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Chapter reports, books for review, problems for solution and solutions to problems, and news items should be mailed directly to the special editors found in this issue under the various sections. Editorial correspondence, including manuscripts should be mailed to THE EDITOR OF THE PI MU EPSILON JOURNAL, Minnemath Center, University of Minnesota, Minneapolis, Minnesota 55455.

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EDITORIAL

The Pi Mu Epsilon Journal consists, mainly, of advanced undergraduates. It is the Editor's feeling that the contributions, also, should come, mainly, from undergraduates.

In the past, the Journal has not succeeded in soliciting very many papers from non-faculty members of Pi Mu Epsilon. This fact is not unrelated to the unfortunate truth that mathematics education in the United States has failed to stimulate and train mathematically-minded students to carry through and to write up investigations more extensive than those suggested in problems of their textbooks. Hopefully, this state of affairs is now changing--especially with the encouragement from the National Science Foundation in the form of financial support for Undergraduate Research Programs.

Thus, dear Undergraduate Reader, consider this a loud call for a paper from you!

GENERALIZED SYNTHETIC DIVISION

F. J. Arena, North Dakota State University

Synthetic division may be defined as a shortened process by which a polynomial is divided by a binomial. Many texts on algebra show how the division is performed when the divisor is of the form $x - c$, but few texts show how to extend the process to **divisors** of degree higher than the **first.**[1] It is the purpose of this paper to review the first case with some modifications and discuss the second case in detail with the hope that synthetic division will be a more useful tool to the student of mathematics.

Now suppose that a **polynomial**[2]

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1} + a_n$$

is divided by the binomial $x - c$. Let the quotient be denoted by

$$Q(x) = b_1x^{n-1} + b_2x^{n-2} + b_3x^{n-3} + \cdots + b_{n-1}x + b_n$$

and the remainder by **R.** Then it follows that

$$f(x) = (x - c)Q(x) + R,$$

or

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n$$

(1) $$= (x - c)(b_1x^{n-1} + b_2x^{n-2} + b_3x^{n-3} + \cdots + b_{n-1}x + b_n) + R \cdots\cdots\cdots\cdots$$

Expanding the right-hand member of equation (1) and equating coefficients of like powers of x, we find that

$$b_1 = a_0,$$
$$b_2 = a_1 + cb_1,$$
$$b_3 = a_2 + cb_2,$$
$$\cdots\cdots\cdots\cdots$$
$$b_{n-1} = a_{n-2} + cb_{n-2}$$
$$b_n = a_{n-1} + cb_{n-1}$$
$$R = a\ \ + cb_n.$$

....

one of the exceptions is **H. S.** Hall and **S. R.** Knight, <u>Higher Algebra</u> (London: **MacMillan** and Co., **1948),** Fourth Edition, pp. 434-435, in which the extension to trinomial divisors is discussed.

[2]It will be assumed throughout this paper that all polynomials have non-zero leading coefficients.

From these equations it is easily seen that each coefficient after the first in the quotient, as well as the remainder, is formed by multiplying the coefficient preceding it by c and adding to this product the next coefficient in the dividend. The process of finding the coefficients in the quotient and the remainder is usually arranged as follows:

$$\begin{array}{c|ccccccc}
c & a_0 & a_1 & a_2 & \cdots\cdots & a_{n-2} & a_{n-1} & a_n \\
 & & cb_1 & cb_2 & \cdots\cdots & cb_{n-2} & cb_{n-1} & cb_n \\
\hline
 & b_1 & b_2 & b_3 & \cdots & b_{n-2} & b_{n-1} \quad b_n & R
\end{array}$$

As examples, let us find the quotient and remainder in each case when $x^3 + 3x^2 - 4$ is divided by $x - 2$ and by $x + 2$.

$$\begin{array}{c|cccc}
2 & 1 & 3 & 0 & -4 \\
 & & 2 & 10 & 20 \\
\hline
 & 1 & 5 & 10 & 16
\end{array}
\qquad\qquad
\begin{array}{c|cccc}
-2 & 1 & 3 & 0 & -4 \\
 & & -2 & -2 & 4 \\
\hline
 & 1 & 1 & -2 & 0
\end{array}$$

So, when $x^3 + 3x^2 - 4$ is divided by $x - 2$, the quotient is $x^2 + 5x + 10$ and the remainder is 16; when it is divided by $x + 2$, the quotient is $x^2 + x - 2$ and the remainder is 0.

If, however, the divisor is of the form $ax - c$, the same process for finding the quotient and remainder can be used with a slight modification. To show this, let $Q(x)$ be the quotient and R the remainder when $f(x)$ is divided by $x - c/a$. Then we can write

(2) $$\frac{f(x)}{x - c/a} = Q(x) + \frac{R}{x - c/a}, \cdots\cdots\cdots\cdots$$

or

$$f(x) = (x - c/a)Q(x) + R.$$

Now, dividing this last equation by $ax - c$, we have

$$\frac{f(x)}{ax - c} = \frac{(x - c/a)Q(x)}{ax - c} + \frac{R}{ax - c}$$
$$= \frac{(x - c/a)Q(x)}{a(x - c/a)} + \frac{R}{ax - c}$$

or

(3) $$\frac{f(x)}{ax - c} = \frac{Q(x)}{a} + \frac{R}{ax - c} \cdots\cdots\cdots\cdots$$

On inspecting equations (2) and (3), we see that the remainder R is unaltered and we can state the following rule: To find the quotient when $f(x)$ is divided by $ax - c$, first divide $f(x)$ by $x - c/a$ and then divide the quotient thus obtained by a.

We now illustrate this rule with some examples. Let us find the quotient and the remainder when $f(x) = 2x^3 - 3x^2 - 3x + 5$ is divided by $2x - 1$. First we divide $f(x)$ by $x - 1/2$ thus:

$$\begin{array}{r|rrrr} 1/2 & 2 & -3 & -3 & 5 \\ & & 1 & -1 & -2 \\ \hline & 2 & -2 & -4 & 3 \end{array}$$

On dividing the quotient $2x^2 - 2x - 4$ by 2, we find that the required quotient is $x^2 - x - 2$ and the remainder is 3. Let us now find the quotient and remainder when $f(x) = 2x^3 - 3x^2 - 3x + 5$ is divided by $x/2 + 1$. First we divide $f(x)$ by $x + 2$ thus:

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -3 & 5 \\ & & -4 & 14 & -22 \\ \hline & 2 & -7 & 11 & -17 \end{array}$$

Sow, dividing the quotient $2x^2 - 7x + 11$ by $1/2$, we find that the required quotient is $4x^2 - 14x + 22$ and the remainder is -17.

This process can easily be extended to divisors of degree higher than the first. The extension will now be made only to divisors of the second degree, since it is similar for divisors of higher degree.

Suppose that a polynomial

$$f(x) = a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1} x + a_n$$

is divided by the trinomial $x^2 - bx - c$. Let

$$Q(x) = b_2 x^{n-2} + b_3 x^{n-3} + b_4 x^{n-4} + \cdots + b_{n-1} x + b_n$$

be the quotient and let $R = px + q$ be the remainder. Then it follows that

$$f(x) = (x^2 - bx - c)Q(x) + R,$$

or

$$a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1} x + a$$

$$= (x^2 - bx - c)(b_2 x^{n-2} + b_3 x^{n-3} + b_4 x^{n-4} + \cdots$$

$$+ b_{n-1} x + b_n) + px + q \qquad (4)$$

Expanding the right-hand member of equation (4) and equating coefficients of like powers of x, we find that

$$\begin{aligned} b_2 &= a_o, \\ b_3 &= a_1 + bb_2, \\ b_4 &= a_2 + bb_3 + cb_2, \\ b_5 &= a_3 + bb_4 + cb_3, \\ &\cdots\cdots\cdots \\ b_{n-1} &= a_{n-3} + bb_{n-2} + cb_{n-3} \\ b_n &= a_{n-2} + bb_{n-1} + cb_{n-2}, \\ p &= a_{n-1} + bb_n + cb_{n-1}, \\ q &= a_n + cb_n. \end{aligned}$$

From these equations it is easily seen that each coefficient after the second in the quotient is formed by multiplying the two preceding coefficients by b and c respectively and adding these products to the next coefficient in the dividend. The process of finding the coefficients in the quotient and remainder can be arranged as follows:

$$\begin{array}{c|cccccccc} b & a_o & a_1 & a_2 & a_3 & \cdots & a_{n-3} & a_{n-2} & a_{n-1} & a_n \\ c & & bb_2 & cb_2 & & & & & & \\ & & & bb_3 & cb_3 & & & & & \\ & & & & bb_4 & \cdots & & & & \\ & & & & & \cdots & cb_{n-3} & & & \\ & & & & & & bb_{n-2} & cb_{n-2} & & \\ & & & & & & & bb_{n-1} & cb_{n-1} & \\ & & & & & & & & bb_n & cb_n \\ \hline & b_2 & b_3 & b_4 & b_5 & \cdots & b_{n-1} & b_n & p & q \end{array}$$

Explanation: First, place the last two coefficients of the divisor with signs changed on the left of the vertical line. Add the first column on the right of the vertical line. This gives a_o or b_2, the first number below the horizontal line. Next, multiply b and c by b_2 and write these products in the second row and in the second and third columns. Next, add the second column. This gives $a + bb_2$ or b_3. Multiply b and c by b_3 and write these products in the third row and in the third and fourth columns. Next, add the third column. This gives $a_2 + bb_3 + cb_2$ or b_4. Continue this process until an entry is made in the last column. Then add the last two columns to find p and q.

We now conclude with a few examples, pointing out that the remarks we made about divisors of the form $ax - c$ also apply to those of the form $ax^2 - bx - c$ and other divisors of the same form of higher degree.

Let us find the quotient and remainder when $x^4 - x^2 + 3x + 4$ is divided by $x^2 - 2x + 2$.

2	1	0	-1	3	4
-2		2	-2		
			4	-4	
				2	-2
	1	2	1	1	2

Hence the quotient is $x^2 + 2x + 1$ and the remainder is $x + 2$.

Let us find the quotient and the remainder when $x^5 - 2x^4 - 4x^3 + 19x^2 - 31x - 12$ is divided by $x^3 - 2x + 3$.

0	1	-2	-4	19	-31	-12
2		0	2	-3		
-3			0	-4	6	
				0	-4	6
	1	-2	-2	12	-29	-6

So the quotient is $x^2 - 2x - 2$ and the remainder is $12x^2 - 29x - 6$.

Let us find the quotient and the remainder when $6x^5 - 3x^4 + x^3 + 11x^2 - 6x + 7$ is divided by $2x^2 - x + 1$.

1/2	6	-3	1	11	-6	7
-1/2		3	-3			
			0	0		
				-1	1	
					5	-5
	6	0	-2	10	0	2

Hence the quotient is $3x^3 - x + 5$ and the remainder is 2.

SUMS OF POWERS OF INTEGERS[1]

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Missouri Gamma

It is well-known that, if n and p are positive integers, then the sum

$$S_p(n) = \sum_{k=1}^{n} k^p \tag{1}$$

can be expressed as a polynomial in n, of degree p+1. Methods for finding these polynomials are numerous, but they usually involve either rather sophisticated concepts such as Bernoulli polynomials [1] or the Euler-Maclaurin formula [2], or else lengthy derivations using elementary finite difference methods. In a recent paper [3], Christiano developed a recursion formula for (1), using only elementary concepts from algebra. In this note, we demonstrate an alternative technique for obtaining a similar recursion formula. Our technique has the advantage that it can be applied equally well to certain sums which are related to (1), but which are not always easily handled by the above methods.

The technique is based on the "summation by parts" formula of the finite difference calculus. This formula can be expressed and derived in a very elementary manner, as follows. Let $a_0, a_1, \ldots, a_n$ and lay, $b_1, \ldots, b_n, b_{n+1}$ be real numbers. Then

$$\sum_{k=0}^{n} a_k(b_{k+1} - b_k) = \sum_{k=0}^{n} a_k b_{k+1} - \sum_{k=0}^{n} a_k b_k$$

$$= \sum_{k=1}^{n+1} a_{k-1} b_k - \sum_{k=0}^{n} a_k b_k \tag{2}$$

$$= a_n b_{n+1} - a_0 b_0 - \sum_{k=1}^{n} b_k(a_k - a_{k-1}),$$

which is the desired result.

....

[1] ~resented at the annual meeting of the Missouri Section of the Mathematical Association of America, Rolla, Missouri, April, 1962.

EDITOR'S NOTE. The referee for this paper suggested that the reader interested in sums of powers of numbers see the recently reprinted [redacted] An Introduction to the Operations with Series. By I. J. Schwatt. Chelsea, New York, 1962. A review, by H. W. Gould, appears in the American matical Monthly, Vol. 70, No. 1, January, 1963.

To obtain our recursion formula for (1), we now let p be a positive integer, and let $a_k = k^p$ and $b_k = k$ in (2), and we have

$$\sum_{k=0}^{n} k^p = n^p(n+1) - \sum_{k=1}^{n} k[k^p - (k-1)^p]$$

$$= n^p(n+1) - \sum_{k=1}^{n} k\Big[\sum_{j=1}^{p} \binom{p}{j}(-1)^{j+1}k^{p-j}\Big]$$

$$= n^p(n+1) - p\sum_{k=1}^{n} k^p - \sum_{j=2}^{p} \binom{p}{j}(-1)^{j+1}\sum_{k=1}^{n} k^{p-j+1}$$

Hence,

$$(p+1)\sum_{k=1}^{n} k^p = n^p(n+1) + \sum_{j=1}^{p-1}\binom{p}{j+1}(-1)^{j+1}\Big[\sum_{k=1}^{n} k^{p-j}\Big],$$

which may be written

$$(3)\qquad (p+1)S_p(n) = n^p(n+1) + \sum_{j=1}^{p-1}\binom{p}{j+1}(-1)^{j+1}S_{p-j}(n).$$

We now consider the sums

$$T_p(n) = \sum_{k=1}^{n}(2k-1)^p;$$

$$S_p^*(n) = \sum_{k=1}^{n}(-1)^k k^p;$$

$$T_p^*(n) = \sum_{k=1}^{n}(-1)^k(2k-1)^p.$$

Each of these can easily be expressed in terms of $S_p(n)$; for example, if $n = 2m$, we have

$$S_p^*(n) = \sum_{k=1}^{m}[(2k)^p - (2k-1)^p]$$

$$= 2^pS_p(m) - \sum_{k=1}^{m}\Big[\sum_{j=0}^{p}\binom{p}{j}(-1)^{p-j}(2k)^j\Big]$$

$$= 2^pS_p(m) - \sum_{j=0}^{p}(-1)^{p-j}\binom{p}{j}2^jS_j(m),$$

and if $n = 2m - 1$, we have

$$S_p^*(n) = \sum_{k=1}^{m}[(2k-2)^p - (2k-1)^p]$$

$$= 2^pS_p(m-1) - \sum_{j=0}^{p}(-1)^{p-j}\binom{p}{j}2^jS_j(m),$$

where $S_0(m) = m$. If we agree that $S_p(\frac{n}{2}) = S_p(\frac{n-1}{2})$ whenever n is odd, we can combine the above results into the single formula

$$S_p^*(n) = 2^pS_p(\tfrac{n}{2}) - \sum_{j=0}^{p}(-1)^{p-j}\binom{p}{j}2^jS_j(\tfrac{n+1}{2}).$$

On the other hand, there is some interest in having each of the sums above expressed recursively, independent of $S_p(n)$. Such expressions are easily derived from (2), following the general procedure used to obtain (3). Thus, letting $a_k = (2k-1)^p$ and $b_k = (2k-1)$ in (2), we obtain

$$(4)\qquad 2(p+1)T_p(n) = (2n-1)^p(2n+1) - (-1)^p + \sum_{j=1}^{p-1}(-2)^{j+1}\binom{p}{j+1}T_{p-j}(n).$$

If we restrict p to positive integers greater than 2, and let $a_k = (-1)^k k^p$ and $b_k = k$ in (2), we obtain

$$(5)\qquad 2S_p^*(n) = (-1)^n n^{p-1}(n+1) + (p-2)S_{p-1}^*(n) - \sum_{j=1}^{p-2}(-1)^{j+1}\binom{p-1}{j+1}S_{p-j-1}^*(n);$$

letting $a_k = (-1)^k(2k-1)^p$ and $b_k = (2k-1)$, we obtain

$$(6)\qquad \begin{aligned} 2T_p^*(n) &= (-1)^n(2n-1)^{p-1}(2n+1) - (-1)^{p-1} + (2p-4)T_{p-1}^*(n) \\ &\quad - \sum_{j=1}^{p-2}(-2)^{j+1}\binom{p-1}{j+1}T_{p-j-1}^*(n). \end{aligned}$$

Several remarks are appropriate at this point. In the first place, it is clear that when $p = 1$ in (3) or (4), or when $p = 2$ in (5) or (6), the sum on the right hand side of each formula is zero. Secondly, it is clear that, from the formulae, $S_p(n)$, $T_p(n)$, $S_p^*(n)$, and $T_p^*(n)$ are all polynomials in n, the first two of degree p+1, and the last two of degree p. Finally, we note that (5) and (6) are incomplete until we append the following results, which are easily proved by induction.

(5a) $$S_1^*(n) = (-1)^n \frac{2n+1}{4} - \frac{1}{4} ;$$

(6a) $$T_1^*(n) = (-1)^n\, n .$$

Formula (2) can be extended in various directions. We give one example. If we replace n with j in (2), and then sum both sides over j, from 0 to n, we obtain

$$(7) \quad \sum_{j=0}^{n} \sum_{k=0}^{j} a_k(b_{k+1} - b_k) = \sum_{j=0}^{n} a_j b_{j+1} - (n+1)a_0 b_0 - \sum_{j=1}^{n} \sum_{k=1}^{j} b_k(a_k - a_{k-1}).$$

In particular, if p is a positive integer, and if we let $a_k = k^p$ and $b_k = k$ in (7), we have

$$(p+1) \sum_{j=1}^{n} S_p(j) = S_{p+1}(n) + S_p(n) + \sum_{q=1}^{p-1} (-1)^{q+1} \binom{p}{q+1} \sum_{j=1}^{n} S_{p-q}(j).$$

These examples should suffice to illustrate the wide range of sums of powers of integers for which recursion formulae can be obtained from the elementary formula (2).

REFERENCES

1. K. S. Miller, Introduction to the Calculus of Finite Differences and Difference Equations. Holt and Company, New York, 1960, pp. 85–94.

2. C. H. Richardson, Introduction to the Calculus of Finite Differences. Van Nostrand, New York, 1954, pp., 82–85.

3. J. G. Christiano, "On the Sum of Powers of Natural Numbers," Amer. Math. Monthly, vol. 68, 1961, pp. 149-151.

ON THE COEFFICIENTS OF $\sum_{x=1}^{n} x^k / \sum_{x=1}^{n} x^m$, WRITTEN IN TERMS OF n

Edgar Karst, Evangel College

For establishing $\sum_{x=1}^{n} x^k$ up to, let's say, k = 21, we use the Euler Numbers E_k and the Bernoulli Numbers B_k with their related coefficients b_k. Since

$$E_k = \frac{(2k)!}{(2k-2)!\,2!} E_{k-1} - \frac{(2k)!}{(2k-4)!\,4!} E_{k-2} + \frac{(2k)!}{(2k-6)!\,6!} E_{k-3} - \cdots$$

$$+ (-1)^{k-1} \frac{(2k)!}{0!\,(2k)!} E_0$$

where $0! = E_0 = 1$ by definition, we receive $E_1 = 1$, $E_2 = 5$, $E_3 = 61$, $E_4 = 1385$, $E_5 = 50521$, $E_6 = 2702765$, $E_7 = 199360981$, $E_8 = 19391512145$, $E_9 = 2404879675441$. Since

$$B_k = \frac{2k}{2^{2k}(2^{2k}-1)} \left[\frac{(2k-1)!}{(2k-2)!\,1!} E_{k-1} - \frac{(2k-1)!}{(2k-4)!\,3!} E_{k-2} + \cdots \right.$$

$$\left. (-1)^{k-1} \frac{(2k-1)!}{0!\,(2k-1)!} E_0 \right]$$

we get further $B_1 = 1/6$, $B_2 = 1/30$, $B_3 = 1/42$, $B_4 = 1/30$, $B_5 = 5/66$, $B_6 = 691/2730$, $B_7 = 7/6$, $B_8 = 3617/510$, $B_9 = 43867/798$, $B_{10} = 174611/330$, which were compared (except B_9 and B_{10}) with the values published by J. S. FRAME [1] and found correct. The numerators are often primes, but 174611 = 283.617 is not.

Then the relations $b_0 = 1$, $b_1 = -1/2$, $b_{2k} = (-1)^{k-1} B_k$, $b_{2k+1} = 0$ for $k > 0$ were used yielding

$b_2 = 1/6$, $b_4 = -1/30$, $b_6 = 1/42$. $b_8 = -1/30$, $b_{10} = 5/66$, $b_{12} = -691/2730$, $b_{14} = 7/6$, $b_{16} = -3617/510$, $b_{18} = 43867/798$, $b_{20} = -174611/330$.

Now the formula developed by J. G. CHRISTIANO [2] was applied

$$\sum_{x=1}^{n} x^k = \frac{n^{k+1}}{k+1} + \frac{n^k}{2} + \sum_{j=2} \frac{1}{j} b_j \binom{k}{j-1} n^{k-j+1} \quad \text{where } \binom{k}{j-1} = \frac{k!}{(j-1)!\,(k-j+1)!}$$

yielding

$$\sum_{x=1}^{n} x = n^2/2 + n/2$$

$$\sum_{x=1}^{n} x^2 = n^3/3 + n^2/2 + n/6$$

$$\sum_{x=1}^{n} x^3 = n^4/4 + n^3/2 + n^2/4$$

$$\sum_{x=1}^{n} x^4 = n^5/5 + n^4/2 + n^3/3 - n/30$$

$$\sum_{x=1}^{n} x^5 = n^6/6 + n^5/2 + 5n^4/12 - n^2/12$$

$$\sum_{x=1}^{n} x^6 = n^7/7 + n^6/2 + n^5/2 - n^3/6 + n/42$$

$$\sum_{x=1}^{n} x^7 = n^8/8 + n^7/2 + 7n^6/12 - 7n^4/24 + n^2/12$$

$$\sum_{x=1}^{n} x^8 = n^9/9 + n^8/2 + 2n^7/3 - 7n^5/15 + 2n^3/9 - n/30$$

$$\sum_{x=1}^{n} x^9 = n^{10}/10 + n^9/2 + 3n^8/4 - 7n^6/10 + n^4/2 - 3n^2/20$$

$$\sum_{x=1}^{n} x^{10} = n^{11}/11 + n^{10}/2 + 5n^9/6 - n^7 + n^5 - n^3/2 + 5n/66$$

$$\sum_{x=1}^{n} x^{11} = n^{12}/12 + n^{11}/2 + 11n^{10}/12 - 11n^8/8 + 11n^6/6 - 11n^4/8 + 5n^2/12$$

$$\sum_{x=1}^{n} x^{12} = n^{13}/13 + n^{12}/2 + n^{11} - 11n^9/6 + 22n^7/7 - 33n^5/10 + 5n^3/3 - 691n/2730$$

$$\sum_{x=1}^{n} x^{13} = n^{14}/14 + n^{13}/2 + 13n^{12}/12 - 143n^{10}/60 + 143n^8/28 - 143n^6/20 + 65n^4/12 - 691n^2/420$$

$$\sum_{x=1}^{n} x^{14} = n^{15}/15 + n^{14}/2 + 7n^{13}/6 - 91n^{11}/30 + 143n^9/18 - 143n^7/10 + 91n^5/6 - 691n^3/90 + 7n/6$$

$$\sum_{x=1}^{n} x^{15} = n^{16}/16 + n^{15}/2 + 5n^{14}/4 - 91n^{12}/24 + 143n^{10}/12 - 429n^8/16 + 455n^6/12 - 691n^4/24 + 35n^2/4$$

$$\sum_{x=1}^{n} x^{16} = n^{17}/17 + n^{16}/2 + 4n^{15}/3 - 14n^{13}/3 + 52n^{11}/3 - 143n^9/3 + 260n^7/3 - 1382n^5/15 + 140n^3/3 - 3617n/510$$

$$\sum_{x=1}^{n} x^{17} = n^{18}/18 + n^{17}/2 + 17n^{16}/12 - 17n^{14}/3 + 221n^{12}/9 - 2431n^{10}/30 + 1105n^8/6 - 11747n^6/45 + 595n^4/3 - 3617n^2/60$$

$$\sum_{x=1}^{n} x^{18} = n^{19}/19 + n^{18}/2 + 3n^{17}/2 - 34n^{15}/5 + 34n^{13} - 663n^{11}/5 + 1105n^9/3 - 23494n^7/35 + 714n^5 - 3617n^3/10 + 43867n/798$$

$$\sum_{x=1}^{n} x^{19} = n^{20}/20 + n^{19}/2 + 19n^{18}/12 - 323n^{16}/40 + 323n^{14}/7 - 4199n^{12}/20 + 4199n^{10}/6 - 223193n^8/140 + 2261n^6 - 68723n^4/40 + 43867n^2/84$$

$$\sum_{x=1}^{n} x^{20} = n^{21}/21 + n^{20}/2 + 5n^{19}/3 - 19n^{17}/2 + 1292n^{15}/21 - 323n^{13} + 41990n^{11}/33 - 223193n^9/63 + 6460n^7 - 68723n^5/10 + 219335n^3/63 - 174611n/330$$

$$\sum_{x=1}^{n} x^{21} = n^{22}/22 + n^{21}/2 + 7n^{20}/4 - 133n^{18}/12 + 323n^{16}/4 - 969n^{14}/2 + 146965n^{12}/66 - 223193n^{10}/30 + 33915n^8/2 - 481061n^6/20 + 219335n^4/12 - 1222277n^2/220$$

If $k > m$ the following theorems can be established.

1. If $m = 1$ then $\sum_{x=1}^{n} x^k / \sum_{x=1}^{n} x^m$ has the remainder $R = 0$, and the coefficients of the quotient Q add up to unity. Example for $n = 1$:

$$\sum_{x=1}^{n} x^{21} / \sum_{x=1}^{n} x = 1/11 + 10/11 + 57/22 - 646/33 + 646/33 + 9367/66 - 9367/66 - 54587/66 + 54587/66 + 79781/22 - 79781/22 - 3713531/330 + 3713531/330 + 7478419/330 - 7478419/330 - 4198297/165 + 4198297/165 + 1222277/110 - 1222277/110 = 1.$$

2. If $m = 1$ and k odd then $\sum_{x=1}^{n} x^k / (\sum_{x=1}^{n} x^m)^2$ has the remainder $R = 0$, and the coefficients of the quotient Q add up to unity. Example for $n = 1$:

$$\sum_{x=1}^{n} x^{21} / (\sum_{x=1}^{n} x)^2 = 2/11 + 18/11 + 39/11 - 96/11 - 1004/33 + 2296/33 + 2357/11 - 16438/33 - 38149/33 + 30912/11 + 48869/11 - 128650/11 - 1783781/165 + 5497312/165 + 660369/55 - 9459526/165 + 1062932/165 + 7333662/165 - 1222277/55 = 1.$$

3. If m = 1 and k even then $\sum_{x=1}^{n} x^k / (\sum_{x=1}^{n} x^m)^2$ has the remainder R # 0, and the coefficients of the quotient Q and the remainder R add up to unity. Example for n = 1: $\sum_{x=1}^{n} x^{20} / (\sum_{x=1}^{n} x)^2 = 4/21 + 34/21$

$+ 68/21 - 170/21 - 526/21 + 1222/21 + 3250/21 - 2574/7 - 2134/3$
$+ 37598/21 + 512882/231 - 1439342/231 - 2723086/693 + 9764198/693$
$+ 367270/231 - 11967818/693 + 18919052/3465 + 349222/55 - 174611/165$
$- 174611/110 - 174611/330 = 1.$

Related topics were found by J. S. FRAME [3].

REFERENCES

1. J. S. Frame, "Bernoulli Numbers Modulo 27000," Amer. Math. Monthly, Feb. 1961, p. 88.

2. J. G. Christiano, "On the Sum of Powers of Natural Numbers," ibid., p 150.

3. J. S. Frame, "Note on the Product of Power Sums," Pi Mu Epsilon Journal, Nov. 1949, p. 21.

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SOME IDENTITIES FOR A GENERALIZED SECOND ORDER RECURRING SEQUENCE

Charles R. Wall, Texas Christian University

In this paper we develop some identities for a yeneralized second order recurring sequence defined by

(1) $$W_{n+2} = gW_{n+1} + hW_n ,$$

with $W_0 = q$, $W_1 = p$ arbitrary. We will also consider the special case h = 1.

Solving the equation associated with (1), namely

$$x^2 - gx - h = 0,$$

we see that its roots are

$$\alpha = \frac{g + \sqrt{(g^2 + 4h)}}{2} \quad \text{and} \quad \beta = \frac{g - \sqrt{(g^2 + 4h)}}{2}$$

We easily verify by induction that

(2) $$W_n = \frac{A\alpha^n - B\beta^n}{\alpha - \beta},$$

where $A = p - q\beta$, $B = p - q\alpha$. Associated with the sequence $\{W_n\}$ are the numbers

$$Y_n = W_{n+1} + hW_{n-1} .$$

We may show that

(3) $$Y_n = A\alpha^n + B\beta^n .$$

Lucas [1, p. 396] considered the numbers U_n and V_n given by the relations

(4) $$U_n = \frac{a^n - \beta^n}{a - 8} \quad \text{and} \quad V_n = a^n + \beta^n .$$

For g = h = 1, U_n and V_n are the n-th Fibonacci and Lucas numbers, respectively, while W_n and Y_n are the n-th generalized Fibonacci and Lucas numbers, respectively.

Two properties of generalized Fibonacci numbers [2] which are easily extended to generalized second order recurring sequences are the following: if W_n # 0, Y_n # 0, from (21, (3), and (4) we have

(5) $$\frac{W_{n+r} + (-h)^r W_{n-r}}{W_n} = V_r ,$$

and

(6) $$\frac{W_{n+r} - (-h)^r W_{n-r}}{Y_n} = U_r .$$

Identities (5) and (6) are rather surprising since, in both cases, the left member of the equation is independent of not only the defining values p and q, but the subscript n as well! Identity (5) for generalized Fibonacci numbers was given by Tagiuri [1, p. 404] and, albeit incorrectly, by Horadam [3, p. 457, property (17)].

By (2) and (3) we readily establish the following de Moivre-type identity, which was given for Fibonacci numbers by Fisk [4]:

$$\left(\frac{Y_n + (\alpha - \beta)W_n}{2A}\right)^m = \frac{Y_{nm} + (\alpha - \beta)W_{nm}}{2A}$$

We now turn our attention to the special case h = 1. Our purpose here is to demonstrate two methods for generating this special case of (1). Let y and 6, with $y = [g + \sqrt{(g^2 + 4)}]/2$, be the roots of

$$x^2 - gx - 1 = 0.$$

Then, if $C = p - q\delta$, $D = p - q\gamma$,

$$J_n = \frac{C\gamma^n - D\delta^n}{\gamma - \delta}, \tag{7}$$

$$K_n = C\gamma^n + D\delta^n, \tag{8}$$

and

$$S_n = \frac{\gamma^n - \delta^n}{\gamma - \delta} \quad \text{and} \quad T_n = \gamma^n + \delta^n \tag{9}$$

correspond to (2), (3), and (4), respectively.

Consider the matrix

$$S = \begin{bmatrix} g & 1 \\ 1 & 0 \end{bmatrix}.$$

By induction we have that

$$S^n = \begin{bmatrix} S_{n+1} & S_n \\ S_n & S_{n-1} \end{bmatrix}. \tag{10}$$

If

$$J = \begin{bmatrix} p & q \\ q & p-gq \end{bmatrix},$$

We may easily verify, since

$$J_n = pS_n + qS_{n-1}, \tag{11}$$

that

$$S^n J = \begin{bmatrix} J_{n+1} & J_n \\ J_n & J_{n-1} \end{bmatrix}.$$

Thus we may generate the sequence $\{J_n\}$ by evaluating powers of the matrix S.

It is obvious from (10) that

$$S_{n+1}S_{n-1} - S_n^2 = (-1)^n,$$

which, for Fibonacci numbers, is the basis for a famous geometrical deception.

It is not the purpose of this paper to investigate at any length the properties which one may derive by consideration of the matrices given above. Let is suffice to say, however, that many identities result from such a study, and that one may, by this approach, avoid many "messy" inductive proofs. In general, one would follow roughly the same procedure as in Basin and Hoggatt [5].

Finally, we remark that it is possible to generate the sequence $\{S_n\}$ -- and, by (11), the sequence $\{J_n\}$ as well -- by the following scheme: we may easily show that the fraction S_{n+2}/S_n is the n-th convergent of the continued fraction

$$g + \cfrac{1}{g + \cfrac{1}{g + \cfrac{1}{g + \cdots}}}$$

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1. L. E. Dickson, History of the Theory of Numbers, Vol. I, Chelsea, 1952.

2. Charles R. Wall, Sums and Differences of Generalized Fibonacci Numbers, to be published in The Fibonacci Quarterly.

3. A. F. Horadam, A Generalized Fibonacci Sequence, Amer. Math. Monthly, 68(1961), 455-59.

4. Stephen Fisk, Problem B-10, The Fibonacci Quarterly, 1(1963), NO. 2, p. 85.

5. S. L. Basin and V. E. Hoggatt, Jr., A Primer on the Fibonacci Sequence, Part 11, The Fibonacci Quarterly, 1(1963), No. 2, pp. 61-68.

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RESEARCH PROBLEMS

This is a new section that will be devoted to suggestions of topics and problems for Undergraduate Research Programs. Address all correspondence to the Editor.

Proposed by M. S. KLAMKIN. Determine an "efficient" computer algorithm for determining the center and radius of the smallest circle which covers a given set of points in a plane. Also, consider extensions to covering with equilateral triangles, squares, ellipses (say of minimum area, minimum major axis, ...) ..., and to higher dimensions.

Proposed by P. C. ROSENBLOOM. If $P(z) = \sum_0^n a_k z^k$ is a polynomial of degree k, then all zeros of P lie in the circle $|z| \le R$, where R is the unique positive solution of $$|a_n| R^n = \sum_0^{n-1} |a_k| R^k.$$

From this, one can obtain many estimates of the zeros of P in terms of the coefficients (see **M. Marden,** <u>Geometry of Zeros of Polynomials in the Complex Domain</u>).

An analogue of the Fundamental Theorem of Algebra is: If

$$P(z_1, z_2) = \sum_{0 \le r+s \le m} a_{rs} z_1^r z_2^s$$

and

$$Q(z_1, z_2) = \sum_{0 \le r+s \le n} b_{rs} z_1^r z_2^s$$

are polynomials in two variables, and their "principal parts"

$$P_1(z_1, z_2) = \sum_{r+s=n} a_{rs} z_1^r z_2^s$$

and

$$Q_1(z_1, z_2) = \sum_{r+s=n} b_{rs} z_1^r z_2^s$$

have no common zero except $z_1 = z_2 = 0$, then the system $P = Q = 0$ has mn common zeros (z_1, z_2) if counted with proper multiplicity.

Can one obtain estimates for these zeros in terms of the coefficients a_{rs} and b_{rs}?

Proposed by **S. SCHUSTER.** Let A and B be non-singular symmetric matrices. Thus, referring to Projective Geometry, they represent polarities. What are the geometric invariants of the pencil of matrices (polarities)

$$A + \lambda B\ ?$$

Reference: Gantmacher, F. R., <u>Applications of the Theory of Matrices</u>.

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PROBLEM DEPARTMENT

Edited by
M. S. Klamkin, University of Minnesota

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication.

An asterisk (*) placed beside a problem number indicates that the problem was submitted without a solution.

Address all communications concerning problems to Professor M. S. Klamkin, Department of Mathematics, University of Minnesota, Minneapolis, Minnesota 55455.

PROBLEMS FOR SOLUTION

163. Proposed by Seymour Schuster, University of Minnesota.
Can any real polynomial be expressed as the difference of two real polynomials each of which having only positive roots?

164. Proposed by F. Zetto, Chicago.
Which numbers of the form $300\cdots007$ are divisible by 37?

165. Proposed by D. J. Newman, Yeshiva University.
Express $\cos\theta$ as a rational function of $\sin^3\theta$ and $\cos^3\theta$.

166. Proposed by Leo Moser, University of Alberta.
Show that 5 points in the interior of a 2x1 rectangle always determine at least one distance less than $\sec 15^\circ$.

167. Proposed by M. S. Klamkin, University of Minnesota.
Given a centrosymmetric strictly convex figure and an intersecting translation of it; show that there is only one common chord and that this chord is mutually bisected by the segment joining the centers.

SOLUTIONS

149. Proposed by John Selfridge, University of Washington.
A game of bridge is dealt and each player has distribution abcd into suits (e.g., each player has 4333). Is each suit distributed abcd among the players? In another deal each player has the same distribution as some suit. Does each suit have the same distribution as some player?

Solution by the proposer.
In the first deal each suit is distributed abcd. Let a, b, c, denote the three of the numbers having the same parity. Then each suit is distributed xyzd. If $a = b = c$, we are done. If not, we may rearrange so that $a \le b < c$ or $a \ge b > c$. Since $x + y + z = a + b + c$, each suit is xycd where $x + y = a + b$. Then x is a or b and y is the other.

In the second deal the answer is in the negative as the suit distributions might have been 4522, 5152, 1345, 3424 and the players either 5431 or 5422.

Also solved by P. Myers, David L. Silverman, M. Wagner and F. Zetto.

151. Proposed by K. S. Murray, New York City.
Three points are chosen at random with a uniform distribution from the three sides of a given triangle (one point to each side). What is the expected value of the area of the random triangle that is formed?

Solution by L. Carlitz (Duke University).
Let (a_1, a_2), (b_1, b_2), (c_1, c_2) denote the vertices of the given triangle and let $(tb_1 + (1-t)c_1,\ tb_2 + (1-t)c_2)$, etc., denote the points on the side. Then the area of the random triangle is equal to

$$\frac{1}{2}\begin{vmatrix} 1 & tb_1 + (1-t)c_1 & tb_2 + (1-t)c_2 \\ 1 & uc_1 + (1-u)a_1 & uc_2 + (1-u)a_2 \\ 1 & va_1 + (1-v)b_1 & va_2 + (1-v)b_2 \end{vmatrix}$$

where $0 \le t, u, v \le 1$. Integrating with respect to t, u, v, we get

$$\frac{1}{2}\begin{vmatrix} 1 & \frac{1}{2}(b_1 + c_1) & \frac{1}{2}(b_2 + c_2) \\ 1 & \frac{1}{2}(c_1 + a_1) & \frac{1}{2}(c_2 + a_2) \\ 1 & \frac{1}{2}(a_1 + b_1) & \frac{1}{2}(a_2 + b_2) \end{vmatrix}$$

or one fourth the area of the given triangle.

Solution by H. Kaye, Brooklyn, N. Y.
The expected area $\bar{A}$ is given by

$$\bar{A} = \Delta - \frac{1}{2abc}\int_0^a\int_0^b\int_0^c [z(b-y)\sin A + x(c-z)\sin B + y(a-x)\sin C]\,dx\,dy\,dz$$

where a, b, c are the sides of the given triangle Δ. Whence, $\bar{A} = \frac{\Delta}{4}$.

Also solved by P. Meyers, M. Wagner and the proposer.

Editorial note: Keep 2 vertices of the random triangle fixed and let the third vertex vary uniformly over its corresponding side. It follows immediately that the average area of this subset of random triangles is obtained when the point is at the midpoint of the given side. Consequently, the expected value of the area for the entire set of random triangles is given by the triangle whose vertices are the three midpoints of the sides.

152. Proposed by Leo Moser, University of Alberta.
If ϕ denotes Euler's totient function, show that in every base

$$\frac{\phi(1)}{1} + \frac{\phi(1+1)}{11} + \frac{\phi(1+1+1)}{111} + \cdots = 1.111\ldots\,.$$

Solution by L. Carlitz, Duke University.
Let b denote the base. Then

$$\sum_{r=1}^{\infty}\frac{\phi(r)}{b^{r-1} + b^{r-2} + \cdots + 1} = (b-1)\sum_{r=1}^{\infty}\frac{\phi(r)}{b^r - 1} = (b-1)\sum_{r=1}^{\infty}\phi(r)\sum_{s=1}^{\infty} b^{-rs}$$

$$= (b-1)\sum_{n=1}^{\infty} b^{-n}\sum_{r|n}\phi(r) = (b-1)\sum_{n=1}^{\infty} nb^{-n} = \frac{1}{1-b^{-1}} = 1 + \frac{1}{b} + \frac{1}{b^2} + \cdots .$$

Also solved by H. Kaye, P. Myers and the proposer.

Editorial note: The given result is a special case of an identity due to Laguerre, i.e.,

$$\sum_{r=1}^{\infty}\frac{F(r)}{x^r - 1} = \sum_{n=1}^{\infty} G(n)x^{-n}$$

where $$G(n) = \sum_{d|n} F(d).$$

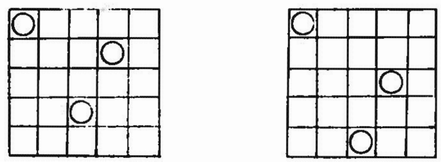
The derivation of this latter identity is contained in Carlitz's solution above.

153. Proposed by M. S. Klamkin, University of Minnesota.
Show that the maximum area ellipse which can be inscribed in an equilateral triangle is the inscribed circle.

Solution by the proposer.
Orthogonally project the configuration of the equilateral triangle and maximum area inscribed ellipse such that the ellipse transforms into a circle. Since the ratio of areas are preserved under the projection, the equilateral triangle will transform into a triangle of minimum area circumscribing the circle. It is a known result that the latter triangle must be also equilateral. Consequently the transformation was the identity one and the ellipse is a circle.

Also solved by L. Carlitz, A. Cohen, H. Kaye, and D. Smith.

122. (Errata)
Joe Konhauser (HRB-Singer, Inc.) has pointed out that 3 queens suffice to cover a 5x5 board as shown in the following figures:



Edited by
Roy B. Deal, Oklahoma State University

Modern Mathematics for the Engineer. Edited by Edwin F. Beckenbach. New York, McGraw-Hill, 1956. xxii + 514 pp., $3.45.

Modern Mathematics for the Engineer, edited by Edwin F. Beckenbach and published by McGraw-Hill (1956), is a series of lectures for a course in mathematics originally organized under the supervision of Dean L.M.K. Boelter of the College of Engineering and Professor Clifford Bell of the Department of Mathematics at the University of California, Los Angeles. The course was presented at this university, at the Corona Laboratories of the National Bureau of Standards, and at the University of California, Berkeley. The book can well serve as collateral reading for similar courses. It is a valuable little encyclopedia for the engineer and for the mathematician interested in the applications of mathematics to physical problems and in the development of mathematics under the stimulation of such problems. The authors are well-known authorities in their fields.

The book is divided into three parts. Part I, "Mathematical Models," deals with physical problems primarily by methods of differential, partial differential, and integral equations. It consists of the chapters: 1. Linear and Nonlinear Oscillations, by Solomon Lefschetz, 2. Equilibrium Analysis: The Stability Theory of Poincare and Liapunw, by Richard Bellman, 3. Exterior Ballistics, by John W. Green, 4. Elements of the Calculus of Variations, by Magnus R. Hestenes, 5. Hyperbolic Partial Differential Equations and Applications, by Richard Courant, 6. Boundary-Value Problems in Elliptic Partial Differential Equations, by Menahem M. Schiffer, 7. The Elastostatic Boundary-Value Problems, by Ivan S. Sokolnikoff. Part 2, "Probabilistic Problems," emphasizes the programming and operational aspects of engineering and the use of stochastic processes in the formulation and solution of problems. Its chapters are: 8. The Theory of Prediction, by Norbert Wiener, 9. The Theory of Games, By Frederic Bohnenblust, 10. Applied Mathematics in Operations Research, by Gilbert W. King, 11. The Theory of Dynamic Programming, by Richard Bellman, 12. Monte Carlo Methods, by George W. Brown. Part 3, "Computational Considerations," is concerned primarily with numerical solutions, and is divided into the chapters: 13. Matrices in Engineering, by Louis A. Pipes, 14. Functional Transformations for Engineering Design, by John L. Barnes, 15. Conformal Mapping Methods, by Edwin F. Beckenbach, 16. Nonlinear Methods, by Charles B. Morrey, Jr., 17. What are Relaxation Methods?, by George E. Forsythe, 18. Methods Of Steep Descent, by Charles B. Tompkins, 19. High-speed Computing Devices and Their Applications, by Derrick H. Lehmer.

university of Illinois — Josephine H. Chanler

Applications of Graph Theory to Group Structure. By C. Flament. Englewood Cliffs, New Jersey; Prentice-Hall; 1963. 142 pp., $6.95.

An Anatomy of Kinship. By H. C. White. Englewood Cliffs, New Jersey; Prentice-Hall; 1963. 180 pp., $6.95.

These are the first two volumes in the Prentice-Hall Series in Mathematical Analysis of Social Behavior. The first volume begins with a compact, informal introduction to the theory of graphs, then treats communications networks, and finally treats balancing processes. An excellent bibliography is attached. The second volume uses graphs, matrix algebra, and group theory to analyze the how and the why of structure in social systems, then applies the theory to develop models of four known tribes. Appendices reprint a famous paper by Andre Weil on the mathematics of kinship and an extension thereof by R. R. Bush.

These two books are important contributions to mathematical sociology. Moreover, they will provide interested mathematical readers with inspiration for further research, not only in mathematical sociology, but also in other areas of physical or natural science where similar structures exist.

University of Illinois — Franz E. Hohn

Elementary Theory of Analytic Functions of One or Several Complex Variables. By Henri Cartan. Reading, Mass., Addison-Wesley, 1963. 227 pp., $10.75.

The licence d'enseignement is a degree roughly comparable to the B.A., but requires essentially only the study of mathematics. This volume is based upon lectures given by the author at the University of Paris in the theory of analytic functions of a complex variable for the requirements of this degree, and is at the advanced undergraduate or beginning graduate level for American Students. The basic concepts of general topology are assumed to be familiar to the reader.

The exposition is clear and concise. All theorems are given exact statements and (with few exceptions) complete proofs. There is very little heuristic argument or general discussion of ideas. The first three chapters are on power series and integral theorems and their applications. The remaining four chapters are on analytic functions of several variables, sequences of holomorphic functions, holomorphic transformations, and holomorphic systems of differential equations. This classical material is given a modern flavor. There are, for example, sections on the topology of the vector space of continuous (complex-valued) functions in an open set (Chapter V), and on the integration of differential forms on a complex manifold (Chapter VI).

This is an excellent book which gives a clear and lucid presentation of these ideas.

University of Illinois — R. P. Jerrard

Calculus of Variations. By I. M. Gelfand and S. V. Fomin. Translated and edited by R. A. Silverman. Englewood Cliffs, N. J., Prentice-Hall, 1963. vii + 232 pp., $7.95.

This is an attractive and modern treatment of the calculus of variations written by two eminent Soviet mathematicians. This (authorized) translation includes exercises and two appendices not in the original Russian edition. It should be very useful for students who wish to learn this interesting and Important field and who have a background in real analysis.

The problems of the calculus of variations are very old and go back to the Bernoullis, Newton, and Euler. Important contributions were later made by Hamilton, Jacobi, Hilbert, and Weierstrass, among others. Yet there is still much research activity on various aspects of the theory.

The main theory of the subject is well-presented. Thus, necessary and sufficient conditions for an extreme are discussed, as are the Hamilton-Jacobi theory, fields and conjugate points, and variational problems involving multiple integrals. Applications to mechanics and physics are also indicated.

One of the most attractive features of this book to this reviewer is that the authors approach the subject from the point of view of modern functional analysis. This sets it off from most other books that are available on this subject.

University of Illinois — Robert G. Bartle

Introduction to Set Theory and Logic. By Robert R. Stoll. San Francisco: W. H. Freeman and Company, 1963. xiv + 474 pp., $9.00.

This book was written as a text book for a year course for advanced undergraduates to give them initial training in the axiomatic method of mathematics. The title notwithstanding, the main focus is upon the real number system. There is ample material for a year course in "The Foundations of Mathematics," so that the instructor may do some selecting, choosing, and emphasizing to suit his own desires. The beginning chapters introduce the three areas of set theory, real number development, and logic. Further chapters go into all three areas much more extensively, and are independent enough so that the book may be used as a one semester text for any of the three areas. The format is very good, with defined words in bold face type and the words "Theorem", "Lemma", etc. in very large and distinctive type. There are numerous examples throughout, and quite a sufficient number of exercises for most sections to be in keeping with the text book aim.

DePauw University — Robert J. Thomas

Sets, Relations, and Functions: A Programmed Unit in Modern Mathematics. By M. McFadden, J. W. Moore, and W. I. Smith. New York, McGraw-Hill Book Company, Inc., 1963. iv + 299 pp., $3.95 paper, $5.95 cloth.

This book is a programmed unit in modern mathematics geared for those who desire to increase their understanding of the "why" in mathematics and for those who have finished two years of high school algebra. The matter is presented in the form of frames: one problem to a frame, with the answer written immediately below in a shaded area. Each frame requires the reader to answer the problem presented either with fill-ins, drawing constructions, or graphs. On the average there are four frames to a page. In all there are 1074 individual frames, not counting the nine self-tests. The answers to these self-tests are given at the end of the book.

The title indicates the three main divisions of the book. In the section on sets are considered the description and notation of sets, operations with sets, and equations and inequalities involving sets. The section on relations treats ordered pairs of numbers, graphs, cartesian product, and binary relations. Various mappings, value of a function, composite function, inverse functions, and various types of correspondences constitute the subject matter of the section on functions.

Each new topic is introduced by giving many examples, and only after these are discussed is a formal definition given. However, some notions are used without any formal definition at all: e.g., a one-to-one correspondence in frame 100 is undefined, although it is used in the definition of equivalent sets; finite and infinite sets in frame 121 are not defined; the relation of "greater than" in frame 318 is left without a definition. The method of introducing ordered pairs might lead one to think that only positive numbers are considered for the elements of each pair, though later on this impression is removed.

Much emphasis is placed on the use of Venn diagrams to exemplify properties of sets and operations with sets. For one using the text without a teacher, false impressions could be readily formed that Venn diagrams prove the various properties of sets. Formal proofs of some mathematical properties are given and occur as problems in one or more successive frames. The review frames and the self-tests are quite appropriate.

There are some misprints which would be obvious to those studying the book with the help of a teacher, but for one going through on his own, they may not be so obvious: e.g., frames 161, 283. Most of the practical applications of the theory occurs in the latter part of the book after functions are treated.

For one who works through the entire book a fine knowledge of sets, relations, and functions with their fundamental properties would be his reward.

Saint Louis University — John F. Daly, S.J.

Probability Theory, Third Edition. By Michael Loeve. Princeton, Van Nostrand, 1963. xvi + 685 pp., $14.75.

This is a revision, with many minor changes and additions, of a well-known, standard work on probability theory. It begins with a 51-page introduction to elementary probability and a 17-page summary of the measure theory required for statistics. Part Two covers general concepts and tools of probability theory. Part Three treats sums of independent random variables and the central limit problems. Part Four treats conditioning, dependence, ergodic theorems, and second order properties. Part Five treats random functions and processes including martingales and Markov processes. The material on martingales has been completely revised.

Earlier reviewers have said: "... an admirable source of deep ideas...", "... the best available advanced book on probability theory...", "... a very scholarly book in the best tradition of analysis...". "Every serious probabilist should, and doubtless will, possess a copy of this important work." These remarks apply equally well to the new edition.

University of Illinois — Franz E. Hohn

Handbook of Mathematical Psychology, Vol. 1. R. D. Luce, R. R. Bush, and E. Galanter, Editors. New York, Wiley, 1963. xii + 491 pp., $10.50. Vol. 2, vii + 606 pp., $11.95.

Readings in Mathematical Psychology, Vol. 1. R. D. Luce, R. R. Bush, and E. Galanter, Editors. New York, Wiley, 1963. ix + 535 pp., $8.95.

The Handbooks are the first two of three volumes. Beginning with a list of books which could form a basic library in mathematical psychology, and which are assumed to be available to the reader, the first volume presents eight expository chapters on measurement and psychophysics. The second volume treats learning theory and social behavior. The third will treat sensory mechanisms and preference. Various topics of which good summaries exist have deliberately not been included. These are, however, appropriately referenced. Finite mathematics, calculus, probability, and statistics, all at an undergraduate level, are the mathematical pre-requisites for this material

Readings is the first of two volumes. It reprints papers, from various professional journals, on measurement, psychophysics, reaction time, learning and stochastic processes.

This series, besides being essential for the library of the student of mathematical psychology, will be useful to those mathematicians who teach courses in mathematics for social scientists, both to provide perspective and to provide significant applications. One also hopes that mathematicians might find here inspiration for research that could enrich both disciplines.

University of Illinois — Franz E. Hohn

computational Methods of Linear Algebra. By D. K. Faddeev and V. N. Faddeeva. Translated from the 1960 Russian edition by R. C. Williams. San Francisco, W. H. Freeman, 1963. xi + 621 pp., $11.50.

This book is a nearly complete collection of methods for numerically solving linear equations, inverting matrices, and finding eigenvalues and eigenvectors of matrices. The theory behind each method is developed rigorously and then the technique is illustrated with a well laid out numerical example. Organization of the work to avoid unnecessary rounding, storing, and computation is also discussed for many methods.

The book opens with a 118 page review of basic linear algebra which retains readability in spite of its conciseness. Chapter 2 covers essentially all variants of direct methods for solving linear equations and inverting matrices. Iterative methods are classified by chapters into methods of successive approximations, methods based on orthogonalization, and gradient methods. The problem of finding all of the eigenvalues of a matrix and that of finding a few (of the largest or smallest) are then discussed separately. In view of the diversity of methods in current use, the book does a good job of classifying them in the chapter structure and underlying the basic similarities. The book closes with an extensive 59 page bibliography which should remain useful for some time (although the original was published in 1960).

The fault in this edition is the excessive number of errors at the typographical level. These are mainly in equations, and, although none were found that were impossible to follow, some required considerable puzzling. Nine errors were counted in the first 32 pages, at which point the count was discontinued, but the reviewer felt that an error rate of one per three and a half pages would be a favorable estimate. This book is not, therefore, a suitable textbook for the unversed student. The concise, unredundant treatment of the material requires some familiarity with the subject, and the high error rate makes it difficult to read. It is, however, a worthwhile reference book, both for the advanced student and the numerical analyst.

University of Illinois — C. W. Gear

~~Methods of~~ Mathematical Analysis and Computation. By J. G. Herriot. New York, Wiley, 1963. xiii + 198 pp., $7.95.

This book is intended for beginning research men and for practicing engineers in the field of structural analysis. It treats interpolation, numerical differentiation and integration, roots of equations, linear algebra and linear computations, and the solution of ordinary and partial differential equations. The emphasis is on numerical procedures appropriate for computers. The exposition is compact but simple and clear. There are examples but no exercises.

University of Illinois — Franz E. Hohn

Matrix Algebra for Social Scientists. By Paul **Horst.** New York; Holt, Rinehart and Winston; 1963. **xxi** + 517 pp., **$10.00.**

This book actually treats matrix computations rather than matrix algebra, and the treatment is initially very slow-moving. The first 308 **pages go** no farther than matrix multiplication. However, a detailed notation is developed and is applied to virtually all conceivable special cases, including partitioned matrices. The **need** for all this specialization seems doubtful to this reviewer.

The remainder of the book treats orthogonal matrices (P_{n+m} is "orthogonal" if P^TP is diagonal), rank, the "basic structure of matrix" (the factorization $X = P \Delta Q^T$ where P and Q are orthogonal and Δ is diagonal with rank the same as that of **S),** inversion, and the solution of linear equations.

Mathematicians who have interests in social science applications may find the latter half of the book useful, as will social scientists with some mathematical training. The long slow introduction is presumably necessary for social scientists with minimal mathematical sophistication. The exposition is clear and generally accurate. Sometimes things are not phrased in approved mathematical style but no such deviation appears serious.

University of Illinois — Franz **E.** Hohn

An Introduction to Linear **Programming** and the Theory of Games. **By** M. Glicksman. New York, Wiley, 1963. x + 131 pp., $2.25 (paper), $4.95 (cloth).

This well-written monograph lives up to the promise of its bright, eye-catching cover. Basic concepts of convex sets, game theory, and linear programming are explained in detail and are illustrated with attractive, simple figures, graphs and tableaux. Written at the sophomore level and using only tools and concepts of algebra and analytic geometry, this book should be of interest, not only to the bright undergraduate mathematics student, but also to social scientists who are interested in a simple, though rigorous, development of applications.

Elementary proofs of the fundamental extreme point theorem for convex polygons, the fundamental duality theorem of linear programming, and its corollary, the minimax theorem, are included. Definitions and theorems are numbered and their use is illustrated. The simplex method in linear programming is used to maximize or minimize functions subject to constraints and to solve m x 2 matrix **games.** The amusing examples and problems help to heighten interest throughout the book.

The only criticisms are the misprints on pages three and four (24 should be substituted for 28) and the author's not discussing dominated strategies in matrix games.

University of Illinois — Leone Y. Low

Advanced Engineering Mathematics. By **Erwin Kreyszig. New** York, **Wiley,** 1962. **xvii** + 856 pp., $10.50.

This book is very well written, and most of the material is integrated into the pattern of the book. It should be a valuable reference for those engaged in engineering work.

Its suitability as a text for a four semester course is, however, open to question. Those needing that much training in mathematics should perhaps have: 1. Advanced Calculus, 1 year (skills type course); 2. Ordinary Differential **Equations, 1** semester; 3. Complex Variables, 1 semester (advanced undergraduate level), which would lead to a much deeper understanding of most of the material covered in this book.

Two areas are emphasized: Differential Equations and Vector **Analysis**-Complex Analysis. The following are the chapter headings: Introduction, Review; Ordinary Differential Equations of the first order; Ordinary Linear Differential Equations; Power Series Solutions of Differential Equations; **Laplace** Transformation; Vector **Analysis;** Line and Surface Integrals; Matrices, Determinants, Systems of Linear Equations; Fourier Series and Integrals; Partial Differential Equations; Complex **Analytic** Functions; Complex Integrals; Conformal Mapping; Complex Analytic Functions and Potential Theory; Special Functions. A lot of care was lavished on the three chapters (about 160 pages) covering ordinary differential equations.

After this, the chapter on **Laplace** Transforms (50 pages) seems abrupt. The beginning of the chapter on Vector Analysis (68 pages) is confusing and seems to be a combination of classical vector analysis, modern linear algebra, and an intuitive idea from the physical world of what a vector should be. If one assumes that the notions of vector, coordinate system, and coordinate can be learned from this, the confusion caused will diminish as one proceeds through the chapter, with the exception of **page** 306. The second displayed expression seems to be wrong.

The chapter on Line Integrals goes up to the theorems of Gauss and Stokes. There follow 80+ pages on Matrices and about 60 pages on Fourier Series. The chapter on Partial Differential Equations (52 pages) goes right into second order equations and uses the method of separation of variables together with Fourier Series almost exclusively. It is highly oriented toward getting an answer as opposed to understanding the theory.

The first two chapters on Complex Functions include Cauchy's Integral Theorem and **Residues.** The next chapter on Conformal Mapping covers material to the maximum modulus principle. In these chapters some of the proofs of basic theorems are incomplete, having been given only for special cases with a statement that they may then be generalized. These three chapters cover about 166 pages.

The last two chapters encompass a conglomeration of things and many specific examples of application.

Many examples have been worked out, and these are frequently non-trivial and interesting. The examples and problems broaden the scope of the book considerably. The footnotes giving historical information about well-known mathematicians are a nice gesture.

The references are arranged in an appendix, grouped according to topic, and mostly refer to standard works or texts.

The index is very good and consists of some 13 or 14 pages.

Applied Physics Laboratory, Johns Hopkins University R. M. Sorensen

Iterative Methods for the Solution of Equations. By J. F. Traub. Englewood Cliffs, N. J., Prentice-Hall, 1964. xviii + 310 pp., $12.50.

This is another unique member of the Prentice-Hall aeries on automatic computation. It organizes for the first time in a single volume a general theory of iteration algorithms for the numerical solution of equations and systems of equations. The treatment is mathematically rigorous but rigor is not the primary aim. Beginning with a general theory of iteration functions, it develops the theory of one-point and multipoint iteration functions without and with memory. A compilation of iteration functions is presented, the literature is thoroughly referenced, and areas for future research are outlined. The book is essential for those working in numerical analysis.

The author is a long-time member of the staff of the Bell Telephone Laboratories. This volume reflects most creditably the contribution of such laboratories to modern scientific progress.

University of Illinois Franz E. Hohn

Plastic Flow and Fracture in Solids. By T. Y. Thomas. New York, Academic Press, 1961. ix + 267 pp., $8.50.

This book is aimed at specialists in mathematical theories of plasticity. An abstract approach to the treatment of fracture is followed throughout. There is no consideration of metallurgical effects or of recent engineering theories of crack propagation. It is essentially restricted to the author's own research.

The book is well written and easily understood, provided the reader has some facility with tensor analysis. It should be very useful to specialists in the field and deserves reading by students of mathematics interested in applications of hyperbolic differential equations. A name index is lacking and reference to other research is not comprehensive.

University of Illinois E. M. Shoemaker

Digital Computer Technology and Design. By Willis H. Ware. New York, Wiley, 1963. Volume I, $7.95; Volume II, $11.75.

This well-written pair of volumes, by an experienced researcher and teacher in the field, can serve as the basis for a two-semester introduction course in digital technology. Volume I, which contains less than one semester's work, presents mathematical topics and the principles of computer operation and programming. It should be accessible to a wide variety of readers. Volume II, the larger volume, treats the "hardware" of digital circuits, and familiarity with basic electronics and electromagnetics is assumed. For one's personal study, or for a two-semester course for science or engineering students, these volumes should be well received. They would ordinarily be too much for a one semester course, particularly for students with scant training in the relevant electrical science. The reviewer would prefer a consecutive numbering of the pages to the pseudo-decimal system which is employed.

University of Illinois Franz E. Hohn

Rounding Errors in Algebraic Processes. By J. H. Wilkinson. Englewood Cliffs, N. J., Prentice-Hall, 1964. vi + 161 pp., $6.00.

The study of the cumulative effect of rounding errors in computations involving a large number of operations has been given expanded interest and significance by the development and increasing use of the digital computer. The present volume, by the leading investigator in the field, presents an elementary introduction to the subject and includes a number of simple analyses presented in a uniform manner. It is the only book of its kind and contains much material not elsewhere available. It is a most welcome addition to Prentice-Hall's series of books on automatic computation.

University of Illinois Franz E. Hohn

Introduction to ALGOL. By R. Baumann, M. Feliciano, F. L. Bauer, and K. Samelson. Englewood Cliffs, N. J., Prentice-Hall, 1964. x + 142 pp., $6.00

This volume of the Prentice-Hall Series in Automatic Computation is a particularly simple, highly readable introduction to the algorithmic language, ALGOL. Exercises are included so that the book is valuable for independent study. The ALGOL 60 Revised Report is included in an appendix.

University of Illinois Franz E. Hohn

College Calculus with Analytic Geometry. By M. H. Protter and C. B. Morrey, Jr. Reading, Mass., Addison-Wesley, 1964. xiv + 897 pp., $11.50.

This is a text designed for the usual three semester course in analytic geometry and calculus. The book does not attempt to be a text for a baby real variables course, but it is, for the most part, a readable text for the typical course in which much emphasis is placed on acquiring i) a strong intuitive grasp of the ideas of calculus and ii) an amount of manipulative skill. The book, as the publishers indicate in their advertising, is not suitable for an honors course in calculus.

On the whole, this book has a good mathematical spirit and flavor, stating theorems with a precision often lacking in calculus books. Unfortunately, the authors have made a number of errors--some of omission and others of commission--and some of these can be very disconcerting to both the student and the inexperienced teacher. For example, the authors define domain and range of a variable but fail to define explicitly domain and range of a function. Also, they show the same confusion shown by many undergraduates in not always properly distinguishing between a relative maximum, for example, and the number at which the function takes its relative maximum. Hopefully, subsequent printings will correct such errors as these.

Since problem sets are so important to a calculus course, it must be pointed out that problems are mainly of a routine numerical nature. The usefulness of this book would be greatly increased by including more challenging problems. In addition, the traditional solutions at the back of the book contain all too many errors.

It is a pleasure, however, to see a book such as this where the text does such a good job of illustrating the essential concepts of calculus. Most students should, once the flaws are removed, be able to read and enjoy this text with little aid from an instructor.

University of Illinois — Hiram Paley

~~Models for~~ Production and Operations Management. By Elwood S. Buffa. New York, John Wiley, 1963. xii + 632 pp., $9.25.

Professor Buffa is a professor of Production Management; the book is written for students of operations research, management science, and industrial engineering; the author deliberately minimizes the use of mathematics. In so far as mathematics is essential to the development of the models, this book can be interpreted as applied mathematics. However, the author emphasizes the empirical data to such an extent that we might prefer to agree with him that this is not a mathematics text.

Champaign, Illinois — Jane I. Robertson

New Directions in Mathematics. J. G. Kemeny, R. Robinson, and R. W. Ritchie, editors. Englewood Cliffs, New Jersey; Prentice-Hall; 1963. ii + 124 pp., $4.95.

This little book is a transcript of the proceedings of a conference entitled "New Directions in Mathematics" held at Dartmouth College on November 3rd and 4th, 1961, at the time of the dedication of the Albert Gradley Center for Mathematics. The conference was organized by Dr. John Kemeny and Dr. Robin Robinson of Dartmouth College.

The book includes thirteen papers by well-known mathematicians on four general topics: New Directions in Secondary School Mathematics, New Directions in College Mathematics, New Directions in Applied Mathematics, and New Directions in Pure Mathematics. Also included are transcripts of the four discussion periods following the presentation of each general topic.

Although it would indeed be difficult to uphold the lofty claim which is made in the review of the book appearing on the book's jacket, that the panels included "every (italics mine) aspect of mathematics from secondary school mathematics education, through applied and pure mathematical research," one does find many interesting and thought provoking statements on each of the four topics considered.

There is something in this book for every individual interested in the future of mathematics and mathematics education. Although the level of mathematical sophistication required to comprehend the details of some of the illustrations in the individual papers is relatively advanced, the central ideas of each paper and each discussion are well within the reach of any interested reader.

Here is a book in which one gets an insight into the thinking of prominent mathematicians relative to topics as varied and as new as the following: eighteen year olds at the second year graduate level (Leon Henkin), six-year Ph.D. programs (J. Laurie Snell), a science fiction calculus course (R. C. Buck), a computer appreciation course (H. O. Pollak), linear programming, and the development of "combinatorial linear algebra" (A. W. Tucker), the place of abstraction in the education of the young (Peter Lax), and many others.

In particular, one gets several insights into the humor of which mathematicians are capable. As one example, this book, in the chapter devoted to New Directions in Applied Mathematics, includes a good deal of discussion on the existence or non-existence of "applied mathematics" which finally leads the moderator to quote the definition of his colleague, Professor Keller, "Pure mathematics is a branch of applied mathematics".

Carleton College — Paul S. Jorgenson

The Algebraic Theory of Measure and Inteqration. By C. Caratheodory; F. E. J. Linton, translator. New York, Chelsea, 1963. 378 pp., $7.50.

The subject of this interesting book is measure theory on a generalized Boolean a-ring. Though the book is written on a level considerably above that of most undergraduates, it can certainly be read profitably by anyone acquainted with the fundamentals of measure theory and modern algebra. Such readers may find this book quite valuable because it is on a subject of considerably contemporary interest and because it is a classic in its field. The author was one of the foremost mathematicians of his time and in fact initiated the study of measures on Boolean a-rings.

The study of such generalized measures arises naturally from the following considerations. A great many results of measure theory actually depend on general theorems from modern algebra, particularly from ring theory and lattice theory. One can simplify measure theory considerably if one applies these general theorems instead of continually reproving special cases of them. But in order to apply modern algebra, one must develop measure theory algebraically, and it then turns out to be not much harder to develop a theory of measures on general Boolean a-rings, the elements of which need not be sets. One then obtains a very neat and elegant testament of a classical subject placed in modern context.

The theory developed by the author makes the proofs of some results of functional analysis more elegant and it also turns out to be a natural setting for the theory of probability. But one of the drawbacks to the present book is that most of the applications presented by the author are to situations involving measures on a-rings of sets, and the reader is left wondering exactly how useful the generalization to Boolean a-rings is. In fact, it has never yet proved essential in either functional analysis or probability to consider measures defined on things other than sets because of the Stone representation theorem--which says that any Boolean a-ring is almost a a-ring of sets. The translator has included several references in the footnotes to this and other representation theorems.

The book has several other drawbacks. Despite the neatness of the overall presentation, the book reads roughly in places which is probably due to the fact that the published version was compiled from notes left after the author's death. There are frequent glaring misprints, some of them in the statements of key theorems and definitions. The treatment is in general much too abstract to provide a good beginning course in measure theory. Finally, the only list of references, except for occasional footnotes, refers only to papers by the author, the last which was published in 1944. However, the book is in general very good, and certainly provides the best treatment in English of its subject.

University of Illinois — Charles W. Neville

Group Theory: The Application to Quantum Mechanics. By P. H. E. Meijer and E. Bauer. New York, Wiley-Interscience, 1963. xi + 288 pp., $9.75.

This little book is mainly a revised translation of a French monograph published in 1933 but with three additional chapters to encompass those areas of application which have been prominent since that time. The result is a book which will prove useful to workers who wish to hybridize the two fields of the book's title. The first three chapters provide a quick introduction to the elements (vector spaces, quantum mechanics, and group theory) which are necessary background to the main portion of the text. The mathematics is given primarily by definition with the occasional proof either being sketched or being done by analogy. In general, the physical scientist will benefit more from these chapters than the mathematician. The older applications are primarily those of the rotation group including a nice treatment of spinors. The new applications contain additional material on the rotation group including Racah coefficients, space groups with direct interest to solid state physics, crystal field chemistry, and the Jahn-Teller effect. The reader should be cognizant of tensor notation, which is often used. In addition, the rapid pace makes for difficult but rewarding reading. Examples are few but excellent and an appendix contains sample problems which will prove useful to those who wish to test their mastery.

TCI--University of Wisconsin — Boris Musulin

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BOOKS RECEIVED FOR REVIEW

A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski: Methods of Quantum Field Theory in Statistical Physics (R. A. Silverman, translator). Englewood Cliffs, N. J., Prentice-Hall, 1963. xv + 352 pp., $16.00.

P. L. Alger: Mathematics for Science and Engineering. New York, McGraw-Hill, 1963. xi + 366 pp., $2.95 (paper).

N. T. J. Bailey: The Elements of Stochastic Processes with Applications to the Natural Sciences. New York, Wiley, 1964. xi + 249 pp., $7.95.

S. F. Barker: Philosophy of Mathematics. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. ix + 111 pp., $1.50.

J. D. Baum: Elements of Point Set Topology. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. x + 150 pp., $5.95.

R. Baumann, M. Feliciano, F. L. Bauer, and K. Samelson: Introduction to ALGOL. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. x + 142 pp., $6.00.

J. D. Baumrin, Editor: Philosophy of Science: The Delaware Seminar, Vol. II. New York, Interscience, 1964. xviii + 551 pp., $14.50.

P. Benacerraf and H. Putnam, editors: Philosophy of Mathematics. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. vii + 536 pp., $8.95.

L. Bers, F. John, and M. Schechter: Partial Differential Equations. New York, Wiley-Interscience, 1964. xiii + 343 pp., $10.70.

H. Cartan: Elementary Theory of Analytic Functions of One or Several Complex Variables. Reading, Mass., Addison-Wesley, 1963. 288 pp., $10.75.

S. Drobot: Real Numbers. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. 102 pp., $3.95.

D. K. Faddeev and V. N. Faddeeva: Computational Methods of Linear Algebra. San Francisco, Freeman, 1963. xi + 621 pp., $11.50.

S. Feferman: Number Systems. Reading, Mass., Addison-Wesley, 1964. xii + 418 pp., $8.75.

B. A. Fuchs and B. V. Shabat: Functions of a Complex Variable and Some of Their Applications, Volume I. Reading, Mass., Addison-Wesley, 1964. xvi + 431 pp., $10.00.

L. Fuchs: Partially Ordered Algebraic Systems. Reading, Mass., Addison-Wesley, 1964. ix + 229 pp., $7.00.

I. M. Gelfand and S. V. Fomin: Calculus of Variations. Englewood Cliffs, N. J., Prentice-Hall, 1963. vii + 232 pp., $7.95.

H. Hadwiger, H. Debrunner, and V. Klee: Combinatorial Geometry in the Plane. New York, Holt, Rinehart and Winston, 1964. vii + 113 pp., $3.75.

F. L. Harmon and D. E. Dupree: Fundamental Concepts of Mathematics. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. ix + 233 pp., $5.95.

C. A. Hayes, Jr: Concepts of Real Analysis. New York, Wiley, 1964. xi + 190 pp., $6.50.

E. M. Hemmerling: Fundamentals of College Geometry. New York, Wiley, 1964. vii + 401 pp., $6.95.

A. M. Hilton: Logic, Computing Machines, and Automation. Cleveland, world, 1964. xxi + 428 pp., $2.95 (paper).

E. I. Jury: Theory and Application of the Z-Transform Method. New York, Wiley, 1964. xiii + 330 pp., $11.50.

J. G. Kemeny: Random Essays on Mathematics, Education, and Computers. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. ix + 163 pp., $4.95.

S. Kobayashi and K. Nomizu: Foundations of Differential Geometry, Vol. I. New York, Wiley, 1963. xi + 329 pp., $15.00.

S. Lang: Algebraic Numbers. Reading, Mass., Addison-Wesley, 1964. ix + 163 pp., $7.00.

S. Lang: A First Course in Calculus. Reading, Mass., Addison-Wesley, 1964. xii + 258 pp., $6.75.

A. H. Lightstone: The Axiomatic Method; An Introduction to Mathematical Logic. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. x + 246 pp., $5.95.

B. E. Meserve and M. A. Sobel: Introduction to Mathematics. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. ix + 290 pp., $5.95.

C. V. Newson: Mathematical Discourses: The Heart of Mathematical Science. Englewood Cliffs, N. J., Prentice-Hall, 1964. 125 pp., $5.00.

J. Plemelj: Problems in the Sense of Riemann and Klein. New York, Wiley-Interscience, 1964. vii + 175 pp., $8.00.

M. H. Protter and C. B. Morrey, Jr.: Modern Mathematical Analysis. x + 790 pp., $10.75.

M. H. Protter and C. B. Morrey, Jr.: College Calculus with Analytic Geometry. Reading Mass., Addison-Wesley, 1964. xiv + 897 pp., $11.50.

P. Ribenboim: Functions, Limits, and Continuity. New York, Wiley, 1964. vii + 140 pp., $5.95.

S. Sternberg: Lectures on Differential Geometry. Englewood cliffs, New Jersey, Prentice-Hall, 1964. xi + 390 pp., $12.00.

D. Thoro: The Second All-Russian Olympiad in Mathematics. Portland, Maine, J. Weston Walch, 1963.

J. F. Traub: Iterative Methods for the Solution of Equations. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. xviii + 310 pp., $12.50.

P. van de Kamp: Elements of Astromechanics. San Francisco, Freeman, 1964. $2.00 (paper), $4.00 (cloth).

J. H. Wilkinson: Rounding Errors in Algebraic Processes. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. vi + 161 pp., $6.00.

E. Williamson and M. H. Bretherton: Tables of the Negative Binomial Probability Distribution. New York, Wiley, 1964. 275 pp., $13.50.

B. K. Youse: Mathematical Induction. Englewood Cliffs, New Jersey, Prentice-Hall, 1964. 55 pp., $2.95.

NOTE: All correspondence concerning reviews and all books for review should be sent to PROFESSOR ROY B. DEAL, DEPARTMENT OF MATHEMATICS, OKLAHOMA STATE UNIVERSITY, STILLWATER, OKLAHOMA, 74075.

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