corps and how you may become a part of it after college. Contact the Peace Corps Liaison Officer on your campus, or write directly to PEACE CORPS, College and University Division, Washington 25, D.C.

## NEW CHAPTERS INSTALLED

On April 19, 1962, Professor J. Sutherland Frame installed Utah Beta Chapter (the eighty-seventh) of Pi Mu Epsilon at Utah State University in Logan, Utah.

The charter members were thirteen initiates, namely, **Antone Bringhurst**, Lavelle Day, Norman Eggert, Julia Frandsen, Danny Goodrich, Robert Hammond, Ivan Keller, Mary Nelson, Janet Olsen, Bruce **Orcutt**, Paul Peterson, Wendell Pope, Wayne Rich, and four former members of the fraternity: Lawrence Cannon, Neville **Hunsaker**, Konrad Suprunowicz, E. E. Underwood.

Before the installation, Professor Frame gave a lecture on "Continued Fractions"; at the banquet he gave a talk about the history of Pi Mu Epsilon and showed some of the historical documents.

The eighty-eighth chapter, Rhode Island Alpha, was installed on April 26, at the University of Rhode Island in Kingston. Professor Frame was again the installing officer. The installation ceremony was at 4:30 p.m., followed by a banquet at 5:30 both events taking place at the Larchmont Inn at nearby Wakefield. Later, Professor Frame gave a talk on "A Bridge to Relativity." Charter members were Harry Bender, Mary Cummings, Albert Bennett, Carl Garabedian, Jerald Greenberg, John James, Clifford **Leitaco**, E. M. J. Pease, Nancy Randall, Jacob Stauffer, **Rammath** Suryanarayan. The following additional nine members were initiated after the presentation of the charter: Richard Bender, Joel Cohen, Diana Drew, James Foster, **Domenick Lombardi**, Mersima Moskos, Maureen **Russo**, Robert Salhany, Ronald **Tourgee**.

Edited By

Houston T. Karnes, Louisiana State University

EDITOR'S NOTE: According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an **annual** report of the chapter activities including programs, results of elections, etc." The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the *Pi Mu Epsilon Journal*. Besides the information listed above, we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Chapter Activities Editor Houston T. **Karnes**, Department of Mathematics, Louisiana State University, Baton Rouge 3, Louisiana. These reports will be published in the chronological order in which they are received.

### REPORTS OF THE CHAPTERS

#### ALPHA OF MARYLAND, University of Maryland

The Maryland Alpha Chapter held nine meetings during the academic year 1961-62. The following papers were presented:

- "**Bourbaki**," by Professor Jean **Dieudonne**, France.
- "Adele Algebra," by Professor Andre Well.
- "The Four Vertex Theorem," by Professor Stanley Jackson.
- "Why Use 29?" by Professor Richard A. Good.
- "Solving Linear Algebraic Systems on a Computer," by L. **Kenton** Meals, Head of Engineering Application Branch at the David Taylor Model Basin
- "Determining  $\pi$  to 100,000 Digits," by Dr. Daniel Shanks, DTMB.
- "Foundations of Probability and Statistics," by Dr. Syski.

Professor Jean Dieudonne gave the initiation dinner address. The title of his paper was "A Panorama of Mathematics." Sixteen new members were initiated. Professor Leon Cohen, Chairman of the Mathematics Department, awarded the Abramowitz prize in Mathematics to an undergraduate student, Mr. Hyman. A scroll was presented to the faculty advisor, Mrs. Dagmar Henny, as a token of appreciation for time and effort given to the Chapter.

Miss Evelyn **Wooley** was the winner of a puzzle-solving contest sponsored by the Chapter.

Officers for 1961-62 were: Director, Howard Wilson; Vice-Director, David Sprecher; Treasurer, Den-ill Bordelon; Secretary, Alan G. Henney; Faculty Advisor, Mrs. Dagmar R. Henney.

#### GAMMA OF MISSOURI, St. Louis University.

The Missouri Gamma Chapter held four meetings during the academic year 1961-62. The following papers were presented:

- "Symmetric and Self-Distributive Systems," by Mr. Patrick Cassens.
- "Group Multiplication by Computers," by Mr. Grattan P. Murphy.
- "Plane **Polygons**," by Mr. Gerald Harshany.
- "What is Topology?" by Dr. Saunders **MacLane**, University of Chicago.

One hundred seventy-two new members were initiated into the Chapter this year.

At the annual banquet Dr. Waldo Vezeau presented the following awards: Pi Mu Epsilon Contest - Senior Division, Mr. **Al** Ciplickas; Junior Division, Mr. Nick **Eissen**. Freshman Achievement Award - Miss Mary Middleton. James W. Garneau Mathematics Award for the Outstanding Senior - Mr. Suthep Chanthrasomsak.

At the first meeting of the year Mr. William Reddy was elected **Vice-Director** to succeed Mr. Richard Doyle and Miss Barbara **Resnik** was elected Secretary to succeed Miss **Marceline** C. Gratia. At the final meeting, Mr. S. Patrick Cassens was elected Director for 1962-63. Dr. Francis **Regan** has

resigned from the position of Faculty Advisor and permanent **Secretary-Treasurer** of the Chapter and Dr. Edwin Eigel, Jr. will succeed him.

#### BETA OF FLORIDA, Florida State University

The Florida Beta Chapter held eight meetings during the academic year 1961-62. The following papers were presented:

- "Fibonacci Sequence," by Mr. **Donadl VanderJagt**.
- "Contributions of Geometry to Mathematics," by Dr. Raymond Wilder, University of **Michigan**.

"A Conjecture of **Borsuk's** - A Problem in n-Dimensional Space," by Dr. R. W. Jollensten, National Security Agency.

"Geometric Construction with Compass Alone," by Mr. Peter Rice.

"Ancient Number Systems," by Mr. Fredrick Zerla.

"Inter-Product Spaces," by Dr. Ralph **McWilliams**.

During the year, twenty-eight new members were initiated into the Chapter.

Officers for 1962-63 are: Director, Peter Rice; Vice-Director, Chris **Schaufele**; Secretary-Treasurer, Sharon Moses; and Faculty Advisor, Dr. **H. C. Griffith**.

#### ALPHA OF GEORGIA, University of Georgia

The Georgia Alpha Chapter held twelve meetings during the academic year 1961-62. The following papers were presented:

- "Why Statistics?" by Dr. C. H. **Kapadia**.
- "**Osgood's** Curve," by Dr. John Jewett.
- "Programming the 1620 IBM Computer," by Mr. J. C. **Fortson**.
- "Join Systems," by Mr. Gene Worth.
- "Dimension Raising Functions," by Mr. Mike **Donahue**.
- "Computer Programming," by Mr. L. M. **Quattelbaum**.
- "Functions Everywhere Continuous and Nowhere Differentiable," by Mr. Julio **Bastida**.

"Implicit Function Theorem," by Mr. Britt Williams.

"Semigroups," by Mr. Chun Jai Rhee.

"Why the Number  $\pi$  is **Transcendental**," by Dr. L. W. Anderson.

"Mathematics of Antiquity," by Dr. R. P. Hunter.

"Odds at Dice," by Dr. J. H. Henkel.

Officers for the spring quarter, 1962 through the winter quarter, 1963 are: Director, Charles Christmas; Vice-Director, Earl Lavendar; Secretary, Saralyn Souter; and Treasurer, John Boyd.

#### GAMMA OF NEW YORK, Brooklyn College

The New York Gamma Chapter held five meetings during the academic year 1961-62. The following papers were presented:

- "Theory of Groups," (a **series** of three lectures) by Robert Kaufman, Nathaniel Riesenber, and Michael Tinkler.
- "Cardinal Numbers," by Nathaniel Riesenber.

"Creativity in Mathematics," by Professor James Singer, Chairman, Department of Mathematics.

Honorable Mention in the National Science Fellowship competition was given to Larry Smith, Nathaniel Riesenber, Charles Rose, and Harvey **Abramson**.

Seven of the Chapter's graduates will be entering graduate schools throughout the country to continue their studies.

Officers for 1961-62 were: Director, Nathaniel Riesenber; Vice-Director, Larry Smith; Treasurer, Robert Kaufman; and Secretary, Harvey D. **Abramson**.

Officers for 1962-63 are: Director, **Shaul** Stahl; Vice-Director, Robert Blumenthal; Secretary, William Messing; and Treasurer, Edward Ross.

#### ALPHA OF LOUISIANA, Louisiana State University

The Louisiana Alpha Chapter held four meetings during the academic year 1961-62. The following papers were presented:

- "The Existence of Fixed Points and Applications," by Dr. James **Keisler**.

"Mathematical Induction," by Dr. Haskell Cohen.

"Pathological Function," by Dr. T. Henry **Hildebrandt**.

"The Structure of the Real Number System," by Dr. Ronald C. **Bzoch**.

Louisiana Alpha held three initiations, one in the fall, spring, and summer. A total of forty students were initiated into the Chapter.

The Annual Honors Examination Awards were presented, at the University's Annual Honors Day Convocation, to the following: **Freshmen** Award to Daniel Elven Jones, and senior award to Charles Sparks Rees.

Officers for 1961-62 were: Director, Kenneth Glen Freeman; Vice-Director, DeWitt Lee Sumners; Secretary, Claire Fasullo; Treasurer, Jeffrey B. **Fariss**; Faculty Advisor, Dr. Haskell Cohen; and Corresponding Secretary, Dr. Houston T. Karnes.

Officers for 1962-63 are: Director, **DeWitt L.** Sumners; Vice-Director, Donald Roy Cowsar; Secretary, Kathleen Dolese; Treasurer, Harold **Reiter**; Faculty Advisor, Dr. Haskell Cohen; and Corresponding Secretary, Dr. Houston T. Karnes.

#### **GAMMA OF LOUISIANA**, Tulane University

The Louisiana Gamma Chapter was installed on November 27, 1961, and held five meetings during the year. The Chapter was installed by the Director General, Dr. J. Sutherland Frame. His installation paper was "Continued Fractions." At the installation dinner Dr. Frame spoke on "The History of Pi Mu Epsilon."

During the balance of the year four meetings were held with the following papers being presented:

"The Axiom of Choice," by Dr. G. S. Young.

"**Packings** of the Plane," by Dr. A. C. Woods.

"The Bieberbach Conjecture," by Dr. A. A. **Armendariz**.

"A Set Similar to the Cantor Set," by Dr. Bruce **Treybig**.

Seven new members were initiated into the Chapter at the spring initiation. Officers for 1961-62 were: Director, George W. Tiller, Vice-Director, Robert **E. Bonini**; Secretary, Sylvia A. **Ibele**; and Treasurer, Charles R. Blackburn, II.

Officers for 1962-63 are: Director, Henry W. **Frantz**; Vice-Director and Treasurer, Donald E. **Ramirez**; and Secretary, Patricia **Merkel**.

#### **ALPHA OF NORTH CAROLINA**, Duke University

The North Carolina Alpha Chapter held two meetings during the 1961-62 academic year. One a business meeting and the other the annual initiation. The initiation address, "Continuous Functions," was given by Professor **F. G. Dressel**. Five students were initiated.

Officers for 1962-63 are: Director, James M. White; Vice-Director, Robert Chapman **Newman**; Secretary, Robert E. Smith; and Treasurer, Dabney W. Townsend, Jr.

#### **EPSILON OF OHIO**, Kent State University

The Ohio Epsilon Chapter held three meetings during the academic year 1961-62. At the annual initiation banquet on April 11, 1962, Professor Earle Bush, Head of the Department of Mathematics at Kent State University spoke on "What Is Mathematics?" Twenty-two members were initiated.

Film's shown at the two business meetings were: *The Earliest Numbers* and *Minute-Men, Missles, and Mission*.

Miss Vera Melinda Chapman received the Pi Mu Epsilon award at the annual Honors Day assembly. She was awarded a plaque and a check for \$40.

Officers for 1962-63 are: Director, Lois Wilson; Vice-Director, James Weaver; Secretary, Barbara Grills; and Treasurer, Richard Schooley.

#### **BETA OF NORTH CAROLINA**, University of North Carolina

The North Carolina Beta Chapter held five meetings during the academic year 1961-62. The following papers were presented:

"Use of the Mathematics-Physics Library," by Dr. A. T. Brauer.

"Some Topics in Integration," by Dr. W. M. **Whyburn**.

"Infinite Processes," by Dr. F. B. Jones.

Nineteen new members were initiated into the Chapter during the year.

Officers for 1961-62 were: Director, Albert Deal; Vice-Director, Clifton **Whyburn**; Secretary, Sandra Ness; Treasurer, Warren Boe.

#### **ALPHA OF NEBRASKA**, University of Nebraska

The Nebraska Alpha Chapter held seven meetings during the academic year 1961-62. The following talks were given:

"Can Mathematicians Know Anything; If So, What?" by Professor Dewey.

"Tunnel **Diode**," by David Gustavson.

"Telemetering of Physiological Responses of Athletes," by Dr. Rose.

Seventeen new members were initiated into the Chapter at the fall initiation and thirty-five were initiated at the spring initiation.

Officers for 1961-62 were: Director, Larry **Dornhoff**; Vice-Director, David Bliss; Secretary, William T. White; Treasurer, Richard Altrock; Faculty Advisor, Dr. Hubert **Hunzeber**.

Officers for 1962-63 are: Director, Jon **Froemke**; Vice-Director, Stephen Lange; Secretary, Robert **Ladd**; Treasurer, Ken Chatfield; Faculty Advisor, Dr. John **Kimber**, Jr.

#### **BETA OF VIRGINIA**, Virginia Polytechnic Institute

The Virginia Beta Chapter held eight meetings during the academic year 1961-62. The following papers were presented:

"Continued Fractions," by Dr. J. Sutherland Frame, Director-General of Pi Mu Epsilon.

"**Riemann** Functions and Their Applications to the Solution of Initial Value Problems in Fluid Dynamics," by Dr. Tom Chang.

"Using a Digital Computer to Obtain Bounds for the Solution of Certain Problems," by Dr. Walter S. Snyder.

"Matrix and Metrics," by Dr. L. B. **Rall**.

Officers for 1961-62 were: Director, Roger Flora; Vice-Director, Robert **Hanson**; Secretary, Janet **Yates**; Treasurer, Fred Patterson.

#### **BETA OF OREGON**, Oregon State University

The Oregon Beta Chapter held three meetings during the academic year 1961-62. No papers were presented at these meetings since the colloquium meetings of the mathematics department as a whole are available to the members of this chapter.

Social activities included a picnic during the fall term and the Initiation Banquet on May 22 at which Mr. David Johnson spoke on the subject of network flow.

A freshman-sophomore competition was held on March 26. The first prize of \$50 was awarded to Stephen P. Ogard and the second prize of \$30 to Gary J. Ford. The four students who received honorable mention and a book were Thomas L. Mundres, Vasilios Papakonstantinou, John **E.** Ferguson, and Janet Allison. Approximately 35 students took the examination.

Officers for 1961-62 were: Director, John **J. Kohfeld**; Vice-Director, Botond G. Eross; Secretary, Betty Kvarda; Corresponding Secretary-Treasurer, Dr. **A. R.** Poole.

Director for 1962-63 will be Botond G. Eross. Other officers will be elected during the fall term.

#### **BETA OF OKLAHOMA**, Oklahoma State University

The Oklahoma Beta Chapter held fourteen meetings during the 1961-62 academic year. The following papers were presented:

"Experiences in Europe," by Professor R. B. Deal.

"Matrix of a Magic Square," by Gene **Sturm**.

"**Applications** of Boolean Algebra to Logic," by Professor R. W. Gibson.

"An Astronomer Looks at Space Travel," by Professor H. S. Mendenhall.

"Orientation," by Louis De **Noya**.



"Finite Differences," by Professor Robert **Hultquist**.

"A Topic from Physics," by Professor **H. P. Hotz**.

"Fixed Point Theorems," by Professor **O. H. Hamilton**.

"Inverse Ratio System of Counting **Ballots**," by Professor Robert **W. Gibson**

"An Extension of Pascal's Triangle," by Jimmie **Lakin**.

"Perfect Numbers" by Mary Johnson.

"A Concept of Time for Scientists," by Mr. Savoe Lottenville.

At the annual banquet on May 4, 1962, Professor L. Wayne Johnson presented the Freshman Mathematics Award to Lynn Carpenter. The banquet address was given by Professor R. B. Deal whose topic was, "A La **Sorbonne**."

Officers for 1961-62 were: Director, Gene **Sturm**; Vice-Director, Stuart Reeves; Secretaries, Mary Lou **Thurman** and Linda Priest West; Treasurer, Dale **Keairns**; and Faculty Advisor, Professor John E. Hoffman.

Officers for 1962-63 are: Director, Jimmie **Lakin**; Vice-Director, Joseph S. Greene; Secretary, Mary Ann Smith; Treasurer, Bruce Edgar; and Faculty Advisor, Professor John E. Hoffman.

#### **ALPHA OF ALABAMA**, University of Alabama

The Alabama Alpha Chapter held five meetings during the academic year 1961-62. The following papers were presented:

"The Seven Bridges of **Königsberg**," by Dr. Charles Neville Maxwell.

"**Non-Interacting** Control Systems," by Dr. O. R. **Ainsworth**.

"Numerical Solutions for Differential Equations," by Dr. Walter L. Wilson.

"Philosophy of Mathematics," by Mr. George W. Roberts.

Dr. Clark, Assistant Dean of the College of Arts and Sciences, gave the annual banquet address; and the annual spring picnic was held in **Moundville**, Alabama.

Two initiations were held, in the fall and spring, and a total of eighty-six students were initiated into the Chapter.

Officers for 1962-63 are: Director, Dieter **Pukatzki**; Vice-Director, J. Z. **Higgs**; Secretary, Patricia Lucas; Treasurer, Nancy Charles Jones; Social Co-Chairmen, **Adonice** Hereford and James **Dixon**; and Faculty Advisor, Dr. Henry Miller.

#### **ZETA OF OHIO**, University of Dayton

The Ohio Zeta Chapter held nine meetings during the academic year 1961-62. The following papers were presented:

"Derivatives of Composite Functions," by Mr. Thomas Grilliot.

"Iterations of the Exponential Function," by Professor **McIntyre**.

"Research and Mathematics," by Professor Buck.

"Structure of Mathematics and Convex Sets," by Professor **Pettis**.

"Applications of Differential Geometry to Hydrodynamics," by Professor **Mishra**.

"Calculus of Variations," by Professor **Cesari**.

"Problems in General Geometrical Dynamics," by Professor **Hlavaty**.

"**Observations** on Recent Trends in School and College Mathematics," by Professor Reingold.

Mr. David Schweickart, a recent initiate, was named as the sophomore who had excelled most in mathematics and was awarded the two volumes of "Differential and Integral Calculus" by Courant.

Officers for 1962-63 are: Director, Mr. Thomas Grilliot; Vice-Director, Mr. David Van **Hausen**; Secretary-Treasurer, Mr. Max **Gruendl**.

#### **GAMMA OF NORTH CAROLINA**, North Carolina State College

The North Carolina Gamma Chapter held fourteen meetings during the 1961-62 academic year. The following papers were presented:

"The Law of Growth and Decay," by Dr. John **W. Cell**, Head, Department of Mathematics.

"Applied Mathematicians and What They Do," by Dr. Horace Trent,

Ballistic Research Laboratories, Washington

"The Changing Undergraduate Curriculum," by Dr. R. Creighton Buck, Chairman, Committee on the Undergraduate Program in Mathematics, University of Wisconsin.

"The Rocket Research Project at North Carolina State College," by Dr. R. C. Bullock.

"A Method of Solving a System of Differential Equations," by Mr. Jafar **Hoomani**.

Officers for 1961-62 were: Director, Walter B. **Cummings**; Vice-Director, Betty G. Harris; Secretary-Treasurer, Philip Nanzetta; and Faculty Adviser, Dr. James B. Wilson.

Officers for 1962-63 are: Director, Thomas H. Banks; Vice-Director, William S. **Guion**; Secretary-Treasurer, Richard Shachtman; and Faculty Adviser, Dr. James B. Wilson

#### **GAMMA OF WASHINGTON**, Seattle University

The Washington Gamma Chapter held four meetings during the academic year 1961-62. The following papers were presented:

"Orthogonal Polynomials," by Dr. Theodore Chihara.

"Conformal Mappings," by Dr. William **Woolf**, University of Washington

"Famous Problems, Solved and Unsolved," by Dr. R. H. Bing, University of Wisconsin.

"Mathematical Curiosities," by Dr. R. H. Bing.

Eleven students were initiated into the Chapter in two initiations held during the year.

Officers for 1961-62 were: Director, Gary Haggard; Secretary-Treasurer, Mary Ann Hoare; and Faculty Advisor, Dr. Theodore Chihara.

#### **MU OF NEW YORK**, Yeshiva University

The New York Mu Chapter held several meetings during the academic year 1961-62. At the final meeting, the following were elected officers for 1962-63: Director, Benjamin **Volk**; Vice-Director, Martin **Braun**; and Secretary-Treasurer, Robert **Reinerman**.

#### **ALPHA OF OKLAHOMA**, University of Oklahoma

The Oklahoma Alpha Chapter held eight meetings during the academic year 1961-62. A paper was presented at each meeting. Dr. N. A. Court gave the annual banquet address. His subject was: "What is Mathematics?" Thirteen students were initiated during the year.

Officers for 1962-63 are: Director, Earl **LaFon**; Vice-Director, Leo Pratte; Secretary-Treasurer, Maureen Pratte; Faculty Sponsor, Dr. Allen Davis; and Faculty Correspondent, Dr. Dora **McFarland**.

#### **BETA OF KANSAS**, Kansas State University

The Kansas Beta Chapter held five meetings during the academic year 1961-62. The following papers were presented:

"Simulation of Fractional Derivative and Integral Operators on an **Analog-Computer**," by Mr. Gordon E. **Carlson**.

"Bernoulli Numbers and Bernoulli Polynomials," by Professor Paul J. **McCarthy**, University of Kansas.

"The Elastic Bar Variational Problem," by Professor Raville. Head, **Department** of Applied Mechanics.

"Rational Function Approximation," by Dr. Henry Thacher, **Argonne** National Laboratory.

"Recent Developments at Kansas State University," by Dean Paul Young. Thirty-seven members were initiated into the Chapter at the spring initiation.

Officers for 1962-63 are: Director, Neal Poland; Vice-Director, Robert Crank, Secretary, Michael Miller; and Treasurer, William Stamey.

#### **ALPHA OF KENTUCKY**, University of Kentucky

The Kentucky Alpha Chapter held eight meetings during the academic year 1961-62. The following papers were presented:

"Mathematics. Computers, and Business," by Dr. J. B. Cornelison.

"Mathematicians and the Computer Revolution," by Dr. Silvio **Navarro**.

An annual banquet was held with Dr. Wendell De Marcus of the Department of Physics giving the address. Dr. De Marcus' topic for the banquet address was, "The Peeled Earth."

Winner of the Pi Mu Epsilon Distinguished Mathematician Book Award was Thomas **Steadman Bagby**. Jim Caveny received the Pi Mu Epsilon Key Award for the best mathematical content seminar paper with **the best** presentation.

Five members of the chapter have received fellowships to aid in their continued studies.

Officers for 1961-62 were: Director, John Pfaltzgraff; Vice-Director, Clifford **Swauger**; Treasurer, **Lael** Kinch; Secretary, Evelyn Rupard; Librarian, Jim Caveny; and Faculty Advisor, Dr. T. J. **Pignani**.

Officers for 1962-63 are: Director, Jackson Lackey; Vice-Director, Evelyn Rupard; Treasurer, Adelbert Roark; Secretary, Mary E. Logan; Librarian, James Miller; and Faculty Advisor, Dr. T. J. **Pignani**.

KAPPA OF NEW YORK, Rensselaer Polytechnic Institute

The New York Kappa Chapter held four meetings during the academic year 1961-62. At the annual banquet Dr. Richard C. **DiPrima** spoke on "The Mathematician in Industry and in the Academic World."

Twelve members were initiated into the Chapter at the fall and spring initiations.

A student-faculty lecture series was established by the Chapter and two lectures were presented in this program.

Officers for 1962-63 are: Director, Richard Mateosian; Vice-Director, Larry Levine; and Secretary-Treasurer, Howard Kushner.

ALPHA OF NEW YORK, Syracuse University

The New York Alpha Chapter held six meetings during the academic year 1961-62. The following papers were presented:

"Proofs of Irrationality," by Dr. Albert Edrei.

"Topology and the Imagination," by Dr. Mark Mahowald.

"Opportunities in Mathematics," by Dr. Erik **Hemmingsen**.

In April the annual banquet was held. Dr. Clyde **Hardin** of the Department of Philosophy gave the banquet address. His topic was, "Can Machines **Think?**" At the banquet, twenty-one students were initiated into the Chapter.

In March the third annual mathematics contest for high school seniors of the Central New York area was sponsored and prepared by the Chapter. Syracuse University donated a full tuition scholarship for one year for the winner and the Chapter purchased other prizes.

Officers for 1961-62 were: Director, Donald V. Davies; Vice-Director, Bradley Fullager; Secretaries, Linda Sullivan and Marilyn **Meinhardt**; and Treasurer, Donald A. Lutz.

Officers for 1962-63 are: Director, Lee Bryant; Vice-Director, David **Kahn**; Secretaries, Leonora **Zebkiw** and Linda **Serednick**; Treasurer, Adrian Wamtz; Faculty Advisor, Professor Mark Mahowald; and Faculty Correspondent, Professor Nancy Cole.

ALPHA OF ARIZONA, University of Arizona

The Arizona Alpha Chapter held eight meetings during the academic year 1961-62. The following papers were presented:

"The Indian Mathematician **Ramanujan**," by Dr. L. J. **Mordell**.

"**Fibonacci** Sequences," by Mr. **Charles Haskell**.

"The Area Theorem," by Dr. L. M. Milne-Thompson.

"**Eigenvalues** of the **Sturm-Liouville** System," by Mr. Joseph Bronder.

Membership requirements were raised during the year and the practice of holding two initiations instead of one was begun. The fall semester's initiation was held at a banquet on January 4, 1962. Dr. John Webb gave the banquet address, "The Mathematician in Society."

Officers for 1962-63 are: Director, Bob **DeVore** and Secretary-Treasurer, John Bunch.

ALPHA OF MONTANA, Montana State University

The Montana Alpha Chapter held twelve meetings during the academic year 1961-62. The following papers were presented:

"What to do About  $J[\sqrt{-5}]$ ?" by Dr. William Ballard.

"**Riemann** Surfaces," by Mrs. Hilda **Haltz**.

"**Aezla's** Generalized Theorem," by Mr. Richard **NanKervis**.

"Uncertainty and Probability," by Mr. George Trickey.

"Number Theoretic Functions," by Mr. Allen Luedecke.

"**Waring's Problem**," by Mr. Fred **DeMarinis**.

"**Buffon's** Needle Problem," by Mr. Richard Konesky.

"Orthonormal Sets," by Mr. Harry Bauer.

"Dimensional Analysis," by Mr. Edward **Kopitzke**.

"Rain on the Roof," by Mr. George **McRae**.

"Two Parameter Eigenvalue Problems," by Dr. Felix **Arscott**.

"Topological Surfaces," by Dr. R. H. **Bing**.

Montana Alpha presented awards in the form of books to Mr. Barry Davis - outstanding student in undergraduate mathematics and Mr. John **Ulvila** - outstanding student in undergraduate physics. The Chapter also contributed \$50 for prize money for the 1962 Montana Science Fair. Winners, and the schools they represented, were each awarded books.

Officers for 1961-62 were: Director, Allen Luedecke; Vice-Director, George Trickey; and Secretary-Treasurer, Robert **Svehla**.

ALPHA OF NEW MEXICO, New Mexico State University

The New Mexico Alpha Chapter held nine meetings during the academic year 1961-62. The following papers were presented:

"**Pontrijagin** Duality for Semigroups," by Mr. Charles Austin, University of Washington.

"A Topic from Statistics," by Dr. Rudolph Borges, University of Cologne.

"**Summability**," by Mr. J. P. **Brannen**, University of Texas.

"Modem Modem Algebra," by Professor S. Eilenberg, Columbia University.

"The **Riemann Hypothesis** and Space **Communication**," by E. C. **Posner**, Jet **Propulsion Laboratory**.

"Generalized Information Theory," by N. **Scarritt**, Purdue University.

"Statistical Metric Spaces," by Professor B. **Schweizer**, University of Arizona.

"Sheaves, Germs, and Lattices," by Professor John D. Thomas.

"**Dedekind** Rings and Linear Algebra," by Professor Robert J. Wisner, Michigan State University, Oakland.

"An Application of Finite Induction to Number Theory," by Professor Robert J. Wisner.

Officers for 1961-62 were: Director, Carol L. Peercy; Vice-Director, Edmund J. Peake, Jr.; Secretary-Treasurer, William L. Caudle. Professor Seymour Goldbert, chapter advisor and permanent faculty correspondent, is on leave of absence during the year 1962-63. **Consequently**, Professor John D. Thomas will assume these duties.

LAMBDA OF NEW YORK, Manhattan College

The New York Lambda Chapter held thirteen meetings during the 1961-62 academic year. The following papers were presented:

"The Mathematics of Computer Programming," by Mr. John **Morrissey**, IBM.

"The **Hierarchy** of Geometry," by Mr. Nicholas De **Lillo**, Fordham **University**.

"The Actuarial Profession," by Mr. Robert Johansen, F. S. A.

"Equivalence Relations," by Mr. Donald **McCarthy**, New York University.

"Orthogonal Function," by Charles Weber.

"Generating Functions," by Jeremiah Kelly.

"Introduction to Infinite Products," by Stanley S. Leory.

"Theory of Numbers," by **Whitney S.** Harris.

Twelve members were initiated into the Chapter at the initiation held in the spring.

Three members of the chapter, Charles Weber, Frank J. Gordon, and John Stout, represented Manhattan College in the Putnam Mathematical Contest and ranked 86th out of the 165 teams entered into the contest.

Five members have received awards which will aid in their continuing studies.

Officers for 1961-62 were: Director, Jeremiah Kelly; Vice-Director, Stanley S. Leroy; and Secretary-Treasurer, Harry J. Neylan.

Officers for 1962-63 are: Director, Thomas G. Walsh; Vice-Director, Gerald Silva; and Secretary-Treasurer, John Stout.

ALPHA OF VIRGINIA, University of Richmond

The Virginia Alpha Chapter held six meetings during the academic year 1961-62. The following papers were presented:

"On Decimal Fractions," by Professor Harley Stafford.

"Programming for Digital Computers," by Professor Frederick Schmidt, Medical College of Virginia.

"A Model for the Real Number System," by Barbara Sue Oglesby.

"The Axiom of Choice," by Kent Wright.

The following awards were issued by the Chapter:

Freshman Mathematics Examination - First Prize (\$10), John Bsiley; Second Prize (\$5), Thomas Hicks. Sophomore Mathematics Examination - First Prize (\$7.50 each), a tie between Van Bowen and Michael Kusheba.

The James D. Crump Prize, a University award for excellence in mathematics, was won by Charlotte Adams.

Officers for 1961-62 were: Director, Paul Cohen; Vice-Director, Alice Hall; Secretary, Charlotte Adams; and Treasurer, Harold Smith.

Officers for 1962-63 are: Director, Kay Koontz; Vice-Director, William R. Tolbert; Secretary, Cecelia Stiff; Treasurer, Hans Carter; Corresponding Secretary, E. Sherman Grable; and Faculty Advisor, H. Pearce Atkins.

ALPHA OF FLORIDA, University of Miami

The Florida Alpha Chapter held eleven meetings during the academic year 1961-62. The following papers were presented:

"Mathematics, Science of Game?" by Professor L. Rosenbaum.

"The Revolution in Geometry," by Professor F. Borges.

"Introduction to Some Recent Concepts in Topology," by Dr. Morton L. Curtis.

"Application of Modular Arithmetic to Computing Machines," by Dr. H. Aiken.

"Complex Numbers and Quaternions," by Dr. Herman Meyer.

"How to Talk to a Computer," by J. B. Kaplan

At the Initiation Banquet Dr. Cesare Emiliani of the University of Miami Institute of Marine Sciences gave the address. His topic was, "The Possibility of Extraterrestrial Life as Evidenced by Certain Types of Meteorites." Thirty-two new members were initiated this year.

A scroll was presented to Mrs. Georgia Del Franco, Faculty Correspondent, who is a charter member of the Florida Alpha Chapter. In addition, winners of the Pi Mu Epsilon Mathematics Contest were: first prize, James W. Rose and second prize, Paul Brown

Officers for 1961-62 were: Director, Lawrence Hawkins; Vice-Director, William Fienning; Secretary-Treasurer, Gloria Cashin; and Faculty Advisor, Miss Patricia Elliott.

Officers for 1962-63 are: Director, Gloria Cashin; Vice-Director, Stephen Love; Secretary-Treasurer, Sofia Pappatheodorou; and Faculty Advisor, J. B. Kaplan.

## INITIATES

ALABAMA ALPHA, University of Alabama (Spring, 1962)

Martha F. Alexander  
Robert D. Alien  
Hollis P. Behannon  
Bettye A. Blake  
James A. Brasher  
Patricia A. Bruening  
James L. Chesser  
Harold L. Daniel  
William A. Dark  
Bernard DiGiorgio  
Eleanor J. Dudley  
Judith K. Dunn  
Ann D. Fite  
Olney R. Fordier

William L. Gamble  
Carolyn A. Gilchrist  
Laura Gonzales  
James A. Guin  
Robert W. Gunderson  
Adonice C. Hereford  
John T. Hubbard, Jr.  
Eddit T. Hunt  
Jane S. Jones  
Jetty V. Lindsey  
William T. Mauldin  
James A. Maxwell  
David N. McNelis

Charles G. Montgomery  
Johanna Nichol  
Gregory T. Ogden  
Jimmy Porter  
Charles Rampack  
Lynn H. Rice  
James E. Simmons  
Kathie Simon  
Thomas C. Smitherman  
Donald Thompson  
John D. Warmbrod  
James W. Wiggins  
Clifton T. Windham

ALABAMA BETA, Auburn University (May 10, 1962)

Hubert R. Adkins  
Robert S. Ballard  
Robert F. Baits  
John E. Burkhalter  
William H. Butler  
Don Chambliss  
Floyd L. Curdie  
Harry L. Defferbach  
Kay Finney  
Sandra Gray

Ann Grogan  
Phillip R. Henderson  
Raymon A. Henton  
Mary E. Hinton  
Stewart V. Horn  
Azalia A. Osborn  
James R. Pitts  
Gerald A. Pounds  
Athanasios Prakouras  
Naomi Robbins

Franke A. Rusche  
Edwin W. Smith  
Thomas H. Springfield  
Lawrence H. Stone  
Jetty F. Thomas  
Martha S. Thomas  
Francis E. Watson  
John P. Weidner  
Jerry F. Williams  
Joe E. Young

ARIZONA ALPHA, University of Arizona (June 4, 1962)

John Louis Bunch

Raphael Finkelstein  
Helen Gin

Bernard A. Fischer

ARKANSAS ALPHA, University of Arkansas (Match 19, 1962)

Donald R. Allen  
Robert Arlington  
Mary K. Beavers  
Nancy W. Brown  
William D. Bushmiae  
Webster T. Gotten  
Bryce E. Curtsinger  
Ronald E. Eddy  
Robert O. Fisher  
Melvin V. Foster  
Rebecca A. Frazier

John K. Harris  
Terry J. Henley  
Ronald E. Hill  
Robert L. Hudson  
James T. Karam  
Marwin K. Kemp  
Joe B. Locke  
Judy M. Loux  
Rex A. Martin  
Phyllis G. McFarland

Kim L. Mitchell  
Jetty K. Ott  
George E. Rouse  
Randall C. Stephens  
Jack B. Swift  
Phillip A. Terry  
George K. Wallace  
Robert J. Welborn  
Leonard A. Wiggins  
Edward G. Woods

CALIFORNIA ALPHA, University of California, Los Angeles (February 4, 1962)

Edwin S. Beckenbach  
Sorrell Berman  
James R. Bock  
Fook Eng  
Robert J. Engert

Motley A. Feldstein  
Allesandro Figa-Talamanca  
Arthur P. Gittleman  
Robert Gold  
Theodore Gold

John S. Lane  
Joseph Motzkin  
Howard Ritea  
Bernard Russo  
Howard B. Vein

CALIFORNIA GAMMA, Sacramento State College (Fall, 1961)

James Chrislock  
Ann Cornelius  
Eric Frerking

Charles Hagopian  
Byron Hendrix  
Barthel W. Huff

Evelyn Larsen  
Ray McAfee  
Cynthia Speed

## DISTRICT of COLUMBIA ALPHA, Howard University (May 19, 1962)

Anjean **B.** Carter  
Basil **Livingston** Cleare  
Willie E. Cook, Jr.

Oscar H. **Criner**  
Eugene **M. DeLoath**  
Robert Gamble  
Ray C. Titus

William Gee  
Sandra Ann Gittens  
James Edward Joseph, Jr.

## FLORIDA ALPHA, University of Miami (May 12, 1962)

Irving Abel  
Richard H. Ault  
John **M.** Blackstock  
Gerald A. Bottorff  
Everett H. Boyd  
Alton T. **Butson**  
John W. Cooper  
William L. Davis, **Jr.**  
James Philip Dean  
John E. Dennis, **Jr.**  
Raymond F. **Dickman**

Edwin **Duda**  
Sue Edwards  
**Delvis** A. Fernandez  
**Jerrold** Gans  
Robert **J.** German  
**Beryle** A. Greenberg  
Harvey **J.** Greenberg  
Giacomo **Grippio**  
Barbara A. Hopf  
Stephen F. Love  
William S. Moore, **III**

Sofia A. **Pappatheodorou**  
James G. **Pardew**  
Fred **Perlove**  
Leonard R. **Rubin**  
Eugene **Sadowski**  
Victor Sung  
**Marinell** Thompson  
Manuel **Villar**  
Thomas **J. Walend**  
Edward R. Young

## FLORIDA BETA, Florida State University (May 15, 1962)

Lee H. **Armstrong**  
**Cavin O.** Cotey  
David P. Hayes  
Martin N. **Heinezer**

Wade T. **Macey**  
Beverly A. Martin  
Stanley E. Payne  
Chris **B. Schaufele**

Harriet H. **Sibley**  
Kurt A. **Snover**  
Janace A. **Speckman**  
Daniel S. Yates

## GEORGE ALPHA, University of Georgia (March 7, 1962)

Alan **Atwood**  
Tommy R. Dell  
John **Goode**

Robert B. Hafer  
Albert G. **Horrocks**  
Larry K. Johnson

Hillary L. Kitchens  
Linda **J.** Sutton  
Susan E. **Tilghman**

## GEORGIA BETA, Georgia Institute of Technology (May 20, 1962)

**Hagood** Bellinger  
Gary D. Bent  
Donald G. **Bodnar**  
David L. Brown

Milton E. **Cram**  
K. **Bruce** Erickson  
James R. **Gard**  
William C. **Lineberger**

Edward B. Saff  
Sanford M. Wiener  
John D. Wright  
**Mehdi S.** Zarghamee

## ILLINOIS ALPHA, University of Illinois (May 2, 1962)

Carol S. Abe  
Muhammad **S. AbuSalih**  
M. Edward Borasky  
Milo F. Byrn  
Thomas A. Bums  
Shu-Park Chan  
Herbert Y. Chang  
Kenneth R. **Conklin**  
Charles G. Denlinger  
Donald **J. DeSmet**  
John H. **Esbin, Jr.**  
Kenneth E. Evans, Jr.  
Robert E. Gaines  
Paul **M.** Grabarkewitz

Frank D. **Grosshans**  
Donald U. Gubser  
Arthur W. **Hammar**  
Robert **M.** Jones  
Robert **J. Kansky**  
Jose Katz  
James **J. Kelleher**  
Robert G. Lange  
Thomas **M.** Latta  
Ralph A. Liguori  
**Lawson** Lobb  
Bernadette G. Londak  
Joseph B. Miles  
Lois **E. Minning**

David D. Morrison  
Billy R. Nail  
Andrew R. Neureuther  
Carol **M.** Perersen  
Edward F. Rittenhouse  
Alexander P. Stone  
Stanley L. **Urwiller**  
**Zalman** Usiskin  
Henry **Waldman**  
**William** Weitzman  
Carroll O. **Wilde**  
Bing Kuen Wong  
Henderson C. H. Yang

## ILLINOIS BETA, Northwestern University (November, 1962)

Vernon L. **Baily**  
Larry W. Broberg  
Roger L. Cooke  
Lee A. **Egherman**  
Stephen P. Fox  
Donald R. Frederick  
Alyce L. **Gagosian**  
Robert L. **Gerlach**  
Roland W. Gubisch  
**William J. Hankley**  
Charles F. Hepner

**Melvyn** E. Huff  
Harvey R. Huttas  
Patricia A. Johnson  
Robert **J.** Kilian  
Sara E. King  
David E. **Kullman**  
John B. **McColly**  
Dennis **J. Mueller**  
**Carol** Muse  
Richard Nielsen

Robert L. Obenchain  
**Deane M.** Peterson  
Robert Rice  
Jeffrey R. Sampson  
John L. **Schofill, Jr.**  
Sherman **M. Shand**  
Richard A. Soderberg  
William F. Stasior  
Robert Streitmatter  
Robert L. Ward

## ILLINOIS DELTA, Southern Illinois University (May 22, 1962)

Allen **R.** Campbell  
Wilbur H. Clark  
Ralph **Czerwinski**  
Lenard A. Defend  
Richard Fulkerson

Marie A. Hughes  
Carolyn **J.** Jarick  
David W. **Kammer**  
Theodore **Kramme**  
Barbara K. **McMillan**

David **C. Mueth**  
Stanley **B.** Poe  
Mary C. Scott  
Charles T. Wright, **Jr.**

## IOWA ALPHA, Iowa State University (Spring, 1962)

Charlotte Louise Ashbaugh  
Geoffrey **S. Boehm**  
Susan **Chamberlin**  
Lloyd Craig Davis  
**Henri** Feiner  
David Leroy Hench  
John H. Hoper  
Richard Ewers Horron  
David Wayne Jackson

Howard Jackson  
Gene **A.** Kemper  
David Kennison  
Mads Ledet  
**Allan** Theodore Leffler, **II**  
Ronald Wilson Moses, Jr.  
Larry L. **Orr**  
Robert Dennis Pedersen

R. Dean Riess  
Steven R. Ritland  
Richard **E.** Robinson  
Marilyn Russell  
Donald E. Schmidt  
Dennis Paul Swain  
**Curran S.** Swift  
James **R.** Veale  
James Brian Wahrenbmck

## KANSAS ALPHA, University of Kansas (April 27, 1962)

Harold W. Breedlove  
Edwin W. Buchert  
Emanuel G. Calys  
Charles **R. Combrink**  
Marvin E. Donaldson  
Roger **Eggerling**  
David L. Erickson  
Robert H. **Feitz**  
Donald L. Foster  
John B. Johnston

James P. Kirk  
Kenneth G. **Klenke**  
David G. Lash  
Floyd D. Lee  
Thomas W. Loewen  
Michael C. Mackey  
Charles D. Marshall **II**  
Philip N. **Merritt**  
Richard A. Moore  
Jean **M. O'Dell**

Kenny L. Peterson  
Frederick **Pilcher**  
Dieter A. Reetz  
Harold Schick  
David C. Scott  
Jeanne L. Sebaugh  
Sara Y. Simcoe  
Lee **M. Sonneborn**  
Urs P. Wild  
Fred L. **Wilson**

## KANSAS BETA, Kansas State University (May 22, 1962)

Frederic C. **Appl**  
Billy H. Bailey  
Gary A. Bogar  
Elmer E. **Branstetter**  
James D. **Callen**  
Kenneth H. Carpenter  
Curt H. Chadwick  
Richard D. **Chelikowsky**  
Kent **Crawford**  
William Davis  
Judith A. **Dreiling**  
Mark **J. Dreiling**  
Gerald W. Ebker

Margaret F. Flynt  
**Mohan** L. Garg  
Everett E. Haft  
John C. **Hassler**  
Thomas **M.** Keegan  
Harry **D. Knostman**  
Dale D. **Koelling**  
Samuel L. **Lesseig**  
Bill **Livesay**  
Francis E. **Masat**  
**Ulrich** Mathis  
Judith A. **Mawdsley**  
Charles H. **Murish**

John H. Nichols  
Larry D. Noble  
**Chia-ven** Pao  
Jerry W. Pence  
David Poon  
Gary S. Spencer  
William W. St. Cyr  
John E. Tuecke  
Hugh **S.** Walker  
Cha Ho Wang  
Wayne L. Woodworth  
Jerald **J.** Wray  
Leroy **J.** Yock

**KANSAS GAMMA**, University of Wichita (May 4, 1962)

Robert L. <b>Blumenshine</b>	James O. <b>Matous</b>	Charles Thomas Snyder
Norbert W. <b>Deneke</b>	Michael R. Mendenhall	Donald H. <b>Wetmore</b>
Ronald E. Dutton	Paul J. Schwindt	Frank L. Wright
Elmer A. Hoyer		

**KENTUCKY ALPHA**, University of Kentucky (May 10, 1962)

Francis R. Clarke	James E. Miller	Saeed <b>Salehi</b>
Bradley B. Cox	Betty B. Robinson	William C. Setzer

**LOUISIANA ALPHA**, Louisiana State University (May 16, 1962)

Charles F. Blank	Donald R. Cowser	Waker H. Grant
<b>Maunsell W. Brousseau III</b>	Kathleen <b>Dolese</b>	Edward J. Pisa
Edward E. <b>Counce, Jr.</b>	Kenneth W. <b>Eiswirth</b>	Richard S. <b>Thomason</b>

**LOUISIANA ALPHA**, Louisiana State University (August 3, 1962)

Sr. Mary Sylvester <b>DeConge</b>	Georgiana Mary Landry	Charles S. Van <b>Arsdall</b>
Charlotte <b>Eloquin</b>	Inez <b>Chappell</b>	Virginia Louise Williams
Margaret Austine Hunter		Joyce Lester Wilson

**LOUISIANA BETA**, Southern University (April 25, 1962)

James Anderson	Kenneth Joseph	Stella Roberson
<b>Montrust Burrell</b>	<b>Noma J. Lewis</b>	Herbert Rolans
Leon <b>Daughtry</b>	Kenneth Maloney	<b>Mendis J. Taylor</b>
Ulyssis Days	Doris <b>Matthews</b>	Henry Thurman
Chester Givens	Ivan <b>McGowan</b>	Doris A. Williams
Archie Harris	Betty J. Neal	Vernon Williams
Colonel Johnson, Jr.	<b>Willis Porter</b>	Austin <b>Woolfolk</b>

**LOUISIANA GAMA**, Tulane University (April 5, 1962)

Henry W. Frantz III	Donald E. <b>Ramirez</b>	Call S. <b>Weisman</b>
Lawrence M. <b>Hanafy</b>	Fred L. Smith, Jr.	

**MARYLAND ALPHA**, University of Maryland (April 29, 1962)

Harry N. Bragg	<b>Barry Kaminsky</b>	Eleanor Schwartz
Charles M. <b>Eckert</b>	Nancy Loweth	Sadegh Siahatgar
Paul A. Gerhard	Leo J. <b>Mueller, Jr.</b>	Nolan Wallach
<b>Ulrich Gerlach</b>	Ronald D. <b>Pittle</b>	William J. Wickless, Jr.
<b>Sallie A. Hartwood</b>	Dickson Preston	Wayne L. <b>Milmot</b>
Daniel M. Hyman		

**MICHIGAN ALPHA**, Michigan State University (March 1, 1962)

William Brewer	Kenneth Malich	Morteza Rahimi
Jerry Chateau	Steve <b>Onderwyzer</b>	Batty Smith
Michael Cooper	Carla Oviatt	James Stewart
William Graham	Gerald Pacholke	Douglas Strickland
<b>Jin-Sheng Huang</b>	Paul Pennock	Tom Tucker
Richard Johnson	<b>Allan Petersen</b>	Sandra <b>Wilcox</b>
James <b>MacKenzie</b>		

**MINNESOTA ALPHA**, Carleton College (June, 1962)

Duane Edwin Anderson	Richard Earl Hammer	Frederick Wilbur <b>Lott, III</b>
Brace David	Douglas Ralph Jones	Felicia Giare <b>Oldfather</b>
George Stanley Eugster	James <b>Erling Johnson</b>	Jonathan Beach Skinner
Alan Dale <b>Fiala</b>	Garry Robert Kampen	Anthony Smith
William Tibbets Ford	John M. Karon	William <b>Sudman</b>
William Russell Gage	Stanley Lewis <b>Klein</b>	Katherine Anne Wier
	George Koehler	

**MISSOURI ALPHA**, University of Missouri (May 1, 1962)

Conrad W. Bowers	Kathryn L. <b>Kordes</b>	Robert F. Randall
Edward K. Bower	Lynn Kuluva	Robert Ritenour
William A. Brock, III	Charles A. Leech, III	Robert M. <b>Sandford</b>
Ernest Brockelmeyer	Fook Wah Leong	Norbert Schmidt
Charles R. Coe	Steve Ligh	Norma Jo <b>Smith</b>
John P. Creason	Mary A. Logan	Randall W. Stone
Ronald L. Cole	Doris R. Long	Floyd G. Teaney
Richard L. Francis	Carl L. Ludwig	Richard D. <b>Teaney</b>
Theodore J. Gibson	David O. Lambeth	Ester I. <b>Tichenor</b>
Jerome M. <b>Ginden</b>	Charles B. <b>McLane</b>	David E. Tomlin
Thomas G. Hallam	<b>Martin J. Megeff</b>	<b>Mathukumalli</b> Vidyasagar
William Herder	John L. Meyer	Marvin <b>Wolfmeyer</b>
Ronald L. Hollrah	Kathy <b>Mussman</b>	<b>Farroll T. Wright</b>
William H. Jennings	Pamela Nett	Robert E. Yorke
<b>Verlin Koper</b>	Nancy L. Nitz	Sergio <b>Lerda-Olberg</b>

**MISSOURI GAMMA**, St. Louis University (April 11, 1962)

Noel J. Abkemeier	Kosta M. Dedo	St. M. de <b>Lourdes</b> Hardesty,
Raymond M. <b>Albers</b>	Earl E. Dee	O.S. F.
Patrick Argos	Julia R. Deimund	Vincent M. Harmon
Virginia A. <b>Arnoldy</b>	William M. Denny	Anthony D. Harris
Joseph H. Austin, Jr.	John M. Desloge	William H. Heidger
Robert Babione	Donald F. Deutsch	James A. Helwig
Victor L. <b>Badillo, S.J.</b>	Neil C. <b>DeVries</b>	Thomas E. Henderson
Charles J. Balsarotti	Andrew J. Dufner, S.J.	Karen Ann <b>Herbst</b>
Helen M. Barren	Patricia J. Durse	Fiances Hewitt
Paul T. Bauer	Donald M. <b>Eisel</b>	Gertrude T. <b>Heyl</b>
S. Louise <b>Beasley</b>	Robert F. Emmett	John T. Hogan
John A. <b>Beuchman</b>	Robert G. Erwin	Edward J. Hoffman
Claire M. Birkbeck	ST. M. Marcelline Falk,	Richard L. Holdener
Gerald Charles Blausey	C. P. S.	Jetty L. <b>Hollingsworth</b>
Jean M. Bordeaux	Fadlullah M. Farouq	Judy A. <b>Hoog</b>
Charles J. Brez	Sr. M. Geraldine Farrant,	<b>John J. Horenkamp</b>
Kathleen P. Brady	S.S.J.	Sr. Elizabeth M. Huber,
Patricia A. Brown	Paul F. <b>Feldker</b>	BBM.
John P. Bufo, Jr.	Thomas D. <b>Fiorino</b>	Jeanne H. Huesemann
Benjamin R. Bullock	Sr. M. Carlita Fitzpatrick,	Wayne L. Humphrey
St. M. L. <b>Burk, O.P.</b>	R.S.M.	Marion A. Joncich, S.J.
Sr. M. J. Carboneau, R.S.M.	Gerald W. Folk	Sr. <b>Martina</b> Kalinosky
Charles P. Casey	John F. Fox	James L. Kappel
Mary J. <b>Chirpich</b>	Joan F. Franz	St. James M. Keezer,
Dorothy Clark	Edward E. Funke	S.U. S.C.
Elizabeth A. <b>Coerver</b>	<b>Darell Gage</b>	Francis X. Ken
Thomas P. Comer	Daniel J. Gannon, S. J.	Joan E. Kessling
Ronald R. <b>Connors</b>	Juan M. Garcia, Jr.	St. Dorothy Kiel,
Sr. M. Coleman <b>Conroy,</b>	Cherie Ann Gass	C. D.P.
O.S. F.	Owen <b>Gleeson</b>	St. M. <b>Fintan Killian,</b>
Major Ralph P. <b>Corbell</b>	Arthur F. Glusenkaup	R.S.M.
John <b>Cortina, S.J.</b>	Michael A. <b>Grayson</b>	Kenneth J. <b>Kintz</b>
Jack Cronin	St. <b>Amrosia Greiner, O.S.F.</b>	Carol Koeppen
Ann <b>Corrigan</b>	James J. <b>Gummels</b>	<b>Orest</b> Koropecy
Patrick Henry <b>Crowe</b>	John B. Haack	Philip M. Krause
Ruth A. David	John A. <b>Haley, S.J.</b>	Joan L. Kristof

Annette **J.** Krygiel  
 Barbara **M.** Kuehne  
 Sr. Lois **M.** Lapeyre, D.C.  
 Frank A. Latuda  
 Sr. **M.** Basil **LeDuc**,  
 O.S.D.  
 Vincent K. Lee, S.J.  
 Joan L. **Leiper**  
 Joan **M.** Leysaght  
 John H. **Liebe**  
 John **Chian** Win Linn  
 Ronald D. Luczak  
 Hubert **J.** Ludwig  
 Nancy L. **M.** Mallonee  
 John E. Mansfield, S.J.  
 Donald **J.** **Manson**, S.J.  
 Madge T. Mao  
 Margo T. Mao  
 St. **DePaul** **Massoni**, D.C.  
 John **J.** Matejcic  
 Matthew N. **McKay**  
 Mark D. **McKenzie**, S.J.  
 Michaela K. **McKittrick**  
 John G. Weber  
 James G. Miller

Mary D. Montie  
 Mary E. Murphy  
 Sara K. Olivier  
 ST. **M.** Patrick **O'Tolle**,  
 S.S.N. D.  
**Joseph M. Paikeday**  
 John **M.** Piet  
 Mary K. **Pisarek**  
 John D. Pope  
 Thomas **J.** Quinn, S.J.  
 Ronald **M.** Reap  
 Thomas **C.** **Rochow**  
 Roy W. Robinson  
 Judy E. Ross  
 Earl **J.** Scalet  
 David G. Schmidt.  
 Raymond **J.** Schmitt  
**R. Gary Schouborg**, S.J.  
 George W. Schroeder  
 James **C.** Schuff  
 Juan Sobrino, S.J.  
 Laurence **J.** Sowash  
 Jerome **E.** Steinke  
 Dennis **E.** Starbuck  
 St. **M.** Barbara **Stastny**, OSF  
 Joyce **Stratman**

Charles D. Stubbs  
 Peter **M.** Szucs  
 Frederick **M.** Tasch  
 Margaret **Thiel**  
 Mary E. Thompson  
**John T. Thompson, S.J.**  
 Anthony P. **Tieber**  
**Joseph A. Tikvart**  
 Mother **M.** Patricia  
 Thro, **RSCJ**  
**Dilawar F.** Uthman  
**Guillermo** Van Hoorde  
 M. Virginia **Vanice**  
 Michael F. Vezeau  
 Wynn A. Volkett  
 Francis **J.** Willy, S.J.  
 Margaret L. Wolf  
 Donald A. Wuebbels  
 Zuhair **Aziz** Yacu  
**Anthony J. Zappanti**  
 Raymond **W.** Zavisla k  
 Eugene A. Zerega  
**Gaylord Zimmerman, Jr.**  
 John H. Zupez, S.J.

#### MONTANA ALPHA, Montana State University (November 14, 1961)

Kenneth V. Bakke  
 David **L.** **Browman**  
 Denny D. Culbertson  
 Barry **P.** Davis

Harlan D. **Dulmage**  
 Jan C. Gerbase  
 Richard **J.** Konesky  
 Edmund **R.** **Kopitzke**

James D. **Mildenberger**  
 Robert L. Svehla  
 Elliot A. Welsh  
 William E. **Whitelaw**

#### MONTANA BETA, Montana State College (May 14, 1962)

Mir Kursheed Ali  
 Bernice **J.** Benson  
 John Bircher  
 David Booth  
 Roswitha **Bullinga**

Brace Furness  
 Gene Leslie Gallagher  
**Maurine** Hager  
 Subodhehen die Mehta  
**Hartvig Melbye**

**Kjell** Nielson  
**Celia** Smith  
 David Stabio  
 Charles **Tolliver**  
 Robert Eugene Tracy

#### NEBRASKA ALPHA, University of Nebraska (May 6, 1962)

Jeanne A. Baird  
**Fredric L.** Bauman  
**Dwain E.** Blum  
**Jerre E.** Bradt  
 Larry G. Bryant  
 Edward L. Calvin  
 Hugh **S.** **Carroll**  
 Marvin E. Criswell  
 James A. Davis  
 Patty A. **Edmiston**  
 Stanley B. **Eliason**  
 Loren **E.** Fairbanks

Anthony E. Hoffman  
 Alan C. Hurd  
 James E. **Huwaldt**  
 Kenneth W. Kaufman  
 William G. Kaufman  
 Stephen G. Kellison  
 Patrick H. Kelly  
 Jin Bai Kim  
 Gary D. Klussman  
 Thomas O. Kotouc  
 Robert **J.** **Kvall**  
 Phillip **M.** Leopold

James W. Lindsay  
 Roger **J.** **Mattson**  
 Robert C. **Miley**, Jr.  
 John I. **Molinder**  
**Mema M.** Prettyman  
 Enid L. Reeder  
 Ronald L. Rogowski  
 Roger H. Schwabauer  
**Mark L.** Teply  
 James R. Wall  
**Khosrow** Hady Youssef

#### NEVADA ALPHA, University of Nevada (April 4, 1962)

Stanley E. Bush  
**Gail M.** Chadwell  
 Robert V. Garcia  
 Charles E. **Gervie**  
 Michael C. **Hislop**  
 Robert D. Horn

James **M.** Hoyt  
 Reginald **Meaker**  
 Kathleen I. Miller  
 Ann L. **Rafetto**  
 Jeanne **M.** **Sadler**

Gary **M.** Tachioires  
 Alan S. Thomas  
 Calvin E. Thompson  
 James D. **Whitlock**  
 Charles W. **Wilmore**, Jr.

#### NEW HAMPSHIRE ALPHA, University of New Hampshire (May 24, 1962)

William A. Cooney  
 Ronald E. Cote  
 Judith A. Flagg  
 John A. Hinchey  
 William E. Lunt

Christine **M.** Malkowski  
 Robert **J.** Moore  
 Elizabeth A. Nichols  
 John L. **Olesniewicz**

Tmng Ngde **Quy**  
 Janet **P.** Ray  
 Charles **M.** Sawyer  
 Marilyn **J.** Staples

#### NEW JERSEY BETA, Douglas College (May 14, 1962)

Marie L. **Crawford**  
 Mary Ann **Ferrarese**  
 Anne L. **Grandin**  
 Elizabeth **M.** Grieder

Marianne Haulenbeck  
 Susan R. Kolba  
 Judith A. **Moraller**

Nancy F. Peters  
 Linda E. Peterson  
 Elaine R. Scheines

#### NEW MEXICO ALPHA, New Mexico State University (May 14, 1962)

Ralph W. Ball  
 Richard K. Fergin  
 Hugh R. Gardner  
 Conrad G. Keyes  
 Maurice **Lipton**

Alfred R. Mitchell  
 Roger W. Mitchell  
 George P. **Mulholland**  
 Charles R. Parsons

Peter **J.** **Ruch**  
 Harold G. Rutherford, II  
 Hal L. Wallace  
 Benjamin Ward, Jr.

#### NEW YORK ALPHA, Syracuse University (April 24, 1962)

Lee T. Bryant  
 Anthony P. **Carpentier**  
 John **J.** Doyle, Jr.  
 James E. **Dunn**  
 Linda **M.** **Dykeman**  
 Jack T. Ferguson  
 Lois Fero

Marcia G. Jackson  
 David F. Johnson  
 David **J.** Kahn  
 Alan Kaplan  
 Carolyn R. Kurgan  
 Vicki **Newman**  
 Jon B. **Pangborn**

Barbara O. Rauch  
 Richard Reed  
 Gary F. Rose  
 Linda E. **Serednick**  
 James P. Walsh  
 Adrian A. **Warntz**  
 Leonora A. **Zobkiw**

#### NEW YORK BETA, Hunter College (April-4, 1962)

George S. **Alland**  
**Ahuva Barnett**  
 Sally A. Burgeson  
 Susan **M.** Dotz  
 Lenore L. Feigelson

Ilene **J.** Goldstein  
 Harriet E. Hayes  
 Susan A. Kahn  
 Helene S. Lessinger

Philomena Russo  
 Susan B. **Sommerfeld**  
**Albert J. Webel**  
 Pauline **Winkelman**

#### NEW YORK GAMMA, Brooklyn College (May 21, 1962)

Harvey **Braverman**  
 Michael **Brozinsky**  
 Bernard **Dickman**

Kenneth **Klein**  
 William Messing  
 Larry Padwa

Charles Rose  
 Edward Ross  
 Jack **Shapiro**

#### NEW YORK DELTA, New York University (February 13, 1962)

Edward H. Bersoff

Michael P. Swirmoff

John T. Wadsworth

#### NEW YORK EPSILON, St. Lawrence University (March 14, 1962)

John Victor Bauer

Lucretia Anne **Cognetta**

#### NEW YORK ETA, University of Buffalo (March 14, 1962)

Judith S. Ackerman  
 Arthur P. **Altman**  
 David V. **Bateman**  
**Robert J.** Buck  
 Ralph L. Disney  
 Call F. Evans

Mary E. Graves  
 Marilyn A. Kanczak  
 Michael D. Kerwan  
 Anthony T. Lauria  
 Gordon L. **Liles**  
 Penelope A. Miller

William E. Price  
 John **Rogozinski**  
**Karl J.** Schmeder  
 Ellen R. Schwartz  
 Irene L. **Sharpe**  
 Marilyn A. Tober

**NEW YORK IOTA**, Polytechnic Institute of Brooklyn (June 12, 1962)

Paul Feder                      Stewart Nagler                      Anthony J. Naro

**NEW YORK KAPPA**, Rensselaer Polytechnic Institute (February 11, 1962)

Alan W. **Adler**                      Irwin Hirsch                      Ralph E. Morganstern  
 Phil J. Best                      Frederick A. Lehrer                      Ralph B. Prescott  
 Stephen H. Davis                      S. Richard Mateosian                      William P. Rogers

**NEW YORK KAPPA**, Rensselaer Polytechnic Institute (June 1, 1962)

George Svetlichny                      Lawrence Elliott Levine                      Robert Frank Anastasi  
 Michael John **Arcidiacono**                                           Howard Butt Kushner

**NEW YORK LAMBDA**, Manhattan College (Spring, 1962)

Kevin C. Anderson                      Whitney S. Harris                      Raymond F. Scanga  
 Joseph L. **Bertorelli**                      John R. Mager                      Gerald S. **Silva**  
 Jack Bova                      Bernard J. **McCabe**                      John Stout  
 Joseph P. Genovese                      Ronald R. Ragonese                      Thomas G. Walsh

**NEW YORK MU**, Yeshiva College (May 1, 1962)

Elliot **Beller**                      Steven E. **Grossman**                      Lawrence Schulman  
 Martin **Braun**                      David Jacobson                      Bert **Sirote**  
 Robert Feinennan                      Ben M. **Schreiber**                      Benjamin Volk  
 Joel **Grossman**

**NORTH CAROLINA ALPHA**; Duke University (Spring, 1962)

Kenneth G. Brown                      Jan M. Hollis                      Hugh H. Mills, Jr.  
 Joan S. **Dimpfl**                      Edwin E. Messikomer

**NORTH CAROLINA BETA**, University of North Carolina (November 20, 1961)

Anne E. Britton                      Richard D. Hofler                      Sue F. Ross  
 Peter J. Brown                      Robert J. Hursey, Jr.                      Charles W. **Shearin**  
 Judith E. Bush                      Knox H. Jones                      Jance C. **Shearin**  
 Bobby F. Caviness                      Elizabeth A. **McLeod**                      John N. **Tunstall**  
 Thomas G. Eason                      Martha A. Myers                      James R. J. **Wadkins**  
 Charles E. Grigsby                      Oscar R. Ogg

**NORTH CAROLINA BETA**, University of North Carolina (February 26, 1962)

Robert Linn **Bernhardt, III**                      Phillip Morris Kannan

**NORTH CAROLINA GAMMA**, North Carolina State University (April 26, 1962)

Robert R. **Allran**                      Thomas C. Harris                      Ralph E. Showalter  
 Anne L. **Fakler**                      James L. Klingennan                      David C. **Shreve**  
 Henry L. Fisher, Jr.                      Bobby E. Phillips                      Jason L. **Sox**  
 John D. Fulton                      Gary D. Richardson                      William J. Tanner  
 Claude D. Greeson                      Robert T. Rood

**OHIO ALPHA**, Ohio State University (May 25, 1962)

Robert D. Baer                      James T. Hanlon                      Charles E. Ryan, Jr.  
 James K. Brooks                      William H. Hosken                      Thurston W. **Shook**  
 Stephen D. Comer                      Kenneth R. **Kimble**                      Allan D. Silver  
 Ralph T. **Compton, Jr.**                      Gary G. **Koch**                      Alan L. Tyree  
 David L. Deever                      Frederick S. Koehl                      Jacob M. **Weinberg**  
 St. M. Noel **Dreska**                      James **Teh-zen** Koo                      Thomas C. Wesselkamper  
 Dorothy S. GUY                      Harry G. Miller

**OHIO BETA**, Ohio Wesleyan University (March 21, 1962)

Linda B. **Connolly**                      Donald S. **Hetzel**                      Betsy L. Rittenhouse  
 Frederick M. **Haney**                      Eric S. Johnson                      Stanley D. **Shawhan**  
 Wilfred James **Hansen**                      John D. Kessler

**OHIO ETA**, Fenn College (June 15, 1962)

James Bevevino                      George Herron                      James Logan  
 Ray **Brezic**                      Kenneth Kasper                      Norman Moore  
 Richard Cerny                      Robert **Kime**                      Robert Weisensed

**OHIO ETA**, Fenn College (July 25, 1962)

F. Norman Lutz                      Joseph Petsche

**OHIO GAMMA**, University of Toledo (May 6, 1962)

Robert S. **Brundage**                      Thomas J. **Mazuchowski**                      Carmen **Vittoria**  
 Ellen C. Brunt                      Susan Nichols                      Stephen M. **Weglian, Jr.**  
 Peggy Loo                      Keith C. Richards                      Carole A. Wright  
 Dennis F. Marvin

**OHIO DELTA**, Miami University (May 1, 1962)

Kenneth R. Adams                      George E. Fredericks                      Lois J. Snyder  
 Nancy W. **Boswell**                      Carol V. Medlar

**OHIO EPSILON**, Kent State University (April 11, 1962)

Joyce L. **Burrell**                      Susan K. Hill                      Barbara L. Pletcher  
 James H. **Buxton**                      Donald L. Hunston                      Robert Schappelle  
 Douglas A. Cope                      Sandra L. Jackson                      Richard G. **Schooley**  
 Dianne E. Coyne                      Neil M. Kettlewell                      Donald B. Siano  
 Clifford H. Curtis                      Carl D. **Lytle**                      James R. Weaver  
 Donald A. Furey                      Glenice A. **Nocjar**                      Richard M. Williams  
 Robert L. Furey                      Mary J. Orovany                      Lois I. Wilson  
 Barbara A. Grills                      Kenneth A. Pew

**OHIO ZETA**, University of Dayton (Spring, 1962)

Michael **Barnoski**                      Laurens W. Houttuin                      Marcella A. Sakalas  
 Joseph P. **Brzozowski**                      James A. Kass                      Thomas E. Scheper, S.M.  
 Thomas S. Clifford, S.M.                      John Edward Kauflin                      Charles D. Schweickart  
 Max G. Gruendel                      Joseph D. Kolesar                      Alan G. Stevens  
 Darrell J. **Horwath**                      Frank T. Kozuh, S.M.                      James D. **VanHusen**

**OHIO ETA**, Fenn College (February 23, 1962)

John E. Conroy                      Bruce Johansen                      Kenneth Schwartzkopf  
 Jack E. Crow                      C. Robert **Klotz**                      David **Sealey**  
 David Herlacher                      Frank **Lozier**                      John J. Whitely  
 Wayne Hudson                      Henry A. **Putre**                      Edgar D. Young

**OKLAHOMA ALPHA**, University of Oklahoma (February 13, 1962)

John R. Lesem	Monte D. Miller	Gerald L. Smith
Charles E. Maudlin, Jr.	Helen M. <b>Murphy</b>	Robert W. Vaughen

**OKLAHOMA ALPHA**, University of Oklahoma (May 4, 1962)

Lynne F. <b>Capehart</b>	Stephen <b>DeCanio</b>	Leo J. Pratte
Charles H. Cook	Gary F. Esch	G. Joseph <b>Wimbish, Jr.</b>
	Forrest R. Miller, Jr.	

**OKLAHOMA BETA**, Oklahoma State University (January 4, 1962)

A. Paul Brokaw	Ann Hemker	Billy G. Monks
David R. Bryan	<b>Parshall L. Howe</b>	Karen Owen
E. Ann Campbell	Jerry Johnson	John R. <b>Perkins</b>
Sue Coates	Mary Johnson	Linda L. Priest
Ralph Ferguson	<b>Jimmie G. Lakin</b>	Robert Sasaki
<b>Darrell Gimlin</b>	James C. <b>Loggins</b>	Margaret <b>Sauls</b>
Joseph S. Greene, Sr.	Lyman L. McDonald	Rose <b>Stuntz</b>

**OKLAHOMA BETA**, Oklahoma State University (April 19, 1962)

Anthony R. Abernethy	Brace C. Edgar	Barton S. <b>Perrinne, Jr.</b>
George W. Brewer	Ronald Gardner	<b>Willa E. Pickering</b>
<b>Jerry W. Brown</b>	Vadis G. <b>Godbey</b>	George F. Reese
Norman Bryant	Glenn Hightower	Jack B. Reid
George M. Butler	William Ralph Lee	Song-yeong Ruo
Burton K. Chambers	<b>Tetral L. McKellips</b>	Many A. Smith
Clifford C. <b>Chrisman</b>	George W. Miller	Wayne R. Stoddard
Boyd A. Christensen, Jr.	Nancy Page	Homer H. Tang
Fred E. <b>Collings</b>	Chin-loh Pan	

**OREGON ALPHA**, University of Oregon (May 29, 1962)

Roger <b>Backen</b>	Jimmy Hinkhouse	Leroy Peterson
Virginia <b>Emilie</b> Brooke	Paul William Janus	Roger Price
Donald Leroy Bmwn	Robert Carl <b>Juola</b>	<b>Darrell Ray Quarles</b>
David <b>Welland</b> Chapman	Karen <b>Kreuder</b> Hoff	Karl <b>Reitz</b>
Jean Therese Collins	Douglas Kelker	Chaika Kim Rhee
Gary Lee <b>Cotliss</b>	Susan Jean Kruttsch	Pamela Robinson
Mary Katherine Davidson	David <b>Leeming</b>	Stewart Sawyer
Kathleen <b>Donaldson</b>	Patricia <b>Jo McCorkle</b>	Robert Siegenthaler
John <b>Edlund</b>	James Wells McCoy	Harry Dean Smith
Barbara Louise <b>Elerath</b>	Allen <b>McDaniels</b>	Delbert Wayne Snyder
Robert Charles Ghent	Gary Hunter Newton	Carol Jean Somekawa
Sharlyn Lee <b>Gillis</b>	Phyllis Nicholson	Larry Virgin
Peggy Lee Huston	Patricia Novak	John Williams

**OREGON BETA**, Oregon State University (May 22, 1962)

Robert J. <b>Albright</b>	Phillip M. <b>Gregg</b>	Sandra Schlinkman
Janet E. Allison	Jerry R. <b>Grunwald</b>	Richard G. Schluter
James J. <b>Auborn</b>	Keith R. Hacker	Arnold Leon Schmeder, Jr.
Rodney <b>Allan</b> Badger	John A. <b>Hocken, Jr.</b>	William R. Scofield
Gerald L. Belt	James H. Husband	George Edward <b>Seymore</b>
<b>Jo Ann Berryman</b>	Ronald E. Johnson	Robin H. Shaylor
Robert Russel <b>Beutler</b>	Kenneth Boyd Knechtel	Richard E. Siemens
Leslie H. <b>Birdsall</b>	Sid G. <b>Knox</b>	Gerald Silke
Chester O. Bishop, Jr.	Lawrence John Koth	Keith B. Snyder
John Amos Boles	David H. Y. <b>Lin</b>	<b>Raj Pal Soni</b>
Jimmie H. <b>Boone</b>	Sharon H. S. Long	<b>J. R. Spoed</b>
Gary L. Brown	Michael H. <b>Malmros</b>	Robert Lee <b>Steele</b>
<b>Manuelita A. Brown</b>	Kaye Kent Manchester	Frederick J. Sterk
William G. Brown	Michael Franklin McCoy	Donald P. Stockard
Thomas <b>Danforth Burnett</b>	Robert W. <b>McCoy</b>	Alan R. Stoebig
Gilbert Wayne Butler	James Lee <b>McElroy</b>	<b>Micael J. Stomp</b>
Don L. Campbell	Roger L. <b>McNair</b>	James <b>Akiyoshi Sunamoto</b>
Theodore W. Cannon	Penelope K. Miller	Patricia Sweet
Jerry Joel <b>Congdon</b>	John L. Morack	Michael T. <b>Sworakowski</b>
Charles J. <b>Conlee</b>	David R. Moser	Michael Allen Talvola
Larry R. Cooper	Mahmoud Abd-elfattah	
Elizabeth Louis <b>Simpson</b>	Moustafa	Alice A. Thompson
Curl	Thomas Munkres	Jay R. <b>Turner</b>
William W. Curtis	Junior A. <b>Nagaki</b>	<b>Sommai Vongsure</b>
Bernard E. Davis	Mark I. Nelsen	James D. Wagner
Arthur F. Deardorff	George Papageorge	Darryl William Walker
Jack D. Dennon	Bill Papakonstantinou	John P. Ward
Sandra D. <b>Dunagan</b>	David Harrison <b>Paup</b>	Michael S. <b>Waterman</b>
Stuart <b>Ames Eide</b>	Dean Edward Perry	Ronald N. Webb
John Alan Evans	John W. Powell, Jr.	Floyd R. Wendlberger
John E. Ferguson, Jr.	Merwyn C. Powell	Jane M. <b>Whitcomb</b>
Lloyd A. Fraser	Arthur Douglas <b>Ritchie</b>	Jean M. Wick
Raymond F. Fry	William John Robertson	John L. Wilkerson
William Earl Gilbert	Eugene R. Royer	Frank O. Williams
David E. <b>Gooding</b>	Michael B. Schartz	James B. Wuori
George Joseph <b>Grammens</b>		James W. York

**PENNSYLVANIA ALPHA**, University of Pennsylvania (Spring, 1962)

Robert <b>Averell</b>	Paul Cotton	Betty <b>Rarig</b>
Elliott M. Badder	Jay E. Israel	Bruce Schoenberg
Melanie Costa	Elliott D. <b>Kieff</b>	Michael Tinkleman
	Eugene H. <b>Poppel</b>	

**PENNSYLVANIA GAMMA**, Lehigh University (November 9, 1961)

Thomas E. Bachman	Charles W. Hofer	Walter H. Nichols
James A. <b>Begley</b>	John P. Janowski	Norman L. <b>Owsley</b>
R. Barry Bischoff	Gary K. <b>Kohler</b>	John R. Pivnichny
Philip H. Brandt	Donald <b>Lookingbill</b>	Thomas A. <b>Reilly</b>
Charles A. <b>Falcone</b>	Thomas G. Ludwig	Thomas A. Slivinski
Leo B. Freeman, Jr.	Michael J. Mendelsohn	Philip H. Swain
Robert J. <b>Galgon</b>	Frank R. Moore	John K. Wagner
Ronald J. <b>Hartmanft</b>	Bernard E. Musch, Jr.	R. Dennis <b>Wayson</b>
G. Raymond <b>Hodil, Jr.</b>	J. Calvin Nafziger	John R. Webb

**PENNSYLVANIA ZETA**, Temple University (March 26, 1962)

Mary <b>Becker</b>	Anne Glass	<b>Herb</b> Silverman
Sue <b>Berwind</b>	Joseph Mandelbaum	Robert Stout
Arthur- <b>Bobrove</b>	Marlene <b>Rosenzweig</b>	Joseph Yeager
Sheldon <b>Eisenberg</b>	Arnold <b>Rubin</b>	



**PENNSYLVANIA DELTA, Pennsylvania State University (May 15, 1962)**

Richard A. **Alo**  
 Mary E. Angstadt  
 James B. **Bartoo**  
 William S. Bickel  
 Dorothy M. **Blankeslee**  
 David L. Briggs  
 John J. Broderick  
 Aviva R. Brown  
 Joan B. **Byers**  
 Judith Q. **Chouinard**  
 Richard L. Clark  
 Edward E. Collins  
 James L. **Cooley**  
 John C. Davis  
 John W. Diercks  
 Lynne E. **Eisenstaedt**

Douglas H. Frank  
 Anthony J. Felice, Jr.  
 Walter L. **Ghering**  
 Carole L. Gibson  
 Charlotte R. Gilson  
 Barry S. **Goluboff**  
 John A. Gustavson  
 Beverly A. **Heckman**  
 Charles E. Houck  
 William M. Houck  
 Robert L. Hunt  
 Leland T. **Mahood**  
 Donald J. **McMahon**  
 George E. Mitchell  
 Robert C. Moore  
 Linda Moritz

Stanislaw Mrowka  
 Edgar P. Niner  
 Robert W. Oliver  
 Carole L. **Pryor**  
 Linda L. Rife  
 Frederick C. Schwere  
 Maria E. Shopay  
 Norma **Shusterman**  
 Jane M. Silverstein  
 Vincent W. Slivinsky  
 Michael J. Smith  
 Rodney L. Smith  
 Edward J. **Unger**  
 Michael **Varano**  
 William E. Wilson

**RHODE ISLAND ALPHA, University of Rhode Island (April 26, 1962)**

Harry A. Bender  
 Richard Bender  
 Albert A. Bennett  
 Joel Cohen  
 Mary H. Cummins  
 Stephen **DeMetrick**  
 Diana Drew  
 James Foster

Carl A. Garabedian  
 Jerald H. Greenberg  
 John T. James, Jr.  
 Clifford D. Leitao  
 Domenick Lombard!  
 Mersina Moskos  
 Edward M. Pease

Nancy E. Randall  
 Maureen Russo  
 Robert J. Salhany  
 Jacob R. Stauffer  
 William Strawderman  
 E. **Ramnath** Suryanarayan  
 Ronald A. **Tourgee**

**SOUTH CAROLINA ALPHA, University of South Carolina (May 1, 1962)**

Robert F. Bradley  
 John G. **Breland, Jr.**  
 Sheang P. Chan  
 Carolyn P. **Crawley**  
 Peggy A. **Crawley**  
 Rodney F. Epting  
 Dianne Fredey  
 George T. **Hawkins**

David B. **Heape**  
 Thomas E. Hogarth  
 Carolyn Honeycurt  
 Barbara **Loewe**  
 Susan **Loewe**  
 Roy Lucas, Jr.  
 Edwina **Rudisill**

Ann Sanders  
 Wei-Mei Shyu  
 Pelham Thomas  
 W. Joseph Van Dyke  
 James Wallace  
 Lindley H. Webb  
 Ronald A. Young

**SOUTH CAROLINA ALPHA, University of South Carolina (August, 1962)**

Nathan E. **Hardwick**

Marshall Pace

Edward C. Thomas

**SOUTH DAKOTA ALPHA, University of South Dakota (May 2, 1962)**

Elwood D. Baas  
 William D. Carda  
 Cleland V. Cook  
 Ervin M. **Eltze**  
 Charles I. **Ernst**  
 James C. **Fargo**  
 Eugene M. Frohling  
 Allison J. Haeder

Joy Hamrin  
 Shapleigh W. Howell  
 Eugene S. Jacobs  
 Helmut B. Nunn  
 Harley M. Oien  
 Joyce E. Palmer  
 John E. Powell  
 Roger F. **Reinking**

Arthur L. **Robertson**  
 Jacqueline M. Rose  
 David E. Rumelhart  
 George D. Schmieg  
 Byron K. Schulke  
 James W. Shouse  
 Robert H. **Sieling**  
 John D. Snyder

**TEXAS ALPHA, Texas Christian University (Spring, 1962)**

Braynard R. Traweek

**VIRGINIA BETA, Virginia Polytechnic Institute (May 15, 1962)**

Jean M. Carr  
 James F. Chew  
 Bryant Chow  
 Dan Chun  
 Bennie A. **Clemmer**  
 Selwyn L. Flournoy  
 John R. Hanson

Margaret A. Hockensmith  
 Merrill W. **Hume**  
 Michael H. Kutner  
 Jack M. **Mendel**  
 William C. Nelson  
 James E. **Norman, Jr.**

Agnes C. Taylor  
 Donald G. Thomas  
 John E. **Tindall**  
 Theodore C. M. Tung  
 Catherine L. Wilson  
 Janet Yates

**WASHINGTON BETA, University of Washington (March 8, 1962)**

Brandt R. Allen  
 Naydene Chadson  
 Gordon P. **Chinn**  
 Peter W. **Cornue**  
 Donald E. **Ekman**  
 M. Owen Englehart  
 William L. Ferris  
 Victor Hasselblad

Thomas D. **Hayward**  
 James P. Jones  
 Robert G. Krause  
 Elizabeth M. Kutter  
 Karen M. **Lenzie**  
 George F. Mason  
 Jean M. **Murakami**  
 Emiko J. Nakamura

Richard A. Owens  
 Robert T. **Ramsay**  
 Richard C. **Reinhold**  
 James A. **Skrivan**  
 Nancy J. **Strother**  
 Joe Tomita  
 Ronald L. **VanEnkevort**

**WASHINGTON BETA, University of Washington (May 22, 1962)**

Dave Barnette  
 John L. Bergaren  
 John R. Cherevnik  
 Janet **Crist**  
 John **Engstrom**  
 Victor I. Francisco  
 Louise B. Haack  
 Pamela Johnson

Duane Alfred King  
 Larry E. Knop  
 Frank **Kukla**  
 Ronald **Lim**  
 John **Lippert**  
 Diana Lubash  
 Gary Miller

Paul Mitts  
 John Reeves  
 Jack Sanford  
 Judith Sleight  
 Douglas Smith  
 Donald Slopier  
 Kuang-Tao Tang  
 Neil W. **Zimmerman**

**WASHINGTON GAMMA, Seattle University (Spring, 1962)**

John E. **Meany**  
 Michael D. Moran

David L. Predeek  
 Leon M. Puzon

Alva M. Wright

**WISCONSIN ALPHA, Marquette University (April 7, 1962)**

Kenneth J. **Bukowski**  
 Karen A. Case  
 Paul S. Cheng  
 Mary E. **DeRosso**  
 Michael H. Egle  
 Richard E. **Fedler**

Susan **Forrestal**  
 Edward J. **LaClare**  
 Robert Stanley **Matulis**  
 Charles J. **Mazza**  
 Robert M. **Nehs**

George M. **Saviello, Jr.**  
 Kristine J. **Skogstrom**  
 Lawrence T. Wachowiak  
 Robert W. **Wacker**  
 Carol A. Wolkerstorfer

**WISCONSIN BETA, University of Wisconsin (February 8, 1962)**

David F. **Appleyard**  
 E. H. **Battistella**  
 Bruce C. Berndt  
 Mary C. Carroll  
 Franklin D. Cheek  
 Nahida H. Gordon

James E. Hall  
 Beverly Henderson  
 David Henderson  
 Kenneth M. **Kapp**  
 Arthur **Knowlton**

Stephen L. Langston  
 Betty M. Lee  
 James M. **O'Connell**  
 Aaron S. Strauss  
 Clyde T. Tahara

**WISCONSIN BETA, University of Wisconsin (May 22, 1962)**

Barbara D. Antolovich  
 Charles K. **Chui**  
 William B. **Dykema**  
 Walter J. **Froehlich**  
 Martin I. Goldstein

Dennis W. Kuba  
 Elizabeth G. Lee  
 Lee C. Marquardt  
 Caryl A. Milkowski

Toby J. Mitchell  
 Claire L. Mount  
 Eunice G. Pollack  
 Stephen M. Robinson  
 Lynn Van **Vleet**

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