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UNDERGRADUATE RESEARCH IN MATHEMATICS AT THE NAVAL ACADEMY

J. C. Abbott, U. S. Naval Academy

In recent years there has been a growing interest in undergraduate research and independent study in mathematics. Much of this new interest is due to the upgrading of both pre-college and college curricula so that students are proceeding at an accelerated rate and, at the same time, are encouraged to take a real interest in "modern mathematics." Further impetus has been supplied by the National Science Foundation through its Undergraduate Science Education Program. This program supplies financial aid to students and schools to encourage gifted students to undertake individual projects apart from the regular curriculum. The principal difficulty in the field of mathematics is that genuine research problems are generally inaccessible to all but advanced graduate students and professional mathematicians. Consequently many of the proposals for undergraduate independent study are apt to degenerate into reading courses not offered in the regular curriculum, or are, at best, library research papers. The purpose of this paper is to describe a rather unique program which has been under development at the Naval Academy on an experimental basis during the past three or four years.

The mission of this program, like many others throughout the country, is to give selected students an early opportunity to see mathematics as a mathematician sees it, as a living, growing science with unexplored frontiers, rather than simply as a hand-maiden for the physical sciences. Too many undergraduates still see mathematics only as a collection of algorithms for solving different typed problems. They seldom have an opportunity to formulate their own concepts and feel the excitement of discovering their own proofs of their own theorems, which is the true fun of mathematics. The program at the Naval Academy is designed to present just such an opportunity to at least, a few students willing to work for the reward of scientific achievement.

The principal innovation of this particular program is its organization as a group activity around a single central theme developed by the students themselves over a period of years. Thus, it is a seminar in the true meaning of the word, a small group of students working together to create new theories in which each is able to make a specific contribution. The program originated three years ago with a single student who spent a year and a half working on an assigned problem in boolean algebra. This project led to a paper which was read to a sectional meeting of the Mathematical Association of America and to the Eastern Colleges Science Conference. Since then two other students took up this same problem and developed it further. They also presented their results to professional audiences. At present two juniors and two seniors are carrying on the work by writing additional papers in allied topics.

The specific topic around which the program has grown has been the development of boolean algebra from a new point of view. The central idea was not unheard of before, and, in fact, was suggested as early as 1880 by C. S. Pierce, but has never been carried out to its logical conclusion. We begin by considering the single set operation known as implication. If A and B are two arbitrary subsets of some universal set, U, then their implication product is the set of elements of U either not in A or in B. In classical notation this is written A' B where ' stands for set complement and is set union. Here we abbreviate it simply as AB and call it implication. (The terminology stems from the fact that, in logic, p implies q means either not p or q.) We now define an implication algebra of sets as any collection of subsets of some universal set, U, which is closed under set implication. It is now easy to verify that this operation satisfies the three following fairly simple laws: P1: $(AB)A = A$, P2: $(AB)B = (BA)A$, P3: $A(BC) = B(AC)$ which we call contraction, quasi-commutativity, and exchange. Using set theory as a model, we now define an abstract implication algebra to be a set, I, of elements, a,b,c,..., closed under a single binary operation which satisfies these three laws as postulates. What is not quite so obvious is that, conversely, these three laws themselves are sufficient to characterize a very

large portion of the algebra of sets. In fact a boolean algebra can be characterized as an implication algebra which satisfies the one additional postulate P4: there exists an element o saitsfying $oa = aa$ for all a in I.

For those who would like to try their hand in the early phases of the subject, we suggest the following: show that the square of any element is a constant in the algebra; we denote it by u. Show that u is a left identity, and, at the same time, a right zero ($au = u$ for all a). Then solve the word problem with two letters, i.e., determine all possible elements that can be generated from two free elements a and b using only P1–P3. Determine their implication table. As a further exercise, define a b if and only if $ab = u$. Show that is a partial order, and that, with respect to this partial order, I is a join-semi-lattice, but is, in general, not closed under greatest lower bounds. This is only the beginning, but will give some idea of the kind of computations that must be performed.

Mathematics abounds in examples of implication algebras. First of all, any boolean algebra is an implication algebra, so that all examples of boolean algebra are candidates. On the other hand, the set of all non-empty subsets of arbitrary set forms an implication algebra which is not a boolean algebra. Furthermore, the operation of set subtraction, A-B, also satisfies P1–P3, so that any collection of sets closed under subtraction is also an implication algebra. For example, the set of finite subsets of any infinite set is such a case. In fact, abstract set theory itself is a second such example, which is again not a boolean algebra. There is no greatest set. On the other hand implication algebra bears the same relation to the positive calculus of proportions (without negation) as does boolean algebra to the full calculus. Hence, logic is a rich field for applications. Finally, for the topologists, if the word neighborhood is taken to mean any set with a non-empty interior, then the set of neighborhoods of a topological space is a further example. In fact, we can use implication algebra to formulate the very term "topological space." Many other specific examples of finite and denumerably infinite algebras have also been concocted to illustrate various aspects of the subject.

Implication algebra is not only rich in applications, but also has a very classical algebraic structure and can therefore be used to illustrate many notions from abstract algebra. In particular, since only a single basic operation is involved, it is natural to turn to group theory as a source of concepts. This is contrary to the popular impression that boolean algebra requires two distinct operations and therefore must be developed either as a special kind of lattice or by analogy with the theory of rings, not groups. Thus, the student can write his own definition of a homomorphism using the same definition as in group theory. He then can define an ideal by analogy with a normal subgroup and a congruence relation in terms of ideals. The basic theorems of elementary group theory can then be restated for implication algebras, but the proofs must be entirely different, since the basic arithmetic is so different. Hence the student is given an incentive to study group theory, not just as a collection of theorems and proofs in a text book, but as a source of concepts needed in order to develop his own theory. The very fact that implication is neither commutative nor associative in itself creates a fascination, while, at the same, the simplicity of the postulates makes it possible for even an undergraduate to achieve new results. On the other hand there is enough challenge in the development of the more sophisticated aspect of the subject to keep the student's interest.

The earliest papers showed the relationship between implication algebra and classical boolean algebra. Specifically, it was shown that every implication algebra can be imbedded in a boolean algebra. This paper required the development of ideal theory which has become one of the major tools for further developments. The latest papers included a representation theory for implication algebra in terms of set theory, essentially an extension of Stone's results for boolean algebra, and a development of the Jordan-Holder Theory for ideals in implication algebras. This final paper also included some new isomorphism theorems which have no counterparts in group theory. Papers underway at the present include a study of free algebras with either a finite or denumerably infinite number of generators, the programming of a computer for the solution of certain word problems, a study of the relation of implication algebra to brouwerian lattices, a completion process using dedekind cuts, applications to logic and topology, etc. Many other topics simply await interested students to investigate them. Perhaps some of the readers of Pi Mu Epsilon may find sufficient interest to enter the field.

We conclude with a few comments about the organization of the program for those who might be interested in forming their own projects. The program is based on a weekly seminar. Students may enroll at the middle of their sophmore year after completing the calculus and one semester of modern algebra. The first phase of the program is devoted to building up a background of a now classical nature. Topics include set theory, relations and functions, partially ordered sets, lattices, the axiom of choice and classical boolean algebra. This part of the program is conducted mostly on a lecture system, but carries no official recognition, has no formal requirements, gives no tests or grades, etc. Reading assignments are suggested and students are encouraged to give lectures themselves on special topics from time to time. The only true incentive is the knowledge gained, and the seminar is open to anyone. Frequently, students complete this phase of the program for the material it contains even though they do not wish to work on a project.

The second phase usually begins by the fall of the junior year and is devoted to a review of the known results in implication algebra as previously obtained by past graduates. During this phase frequent side issues arise which give the students their first opportunity to work out some new theorem, or reformulate past results. This gives the student his first opportunity to present results to the rest of the seminar for discussion and suggestions. Results obtained are often the joint efforts of more than one student. These results are then written up into the notes for the use of future members. Again, this phase lasts for approximately one semester.

The final phase consists of work on individual projects which may take anywhere from a year to a year and a half to complete. Successful projects terminate in the writing of a paper which, if acceptable, is awarded a three semester hour credit, the only official recognition of the program. During this phase, the seminar continues to meet regularly for discussion of the various projects, giving all the students an opportunity to keep abreast of the others and permitting mutual criticism and suggestions from the group. Student activity now becomes the heart of the program rather than formal lectures.

Hence, the goal of the program is met when undergraduates are given a chance to read mathematics, to talk mathematics and to write mathematics, i.e., to act like mathematicians. They learn that mathematics has unsolved and unsolvable problems and they gain a sense of mathematical maturity not generally available to undergraduates. The success of the program is the enthusiasm of the students themselves who all acclaim, universally, that this program is the salient feature of their undergraduate education. It is hoped that this success may encourage others elsewhere to undertake similar projects.

THE ABSOLUTE VALUE OF A MATRIX*

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Introduction. Function of a Matrix. The idea of a function with a matrix argument is not new. For the matrix A, matrix expressions for e^A, the Taylor series in A, polynomials in A, and transcendental functions of A are well known. See [2].

The absolute value of a matrix $A = [a_{pq}]$ whose elements are complex numbers of functions was defined by J. H. M. Wedderburn [4] to be the scalar $\sum_{p,q} a_{pq} \cdot \bar{a}_{pq}$. The definition to be given in this paper is motivated by a different view. We shall give a matrix expression for abs A by using the absolute value function to define a mapping of matrices onto matrices. Then, the properties of the function matrix will be discussed and the paper will conclude with an interesting result which can be seen analogous to the polar factorization of a complex number.

Before discussing the absolute value function, a discussion of the general definition of functions of matrices to be applied is in order. The definition of a function of a matrix most useful to the purpose of this paper is given by Gantmacher [1] and is summarized below.

Let A be a square matrix of order n and $f(x)$ a function of a scalar argument x. We wish to define what is meant by $f(A)$; that is, we wish to extend the function $f(x)$ to a matrix value of the argument.

If $f(x)$ is a polynomial,

$$f(x) = a_t x^t + a_{t-1} x^{t-1} + \dots + a ,$$

we define $f(A)$ to be the matrix

$$f(A) = a_t A^t + a_{t-1} A^{t-1} + \dots + a_o I. \tag{1}$$

*This paper was written under the supervision of J. L. Zemmer and submitted to the Honors Council of the College of Arts and Science, University of Missouri, as part of the requirement for the B. A degree with Honors in Mathematics.

Using this special function of a matrix as a basis, we can obtain a definition of $f(A)$ when $f(x)$ is not necessarily a polynomial but an arbitrary function. To do this certain terms defined in the following paragraphs will be used.

<u>Definition 1</u>. A scalar polynomial $\emptyset(x)$ is called an annihilating polynomial of the square matrix A if $\emptyset(A) = 0$. An annihilating polynomial $u(x)$ of least degree and highest coefficient one is called a *-<u>mal polynomial of</u> A. If $p(x)$ is any annihilating polynomial, then ~ (x I is divisible by $u(x)$.

By the well-known Cayley-Hamilton Theorem, the characteristic polynomial of A, $\Delta(x) = \det(A - xI)$ is an annihilating polynomial of A but it is not, in general, a minimal polynomial. Let

(2) $$u(x) = (x - k_1)^{m_1}(x - k_2)^{m_2} \cdots (x - k_s)^{m_s}$$

be the minimal polynomial of A where $k_1, k_2, \ldots, k_s$ are the characteristic roots of A and the degree of the polynomial is the sum of the multiplicities of the roots,

$$m = \sum_{i=1}^{s} m_i .$$

<u>Definition 2</u>. Given the arbitrary function $f(x)$, consider

(3) $$f(k_i),\ f'(k_i),\ f''(k_i),\ \ldots,\ f^{(m_i-1)}(k_i),\quad i = 1, 2, \ldots, s,$$

and m_i is the multiplicity of k_i in the minimal polynomial of A, (2). The m numbers in (3) will be called the <u>values of the function $f(x)$ on the spectrum of the matrix A</u>, denoted by $f(X_A)$.

We may now proceed to prove a valuable theorem:

THEOREM 1. If $p(x)$ and $q(x)$ are polynomials which assume the same values on the spectrum of A, $p(X_A) = q(X_A)$, then

$$p(A) = q(A).$$

<u>Proof</u>. We will use the following well known result from a theorem in the theory of equations: If $h(x)$ is a polynomial, then k is a root of $h(x)$ of multiplicity m if and only if $h(k) = h'(k) = \ldots = h^{(m-1)}(k) = 0$. To use this result: Consider the difference $d(x) \overset{D}{=} p(x) - q(x)$ of the two polynomials above. Since p and q have the same values on the spectrum of A, $d(k_i) = d'(k_i) = \ldots = d^{(m_i-1)}(k_i) = 0$ for $i = 1, 2, \ldots, s$. Then, by the theorem cited above, $k_1, k_2, \ldots, k_s$ are roots of $d(x)$ and $d(x)$ may be written $d(x) = (x - k_1)^{m_1}(x - k_2)^{m_2} \cdots (x - k_s)^{m_s} \cdot t(x)$ or from (2), the definition of the minimal polynomial, $d(x) = u(x)t(x)$. Since $u(A) = 0$, it is seen that $d(A) \overset{D}{=} p(A) - q(A) = 0 \cdot t(A) = 0$, and $p(A) = q(A)$ as was to be shown.

The definition of $f(A)$ in the general case can be made subject to the principle of the above theorem. That is, the values of the function $f(x)$ must determine $f(A)$ completely, or, in other words, all functions $f_i(x)$ having the same value on the spectrum of A must have the same matrix value, $f(A)$.

<u>Definition</u> 3. If the function $f(x)$ is defined on the spectrum of the matrix A, and

$$f(X_A) = p(X_A)$$

where $p(x)$ is an arbitrary polynomial that assumes on the spectrum of A the same values as does $f(x)$:

$$f(A) \overset{D}{=} p(A).$$

It can be shown [1] that for any function, defined on the spectrum of a matrix, there exists a polynomial having the same values on the spectrum of the matrix. Thus, given an arbitrary function, it is sufficient to look for the polynomial $p(x)$ that assumes the same spectral values as the function and define the function of the matrix as above. e^A, cos A, sin A and other analytic functions are defined by using their Taylor series expansions for the $p(x)$ above.

The following example is stated to show the application of Definition 3 to the absolute value function.

Example. Consider the two by two matrix

$$A = \begin{bmatrix} 2 & 3/2 \\ 2 & 1 \end{bmatrix}.$$

The characteristic roots of A (and hence the roots of the minimal polynomial) are

$$k_1 = \frac{3+\sqrt{13}}{2} \quad \text{and} \quad k_2 = \frac{3-\sqrt{13}}{2}.$$

The polynomial $3/\sqrt{13}\, x + 2/\sqrt{13}$ has the same value as $f(x)$ on the spectrum of A so

$$\text{abs } A = 3/\sqrt{13}\, A + 2/\sqrt{13}\, I = 1/\sqrt{13} \begin{bmatrix} 8 & 9/2 \\ 6 & 5 \end{bmatrix}.$$

Before studying the properties of the absolute value matrix further, several basic theorems which will make our work easier will be stated. The proofs can be found in Murdock [3].

SUMMARY OF SIMILARITY THEOREMS.

(4) A matrix R is said to be similar to a matrix S if there exists a nonsingular matrix P such that $R = P^{-1}SP$. The passage from S to $P^{-1}SP$ is called a similarity transformation.

(5) Similar matrices have equal determinants, the same characteristic equations, and the same characteristic roots.

(6) If the characteristic roots of a matrix are distinct, it can be shown that the matrix is similar to a diagonal matrix, the diagonal elements being the characteristic roots of the matrix. If the characteristic roots are not distinct, the matrix may not be similar to a diagonal matrix but every matrix is similar to a triangular matrix, that is, a matrix with only zeros below (or above) the principal diagonal. The elements on the principal diagonal are obviously the characteristic roots of the triangular matrix and hence (by 5) of the transformed matrix.

(7) If $k_1, k_2, \ldots, k_r$ are the characteristic roots of the matrix A and $p(x)$ is any polynomial, the characteristic roots of $p(A)$ are $p(k_1), p(k_2), \ldots, p(k_r)$.

(8) Every matrix A is similar to a triangular matrix of the form

$$TAT^{-1} = J = \begin{bmatrix} F_1 & & & \\ & F_2 & & \\ & & \ddots & \\ & & & F_r \end{bmatrix}$$

where each F_i on the diagonal has the form,

$$F_i = \begin{bmatrix} k_i & 1 & & \\ & k_i & 1 & \\ & & \ddots & 1 \\ & & & k_i \end{bmatrix}$$

and $k_1, k_2, \ldots, k_r$ are the characteristic roots of A but are not necessarily distinct. J is called the classical or Jordan canonical form of A and two matrices are similar if and only if they have the same Jordan form except possibly for the order in which the matrices F_i occur in the diagonal of J. The Jordan canonical form is a diagonal matrix if each of the submatrices has order one. This is the case for F_j if the characteristic root k_j has multiplicity one in the minimal polynomial. F_j then is simply the element k_j.

An important theorem can now be given:

THEOREM 2. If $G = \begin{bmatrix} H_1 & & & \\ & H_2 & & \\ & & \ddots & \\ & & & H_v \end{bmatrix}$ is any matrix with v square submatrices $H_1, H_2, \ldots, H_v$ along the diagonal then $f(G) = \begin{bmatrix} f(H_1) & & & \\ & f(H_2) & & \\ & & \ddots & \\ & & & f(H_v) \end{bmatrix}$ where $f(x)$ is an arbitrary scalar function.

Proof. Consider the characteristic equation $\Delta(x)$ of G

$\Delta(x) = \det(G - xI) = \det(H_1 - xI)\cdot\det(H_2 - xI) \ldots \det(H_v - xI) = 0.$

It is apparent that any characteristic root (which is also a root of the minimal polynomial) of one of the submatrices H_i is also a characteristic root of the matrix G.

Let $f(x)$ be an arbitrary function defined on the spectrum of G. There exists a polynomial $p(x)$ such that $f(X_G) = p(X_G)$. By Definition 3, $f(G) \overset{D}{\equiv} p(G)$.

Since $G^u = \begin{bmatrix} H_1^u & & & \\ & H_2^u & & \\ & & \ddots & \\ & & & H_v^u \end{bmatrix}$ for any exponent u, $p(G) = \begin{bmatrix} p(H_1) & & & \\ & p(H_2) & & \\ & & \ddots & \\ & & & p(H_v) \end{bmatrix}$

See equation (1).

Now, consider $p(H_i)$. In order to define $f(H_i)$, let $p_i(x)$ be a polynomial such that $f(X_{H_i}) = p_i(X_{H_i})$. Since the spectrum of H_i was seen above to be a subset of the spectrum of G, $p(X_{H_i}) = p_i(X_{H_i}) = f(X_{H_i})$; so from THEOREM 1 and Definition 3,

$$(9) \qquad f(H_i) \overset{D}{\equiv} p(H_i) = p_i(H_i) \qquad \text{for } i = 1, 2, \ldots, v.$$

Since $f(G)$ and $f(H_i)$ are defined as $p(G)$ and $p(H_i)$, a glance at the $p(G)$ matrix in the paragraph above shows THEOREM 2 is now proved.

Now, by the theorem just proved and (8), the Jordan form theorem, we see that for any matrix A, there exists a nonsingular matrix T such that $A = T^{-1}JT$, where J is the Jordan form of A and thus

$$(10) \qquad f(A) \overset{D}{\equiv} p(A) = T^{-1}p(J)T = T^{-1}f(J)T = T^{-1}\begin{bmatrix} f(F_1) & & & \\ & f(F_2) & & \\ & & \ddots & \\ & & & f(F_r) \end{bmatrix} T.$$

If A, and hence (by 5) also J, has distinct characteristic roots, J is a simple diagonal matrix with the characteristic roots as the diagonal elements. The i^{th} element of $f(J)$ as seen above would be in this case simply $f(k_i)$ and $f(J)$ is diagonal in this form.

<u>The Absolute Value Function</u>, <u>abs A</u>. In the introduction, the general concept of a function matrix was defined. If $f(x) = |x|$, $f(A) = \text{abs } A$ is defined if $|x|$ is defined on the spectrum of the matrix A. We will restrict our attention to square matrices with real elements. Since the derivatives of $f(x) = |x|$ are not defined for zero or for a complex value of x, we must further restrict A to be a nonsingular matrix (a nonsingular matrix has nonzero characteristic roots) having no repeated complex roots in its minimal polynomial.

The function matrix for a real nonsingular n by n matrix A is by (10)

$$(11) \qquad \text{abs } A = T^{-1}p(J)T = T^{-1}\begin{bmatrix} \text{abs } F_1 & & \\ & \text{abs } F_2 & \\ & & \ddots \\ & & & \text{abs } F_r \end{bmatrix} T.$$

To define abs F_i we must find the polynomial $p_i(x)$ such that $p_i(x)$ assumes the same values as $f(x) = |x|$ on the spectrum of F_i, that is

$$p_i(k_i) = |k_i|$$

$$p_i'(k_i) = \begin{cases} 1 & \text{if } k_i > 0 \\ -1 & \text{if } k_i < 0 \end{cases}$$

$$p_i''(k_i) = \ldots = p_i^{(m_i - 1)}(k_i) = 0, \quad i = 1, 2, \ldots, r.$$

Such a polynomial is simply $p_i(x) = \dfrac{|k_i|}{k_i}\,x$. From (9), since $p(X_{F_i}) = p_i(X_{F_i}) = f(X_{F_i})$, $\text{abs } F_i \overset{D}{\equiv} p(F_i) = p_i(X_{F_i}) = \dfrac{|k_i|}{k_i} F_i$ for $i = 1, 2, \ldots, r$. Filling in (11) then, we have

$$(12) \qquad \text{abs } A = T^{-1}\begin{bmatrix} \frac{|k_1|}{k_1}F_1 & & & \\ & \frac{|k_2|}{k_2}F_2 & & \\ & & \ddots & \\ & & & \frac{|k_r|}{k_r}F_r \end{bmatrix} T.$$

Several theorems which state important properties of the absolute value matrix will now be given.

THEOREM 3. If $k_1, k_2, \ldots, k_r$ are characteristic roots of A then $|k_1|, |k_2|, \ldots, |k_r|$ are the characteristic roots of abs A.

Proof. Consider an arbitrary submatrix F_i of abs J in (12):

$$\frac{|k_i|}{k_i} F_i = \frac{|k_i|}{k_i}\begin{bmatrix} k_i & 1 & & \\ & k_i & 1 & \\ & & \ddots & 1 \\ & & & k_i \end{bmatrix}.$$

Carrying out the multiplication of the matrix by the scalar, we see that the elements on the principal diagonal of the above matrix are $|k_i|$'s. Thus, the characteristic roots of abs J by (6) and thus by (5) the characteristic roots of abs A are $|k_1|, |k_2|, \ldots, |k_r|$ which was to be shown.

COROLLARY 3a. If abs A is defined $|\det(A)| = \det(\text{abs } A)$.

Proof. Since they are similar matrices, $\det(J) = \det(A)$ and also $\det(\text{abs } J) = \det(\text{abs } A)$. Now, $\det(J)$ is simply the product of its diagonal elements by (6) as is $\det(\text{abs } J)$:

$$|\det(A)| = |\det(J)| = \left|\prod_{j=1}^{r} k_j^{n_j}\right| = \prod_{j=1}^{r} |k_j|^{n_j} = \det(\text{abs } J) = \det(\text{abs } A).$$

COROLLARY 3b. If A is nonsingular then abs A is nonsingular.

Proof. This follows immediately from COROLLARY 3a since $\det(A)$ is not zero because A is nonsingular. $\det(\text{abs } A) \neq 0$ implies abs A's non-singularity.

THEOREM 4. If abs A is defined and d is an arbitrary real number, then abs $dA = |d| \cdot$ abs A.

Proof. From (8), $T(dA)T^{-1} = d\,TAT^{-1} = dJ$ and hence,

$$dA = T^{-1}\begin{bmatrix} dF_1 & & & \\ & dF_2 & & \\ & & \ddots & \\ & & & dF_r \end{bmatrix} T.$$

By (11)

$$\text{abs } dA = p(dA) = T^{-1}\begin{bmatrix} \text{abs } dF_1 & & & \\ & \text{abs } dF_2 & & \\ & & \ddots & \\ & & & \text{abs } dF_r \end{bmatrix} T.$$

The polynomial $p_i(x)$ which has the same values as x on the spectrum of dF_i is $p_i(x) = \frac{|dk_i|}{dk_i} x$. Thus by (9), abs $dF_i \overset{D}{=} p(dF_i) = p_i(dF_i) = |d| \frac{|k_i|}{k_i} F_i$. Therefore

$$\text{abs } dA = T^{-1}\begin{bmatrix} |d|\frac{|k_1|}{k_1}F_1 & & & \\ & |d|\frac{|k_2|}{k_2}F_2 & & \\ & & \ddots & \\ & & & |d|\frac{|k_r|}{k_r}F_r \end{bmatrix} T = |d|T^{-1}\begin{bmatrix} \frac{|k_1|}{k_1}F_1 & & \\ & \ddots & \\ & & \frac{|k_r|}{k_r}F_r \end{bmatrix} T =$$

$|d| \cdot$ abs A.

THEOREM 5. If A is nonsingular and abs A is defined, then

$$\text{abs } A^{-1} = (\text{abs } A)^{-1}.$$

Proof. We know from (12) that abs $A = T^{-1}(\text{abs } J)T$ and thus $(\text{abs } A)^{-1} = T^{-1}(\text{abs } J)^{-1}T$, and we know from (8) that $A = T^{-1}JT$ or $A^{-1} = T^{-1}J^{-1}T$. Also, if k_i is a characteristic root of a nonsingular matrix A, then $\frac{1}{k_i}$ $(= k_i^{-1})$ is a characteristic root of A^{-1}. Thus,

$$(13) \qquad (\text{abs } A)^{-1} = T^{-1}(\text{abs } J)^{-1}T = T^{-1}\begin{bmatrix} \frac{k_1}{|k_1|}F_1^{-1} & & \\ & \ddots & \\ & & \frac{k_r}{|k_r|}F_r^{-1} \end{bmatrix} T.$$

Now for abs A^{-1}, from (11):

$$\text{abs } A^{-1} = T^{-1}(\text{abs } J^{-1})T = T^{-1}\begin{bmatrix} \text{abs } F_1^{-1} & & \\ & \ddots & \\ & & \text{abs } F_r^{-1} \end{bmatrix} T.$$

F_i^{-1} has a single characteristic root k_i repeated n_i times on its principal diagonal. Hence, in defining abs F_i^{-1}, we must find a polynomial $p_i(x)$ such that

$$p_i\left(\frac{1}{k_i}\right) = \frac{1}{|k_i|}$$

$$p_i'\left(\frac{1}{k_i}\right) = \begin{cases} 1 & \text{if } k_i > 0 \\ -1 & \text{if } k_i < 0 \end{cases}$$

$$p_i''\left(\frac{1}{k_i}\right) = \ldots = p_i^{(m_i - 1)}\left(\frac{1}{k_i}\right) = 0,$$

where m_i is the multiplicity of k_i in the minimal polynomial of F,. Such a polynomial is $p_i(x) = \frac{\quad}{|k_i|} x$ so by (10), abs $F_i^{-1} = p(F_i^{-1}) = p_i(F_i^{-1}) = \frac{k_i}{k_i} F_i^{-1}$ so

$$\text{abs } A^{-1} = T^{-1} \begin{bmatrix} \frac{k_1}{|k_1|} F_1^{-1} & & \\ & \ddots & \\ & & \frac{k_r}{|k_r|} F_r^{-1} \end{bmatrix} T.$$

This is the same as (13), so THEOREM 5 is proved.

The following is the major theorem of this paper.

THEOREM 6. Any real nonsingular square matrix A whose absolute value is defined can be written as the product of its absolute value and a matrix whose absolute value is the unity matrix, that is

$$A = \text{abs } A \cdot B$$

where abs B = I.

Proof. The proof will consist of showing abs B = abs [(abs A) - A] = I. From (13) and (8),

$$\text{(14)} \qquad B = (\text{abs } A)^{-1} \cdot A = A \cdot (\text{abs } A)^{-1} = T^{-1} \begin{bmatrix} \frac{k_1}{k_1} & & & \\ & \frac{k_2}{k_2} & & \\ & & \ddots & \\ & & & \frac{k_r}{k_r} \end{bmatrix} T.$$

$$= T^{-1} D T.$$

Every element of the diagonal matrix D in (14) is either +1 or –1 and these elements are the characteristic roots of D. By THEOREM 2 and THEOREM 3, the characteristic roots and elements of the diagonal matrix abs D are equal to 1. Hence abs D is the identity matrix and thus

abs B = $T^{-1} I T$ = I. Q.E.D.

It is noted in linear algebra and matrix theory that every real nonsingular matrix A can be written as a product A = S·0 where S is a positive definite symmetric matrix and 0 is an orthogonal matrix. Generalizing to complex matrices: A complex nonsingular matrix can be written as a product of a Hermitian matrix and a unitary matrix. See [3]. These facts are often cited in analogy to the polar factorization of a complex number $z = x + iy$ $(i = \sqrt{-1})$,

$z = |z| \cdot (a + ib)$, where $|a + ib| = 1$

$a = \cos 0$
$b = \sin \theta$
where $\theta = \tan^{1} \frac{y}{x}$
and $z = \sqrt{x^2 + y^2}$

and are called "polar factorization of a matrix."

THEOREM 6 might be called a "polar factorization" theorem also by analogy to polar factorization of the complex numbers and it is an even more direct analogy. Thus, the concept of the absolute value matrix has proved to have interesting and useful properties.

References

1. Gantmacher, F. R. The Theory of Matrices, Volume I, pp. 95-103, New York, 1959.

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3. Murdoch, D. C. Linear Algebra for Undergraduates, pp. 130-137, New York, 1957.

4. Wedderburn, J. H. M. Bull. Amer. Math. Soc., Volume 31 (1925), pp. 304-308.

X IS π

Little Jack Horner sat in a corner
Repeating that 2 times π
Into a circumference avoids the encumbrance
Of measuring radii.

Marlow Sholander

PROBLEM DEPARTMENT

Edited by

M. S. Klamkin

State University of New York
at Buffalo

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication. Address all communications concerning problems to Professor M. S. Klamkin, Division of Interdisciplinary Studies, University of Buffalo, Buffalo 14, New York.

PROBLEMS FOR SOLUTION

154. Proposed by Kenneth Kloss, Carnegie Institute of Technology.

For a number in (0,1), does there exist a base so that in this new system of enumeration the first two digits are the same?

155. Proposed by William J. LeVeque, University of Colorado.

Two mountain climbers start together at the base of a mountain and climb along two different paths to the summit. Show that it is always possible for the two climbers to be at the same altitudes during the entire trip (assuming each path has on it a finite number of local maxima and minima).

Editorial Note: The proposer notes that the problem is not original with him and he does not know the original proposer.

141. Proposed by D. J. **Newman**, Yeshiva University.

Determine conditions on the sides a and b of a rectangle in order that it can be imbedded in a square.

Solution by David L. Silvennan, Beverly Hills, California.

Imbedding is simple if the longer side does not exceed unity. Otherwise it is necessary and sufficient that the sum of the sides does not exceed the diagonal of the square.

These conditions are summed up in the inequality

$$\min\left\{\frac{a+b}{2}, \quad \max(a,b)\right\} \leq 1.$$

Also solved by George E. Andrews, Michael Goldberg, H. Kaye, L. Smith, M. Wagner, J. E. Yeager, and the proposer.

142. Proposed by Pedro A. **Piza** (posthumously), San Juan, Puerto Rico.

Show that unity can be expressed as the sum of four squares less the sum of four squares (all squares distinct) in an infinitude of ways.

Solution by George E. Andrews, Philadelphia, Pennsylvania.

$$1 = (a^2+b^2)^2 + (m^2+n^2)^2 + 1^2 + 0^2 - (2ab)^2 - (a^2-b^2)^2$$
$$- (2mn)^2 - (m^2-n^2)^2.$$

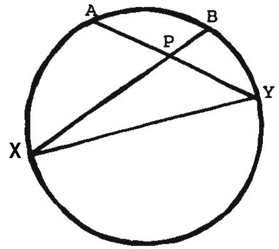
Also solved by John E. **Fergurson**, Theodore Jungreis, David L. **Silverman**, L. smith, M. Wagner, and the proposer.

Editorial Note: Another four parameter solution is given by

$$1 = (x^2-y^2-u^2-v^2)^2 + 4x^2y^2 + 4x^2u^2 + 4x^2v^2 - 1 - (x^2-y^2-u^2+v^2)^2$$
$$- 4(xy-uv)^2 - 4(xu+yv)^2.$$

156. Proposed by K. S. Murray, New York City.

If A and B are fixed points on a given circle and XY is a variable diameter, find the locus of point P.



157. Proposed by John Selfridge, Ohio State University.

Prove $n^{2^{2^2}} - n^{2^2}$ is divisible by $2^{2^{2^2}} - 2^{2^2}$.

158. Proposed by M. S. **Klamkin**, State University of New York at Buffalo.

If $P(x)$ is an nth order polynomial such that $P(x) = 2$ for $x = 1, 2, 3, \ldots, n+1$, find $P(n+2)$.

SOLUTIONS

137. Proposed by Leo Moser, University of Alberta.

Show that squares of sides $1/2, 1/3, \ldots, 1/n, \ldots$ can all be placed without overlap inside a unit square.

Solution by Michael Goldberg, Washington, D. C.

The number of terms in the sequence beginning with $1/2^n$ and ending with $1/(2^{n+1} - 1)$ is 2^n. Hence, the geometric squares with these terms as edges can be included in a strip of width $1/2^n$ and unit length. Hence, all the squares can be included in the strips of width $1/2, 1/4, 1/8, \ldots, 1/2^n, \ldots$ and a unit length to make up the unit square.

Since $\sum_1^\infty n^{-2} = \pi^2/6$, the coverage of the unit square is $\pi^2/6 - 1$ or 64.5%.

Also solved by H. Kaye, P. Myers, L. Smith and M. Wagner.

138. Proposed by David L. Silverman, Beverly Hills, California.

The points of the plane are divided into two sets. Prove at least one set contains the vertices of a rectangle.

Solution by John E. Ferguson, Oregon State University.

In any straight line there are at least four points P_1, P_2, P_3, P_4 in one of the sets say S_1. Now consider the following configuration:

P_1.	A_1.	B_1.	
P_2.	A_2.	B_2.	(Note: The distances between points are out of scale.)
P_3.	A_3.	B_3.	
P_4.	A_4.	B_4.	

In order not to form an S_1 rectangle, at least 3 points of the A group and at least 3 points of the B group must belong to the other set S_2. It then follows immediately that there is an S_2 rectangle.

Also solved by George E. Andrews, P. Myers, Charles S. Rose, L. Smith, M. Wagner, and the proposer.

Editorial Note: The same result will hold if we restrict the points of the plane to being lattice points. This problem is related to Van Der Waerden's Theorem on arithmetic progressions, i.e., if we divide the natural numbers into k classes, then an arithmetic progression of arbitrary length can be found in at least one of the classes. Another associated problem here would be to find the smallest square of lattice points which must contain a rectangle (or some other possible figure). The four vertices of the rectangle must belong to one of the classes into which the lattice points have been divided.

139. Proposed by Leo Moser, University of Alberta.

Show that there exists a unique sequence of non-negative integers, $\{a_i\}$, such that every non-negative number n can be expressed uniquely in the form $n = a_i + 2a_j$.

Solution by L. Carlitz, Duke University.

If we let $f(x) = \sum_i x^{a_i}$, then the statement that every $n \geq 0$ can be expressed uniquely in the form $n = a_i + 2a_j$ is equivalent to

$$f(x)\, f(x^2) = 1 + x + x^2 + \ldots = \frac{1}{1-x}\,.$$

Thus, $f(x) = \dfrac{1}{(1-x)f(x^2)} = \dfrac{(1-x^2)f(x^4)}{1-x} = (1+x)f(x^4)$.

Whence, $f(x) = (1+x)(1+x^4)(1+x^{16}) \ldots$.

Therefore, the $\{a_i\}$ are the numbers of the form

$$c_0 + c_1 4 + c_2 4^2 + \ldots + c_k 4^k$$

where $k \geq 0$ and each $c_r = 0$ or 1.

Solution by Charles S. Rose, Brooklyn College.

The required sequence is formed by the non-negative integers that can be expressed in the base four using only zeros and ones. It is obvious how to produce 0, 1, 2, and 3. From these, the unique expression of n follows. For example:

$3102_4 = (1100_4) + 2\cdot(1001_4)$. The production of no carrys shows that the expression of n is unique and that none of the a_i can be produced from the others. If another sequence were formed, it must contain the a_i; since any other member a* could then be represented as its expression in the a_i and distinctly as $a^* = a^* + 2\cdot 0_4$, the sequence a_i is unique.

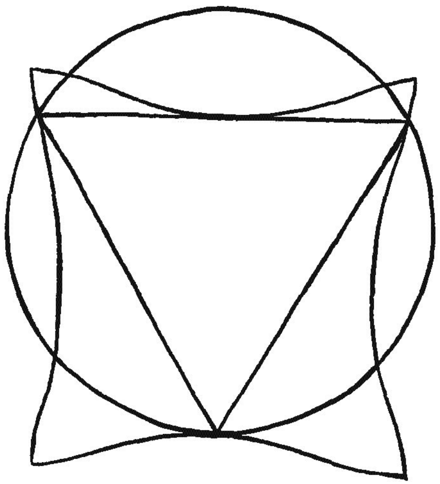
Also solved by George E. Andrews, P. Myers, David L. Silverman, L. Smith and the proposer.

140. Proposed by Michael Goldberg, Washington, D. C.

What is the smallest area within which an equilateral triangle can be turned continuously through all orientations in the plane? This problem is unsolved and similar unsolved ones exist for the square and other regular polygons.

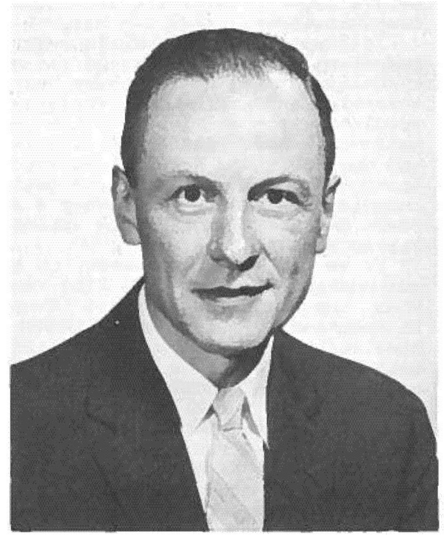
Partial solution by the proposer.

The problem is still unsolved. It is obvious that the triangle can be rotated within its circumscribing circle. However, the smaller area described in the proposer's paper "N-gons making n + 1 contacts with fixed simple curves," American Mathematical Monthly, July, 1962, has an area equal to approximately 79% of the area of the circle. The exact minimum of the infinite family of curves described is not known. Furthermore, there may be other curves of even lesser area.



The figure shows the triangle with its circumscribing circle and the smaller four-lobed curve within which the triangle may also be rotated.

UNIQUE RESEARCH ROLE FOR MATHEMATICIANS



DR. LOUIS W. TORDELLA
Deputy Director
National Security Agency
Fort George G. Meade, Maryland

Mathematicians will find an increasing number and diversity of professional opportunities in Government. These include many fine opportunities to contribute significantly to major technical programs or to attain high-level managerial positions.

Although little known outside the circles of a select portion of the American scientific community, the National Security Agency has existed for many years as a leading research and development activity of the U. S. Department of Defense. The work of the Agency is founded on science and technology, which, in their constantly advancing state, make increasing demands on the capabilities of scientists in many fields. Mathematicians are key members of this scientific fraternity. They are assigned both individually and in groups throughout the Agency's extensive laboratories and research facilities.

NSA mathematicians are concerned mainly with problems which must be solved in support of the communications requirements of the United States Government. These requirements are for general and special-purpose communications equipments. The total systems encompass transmitters, receivers, antennas, terminal units to handle all types of information transmission, and recording and information storage devices. In addition to the hardware which is employed in U. S. communications, NSA must also provide highly esoteric principles to insure the invulnerability of classified governmental information. Together, the foregoing requirements present a variety of interesting challenges, and their successful prosecution is gratifying in a sense beyond the ordinary.

The solution of U. S. communications problems involves, among other things, the statistical analysis of data forcausal significance, probability theory, the statistical design of experiments, and Fourier analysis. Some of the problems stemming from systems design require extensive research and the application of statistics, modern algebra, linear algebra, and information theory. Here, too, we find useful such tools as groups, Galois fields, matrices, number theory, and stochastic processes. Many of the mathematical problems are by nature urgent, but there is also much long-range research in general communications.

In support of the work in communications, NSA maintains a fine computing facility employing the most advanced systems and computing techniques. As machines have become the "slide-rules" of the scientist and engineer, a whole array of intriguing problems have resulted which challenge to the utmost a growing breed of computer mathematicians. These individuals work closely with the physicists and engineers who develop new concepts and circuit devices to be incorporated in the logic and memory elements of faster and more versatile computers. Indeed, the Agency's research and development program in this field has had a significant influence on computer development in the United States.

It is not enough, however, to have better machines. The computer mathematician is asked to find newer and more efficient ways of using them, the "software" side of the picture. This leads to interesting if sometimes bewildering problems in automatic coding and in programming languages, speech recognition, pattern recognition, and the mathematical analysis associated with learning machines. The latter are machines that are programmed not just to do a job, but to learn how to do it. Much of this work emphasizes the solution of logical problems rather than numerical analysis.

There are, of course, other exciting areas of concentration for mathematicians at NSA. These problems are of a high order of difficulty and require an uncommon amount of ingenuity. In fact, they have led to the development of an entirely new mathematical science. It is a stimulating experience to become acquainted with the language and techniques of this science and to see practical applications of some hitherto purely academic branches of mathematics. Moreover, many branches of mathematics which could be fruitfully used await the necessary capability and interest of new mathematicians. For example, many of the combinatorial problems would be challenging to a mathematician interested in graph theory, operations research, information theory, or organization theory. In short, a mathematician at NSA will use as much mathematics as he is both inclined and capable of using.

The present state of knowledge in certain fields of mathematics is not sufficiently advanced to satisfy NSA requirements, and it is therefore necessary to undertake theoretical research in these fields. Those individuals who are interested in doing this type of work, and who have the competence, are encouraged to engage in independent research. In addition, there is considerable opportunity to make substantial scientific contributions in bridging the gap between theoretical investigations and practical applications.

There is an additional point to be made about the work at NSA. It involves the lesson which must be learned by all mathematicians, namely: that problems are seldom, if ever, formulated and handed to the mathematician for solution. Instead, he must help to define the problem by observing its origin and characteristics, and the trends of any data associated with it. Then he must determine whether the problem and the data are susceptible of mathematical treatment, and if so, how. As he grows in his appreciation of this approach to mathematical problems, and the relationship of his academic field to non-mathematical subject matter, both his personal satisfaction and his value to the profession will increase.

As a result of contacts with many distinguished consultants at the colleges and universities, and a close and continuing relationship with numerous industrial laboratories, it is quite apparent that there is no dearth of opportunities for mathematicians. Nevertheless, the common denominator of the nation's total need for these professionals is quality. There are indeed many opportunities for mathematicians of every calibre and field of specialization; but, in an organization which relies heavily on mathematics, it is the versatile and imaginative mathematician who contributes most effectively.

MATHEMATICS TEACHERS NEEDED OVERSEAS

Washington, D.C. – The Peace Corps estimates that during 1964 more than 5,000 teachers will be required to meet the requests coming to it from 48 countries throughout Latin America, Africa and Asia. These teachers will instruct on the elementary, secondary and college levels. More than 1,000 of these teachers have been requested to teach on the secondary and college levels in the fields of science and mathematics--650 in general science, physics, biology, chemistry, botany and zoology, and 350 in mathematics. The major requests have come from Bolivia, Ethiopia, Ghana, India, Liberia, Malaysia, Nigeria, Philippines, Sierra Leone and Turkey.

Teachers who can qualify and desire to secure one of these interesting overseas posts at the end of the current school year should file an application at an early date. Full details and an application form may be secured by writing the Division of Recruiting, Peace Corps, Washington, D.C. 20525.

BOOK REVIEWS

Edited by

Franz E. Hohn, University of Illinois

Elements of Algebra, Fourth Edition. By H. Levi. New York, Chelsea, 1961. 189 pp., $3.25.

This book was written as a textbook for an introductory course leading to more advanced and abstract mathematical courses, and to expose nonmathematical students to genuine mathematical problems and procedures. It presupposes no mathematical training beyond arithmetic, but does require the ability to master moderately subtle concepts and arguments. The book carries out the construction of the natural numbers, the integers, the rationals, and the reals. It develops the algebra appropriate to each of the number systems. The terms used are clearly defined and usually are followed by an example demonstrating the term but are explained in more everyday language to give them meaning.

The book fulfills its original aims. It also is an excellent book for introducing science students with a background in applied mathematics to the subject of abstract mathematics.

Urbana, Illinois — George Kvitek

Elements of Finite Mathematics. By Francis J. Scheid. Reading, Mass.; Addison-Wesley: 1962. vii + 279 pp., $6.75.

Professor Scheid has written this book to illustrate the use of mathematical abstraction for readers acquainted with high school algebra. He presents four major illustrative topics. These topics are Boolean algebra, the concept of number, combinational analysis, and probability. One quarter of the book is devoted to each of these topics.

The author begins the book by pointing out that mathematical formulations are required to solve real-life problems. He then develops the formulations necessary for the solution of simple problems in the above-mentioned four topics. This he does by a clear but abstract development of the needed mathematical structures. Many unusual and interesting problems are solved as examples, others are left for the reader. In addition, an appendix details the elementary programming of a digital computer.

This book is a fine text for an introductory cultural mathematics course for liberal arts students and would be enjoyed by the amateur mathematician. It would, however, be too simple for the serious mathematics Student or for the mathematical education of the scientist and engineer.

Monsanto Research Corporation--Mound Laboratory — L. A. Weller

Play Mathematics. By Harry Langman. New York, Hafner 1962. 216 pp., $4.95.

This book contains a vast collection of mathematical puzzles, almost all of which are original with the author. Very few of the standard problems of recreational mathematics are included although many variations of standard types do appear. There are number tricks, many kinds of geometrical problems, cryptarithms, magic arrangements, combinatorial problems, etc. There is no bibliography and there are no answers or solutions.

The textual material forms only a minor part of the book and does not pretend to offer a complete introduction to the various problem types that appear. This is clearly a problem book and not an expository text. Occasional sentences are obscurely phrased and some of the arguments are needlessly hard to follow. Chapter X, which presents tedious numerical methods of solving problems that could be solved more directly with the aid of systems of linear equations, will probably not appeal to most readers. The exercises of Chapter X can, of course, be solved by more familiar methods. Despite the fact that good exposition is not an outstanding feature of the book, there are so many hundreds of tempting problems and puzzles here that the book is well worth its price to any puzzle enthusiast.

University of Illinois — Franz E. Hohn

Ordinary Differential Equations. By Garrett Birkhoff and Gian-Carlo Rota. Boston, Ginn, 1962. vi + 318 pp., $8.50.

As stated in the preface, one of the chief objectives of this book is to bridge the gap between the usual material treatedin afirst course, and the study of advanced methods and techniques. The book amply meets this objective. The background expected of the reader is, in addition to the usual first course in differential equations, a thorough grasp of the major ideas and methods given in a sound course in advanced calculus, and some knowledge of vectors, matrices, and elementary complex variable theory.

The first four chapters review the methods usually covered in a first course, and also include careful discussions of many theoretical questions, and some new techniques. Chapters V through VIII deal with nonlinear systems, while Chapters IX through XI treat second order linear differential equations. An additional indication of the subject matter treated is given by the chapter headings of these latter chapters. V--Existence and Uniqueness Theorems. VI--Plane Autonomous Systems. VII--Approximate Solutions. VIII--Efficient Numerical Integration. IX--Regular Singular Points. X--Sturm-Liouville Systems. XI--Expansions in Eigenfunctions.

Important special functions are defined and studied by means of their defining differential equations and boundary conditions. The book is suitable for a year's work; or parts of it, as suggested in the preface, can be used for a semester course.

The choice of subject matter is excellent and the exposition is clear. There is a thoroughly adequate set of problems. The authors are to be congratulated on having made a substantial educational contribution to the field involved.

St. Louis University — J. D. Elder

The authors state that the book was not written to be used as a textbook. No exercises are included. The book may be used, however, as a supplement for a course in multivariate analysis.

Perhaps the book will be most useful to research workers in the behavioral sciences at installations which have not yet acquired a library of behavioral science programs. These persons can merely copy the programs and thus obtain immediately a small basic library. The authors state that each program has been test& on-an IBM 709 and proven to be correct.

University of Illinois — Kern W. Dickman

Partial Differential Equations, an Introduction. By Bernard Epstein. New York: McGraw-Hill, 1962. x + 273 pp., $9.50.

The subject of partial differential equations is one which frequently gets slighted in the training of students of mathematics. The reason for this is very simple: it is a vast and difficult field which has its roots deep underground and has its head in the stars. It cuts across almost all mathematical fields--starting, perhaps, in mathematical physics, through complex and real analysis into functional analysis, through differential geometry into group theory, whence it re-enters into physics. Because of its breadth, no mathematics student should be innocent of some of the main results in this subject. However, because of its depth (and difficulty), an unfortunately large percentaqe of such students are not introduced to these results.

The reviewer hopes and feels that this book may help to improve the situation, for it is written as an introduction to this rich subject. Unlike some books, it treats partial differential equations as a branch of mathematics (rather than engineering). Anyone can quibble over the content, but it is unquestionable that what is treated, is done well.

After an introductory chapter which discusses the Ascoli Theorem, Weierstrass Approximation Theorem, Fourier integral, etc., the author gives a very clean presentation of first order equations. Next he discusses the Cauchy problem and the wave equation. Two long chapters on the theory of operators in Banach and Hilbert Spaces are then included, followed by a good treatment of potential theory and various approaches to the Dirichlet problem. The book ends with one brief chapter on the heat equation and one on Green functions.

Although this reviewer feels that there is more of the Banach and Hilbert space theory than is justified by the applications given in this text, he unhesitatingly recommends it to any mathematics student as a useful and interesting book.

University of Illinois — Robert G. Bartle

Linear Algebra and Geometry. By Nicolaas H. Kuiper. Amsterdam: North-Holland Publishing Company, 1962. viii + 285 pp., $8.25.

This well-written book, essentially a translation from the Dutch by A. van der Sluis, gives an excellent treatment of linear algebra and geometry from a somewhat higher standpoint. It will be highly useful

as background material for college instructors teaching linear algebra or advanced analytic geometry, because of its depth, breadth, and modern flavor, and it might be very suitable for an honor's course on the junior or senior level as well as for a high school teachers' institute. For graduate students it may be recommended as an interesting and eminently readable introduction to advanced materials.

After short chapters on geometric vectors in the classical sense and on the elementary set-theoretic notions, the author introduces the n-dimensional tuple space V^n and makes use of it in a preliminary definition of the n-dimensional affine space A^n. After a discussion of some algebraic and geometric notions and their properties, including the dual space and the cobasis, the affine space A^n is now defined as a set of elements called points with an atlas of one-to-one correspondences k: $P \to k(P)$ of A^n onto V^n such that $k(P)P = 0$ and $k(P)k^{-1}(Q)$ is a translation. This leads to the definition of a linear m-variety. The classical geometric theorems are presented, homomorphisms and their duals are studies in detail, matrices are introduced as their representations, systems of linear equations are solved, determinants are treated and applied to geometry, endomorphisms are classified, quadratic and bilinear functions as well as quadratic varieties in Euclidean spaces are investigated. Special mention should be given to a chapter on applications to statistics, including the method of least squares, linear adjustment, regression, and the correlation coefficient. The book ends with chapters on Motions and Affinities, Projective Geometry, Non-Euclidean Planes, and some topological remarks. The author is very successful in keeping a healthy balance between geometry and algebra.

University of Cincinnati — Arno Jaeger

Diophantine Approximations. By Ivan Niven. Interscience Tracts in Pure and Applied Mathematics, Number 12. New York, John Wiley, 1963. viii + 68 pp., $5.00.

The inclusion of this book in the series of Interscience Tracts in Pure and Applied Mathematics is somewhat surprising. The advertising for the Interscience Tracts says, "The presentation is on an advanced level." Actually the presentation of this book is on a very elementary level indeed, for it requires only a smattering of elementary number theory and a knowledge of the basic facts about inequalities. While it certainly is no tragedy that this author has produced a very accessible book, it must be admitted that the difference in level between Niven's book and its predecessor in the series is practically infinite!

Diophantine approximation deals with the approximation of real numbers by rationals and, more generally, with the solution of conditional inequalities in integers. As already indicated, the author discusses only certain facets of the subject which are susceptible of an elementary treatment. The exposition is very clear and well-arranged, and the book should be within the reach of any serious undergraduate mathematics student. As a result, the book is sure to be welcomed by those running independent study programs for undergraduates, for it is ideal for such a purpose.

The only reasonable criticism of the book is that it does not go far enough. Personally, the reviewer was somewhat disappointed by its relatively narrow compass. The reader would certainly get a more balanced view of the subject by reading the relevant chapters in Hardy and Wright's Theory of Numbers. The novice reading Niven's book could easily come to the false conclusion that Diophantine Approximation consists solely of elementary manipulations with inequalities. Actually, Diophantine Approximation can serve as a scenic path on which to lead the reader into deeper mathematics, such as Fourier analysis, measure theory, probability, convex sets, geometry of numbers, algebraic number theory, valuation theory, and so on. The reviewer regrets that the author did not use his expository talents for such a program. However, within the narrow limitations which he has set for himself, the author has produced a first-rate book.

University of Illinois — Paul T. Bateman

A New Journal, *THE* FIBONACCI QUARTERLY

The Fibonacci Quarterly is a journal "devoted to the study of integers with special properties." It is under the general editorship of Verner E. Hoggatt, Jr. It serves as an outlet for serious elementary as well as advanced papers, also includes both elementary and advanced problems. The level of expository quality of the papers is kept high so as to make the results widely available to students at all levels, whether mathematically sophisticated or not. The journal should provide a great deal of inspiration and enjoyment to all of those interested in that part of number theory which deals with "integers with special properties."

The page size is 7 x 10½. Vol. 1, No. 1 contains 75 pages. The subscription rate is $4.00 per year. Subscriptions are to be addressed to Brother U. Alfred, St. Mary's College Post Office, California.

University of Illinois — Franz E. Hohn

NOTE: All correspondence concerning reviews and all books for review should be sent to PROFESSOR FRANZ E. HOHN, 375 ALTGELD HALL, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

BOOKS RECEIVED FOR REVIEW

Edited by

Franz E. Hohn, University of Illinois

R. L. Ackoff and P. Rivett: A Manager's Guide to Operations Research. New York, Wiley, 1963. x + 107 pp., $4.25.

L. J. Adams: Modem Business Mathematics. New York; Holt, Rinehart, and Winston; 1963. x + 348 pp., $5.75.

L. J. Adams: Applied Calculus. New York, Wiley, 1963. ix + 278 pp., $5.95.

*A. A Albert (Editor): Studies in Modern Algebra (Studies in Mathematics, Volume 2). Englewood Cliffs, New Jersey: Prentice-Hall; 1963. 190 pp., $4.00.

*R. W. Ball: Principles of Abstract Algebra. New York; Holt, Rinehart and Winston; 1963. ix + 290 pp., $6.00.

B. Baumrin (Editor): The Philosophy of Science: The Delaware Seminar, Volume I. New York, Wiley, 1963. xvi + 370 pp., $9.75.

*V. E. Benes: General Stochastic Processes in the Theory of Queues. Reading, Mass.; Addison-Wesley; 1963. xiii + 88 pp., $5.75.

H. Boemer: Representations of Groups. New York, Wiley, 1963. xii + 325 pp., $13.50.

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*C. W. Curtis and I. Reiner: Representation Theory of Finite Groups and Associative Algebras. New York, Wiley, 1963. xiv + 686 pp., $20.00.

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*A. Friedman: Generalized Functions and Partial Differential Equations. Englewood Cliffs, N. J., Prentice-Hall, 1963. xii + 340 pp., $7.50.

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*A. W. Glicksman: Linear Programming and the Theory of Games. New York, Wiley, 1963. x + 131 pp., $2.25 (paper), $4.95 (cloth).

*R. F. Graesser: Understanding the Slide Rule. Paterson, N. J.; Littlefield, Adams and Co.; 1963. ix + 141 pp., $1.50.

J. G. Herriot: Methods of Mathematical Analysis and Computation. New York, Wiley, 1963. xiii + 198 pp., $7.95.

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*J. A. H. Hunter and J. S. Madachy: Mathematical Diversions. Princeton, Van Nostrand, 1963. vii + 178 pp., $4.95.

R. C. James: University Mathematics. Belmont, Calif.; Wadsworth; 1963. xiii + 924 pp. No price provided.

F. L. Juszli: Analytic Geometry and Calculus. Englewood Cliffs, N. J.; Prentice-Hall; 1963. xii + 178 pp., $4.95.

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*H. Langman: Play Mathematics. New York, Hafner, 1962. 216 pp., $4.95.

*S. Lefschetz: Differential Equations - Geometric Theory, Second Edition. New York, Wiley - Interscience, 1963. x + 390 pp., $10.00.

C. W. Leininger: Differential Equations. New York, Harper, 1962. x + 271 pp., $6.00.

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R. D. Luce, R. R. Bush, and E. Galanter (Editors): Readings in Mathematical Psychology, Vol. I. New York, Wiley, 1963. ix + 535 pp., $8.95.

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A. I Mal'cev: Foundations of Linear Algebra. San Francisco, Freeman, 1963. xi + 304 pp., $7.50.

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*M. Nagata: Local Rings. New York, Wiley (Interscience), 1962. xiii + 234 pp., $11.00.

*I. Niven: Diophantine Approximations. New York, Wiley (Interscience), 1963. ix + 68 pp., $5.00.

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M. Rosenblatt (Editor): Proceedings of the Symposium on Time Series Analysis. New York, Wiley, 1963. xiv + 497 pp., $16.50.

H. J. Ryser: Combinatorial Mathematics, Carus Monograph No. 14. New York, Wiley, 1963. xiv + 154 pp., $4.00.

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*I. N. Vekua: Generalized Analytic Functions. Reading, Mass., Additon-Wesley, 1962. xxix + 668 pp., $14.75.

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H. C. White: An Anatomy of Kinship. Englewood Cliffs, N. J.; Prentice-Hall; 1963. 180 pp., $6.95.

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California

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*See review, this issue.

ALABAMA BETA, Auburn University (May 16, 1963)

Lynda C. Arnold
William V. Barber, Jr.
Robert McArthur Beard
Robert Earl Blankenship
Charles B. Boardman
William H. Boykin, Jr.
Lawrence Owen Brown
Jim Allen Burton
Mary Ann Cahoon
Albert Steven Cain
Thomas Rush Clements
Trson S. Craven
Judy Davidson
William Byrd Day
Clyde Patrick Drewett
James W. Dumas
Richard E. Fast
Daniel M. Fredrick
Clay Gibson Griffin
Douglas Van Hale
Julie Hoffman
Daniel C. Holsenbeck
George A. Howell
John C. Ingram
Sarah A. Jackson
William Douglas Jackson
James Cecil Johnson
Fred N. Kleckley, Jr.
William W. Lazenby
James T. Lewis
Donald W. Lynn
William C. Mayrose
Roy W. McAuley
Wilson S. McClellan
Bryant E. McDonald
Penn E. Mullowney, Jr.
Bobby C. Myhand
Marino J. Niccolai
Lowell W. Patak
Ben Starling Pearson
Charles F. Perkins, Jr.
Mickie N. Porch
James Wood Price
Tommy Jay Richards
Fred Randolph Robnett
Russell H. Ryder, Jr.
C. D. Scarbourough
Paul Burton Sigrest
John D. Skeparnias
Marsha Stanley
James R. Thomas
Pamela D. Turvey
John T. VanCleave
Alice Marie Venable
Barbara G. Wallace
David J. Wilson, Jr.
Shelby Davis Worley
Philip J. Young

ARIZONA ALPHA, University of Arizona (Spring 1963)

Margaret L. Cadmus
Leroy J. Dickey
Clarence K. Hutchinson
Peter B. Lyons
Demir Ozdes
Harry L. Rosenzweig
Gerald R. Sprouse
Helen Wong

ARKANSAS ALPHA, University of Arkansas (October 11, 1963)

Margaret A. Atkinson
Sam Ray Bailey
Bennie F. Blackwell
Dale Keith Cabbiness
Roger Clyde Clubbs
Franklin H. Cochran
Lawrence Davenport
Donald D. Dillard
Donald S. Douglas
Abdul Wadud Draki
Ronald Gene Embry
Ronald Wayne Glass
Lawson Edward Glover
Carl Edwin Halford
Travis E. Harrell
Richard F. Hatfield
Thomas Wagner Hogan
Mary Sue Hornor
Robert Denham Hurley
George Jew
John B. Luce, Jr.
Joyce Ann Mikeska
Thomas Stephen Moore
Walter T. Murphy
Ted Kazuo Nakamuro
Jerry Lee Parker
John W. Perry
Michael R. Platt
Richard D. Remke
Mehdi Sadr
James W. Seay
Charles Paul Sisco
Kenneth R. Skillern
David E. Standley
Clifton C. Stewart, Jr.
Michael T. Taylor
James T. Womble
Kenneth Elmer Wood
Jo Ellen Woody

ARKANSAS ALPHA, University of Arkansas (March 7, 1963)

Michael C. Carter
Onis J. Cogburn
Dwight Arles DeBow
Nina L. Fisher
Troy Floyd Henson
Raymond Higdon
Tim C. Hinkle
William A. Jasper
Lynn Morris Leek
Charles B. Martin
John George Weber

CALIFORNIA GAMMA, Sacramento State College (Fall 1962)

James Daniel
Janet Snyder

CALIFORNIA DELTA, University of California, Santa Barbara
(Charter Members) (May 23, 1963)

Charles Huff
Marvin Marcus
Dan Moore
Susan Moore
Robert Newcomb
Judith Paige
Don Potts
James Sloss
Donald Stice
Eric Stolz
William Watkins
Adil Yaqub

CALIFORNIA EPSILON, The Claremont Colleges (Spring 1963)

Richard Abel
David A. Angst
Robert E. Beck
Victor Buhler
Jon Bushnell
Michale Chamberlain
Courtney Coleman
Kenneth L. Cooke
Mary Kay Emery
James Enstrom
John A Ferling
Donald Fox
Judith Frye
Ross Goodell
John Greever
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David G. Haut
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Richard L. Hawley
J. Philip Huneke
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Robert C. James
David V. Jensen
Alan Kirschbaum
Beverly P. Lientz
G. John Lucas
Thomas W. Moran
Janet Myhre
Karen Nicholson
Norman Nielsen
Ronald M. Oehm
Alden F. Pixley
Evan L. Porteus
James Ritter
Richard W. Rosin
Steve D. Silbert
Elmer B. Tolsted
Barbara Waite
Herbert Walum
Jane Wheelock
Alvin White
Edward Wilson
Mary Worrell
David Young

CONNECTICUT ALPHA, University of Connecticut (May 18, 1963)

Estelle Chmura
Ronald W. DeGray
Edward Fawcett
Raymond Ferris
Ann Foley
Marilee Goldfarb
Lawrence C. House
Janice Ingrain
Katherine Lehmann
Patricia McHugh
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Diane Rose Nelson
Leonard Orzech
Edward Lee Putman
Elizabeth A. Regan
Helen G. Roberts
Lydia Rufleth
David Sleeper
Regina N. Slivinskas
Holley Hewitt Ulbrich
Dorothy Volosin
Sherman Wolff
Della Joanne Zera

WASHINGTON, D. C., ALPHA, Howard University (June 1963)

Goldie Lee Battle
Jean Lloyd Blake
George Gardner
Marvie De Lee
Mary Ann McAlister
Gloria Prather
Edward Singletary

FLORIDA ALPHA, University of Miami (April 28, 1963)

William C. Brown, Jr.
Hazel Alice Cohen
N. Abraham Glatzer
Miguel C. Guerrero
Maria Auxiliadora Hernández
James Edgar Keesling
Brita Laux
Gisela Rosch
Michael Ira Sidrow
Douglas R. Skuce
Albert J. Storey

FLORIDA BETA, Florida State University (April 6, 1963)

James William Brewer
Theodore H. Brittan
Barbara Carroll Brogden
Simcha Brudro
Robert G. Carson
Richard G. Cornell
Bill Dahl
Richard Henry Goodell
Horace Benton Gray, Jr.
John F. Hannigan, Jr.
Franklin Robert Hartranft
William James Heinzer
Kenneth Clayton Hepfer
Leonard R. Howell, Jr.
James Ralph Hughes
Rhonald M. Jenkins
Richard Alan Jensen
Oscar Taylor Jones
Clinton W. Kennel
Connie Clarke Kimbrough
William E. Lever
Norman H. Magee, Jr.
David Lee Neuhouser
James Wilson Newman, Jr.
Lloyd Nathan Nye
Matthew Joseph O'Malley
Richard Murdoch Root
Ronald Albert Schmidt
Linda Marceline Spaugh
Ronald Andrew Sweet
Frank Wilcoxin
Craig Adams Wood

GEORGIA BETA, Georgia Institute of Technology (May 26, 1963)

Garth Russell Akridge
James Lucius Grant
Harry Adelbert Guess, Jr.
E. Dennis Huthnance
Lawrence P. Staunton
Doris Alexandria Truitt
Woodson Dale Wynn

ILLINOIS DELTA, Southern Illinois University (May 24, 1963)

Richard Dean Daily
Marian Dean
Larry Ramon Diesen
Victor H. Gummersheimer
John Paul Helm
William Gerry Howe
Ronald E. Hunt
Gary D. Jones
Judith D. Kistner
Robert A. McCoy
John Clement McNeil
Carol Ann Mills
S. Burkett Milner
Mary Jane Prange
Robert Curtis Profilet
James D. Snyder
William J. Spicer
William Paul Wake
Charles Russell Weber
Ella L. Weitkamp
James S. Younker, Jr.

KANSAS ALPHA, University of Kansas (March 18, 1963)

Stephen J. Bozich
Woodrow Dale Brownawell
Donovan E. Cassatt
Karin VanTuyl Chess
James S. Dukelow, Jr.
Kenneth C. Ford
Warren D. Keller
Max Dean Larsen
Harold W. Mick
Edwin Alan Nordstrom
Michael J. O'Neill
Franklin D. Shobe
William P. Vale
Tara Vedanthan
John T. White

KANSAS ALPHA, University of Kansas (April 22, 1963)

William M. Causey
Donald R. Dittmer
David Edward Fischer
Sally Foote
Dorothy Jean Hain
James Dean Harris
Thomas J. Henninger
William R. Jines
William H. Jobe
Robert L. Johnson
Phyllis M. Lukehart
Judith Jane Moats
Edward William Munster
John A. Mura
Elbert M. Pirtle
Dennis Harold Schnack
Shirley E. Scott
Gary Alan Smith
Donald F. St. Mary
Richard F. Taylor
James Madison Tilford
Bette K. Weinshilboum
Louis H. Whitehair
Carl Scott Zimmerman

KANSAS ALPHA, University of Kansas (June 1, 1963)

John J. Hutchinson

KANSAS GAMMA, University of Wichita (December 14, 1963)

Marion G. Speer

KANSAS BETA, Kansas State University (May 8, 1963)

Judith I. Brandt
Janice Caldwell
James West Calvert
John W. Carlson
An-Ti Chai
Melvin C. Cottom
David A. Draegert
David J. Edelblute
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H. K. Huang
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Clyde Sprague
Sumpunt Vimolchalao
Ray A. Waller
Chee Gen Wan
Chester C. Wilcox
William K. Winters
Mary Louise Zavesky

KANSAS BETA, Kansas State University (May 16, 1963)

George Dailey

KANSAS GAMMA, University of Wichita (June **5, 1963)**

Judith A. Coombs
Donald Franklin **Cowgill**
Ted Davis
J. Fred Giertz
Samuel Dale Gill
Donald L. Hull
Thomas George Klem
Joanne V. Larson
Wilbur **J.** Lewis
Samuel A. Lynch
Toma I. Sara
David T. Sawdy
Frank Wilson
Robert Ernest Young

KENTUCKY ALPHA, University of Kentucky (May **9, 1963)**

Austin W. Barrows
Joseph Lawrence Beach
William L. Crutcher
Nancy Rodgers Dykes
Ronald C. Glidden
Paul Martin Ross
W. Prentice Smith

LOUISIANA BETA, Southern Louisiana (May 8, **1963)**

Mrs. Prince Armstrong
Talmadge Bursh
Mary Ann Coleman
Oscar Ray Jackson
Glendell Kirk
Jo **McCray**
Roosevelt Peters
Joan Faye Perry
Winfield Reynolds
John Stills

MARYLAND ALPHA, University of Maryland (May **15, 1963)**

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Nicholas **Cianos**
Lawrence Edelman
Joseph F. Escatell
Paul **Gammel**
Alvan M. Holston
William A. Horn
Jyun **J.** Kim
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Margarita **C. Sotolongo**
Garrett Oliver **VanMeter, II**
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David Weiss
George **Westwick**
David Louis Wilson
George Wilson

MICHIGAN ALPHA, Michigan State University (February **28, 1963)**

Carolyn A. Burk
John K. Cooper, **Jr.**
Stephen E. Crick, **Jr.**
John **R.** Faulkner
Nancy **J.** Fitchett
Mary Ellen Greene
Ernest **S.** Grush
Jeffrey **I.** Hack
Gail E. Haske
Joanne L. Holdsworth
Lawrence **Leftoff**
Ralph **B.** Leonard
Margaret **M. Loomis**
Edna E. Madison
Arnold **R.** Naiman
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Angela M. **O'Neill**
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John M. Rawls
Peter H. Rheinstein
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Walter **N.** Schreiner
Diane K. Sovey
M. C. Trivedi
William A. Webb
Barbara A. Weeks
Ronald H. Wenger
James R. **Whitney**
Deborah A. Williams

MISSOURI ALPHA, University of Missouri (May 8, **1963)**

Ramzia M. Abdulnour
John **W.** Alspaugh
Fenson N. Anadu
Robert Lee Beneditti
William M. Bolstad
William Paisley Brown
Joseph Kent Bryan
Carol Calhoun
Chi Cheng Chen
Larry **Claypool**
Ronald **W.** Friesz
Larry **E.** Halliburton
Monty **J. Heying**
William D. Hibler, **III**
Raymond A. Hicklin
David Howe
James Mark Hunt
John Irvin Israel
June Jenny
Robert Jordan
Donald G. Kaiser
Udo Karst
Randolph **H.** Knapp
Ester **Lorah**
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Wayne **D.** Meyer
John W. Neubauer
Richard L. Norman
Neal **F. Peterman**
Samuel T. Picraux
Norman Recknor
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Slade W. Skipper
Elvin **B.** Standrich
Joseph **S.** Starr
Fred Stroup
Albert **L.** Tryee
Ning Sang Wong
Scott **Yeargain**
Paul **J.** Ziegelkin

MISSOURI GAMMA, St. Louis University (April **25, 1963)**

Lawrence W. Albus
James F. Aldrich
Barbara Lee Bacon
Larry **G.** Bauer
William F. Bayer, **Jr.**
Nathaniel A. Boclair, **Jr.**
Enrique **Bolanos**
Francis **R.** Boman
Edward M. Boule
Marilyn L. Boxdorfer
Sister Duns **Scotus** Breitbart
Anne Brightwell
Richard **B.** Brown
Cathleen Adelaide **Callahan**
Robert L. **Carberry**
Barbara A. Carpenter
Mary Kezia Carrothers
Feng-Keng Chang
Quiza Chang
Lurelle K. Coddington
Sister Joseph Norbert Crete
Timothy **J.** Cronin, **S.J.**
Lames Leo Daly, **S.J.**
Susan **M.** Davidson
Maria Davis
Paul **R.** Dixon
Rev. Evan **T.** Eckhoff, **O.F.M.**
Nicolaas **W.** Eissen
Ronald **F.** Eldringhoff
Mary Rose Enderlin
Fred **K.** Enseki
Kenneth **J.** Feuerborn
John F. Fischer
Patricia **R.** Flannery
Joseph M. Fouquet
Martin D. Fraser
Sheila M. Gallagher
Donald H. Galli
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Gerald A. Geppert
Edward O. **Gotway, Jr.**
James **M.** Guida
Joyce C. Gunnels
Mary Kathryn **Haas**
Thomas **J. Higgins**
Barbara M. Holtkamp
Paul **B.** Hugge
Judith L. Huntington
Dale N. Jones
Kathryn **M.** Keller
Katharine **J.** Kharas
Jane **M.** Klein
Fred **J.** Kovar
Robert G. Kribs
Elmer A. Krussel
Carolyn L. **Kuciejczyk**
Carl **R. LaForge**
Linda Lee Leech
Stevenson **Dun-Pok** Mack
James **E.** Maletich
Elmer **E.** Marx
Jacqueline McCoy
James **T.** Melka
Carl F. Meyer
Susan D. Miller
James **G.** Monika, **S.J.**
Thomas **E.** Moore
Robert **J.** Muccie
LaVerne S. Oakes
Joan **M.** Oliver
Thad P. Pawlikowski
Michael W. **Pieper**
Geraldine C. Pisarek
Randolph **C. Reitz**
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J. Mark Robinson
Kathleen A. Rohan
Juliana **Rohling**
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Lawrence A. Ruski
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David A. **Schmitt**
Ellen M. Schroeder
Carol Patricia Sipe
Mary Ann **Smola**
Sally A. Snyder
Sue Elaine Snyder
Louise Speh
Judith Anne **Stute**
John **F.** Suehr, **S.J.**
Frank **H.** Tubbesing
William **B.** Walker
Edward **J. Wegman**
Mary Anita **Weis**
Mary Anna Wehner
David J. White
Rosemary H. Winterer
Ronald **E.** Yanko
Dennis L. Young

MONTANA ALPHA, Montana State University (October **31, 1962)**

Carl Cain
William Gregg
Margaret Kem
Anton Kraft
Kenneth Osher
Robert Vosburgh

MONTANA BETA, Montana State College (May **20, 1963)**

Patrick Arthur **Cowley**
Minerva Rae **Hodis**
Donald James Hurd
Glenn **R. Ingram**
Leon Eugene **Mattics**
Dean Paul **McCullough**
Judith Remington Schagunn
Robert Frank Sikonia
William George Sikonia
George Henry Spanqrude
William A. **Stannard**
Raymond Clayton Suiter
Gloria Eileen Wheeler

NEBRASKA ALPHA, University of Nebraska (May **19, 1963)**

Robert Wesley Brightfelt
Theron David **Carlson**
Richard Corrill **Conover**
Rodney Dean **Crampton**
Stephen Paul Davis
Richard Victor **Denton**
Lyal Val Gustafson
Daniel **B.** Howell
Kenneth Francis **Hurst**
Helen **J.** James
James Lee Jorgensen
James Henry Kahrl
Gary **Samuel Kearney**
Robert Dean **Lott**
Rodney Lee Marshall
James Pougal **McCall, Jr.**
Robert Joseph **McKee, Jr.**
William Howard **Odell**
Allen Arthur **Otte**
Carol Ann Phelps
Donald Howard Schroeder
Ann Marie **Senis**
Earl Eugene Senseney
Richard Paul Smith
Harold **D.** Spidle
Daryl Andrew Travnicek
Karen Mary **Woodward**

NEW HAMPSHIRE ALPHA, University of New Hampshire (May 30, 1963)

Richard B. Aldrich
Sidney Einbinder
Robert L. Rascoe
Vincent B. Roberts
Paul W. Stanton
Frank D. Szachta
George N. Yamamoto
John A. Wilhelm

NEW JERSEY ALPHA, Rutgers, The State University2 (December 16, 1962)

William J. Culverhouse
Peter R. Mumber

NEW JERSEY BETA, Douglass College (March 18, 1963)

Mary Janet Casciano
Anne T. Crumpacker
Joyce Danziger
Barbara Lee Elcome
Judith E. Fischer
Judith Diane Flaxman
Eleanore Ann Geary
Prances H. Griffith
Gloria Herships
Janet Lynne Johnston
Carol Shapiro Lessinger
Roberta Neslanik
Carolyn Clark Palmer
Arlene R. Silverman

NEW MEXICO ALPHA, New Mexico State University (May 29, 1963)

David R. Arterburn
Michael Carroll
Richard Ed Davies
Robert W. Deming
Ernest E. Denby
Frances Hammer
Edgar Howard
Adolf Mader
Thomas Meaders
Laurel Ruch
Ronald Stoltenberg
Gary N. Smith
Gregory Trachta
Charles Ward
Robert Whitley
Nathan Williamson

NEW YORK ALPHA, Syracuse University (April 24, 1963)

Richard R. Bates
Jean H. Becker
William Paul Blake
John E. Bothwell
Richard Brandshaft
Raymond Caputo
Nicholas Celenza
William Haldance Courage
Howard L. Empie
Robert B. Fletcher
William Garrett
Patricia Ann Gawarecki
Robert James Heins
Elizabeth Hufnagel
Beverly Anne Kaupa
Stephen B. Kazin
Carol Kwietniak
Tanya Francine Landau
Richard C. Lessmann
Francine Sue Librach
Paul Lovecchio
Sheila Magaziner
Barbara Micski
Donald Charles Miller
Barbara Adele Morgenroth
Thomas J. Riding
Diane Schlieckert
Sheridan Gilmore Smith
Charles J. Stemples
Richard B. Stock
Ann R. Tierney
Judith Helen Yavner
Paula Zak
Marsha Ellen Zanville
Ronald Charles Zimmerman

NEVADA ALPHA, University of Nevada (April 26, 1963)

Ernest Samuel Berney,III
Betty Jo Cosby
Joseph N. Fiore
James A. Hammond
James R. Herz, Jr.
Wendell A. Johnson
Ronald A. Jeuning
Gordon L. Nelson

NEW HAMPSHIRE ALPHA, University of New Hampshire (May 16, 1963)

Jack L. Baker
Raoul S. Barker
Robert E. Bennett
Robert G. Drever
Virginia Ann Gross
Paul L. Hardy
Donald Wayne Hodge
Charles E. Horne
Curtis S. Morse
Robert J. Oelke
Beverly S. Payne
James L. Priest
Robbin Roberts
Walter J. Savitch
William H. Weaver
Roberta S. Wright
Edwards H. Veech

NEW YORK BETA, Hunter College (March 31, 1963)

Elaine Akst
John Altson
Eleanor Barnabic
Elaine Baron
Rhoda Goldwein
Lillian Heim
George Levine
Daniel Lieman
Stephen Lieman
Barbara Nissel
Camille Volence
Carol Vollmer
Myra Zeleznik

NEW YORK GAMMA, Brooklyn College (April 25, 1963)

Jack M. Arnow
Milton K. Benjamin
Paula Dousk
Stephen Druger
Robert M. Elisofon
Allan S. Gotthelf
Marian C. Gunsher
Kenneth Kalmanson
Kenneth D. Klein
Jeffrey M. Lehr
Arline Levine
Deborah Lewittes
Harry L. Nagel
Margery Puretz
Marvin Ratner
Bayle Schorr
Sheldon Teichman

NEW YORK DELTA, New York University (February 25, 1963)

Susan Feinberg

NEW YORK EPSILON, St. Lawrence University (February 6, 1963)

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Mary Justine Coss
Wayne Lloyd Huntress

NEW YORK ETA, University of Buffalo (April 3, 1963)

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NEW YORK IOTA, Polytechnic Institute of Brooklyn (May 15, 1963)

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Donald Neil Levine
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NEW YORK KAPPA, Rensselaer Polytechnic Institute

(May 7, 1963)

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(June 1, 1963)

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Nicholas T. Losito
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James H. McMahon
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Jerry Robert Hobbs
Wayne Terryl Peterson
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Elizabeth Anne Walris
Donald F. Young

NORTH CAROLINA BETA, University of North Carolina (May 27, 1963)

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Nelson F. Page
Robert L. Peek
Thomas F. Reid
Frank A. Roescher
Ann R. Sarratt
David W. Showalter
Melba Donnell Smith

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Stavros John Stephanakis
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Charles Newton Winton
James Adams Woodward

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Thomas S. Graham
Albert F. Hanken
Del William Heuser
Joseph J. Y. Liang
Joseph Meeks
Randal P. Miller
Roger Jeffrey McNichols
Randolph H. Ott
Daryl J. Rinehart
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Robert Willard Scott
David M. Thompson
Bert K. Waits

OHIO BETA, Ohio Wesleyan University (April 25, 1963)

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Gerald William Boston
Nancy Alice Lange
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Dennis Lee Orphal
James Eldon Wiant
William Aaron Woods, Jr.

OHIO EPSILON, Kent State University (Spring 1963)

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Ann Ayres
George R. Brulin
Lowell N. Cannon
Charles Cole
Michael Habenschuss
Thomas Hinks
Paul N. Iwanchuck
Olga Kitrinou
Kenneth W. Klouda
Geraldin Kucinski
Constance Lindquist
peter A. Lindstrom
Yih Tang Ling
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Suzanne M. Pauline
Bonnie Pentz
Duane L. Shie
Dorothy L. Shipman
Karen K. Stein
Eric J. Thompson
Nola J. Troxell
Anka M. Vaneff
Sigrid E. Wagner
Marion B. Walker
Anne Way

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Joseph Diestel
Roger F. Ferry
Richard J. Fox
John T. Herman
Alex I. Koler
Martin R. Kraimer
William T. Marquitz
Henry J. Prince
Gerald J. Shaughnessy
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Gerard O. Wunderly

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Timothy R. Buhl
David S. Chandler
Frank E. Hess
David D. McFarland
Kamran Mokhtarhan
Edward W. Rummel
William J. Scarff
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Linda Voorhis

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Lynn A. Carpenter
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Thomas E. Ikard
James P. Johnson
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Beverly Mitchell
Wayne Otsuki
Donald L. Stout
Donald Lee Williams

(April 25, 1963)

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David Bagwell
Arleen M. Carr
Peter W. Cowling
Michael Lee Gentry
David W. Gibson
Carl Edward Hittle
Alan M. Klein
Bobby Joe Lane
Carleton Yu-Wei Ma
Robert Owen Morris
Hoang Duc Nha
Nabi M. Rafiq
James Stephen Randles
William A. Thedford
Bruce E. Wiancko

OREGON ALPHA, University of Oregon (May 9, 1963)

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George F. Bachelis
Charles Burke
Pamela S. Charles
Paul Chern
John P. Colvin
William L. Cranor
Pamela R. Delany
Mary L. Eagleson
Barbara Edwards
Leland S. Endres
Michael G. Engel
Joseph K. Fang
Terry J. Forsyth
Kent R. Fuller
LeRoy G. Haggmark
Raymond W. Honerlah
Donald Richard Iltis
Juanita Rae Johnston
Edward J. Kushner
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Paul B. Martz
Tom H. May
Robert A. Osborne
James A. Paulson
Rhomas M. Poitras
Paul A. Robisch
Craig T. Romney
AhhadRSaeeds
Wilhiam Sprsevage
Laverne W. Stanton
George H. Starr
Ragetame Jattennent
Billy E. Vertrees
Bruce A. Vik
Thomas J. Warner
Michael B. Woodroofe
Robert V. Youdi
Lee H. Ziegler

OREGON BETA, Oregon State University (May 9, 1963)

Gerald Lee Caton
Chi-Ming Chow
Allen R. Freedman
James W. Green
Terry B. Hinrich
Robert Carl Johnson
John William Kjos
Richard Bruce McFarling
James Terry McGill
Arthur Eugene Olson, Jr.
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Kenneth Vance Smith

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Michael D. Fitzpatrick
Stephen L. Ginsburg
Jarvis E. Kerr
Kathryn A. Kneen
Linda A. Larson
Nancy L. Rodenberger
Donna L. Sirinek
John E. Tozier
Harrison D. Weed, Jr.
Guy E. Witman

(April 2, 1963)

Daniel Motill

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Rocco David Walker
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(May 10, 1963)

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Marilyn Leonard
Stephen Nemorufsky
Judith Ravitz
Ronald Sheinson
Eileen Silo
David Tipper
Miles N. Wrigley
Sheppard Yarrow
David Zitarelli

(June 6, 1963)

Allan Becker
Alan Cutler
Gail Forman
Joel Greene
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Lowell Nerenberg
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Arthur Rosenthal
Stephen Arthur Schneller
Francis Joseph Smaka
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David E. Tepper
Sandra Volowitz

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David Lee Gray
Judith A. Holshouser
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Larry Harold Kline
Cheri Anne Moore
Richard Allen Myers
Thomas Gold Owen
David Roger Roth
Kelly F. Shippey
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Edwin C. Strother
William F. Wheeler
Morton N. Winter

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Theodore Stanley Erickson
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Donald Robert Greenwaldt
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George Solomon Keil
Ronald James Leidholm
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Nelontine Maria Maxwell
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(May 22, 1963)

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J. Michael Gray
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John C. Knowles, Jr.
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James C. Nicholson
Donald George Pray
Walter J. Rainwater, Jr
John Duncan Raithel
Randa Suzanne Randolph
Grady Roberts
Woodlea Sconyers
William B. Self
William A. Sisk

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Charles Bentley
Austin F. Bishop
Eddie George Chaffee
K. Michael Day
Lynn E. Gamer
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Hugh Bradley Hales
V. Ronald Halliday
Elbert Troy Hatley
Joseph Taylor Hollist
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Stanley M. Jencks
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Willes L. Werner
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Quinn Ernest Whiting
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James Arthur Wixom
James H. Wolfe

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Richard Henry Lee Mark.
Sara Janet Renshaw

VIRGINIA BETA, Virginia Polytechnic Institute (May 15, 1963)

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George Christopher Canavos
Cecile Korsmeyer Cotton
Harvey Arlen Dane
Patrick H. Doyle
John Richard Hebel
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Robert Heath Tolson
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Donald Womeldorph

WASHINGTON BETA, University of Washington (January 30, 1963)

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Kay Harding
Chung-Wu Ho
James E. Hoard
Joyce E. Imus
Leslie A. Fox
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Rodney B. Thorn
Caroline Wiles
Richard Tse-Hung Woo

(Spring 1963)

Brian K. Bryans
Kristen Cederwall
Paul P. Chen
Barton H. Clennan
Raymond L. Ostling
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Ann L. Schultz
Noel W. Vencil
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Mary Ann Kertes
Howard Frank Matthews
Douglas Arthur Ross
Nevada Lee Sample
John Michael Stachurski

WASHINGTON DELTA, Western Washington State College (July 5, 1963)

David Arthur Ault
Robert Myran Chandler
Orin Francis Dutton
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Ronald Joe Saltis

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Loretta Mary Buttice
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Maria Elena Stanislawski
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Robert L. Tatalovich
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WISCONSIN BETA, University of Wisconsin (May 20, 1963)

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