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GENERAL TERMS FOR REPEATING SEQUENCES

Steven Minsker, Brooklyn College

One of the many beauties of mathematics is that, as a language, it can state succinctly that which otherwise could not be concisely put. The general (or n^{th}) term particularly illustrates this. For instance, instead of exhibiting a sequence to be 0, 1, 0, 1, 0, 1, ..., we can simply say $T_n = \frac{1}{2}[1 + (-1)^n]$, where T_n will henceforth denote the n^{th} term of the sequence in question, or, instead of exhibiting the sequence 1, 5, 7, 31, 65, ... and then having to explain that the odd-numbered terms are equal to one less than two raised to that odd number while the even-numbered terms are equal to one more than two raised to that even number, we may write $T_n = 2^n + (-1)^n$.

The object of this paper is to find, in as "neat" a form as possible, the general term for the repeating sequence:

$$b_1, b_2, \ldots, b_j, a_1, a_2, \ldots, a_k, a_1, a_2, \ldots, a_k, a_1, a_2, \ldots, a_k, \ldots$$

where, after a finite number ($= j$) of terms, the sequence repeats itself in blocks of k terms. We shall choose the b's and a's so that they be real numbers. However, our conclusions hold equally well if they are complex numbers. [Note: I have chosen to call a sequence of this type "repeating" and not "recurring" because a recurring sequence has already been understood to be one in which the n^{th} term is a certain linear combination of a fixed number of directly preceding terms, such as Fibonacci's sequence = 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ... where $T_n = T_{n-1} + T_{n-2}$, $T_1 = 0$, $T_2 = 1$. To find a general term for such a sequence (for the Fibonacci sequence, $T_n = \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^{n-1} - (\frac{1-\sqrt{5}}{2})^{n-1}]$), involves a standard method using scales of relation, which is discussed in "Higher Algebra" by Hall and Knight. Enough said on this topic; however, we may ask the interesting question: If we take Fibonacci's sequence, and from it form a sequence using the last digits of the numbers of the original sequence, that is, 0, 1, 1, 2, 3, 5, 8, 3, 1, 4, ..., what type of sequence is obtained? Readers who like arithmetic may begin by adding.]

Before attacking the problem firsthand, let us consider three examples, the results of which will be useful later on.

__Example__ I. Sequence is $a_1, a_2, a_1, a_2, a_1, a_2, \ldots$

What is needed here is some operation which distinguishes between the cases n odd and n even; such an operation is $(-1)^n$. A little thought yields the result

$$T_n = \frac{1}{2}[1 + (-1)^n]a_2 + \frac{1}{2}[1 - (-1)^n]a_1$$

$$= \frac{a_1 + a_2}{2} + (-1)^n(\frac{a_2 - a_1}{2})$$

Example **II.** Sequence is a_1, a,, a_2, a,, a,, a_2, ...

What is needed is an operation which distinguishes between the cases $n = 1$ and $n > 1$. Such an operation is 2^{n-1}, which yields an odd number for $n = 1$, an even number for $n > 1$. Hence, $(-1)^{2^{n-1}}$ does the trick:

$$T_n = (1 + (-1)^{2^{n-1}})\frac{a_2}{2} + (1 - (-1)^{2^{n-1}})\frac{a_1}{2}$$

$$= \frac{a_1 + a_2}{2} + (-1)^{2^{n-1}}\left(\frac{a_2 - a_1}{2}\right).$$

If we introduce the operation [Z], where [Z] is defined as an integer such that $[z] \leq z < [z] + 1$, we can get a different n^{th} term above. Considering the sequence 1, 0, 0, 0, ..., we see that the n^{th} term here = $[\frac{1}{n}]$. Since the sequence a_1, a,, a,, a,, ... may be obtained from 1, 0, 0, 0, ... by multiplying each term of the latter by (a, − a,) and adding a,, we get $T_n = (a_1 - a_2)[\frac{1}{n}] + a_2$.

It must be added that the operation [Z] should be used only reluctantly, since, when it is used, one must take pains to say "where [Z] is the greatest integer contained in Z." In fact, whenever possible, use more "commonly known" operations in place of [Z], such as absolute value, as will be presently done.

Example **III.** Sequence is $\underbrace{a_1, a_1, a_1, \ldots, a_1}_{j \text{ terms}}, a_2, a_2, a_2, \ldots$ where $j \geq 2$

Call the n^{th} term of this sequence T_n', and consider

$\underbrace{0, 0, 0, \ldots, 0}_{j \text{ terms}}, 1, 1, 1, \ldots$

Call the n^{th} term of this sequence T_n. After some thought, we realize that $[\frac{n-1}{j}]$ has value 0 for $n \leq j$ and equals some positive integer for $n > j$, while $[\frac{j}{n}]$ has value 0 for $n > j$ and equals some positive integer for $n \leq j$. Hence,

$$T_n = \frac{[\frac{n-1}{j}]}{[\frac{n-1}{j}] + [\frac{j}{n}]}.$$

However, another operation which distinguishes between the cases $n \leq j$ and $n > j$ is $\frac{|n - j - \frac{1}{2}|}{n - j - \frac{1}{2}}$, which equals 1 for $n > j$, −1 for $n \leq j$. The factor $\frac{1}{2}$ is introduced to avoid a zero denominator when $n = j$. We thus obtain

$$T_n = \frac{1}{2}\left(1 + \frac{|n - j - \frac{1}{2}|}{n - j - \frac{1}{2}}\right).$$

To obtain the sequence a,, a,, ..., a,, a_2, a_2, ... from 0, 0, ... 0, 1, 1, ... we multiply each term by $(a_2 - a_1)$ and add a_1, analogous to what was done in Example **II**, so that Ti! = (a, − $a_2)T_n$ + a,.

We are now ready to attack the general repeating series. If we can find an n^{th} term for the sequence

$$\underbrace{0, 0, 0, \ldots, 0, 1}_{k \text{ terms}}, 0, 0, 0, \ldots, 0, 1, 0, 0, 0, 0, \ldots, 0, 1, \ldots$$

the rest will follow, as we shall see later.

When $k = 2$, we have already seen that $T_n = \frac{1}{2}[1 + (-1)^n]$ in the opening paragraph. consider now $k = 4$; that is 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, ... Without exploring the motivation we can say that, experimenting with (−1) raised to odd and even powers, we come up with the result:

$$T_n = \frac{1}{4}[(-1)^{\frac{n(n-1)}{2}} + (-1)^{\frac{n(n+1)}{2}} + (-1)^n + 1].$$

The reader may verify the validity of this general term. The important point, however, is that this result is rather isolated, and the method of using (−1) to various powers does not readily resolve other cases, in particular $k = 3$. Thus, this method will be abandoned in favor of three more general approaches.

A. Observe that $[\frac{n}{k}] = \frac{n}{k}$ if and only if k divides n. If k does not divide n, then $[\frac{n}{k}] < \frac{n}{k}$. Since the given sequence is such that $T_n = 0$ for n not divisible by k and $T_n = 1$ for n divisible by k, then $T_n = [\frac{[\frac{n}{k}]}{\frac{n}{k}}]$.

Although this is a nicely compact statement, we shall look for other methods for the reasons mentioned in Example **II**.

B. We must find an operation which distinguishes between the cases n divisible by k and n not divisible by k. The desired operation, resorting to trigonometry, is $\sin(\frac{n\pi}{k})$. Assume $k \geq 2$, since the case $k = 1$ (sequence = 1, 1, 1, 1, 1, ...) is solved immediately by $T_n = 1$. Consider:

$$\frac{\sin[(n+1)\frac{\pi}{k}]\sin[(n+2)\frac{\pi}{k}]\cdots\sin[(n+k-1)\frac{\pi}{k}]}{\sin(\frac{\pi}{k})\sin(\frac{2\pi}{k})\cdots\sin(\frac{(k-1)\pi}{k})}$$

If n is not divisible by k, then one of the terms $(n+1)$, $(n+2)$, ..., $(n+k-1)$ must be divisible by k, so that the above expression equals 0. If n is divisible by k (let n = pk), we see that $\sin[(n+1)\frac{\pi}{k}] = \cos p\pi \sin(\frac{i\pi}{k}) = \pm \sin(\frac{i\pi}{k})$ if p is even or odd respectively. If p, therefore, is odd, and k happens to be even, the above expression has value $= -1$. Since, under the circumstances, we want a value $= 1$, we can rectify the situation by taking an absolute value of the expression or by squaring it. (It will suffice to do this just to the numerator, but for purposes of symmetry, we will do this to the whole expression.) Squaring, and using product notation:

$$T_n = \prod_{\ell=1}^{k-1} \frac{\sin^a[(n+\ell)\frac{\pi}{k}]}{\sin^2[\ell \cdot \frac{\pi}{k}]}$$

C. A third method involves complex numbers. If we take the first of the k^{th} roots of unity $= w = \cos\frac{2\pi}{k} + i\sin\frac{2\pi}{k}$, then $w^n = 1$ if and only if n is a multiple of k. (This follows from De Moiore's Theorem.) It immediately becomes evident that:

$$T_n = \frac{(w^n - w)(w^n - w^2)\cdots(w^n - w^{k-1})}{(1-w)(1-w^2)\cdots(1-w^{k-1})}$$

If n is not a multiple of k, one of the terms in the numerator must equal 0; if n is a multiple of k, then $w^n = 1$ and $T_n = 1$.

The remaining problem is one of simplification. The numerator is a polynomial in the variable w^n with roots $= w, w^2, \ldots, w^{k-1}$. From the theory of complex variables, $(w^n)^k - 1 = 0$ is the polynomial in the variable w^n with roots $= w, {}^2, \ldots, w^{k-1}, 1$. Dividing synthetically by 1, we can obtain the polynomial with roots $= w, w^2, \ldots, w^{k-1}$:

$$\begin{array}{cccccccc|l} 1 & 0 & 0 & 0 & \ldots & 0 & -1 & & \underline{1} \\ & 1 & 1 & 1 & \ldots & 1 & 1 & & \\ \hline 1 & 1 & 1 & 1 & \ldots & 1 & \boxed{0} & & \end{array}$$

Hence the numerator of the general term $= (w^n)^{k-1} + (w^n)^{k-2} + \cdots + w^n + 1$. The denominator is obtained by replacing w^n by 1, so that it $= 1^{k-1} + 1^{k-2} + \cdots + 1^1 + 1 = k$.

In place of the "1" in the numerator, write w^0 for symmetry, so that:

$$T_n = \frac{1}{k}\sum_{\ell=0}^{k-1}(\cos\frac{2\pi}{k} + i\sin\frac{2\pi}{k})^{\ell n}.$$

But by De Moiore's Theorem:

$$T_n = \frac{1}{k}\sum_{\ell=0}^{k-1}\cos\frac{2\pi\ell n}{k} + \frac{i}{k}\sum_{\ell=0}^{k-1}\sin\frac{2\pi\ell n}{k}$$

For N a positive integer, T_n must be real, so that $\sum_{\ell=0}^{k-1}\sin\frac{2\pi\ell n}{k} = 0$ for such n. It can, in fact, be shown that this sum $\equiv 0$ for all real n. We may thus write:

$$T_n = \frac{1}{k}\sum_{\ell=0}^{k-1}\cos\frac{2\pi\ell n}{k}$$

The reader, using the identity

$$2\sin\frac{1}{2}\theta\,[\sum_{\ell=0}^{k-1}\cos\ell\theta] = \sin(k-\frac{1}{2})\theta + \sin\frac{1}{2}\theta$$

can easily verify this qeneral term.

We now want the general term for:

$$a_1, a_2, \ldots, a_k, a_1, a_2, \ldots, a_k, a_1, a_2, \ldots, a_k, \ldots$$

Call the qeneral term T_n^*. Note that, in any sequence's general term, substituting $(n+1)$ in place of n shifts the sequence one term to the left; that is, for example, if T_n is the n^{th} term of

$$\underbrace{0, 0, \ldots, 0, 1,}_{k \text{ terms}} 0, 0, \ldots, 0, 1, 0, 0, \ldots, 0, 1, \ldots$$

then the general term of

$$\underbrace{0, 0, \ldots, 0, 1, 0,}_{k \text{ terms}} 0, 0, \ldots, 0, 1, 0, 0, 0, \ldots, 0, 1, 0, \ldots$$

is $T_{(n+1)}$. Since

$$(a_1, a_2, \ldots, a_k)\ldots = a_k(0,0,\ldots,0,1)\ldots + a_{k-1}(0,0,\ldots,1,0)\ldots + \cdot\cdot\cdot + a_2(0,1,\ldots,0,0)\ldots + a_1(1,0,\ldots,0,0)\ldots,$$

it follows that

$$T_n^* = a_k T_n + a_{k-1}T_{n+1} + \cdot\cdot\cdot + a_2 T_{n+k-2} + a_1 T_{n+k-1}$$

We have already determined T_n in sections B and C.

Using the value obtained in section B, we get:

$$T_n^* = \sum_{i=1}^{k}\left[a_i \prod_{\ell=1}^{k-1} \frac{\sin^2[(n+\ell+k-i)\frac{\pi}{k}]}{\sin^2[\ell\frac{\pi}{k}]}\right].$$

However, $\sin[(n+\ell+k-i)\frac{\pi}{k}] \equiv -\sin[n+\ell-i)\frac{\pi}{k}]$ so

$$\text{(I)} \qquad T_n^* = \sum_{i=1}^{k}\left[a_i \prod_{\ell=1}^{k-1} \frac{\sin^2[(n+\ell-i)\frac{\pi}{k}]}{\sin^2[\ell\frac{\pi}{k}]}\right].$$

Or, in a completely analogous fashion, using the result of section C:

$$\text{(II)} \qquad T_n^* = \frac{1}{k}\sum_{i=1}^{k}\left[a_i \sum^{k-1} \cos\left[\frac{2\pi\ell(n-i)}{k}\right]\right]$$

At this point, the reader may wish to verify that the sequence $a_1, a_2, a_1, a_2, \ldots$ (that is, the case $k = 2$) has the general term (using equation I) of:

$$a_1 \sin^2[\frac{\pi n}{2}] + a_2 \sin^2[\frac{(n-1)\pi}{2}]$$

$$= a_1 \sin^2(\frac{\pi n}{2}) + a_2 \cos^2(\frac{\pi n}{2})$$

and (using equation II) of:

$$\frac{a_1 + a_2}{2} + [\cos(\pi n)] \cdot (\frac{a_2 - a_1}{2})$$

Compare these with Example I.

Now consider the final result desired, the n^{th} term of $b_1, b_2, \ldots, b_j, a_1, a_2, \ldots, a_k, a_1, a_2, \ldots, a_k, \ldots$ We already know how to distinguish between the cases $n > j$ and $n \leq j$ (See Example III). For now, consider the individual cases $n = 1, 2, \ldots, j$. We first desire an expression which equals 1 when $n = 1$ and which equals 0 for $n = 2, 3, \ldots, j$. This requires no new ingenuity; such an expression is similar in form to the ones originally formed in sections B and C:

$$\frac{(n-2)(n-3)\cdot\cdot\cdot(n-j)}{(1-2)(1-3)\cdot\cdot\cdot(1-j)}$$

(If $j = 1$, then let this expression be 1 identically.)

Similarly, for the particular case $n = 2$, consider:

$$\frac{(n-1)(n-3)(n-4)\cdot\cdot\cdot(n-j)}{(2-1)(2-3)(2-4)\cdot\cdot\cdot(2-j)}$$

In an analogous fashion, we develop "coefficients" for the cases $n = 3, 4, \ldots, j$. Now, starting with $n = j+1$, we wish to obtain the sequence $a_1, a_2, a_3, \ldots, a_1, a_1, a_2, \ldots, a_k, \ldots$ This can be easily accomplished, however, by substituting $(n-j)$ for n in either equation I or II. After such a substitution, we will get $T^*_{(n-j)} = a_1$ when $n = j+1$, $T^*_{(n-j)} = a_2$ when $n = j+2$, etc. When $n \leq j$, we get $T^*_{(\text{non-positive integer})}$ but T_n^* obviously exists for negative n and, moreover, we will have a multiplier in front which equals 0 for $n < j$.

Thus, we are now ready to establish the final result. The general (n^{th}) term for the sequence $b_1, b_2, \ldots, b_j, a_1, a_2, \ldots, a_k, a_1, a_2, \ldots, a_k, \ldots$, which is denoted by T_n, is given by the equation

$$T_n = \frac{1}{2}\left(1 - \frac{|n-j-\frac{1}{2}|}{n-j-\frac{1}{2}}\right)\left(\frac{(n-2)(n-3)\cdots(n-j)}{(1-2)(1-3)\cdots(1-j)}\, b_1 + \frac{(n-1)(n-3)\cdots(n-j)}{(2-1)(2-3)\cdots(2-j)}\, b_2 + \cdot\cdot\cdot + \frac{(n-1)(n-2)\cdots(n-[j-1])}{(j-1)(j-2)\cdots(j-[j-1])}\, b_j\right) + \frac{1}{2}\left(1 + \frac{|n-j-\frac{1}{2}|}{n-j-\frac{1}{2}}\right) T^*_{(n-j)}$$

where T_n^* is defined by either equation I or II

THE SCHRÖDER-BERNSTEIN THEOREM

Frank Bernhart, The University of Oklahoma

A cardinal number is any number which is used to answer the question "How many?" A good part of the mathematical history of numbers has been taken up with finding adequate symbolization for expressing finite cardinal numbers. The numerals used in counting form an ordered array, 1, 2, 3, ..., but they all belong to the set N of natural numbers. But the awareness of the natural numbers as a group invites us to ask 'How many natural numbers are there?', and the answer is not a natural number! We might also ask how many real numbers there are, and whether there are more real numbers than natural numbers. To say only 'They are infinite' is a weak answer to 'How many' and hardly touches the question 'Is this infinity more than that?'.

The modern notion of a cardinal number runs something like this:

1. We talk about sets, especially sets of numbers. To every set there should be associated a cardinal number. It is intended that two sets should have the same cardinal number if and only if they have the "same number" of elements.

2. If sets can be organized into groups of the "same number" without using anything called a cardinal number, then it will be possible to assign a cardinal number to the sets of each group. What these cardinal numbers are is of no particular importance if they can easily be identified with the proper group, and can be distinguished among themselves.

3. The usual choice for cardinal number of a class of sets is that class itself. For example, the cardinal 1 is just the class of all sets which contain but a single element. To be sure, other definitions of cardinal number may serve as well. The cardinal number of a class of sets might be one of those sets, picked to represent the class.

4. We have put off discussing what we could use to define the groupings. The criterion of "sameness" or equivalence, as it is more often called, is this: If there exists a way to pair off the elements in one set with the elements of another set so that each element in each set is matched with exactly one element of the other, the sets are equivalent. In other words, to say that sets X and Y are equivalent, written $(X \sim Y)$ means <u>there exists</u> a one-to-one mapping, or function, from X <u>onto</u> Y. It may seem perhaps strange that mappings are considered more basic than cardinal numbers, but that is the modern viewpoint, and it works very well.

5. Equivalence $(X \sim Y)$ defines a relation between sets. It has the three properties we would expect of an equivalence relation.

   (E1) $X \sim X$ (for all X). Reflexivity.
   (E2) If $X \sim Y$, then $Y \sim X$. Symmetry.
   (E3) If $X \sim Y$, and $Y \sim Z$, then $X \sim Z$. Transitivity.

   These properties are sufficient to separate all sets into equivalence classes (or cardinal numbers). Essentially, $X \sim Y$ if and only if X and Y belong to the same cardinal number.

Nothing more than the above idea would be needed if the relation $X \sim Y$ was all we had to contend with. We could take the infinite sets we know and show which were equivalent and which were not. But let us now ask how to tell when one cardinal number is <u>more</u> than another. Consider the following mapping from N <u>onto</u> a subset of N.

$f(n) = n + 3$

n	1	2	3	4	5	· · ·
f(n)	4	5	6	7	8	· · ·

If N^* is the set of values $f(n)$, then $N \sim N^*$, even though N^* is a proper subset of N. Now Cantor showed that $R \sim N$ is false, where R denotes the set of real numbers. But the mapping $f(n) = \pi^n$ from N to R identifies N with, or carries N <u>onto</u> a proper subset R^* of R, and is one-to-one. That is, $N \sim R^*$ and $R^* \subset R$. What do we wish to conclude? Simply that if $X \sim Y^*$ where Y^* is a subset of Y, then X certainly cannot have a bigger cardinal number than Y. Let $\bar{X}$ denote the cardinal number to which X belongs, and similarly for $\bar{Y}$, etc. We have seen that $\bar{X} = \bar{Y}$ if and only if $X \sim Y$. We are now defining $\bar{X} \leq \bar{Y}$ to mean that there is a subset Y^* of Y and $X \sim Y^*$. Actually the properties of sets should be invoked to show that the ordering of cardinal numbers $\bar{X}$, $\bar{Y}$ does not depend on the particular sets X, Y, representative of $\bar{X}$ and $\bar{Y}$, that are chosen. But it is true, and we will now simply write $X \leq y$ to save on notation. So we define $X \leq Y$ to mean there exists a one-to-one mapping from X <u>onto</u> a subset of Y (into Y). Concisely $X \leq Y$ to means "there is some Y^* and $X \sim Y^* \subset Y$".

We have suggested that with the relation $X \leq Y$ cardinal numbers can be ordered. Let us examine that supposition by looking at the properties order must have. (Since we are using sets instead of the cardinal numbers, $X \sim Y$ will occur where we would expect $X = Y$.)

(O1) $X \leq X$ (for all X). Reflexivity.
(O2) If $X \leq Y$ and $Y \leq Z$, then $X \leq Z$. Transitivity.
(O3) If $X \leq Y$ and $Y \leq X$, then $X \sim Y$. Anti-symmetry.
(O4) $X \leq Y$ and $Y \leq X$ not both false. (Used only for strong order.)

As it turns out, $\leq$ is a well-defined order, for all four properties are true--however, (O1) and (O2) are trivial, but (O3) and (O4) are not. We will say no more about (O4) since it requires the Axiom of Choice before it can be proved. It is the difficulty of (O3) that interests us. Let us try to see it more clearly with an example. Let Q denote the positive rational numbers.

$f:N \to Q \quad f(n) = n$ Therefore $N \leq Q$ (Both f and g are one-to-one)

$g:Q \sim N \quad g(a/b) = 2^a 3^b$ Therefore $Q \leq N$

We see that neither f nor g will do to establish $N \sim Q$. We need another mapping h, from N onto, not into, Q. To prove property (03) we need to show that given f and g ($X \leq Y$ and $Y \leq X$), it is always possible to find an h such that $X \sim Y$. The proof of 'If $X \leq Y$ and $Y \leq X$, then $X \sim Y$' is called the Schröder-Bernstein Theorem. It is a notable theorem of elementary cardinal number theory. Like many theorems in mathematics, it establishes a result that seems plausible or even obvious, but really requires some ingenuity to prove. The converse of the Schröder-Bernstein, 'If $X \sim Y$, then $X \leq Y$ and $Y \leq X$', is trivial, since $X \sim Y \subset Y$ and $Y \sim X \subset X$, X and Y being subsets of themselves.

proof of the Schroder-Bernstein Theorem.

Let us write $(X_1, X_2, X_3, \ldots) \sim (Y_1, Y_2, Y_3, \ldots)$ to mean $X_1 \sim Y_1$, $X_2 \sim Y_2$, $X_3 \sim Y_3$, etc and write $(-)(X_1, X_2, X_3, X_4, \ldots)$ to mean that $X_i \sim X_j$ for every i and j.

To show 'If $X \leq Y$ and $Y \leq X$, then $X \sim Y$.'

Let $A, B_1, B_2, B_3, \ldots C_1, C_2, \ldots$ be any collection of disjoint sets and likewise $A', B_1', B_2', B_3', \ldots C_1', C_2', \ldots$, such that $(-)(C_1, C_1', C_2, C_2', C_3, \ldots)$, $(\sim)(B_1, B_1', B_2, B_2', B_3, \ldots)$ and $A \sim A'$. Then obviously $(C_1, C_2, \ldots) \sim (C_1', C_2', \ldots)$ and $(B_1, B_2, \ldots) \sim (B_1', B_2', \ldots)$. Let $C = \cup(C_k)$, $B = \cup(B_k)$, $B' = \cup(B_k')$, and $C' = \cup(C_k')$. And let $X^* = A \cup B \cup C$ and $Y^* = A' \cup B' \cup C'$. consider the two mappings, f and g, defined as follows.

$f:X^* \to Y^*$ by $(A, B_1, C_1, B_2, C_2, \ldots) \sim (A', B_1', C_2', B_2', C_3', \ldots)$
$g:Y^* \to X^*$ by $(A', B_1', C_1', B_2', C_2', \ldots) \sim (A, B_2, C_1, B_3, C_2, \ldots)$

or $\quad f:A \sim A' \quad B_k \to B_k', \; C_k \to C_{k+1}'$, For $k = 1,2,3,4,5,6,\ldots$
$\quad g: A' \sim A, \; B_k' \sim B_{k+1}, \; C_k' \sim C_k$.

The sum of this information can be visualized in Fig. 1. We see that by $f:X^* \sim (Y^* - C_1')$, or $X^* \leq Y^*$; also by $g:Y^* \sim (X^* - B_1)$ or $Y^* \leq X^*$.

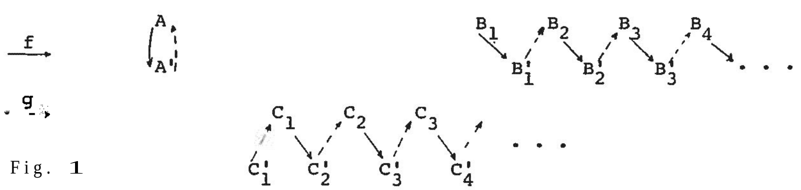


Fig. 1

Now define a function h from X^* to Y^* (See Fig. 2).

$$h(x) = \begin{cases} f(x) & \text{if } x \in A \text{ or } x \in B \\ y, \text{ where } y \text{ is the one element of } Y^* \text{ such that } g(y) = x, & \text{if } x \in C \end{cases}$$

That is, h is the function indicated by $(A, B_1, C_1, B_2, C_2, B_3, \ldots) \sim (A', B_1', C_1', B_2', C_2', B_3', \ldots)$. Clearly h gives $X^* \sim Y^*$.

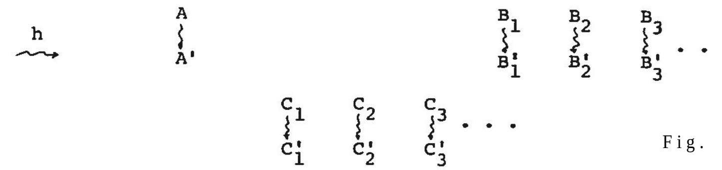


Fig. 2

The results so far are suspiciously like what we wish to prove for X and Y. The space taken with X^* and Y^* will be justified if we show that $X = X^*$ and $Y = Y^*$ for suitably chosen sets A, A', B_1, B_1', etc.

Let f and g be the mappings which exist by virtue of $X \leq Y$ and $Y \leq X$. Let us suppose, as we may, that $f:X \sim Y_1 \subset Y$, and $g:Y \sim X_1 \subset X$. Define $B_1 = (X - X_1)$; $C_1' = (Y - Y_1)$. Then define inductively the other sets in the B, C, B', C', series by the formulas:

$\{f:B_k \to B_k' \quad \{g:C_k' \to C_k \quad k = 1,2,3,4,$
$\{g:B_k' \to B_{k+1} \quad \{f:C_1 \to C_{k+1}' \quad 5,6,7,\ldots$

All the sets defined are distinct, because f and g are one-to-one functions. Otherwise, working backward in the formulas, if B_{j+1} were not distinct from B_{j+k+1}, then B_j' and B_{j+k}' would have overlapped too, and then B_j and B_{j+k}, and so on until we found that B_1 and B_k had elements in common. But this is impossible, since B_k, $k \# 1$, was defined by the mapping g, whereas B_1 is not included in the mapping g by definition. That is, $B_1 = X - X_1$, but all other $B_k \subset X_1$. The same tale can be told to cover all possibilities, so that we are sure of $A, B_1, C_1, B_2, \ldots$ and $A', B_1', C_1', B_j, \ldots$ each being distinct collections.

As an easy corollary to these definitions we see that $C_k' \sim C_k \sim C_{k+1}'$ and $B_k \sim B_k' \sim B_{k+1}$. By repeated use of property (E3) of the relation $\sim$ we establish that $(\sim)(B_1, B_1', B_2, B_2', B_3, \ldots)$ and $(\sim)(C_1, C_1', C_2, C_2', C_3, \ldots)$. Let $B = \cup(B_k)$, $C = \cup(C_k)$, $B' = \cup(B_k')$, and $C' = \cup(C_k')$. Then we see that if we let $A = X - (B \cup C)$ and $A' = Y - \cup(C_k')$. Then we see that if we let $A = X - (B \cup C)$ and $A' = Y - (B' \cup C')$, it must be true that $f:A \sim A'$ and $g:A' \sim A$, hence $A \sim A'$.

The assumption that $X = X^*$ and $Y = Y^*$ is now justified by the choice of the sets A, A', etc. and the demonstration that the choice was in line with the requirements on these sets at the beginning of the proof. The Theorem is now proved.

The Schröder-Bernstein Theorem is not hard to prove, once it is seen that the mappings f and g derived from $X \leq Y$ and $Y \leq X$ actually disect X and Y into parts precisely as is displayed in Fig. 1. For it is an easy step from Fig. 1 to Fig. 2. This seems long because it takes space to write down all the information contained in Fig. 1 in a rigorous manner.

There is one other reason why the proof was offered in more detail than necessary. It is important to see how this trick with prechosen sets characterizes and makes transparent the core of the proof. Over the years, since proof was first supplied independently by Schröder and by Bernstein in 1896 and 1899, various proofs have been found which are in the literature. But these have the disadvantage of appearing to use independent means of arriving at the conclusion, and both the proofs themselves, and their interrelations are not always obvious. The proof given here was developed from a comparative study, and should not only show the method clearly, but by comparison reveal that all the proofs in the literature studied by this author (about 5) are totally equivalent. All produce exactly the same mapping h. (Any property of the elements under the mapping which allow A, B, C and A', B', C' to be distinguished will provide a proof. A study of Fig. 1 can suggest several such properties, such as the "chain" idea used by Birkhoff-Maclane.) To my knowledge this powerful observation seems not to have been made explicitly.

Reading Suggestions

1. Brewer, Joseph, Introduction to the Theory of Sets (page 41).
2. Stoll, Robert, Set Theory and Logic (page 81).
3. Suppes, Patrick, Axiomatic Set Theory (page 94).
4. Quine, W. v. O., Set Theory and its Logic (page 204).

* * *

UNDERGRADUATE RESEARCH PROJECT

Proposed by PAUL SAMUEL, Minneapolis.

H. S. M. Coxeter, in his Real Projective plane, shows how to modify his axiom system in order to develop Complex Plane Projective Geometry. Carry out this synthetic development (or a portion thereof) and contrast it with the development in projective Geometry by Veblen and Young. What has to be added to Coxeter's axiom system in order to extend it to three-dimensional Complex Projective Geometry? Are any of his axioms rendered dependent by the ones you have added?

PROBLEM DEPARTMENT

Edited by
M. S. Klamkin, Ford Scientific Laboratory

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication.

An asterisk (*) placed beside a problem number indicates that the problem was submitted without a solution.

Address all communications concerning problems to M. S. Klamkin, Ford Scientific Laboratory, P. O. Box 2053, Dearborn, Michigan 48121.

PROBLEMS FOR SOLUTION

182. Proposed by GABRIEL ROSENBERG, Temple University.

Prove that an odd perfect number cannot be a perfect square.

Editorial Note: Although Euler established the more general result that every odd perfect number must be of the form $p^{4a+1}k^2$ where p is a prime of the form $4n + 1$, the above problem is given since it has an elegant solution in addition to the fact that there is a shortage of proposals.

183. Proposed by R. PENNEY, Ford Scientific Laboratory.

If

$$r_0(r_1r_2 + r_2r_3 + r_3r_1) = 0 \quad ,$$

$$r_1(r_0r_2 + r_0r_3 - r_2r_3) = 0 \quad ,$$

$$r_2(r_0r_3 + r_0r_1 - r_3r_1) = 0 \quad ,$$

$$r_3(r_0r_1 + r_0r_2 - r_1r_2) = 0 \quad ,$$

show that at least one of the quantities r_0, r_1, r_2, r_3 vanishes.

184. Proposed by ALAN LAMBERT, University of Miami.

Find a parametric representation for the transcendental curve given by the equation

$$x^y = y^x .$$

185. Proposedby A. E. LIVINGSTON and L. MOSER, University of Alberta, Edmonton.

Given: $f(n) = f(n+3), \quad n = 1,2,\ldots,$

$$f(1) = 1 \quad , \quad \sum_{n=1}^{\infty} f(n)/n = 0 \; .$$

Find $f(2)$.

186. Proposed by LT. JONES-BATEMAN, U.S.C.G. and M. S. KLAMKIN, Ford Scientific Laboratory.

Two ships are steaming along with constant velocities. What is the minimum number of bearings necessary to be taken by one ship in order to determine the velocity of the other ship? Given this requisite number of bearings, show how to determine the other ship's velocity. (It is assumed that range-finding equipment is either non-existent or non-operative.)

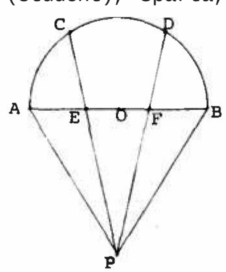
SOLUTIONS

172. Proposed by JOHN BAUDHUIN, Sparta High School (student), Sparta, Wisconsin.

Given: Semi-circle O with diameter AB and equilateral triangle PAB; C and D are trisection points of $\overset{\frown}{AB}$ (i.e., $\overset{\frown}{AC} = \overset{\frown}{CD} = \overset{\frown}{DB}$).

Prove: E and F are trisection points of $\overline{AB}$.

Note: A synthetic proof is desired.



Solution by Gabriel Rosenberg, Temple University

By Symmetry, AE = FB and OE = OF. Since $\angle$ AOC = $\angle$ EAB = 60°, Δ AEP ~ Δ EOC. Since also, CO = $\frac{1}{2}$ AP, OE = $\frac{1}{2}$ AE and, thus, E and F are trisection points of AB.

Joe Konhauser, University of Minnesota, notes that if $\overline{AB}$ is divided into n equal segments and then equilateral triangles are erected on these segments as bases, the lines joining P to the vertices off $\overset{\frown}{AB}$ will (n + 1)-sect $\overline{AB}$. The proposed problem corresponds to n = 2.

He also notes that the problem has appeared previously in the Mathematics Teacher, Nov. 1953, p. 524. There the construction was submitted by R. J. Orr, Iowa State Teachers College and was originated and solved by Patty Hake, a student.

Also solved by P. J. Campbell (University of Dayton), R. C. Gebhardt (Parsippany, N. J.), R. Prielipp (University of Wisconsin), W. J. Rickert (Rutgers - The State University), N. H. Sleep (Michigan State University), S. Spital (California State Polytechnic college), C. W. Trigg (San Diego, Cal.) and the proposer.

Editorial **Note:** If the semi-circle AB and diameter AB are both n-sected and corresponding points are joined by straight lines, the lines will only be concurrent for n = 3 or 4.

173. Proposed by K. S. MURRAY, Ann Arbor, Michigan.

If

$$D^n[\Phi(x)/x] = \Psi_n(x)/x^{n+1} \; ,$$

show that

$$D\Psi_n(x) = x^n D^{n+1}\Phi(x) \; .$$

Solution by J. R. Kuttler and N. Rubinstein, University of Maryland

$$D^{n+1}[\Phi(x)/x] = \Psi_n'/x^{n+1} - (n+1)\Psi_n/x^{n+2}$$

or

$$\Psi_n' = x^{n+1}D^{n+1}[\Phi(x)/x] + (n+1)x^n D^n[\Phi(x)/x] \; .$$

Then by Leibniz's rule for the n^{th} derivative of the product $x \cdot \Phi(x)/x$,

$$D\Psi_n(x) = x^n D^{n+1}\Phi(x) \; .$$

Also solved by H. Kaye (Brooklyn, N.Y.), and the proposer.

Editorial Note: Also by Leibniz's rule,

$$\Psi_n(x) = \sum_{r=0}^{n} (-1)^r \binom{n}{r} x^{n-r} D^{n-r} \Phi(x)$$

and the result follows by differentiation.

Additionally, since

$$\Psi_{n+1}' = x\Psi_n'' - n\Psi_n' \; ,$$

the solution of the differential-differenceequation

$$F_{n+1} = xF_1' - nF_1 \quad , \quad F_0 = F_0(x),$$

is given by

$$F_n(x) = x^n D^n F_0(x).$$

174. Proposed by C. S. VENKATARAMAN, Sree Kerzala Varma College, Trichur, South India.

Find the locus of a point which moves such that the squares of the lengths of the tangents from it to three coplanar circles are in arithmetic progression.

Solution by Charles W. Trigg, San Diego, California.

Let the circles be

$$x^2 + y^2 + a_i x + b_i y + c_i = 0 \quad , \quad i = 1,2,3.$$

Then the tangents from the point $P(x,y)$ to the circles are given by

$$t_i^2 = x^2 + y^2 + a_i x + b_i y + c_i .$$

Since the squares of the tangents are to be in arithmetic progression,

$$2t_2^2 = t_1^2 + t_3^2 .$$

Thus the locus of P is the straight line

$$(2a_2 - a_1 - a_3)x + (2b_2 - b_1 - b_3)y + 2c_2 - c_1 - c_3 = 0 .$$

Also solved by P. Meyers (Philadelphia, Pa.), M. Wagner (N.Y.C.), F. Zetto (Chicago, 111.) and the proposer.

175. Proposed by R. C. GEBHARDT, Parsippany, New Jersey.

The twenty-one dominoes of a set may be denoted by (1,1), (1,2), ••• (1,6), (2,6), •••, (6,6).

(a) Is there any arrangement of these, end-to-end with adjacent ends matching, such as ··· (3,1) (1,1) (1,6) (6,4) ···, such that all twenty-one dominoes may be involved?

(b) What conditions must a general set of dominoes satisfy in order that such an arrangement in (a) exists?

Editorial Note: A related problem would be to find the largest and the smallest completed chain which can be formed with a given set of general dominoes.

Solution by Craig A. Kalicki, John Carroll University.

Each digit appears seven times. Any digits that are strictly interior to the chain must occur an even number of times. Thus if a given digit is to appear an odd number of times it must occur at an end point. Since there are only two end points, there is no such arrangement.

In general, the chain can be constructed from the set (1,1),···,(n,n) if and only if n is odd or $n = 2$.

Solution by Neal Felsinger, SUNY at Buffalo.

Consider a set of points P_r $(r = 1,2,...,n)$ in general position in space. If (i,j) is a domino used in the arrangement, draw a line between P_i and P_j. For the domino (i,i), draw a line from P_i to itself. The stated problem has now been reduced to traversing the network of lines such that each line is traced exactly once. Using Euler's results, the network must be connected and at most two vertices can be odd. Translated to dominoes, you must be able to get from one number to any other number by placing dominoes end to end (connectiveness) and there can be at most two numbers occurring an odd number of times. For the set of dominoes (1,1), •••, (n,n), this is possible, if and only if, n is odd or $n = 2$.

Also solved by C. W. Trigg (San Diego, Cal.), M. Wagner,(N.Y.C.) and the proposer.

Editorial Note: The "smallest" chain that can be formed from the dominoes (1,1), •••, (n,n) is $2n-2$ (n-odd) and $2n-1$ (n-even) (Note that a chain is incomplete if another domino can be added).

The problem of determining the largest chain is equivalent to finding the maximum number of edges in an Eulerian tour of a complete graph of n vertices. If n is odd it follows from above that the number is $\binom{n}{2}$, the entire set of edges. If n is even, the number is $\binom{n}{2} - (\frac{n}{2} - 1)$.

176. Proposed by M. S. KLAMKIN, Ford Scientific Laboratory.

Determine all continuous functions $F(x)$ in $[0,1]$, if possible, such that $F(x^2) = F(x)^2$ and

(a) $F(0) = F(1) = 0$,

(b) $F(0) = F(1) = 1$,

(d) $F(0) = 0, \quad F(1) = 1$

(d) $F(0) = 1, \quad F(1) = 0$.

Solution by K. S. Murray, Ann Arbor, Michigan.

Clearly, $F(x^2) = F(x)^2$.

Thus for $0 \le x \le 1$,

$$\lim_{n\to\infty} x^{2^n} = 0 \ , \qquad \lim_{n\to\infty} x^{2^{-n}} = 1 \ ,$$

$$\lim_{n\to\infty} F(x)^{2^n} = \begin{cases} \infty \text{ if } |F(x)| > 1 \ , \\ 1 \text{ if } |F(x)| = 1 \ , \\ 0 \text{ if } |F(x)| < 1 \ , \end{cases}$$

$$\lim_{n\to\infty} F(x)^{2^{-n}} = 1 \text{ if } F(x) > 0 \ .$$

Since $F(x)$ is continuous, we then must have

(a) $F(x) = 0$,

(b) $F(x) = 1$,

(d) impossible.

For case (c), there are infinitely many functions satisfying the requirements. One simple example is $F(x) = x^c$. For a wider class of functions, arbitrarily define $F(x)$ in the interval $[\frac{1}{n^2}, \frac{1}{n}]$ $(n > 1)$ such that it is continuous and that $F(1/n^2) = F(1/n)^2$. Then, $F(x)$ is continuous and defined over the interval $[0,1]$ by means of the functional relation $F(x^2) = F(x)^2$.

Also solved by R. C. Gebhardt (Parsippany, N. J.), and F. Zetto (Chicago, 111.) and the proposer.

1967 MEETING

Your chapter is encouraged to nominate your best speaker who will HOT have a masters degree by April, 1967, as a speaker for the Pi Mu Epsilon meetings to be held in Toronto, Canada, August 28-29, 1967. As usual the national office will pay partial expenses for speakers. Nominations should be submitted by February 25, 1967, and mailed to Dr. Richard V. Andree, Pi Mu Epsilon, The University of Oklahoma, Norman, Oklahoma 73069.

BOOK REVIEWS

Edited by
Roy B. Deal, Oklahoma State University

Approximation of Functions. By G. G. Lorentz. Holt, Rinehart and Winston, New York, Athena Series.

An excellent book for Pi Mu Epsilon members who know some advanced calculus and can pick up, occasionally, some elementary theory of functions of a real variable, having interest for many mathematics, physical science, and engineering majors. It has modern results and modern treatments of classical results in approximations of functions, mostly real, with elegant, simple proofs and excellent exposition, including enlightening, historical notes or concept remarks, interesting exercises, and a good bibliography. Except for some results on Chebyshev systems, Kolmogorov's concept of entropy, and a few auxiliary notions on polynomials and moduli of continuity the central notion is that of degree of approximation.

Square Summable Power Series. By de Branges and James Rovnyak. New York, Holt, Rinehart and Winston, Athena Series, 1966. viii + 104 pp.

An introduction to complex complete vector spaces with scalar products (Hilbert spaces). In particular the so called l_2 space of sequences for which the sum of squares of the absolute values converges, with a fairly extensive treatment of invariant subspaces using the concepts developed by considering the sequences as coefficients of a formal power series. This leads to a fairly complete introduction to complex function theory in the second part of the book. The approach is basically to develop the material through an interesting selection of problems which is good for self-study for those with the motivation and perseverance. It is a necessary prerequisite to the paper on "Canonical Models in Quantum Scattering Theory" in the proceedings on "Peturbation Theory and its Applications in Quantum Mechanics" Listed in the "Books Received" list in this journal.

The Elements of Integration. By Robert G. Bartle. New York, John Wiley and Sons, Inc., 1966. x + 129 pp., $6.95.

A well written "short but thorough treatment of Lebesgue integration", containing its own development of measure concepts needed, for the reader with an advanced calculus background.

Elementary Differential Geometry. By Barrett O'Neill. New York, Academic Press, Inc., 1966. viii + 411.

A study of the classical geometry of curves and surfaces designed to introduce the reader to some basic ideas in the rapidly growing fields of modern differential geometry and differential topology. Except for the maturity sufficient to accept an abstract definition of a familiar concept, a strong elementary calculus course and some linear algebra provides an adequate background.

Theory and Problems of General Topology. By Seymour Lipschutz. New York, Schaum Publishing Company, 1965. 239 pp., $3.50.

With the exceptions of filters, nets, and uniform spaces (uniform concepts in metric and normed spaces are included) this book provides a rather thorough self-study introduction to the general topology necessary to studying analysis including some function space theory. It is within the reach of any well motivated undergraduate and provides an excellent background for advanced calculus.

Elementary Probability. By Edward O. Thorp. New York, John Wiley and Sons, Inc., 1966. x + 152 pp., $4.95.

A brief but excellent introduction to probability theory by the professor famous for beating the Nevada casinos at blackjack. The first three chapters provide the fundamental concepts in the discrete case with many interesting examples and exercises, requiring very little mathematical background. Elementary calculus suffices for the last chapter which gives a short provocative basic introduction to continuous probability. problems designed for computer use are included with very little programming knowledge required.

Linear Geometry. By Rafael Artzy. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1965. ix + 273 pp.

The Geometry of Incidence. By Harold L. Dorwart. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1966. xvii + 156 pp.

Introduction to Projective Geometry and Modern Algebra. By Robert A. Rosenbaum. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc 1963. viii + 344 pp.

Three interesting books on geometry. Dorwart's book is a brief expository and historical account of the fundamental concepts of incidence in classical projective geometry. Artzy and Rosenbaum skillfully interweave modern linear algebra into the study of classical geometry. Of these two Rosenbaum's is the more elementary, using matrices more extensively than vector spaces in the computations, providing more algebraic detail, and sticking primarily to properties of projective geometry, with the exception of conics, which hold in all finite dimensions. Artzy uses vector spaces for these studies and in addition devotes about half of the book to non-Euclidean geometries, Cayley-Klein models, Cayley algebras, quaternions, elliptic three-space, and the often avoided coordinatization in nondesarguesian planes, with Veblen-Wedderburn planes, Monfang planes, Desarguesian planes, Pappian planes and other axiomatics. Artzy has more affine and Euclidean geometry but presupposes, for ease in reading, some familiarity with modern algebra whereas Dorwart and Rosenbaum are accessible to any reader with high school geometry and a little analytics.

Discrete and Continuous Methods in Applied Mathematics. By. J. C. Mathews and C. E. Langenhop. New York, John Wiley and Sons, Inc., 1966. xiii + 525 pp., $10.95.

The chapter headings indicate the modern selection of topics in applied mathematics interwoven as indicated in the title. It presupposes a standard elementary calculus background. The headings in order are: Discrete Probability Theory, Linear Algebra, Finite Markov Chains, Linear Programming, Capacitated Transport Networks, Ordinary Differential Equations, Linear Systems of Differential Equations, Nonlinear Differential Equations and Stability, and Stochastic Processes.

Variational Principles. By. B. L. Moiseiwitsch. New York, Interscience Publishers, a division of John Wiley and Sons, Inc., 1966. x + 310 pp., $14.00.

By writing succinctly and elegantly in the mathematical physics style the author is able to provide the reader who has a background of advanced calculus and three or four non-elementary undergraduate physics courses with an excellent selection of results in analytical dynamics, optics, wave mechanics, quantum mechanics, field equations, eigenvalue problems, and scattering theory within the unifying concepts of calculus of variations.

$100 cash prize award

for the best expository paper published in the Pi Mu Epsilon Journal by an undergraduate student goes to:

James Williams
Carleton College

for his article on A MULTIPLICATION-FREE CHARACTERIZATION OF RECIPROCAL ADDITION published in Volume 4 Number 4, Spring 1966, pp. 168 - 176.

BOOKS RECEIVED FOR REVIEW

Richard G. Cooke. Infinite Matrices and Sequence Spaces. New York, Dover Publications, Inc., 1955. xiii + 347 pp., $2.50.

F. N. David. Research Papers in Statistics. New York, John Wiley and Sons, Inc., 1966. viii + 468 pp., $16.75.

Henry E. Fettis and James C. Caslin. Ten Place Tables of the Jacobian Elliptic Functions. Aerospace Research Laboratories, United States Air Force, Wright-Patterson Air Force Base, Ohio, 1966. iv + 109 pp.

Daniel T. Finkbeiner, 11. Introduction to Matrices and Linear Transformations, Second Edition. San Francisco, California, W. H. Freeman and Company, 1966. xi + 297 pp., $7.75.

Andrew Gray and G. B. Mathews. A Treatise on Bessel Functions and Their Applications to Physics. New York, Dover Publications, Inc., 1966. xiii + 327 pp., $2.25.

J. Hartmanis and R. E. Stearns. Algebraic Structure Theory of Sequential Machines. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1966. viii + 211 pp., $12.00.

John B. Harvill, Basic Fortran Programming. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1966. 264 pp.

Banesh Hoffmann. About Vectors. Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1966. ix + 134 pp., $3.25.

J. A. Hummel. Vectors. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1965. vii + 108 pp.

Robert C. James. Advanced Calculus. Belmont, California, Wadsworth Publishing Company, Inc., 1966. x + 646 pp., $12.50 (Text).

Samuel J. Karlin and William J. Studden. Tchebycheff Systems: With Applications in Analysis and Statistics. New York, John Wiley and Sons, Inc., 1966. xviii + 586 pp., $16.00.

Serge Lang. A Second Course in Calculus. Reading, Massachusetts, Addison-Wesley Publishing Company, Inc., 1964. xii + 242 pp.

Cyrus Colton MacDuffee. & Introduction & Abstract Algebra. New York, Dover Publications, Inc., 1966. vii + 303 pp., $2.25.

Edwin E. Moise. Calculus: Part I. Reading, Massachusetts, Addison-Wesley Publishing company, Inc., 1966. ix + 498 pp.

Charles B. Morrey, Jr. University Calculus with Analytic Geometry. Reading, Massachusetts, Addison-Wesley publishing Company, Inc., 1962. xiv + 754 pp.

Ivan Niven and Herbert S. Zuckerman. An Introduction to the Theory of Numbers, 2nd Ed. New York, John Wiley and Sons, Inc., 1966. viii + 280 pp., $7.95.

John M. H. Olmsted. Calculus With Analytic Geometry. New York, Appleton-Century-Crofts, 1966. xiv + 655 pp., $7.00.

Sam Perlis. Introduction to Algebra. Waltham, Massachusetts, Blaisdell Publishing Company, 1966. xx + 440 pp., $9.50.

L. B. Rall. Error in Digital Computation, Volume 11. New York, John Wiley and Sons, Inc., 1965. x + 288 pp., $6.75.

Shepley L. Ross. Introduction to ordinary Differential Equations. Waltham, Massachusetts, Blaisdell Publishing Company, 1966. viii + 337 pp., $7.50.

Seaton E. Smith, Jr. Explorations in Elementary Mathematics. Englewood cliffs, New Jersey, Prentice-Hall, Inc., 1966. xv + 280 pp.

Paul A. White. Linear Algebra. Belmont, California, Dickenson Publishing Company, Inc., 1966. x + 323 pp.

Calvin H. Wilcox. Perturbation Theory and its Applications in Quantum Mechanics. New York, John Wiley and Sons, Inc., 1966. x + 428 pp., $6.95.

NOTE: All correspondence conerning reviews and all books for review should be sent to PROFESSOR ROY B. DEAL, DEPARTMENT OF MATHEMATICS, OKLAHOMA STATE UNIVERSITY, STILLWATER, OKLAHOMA, 74075.

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