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A FORMULA FOR THE DERIVATIVE OF THE ABSOLUTE VALUE OF A POLYNOMIAL

Margaret R. Wiscamb

The University of St. Thomas, Houston, Texas

In a calculus class a student noticed that $D_x|x| = \frac{|x|}{x}$ and $D_x|x^2 - 4| = \frac{|x^2 - 4|}{x^2 - 4}(2x)$.

This led to the question:
If $P(x)$ is a polynomial, is the following formula valid?

$$D_x|P(x)| = \frac{|P(x)|}{P(x)} \cdot P'(x)$$

The class quickly found a counterexample. If $P(x) = x^3$, then $|P(x)|$ has a derivative everywhere, hence the formula, which would indicate that the derivative did not exist at the origin, is not valid in this case.

This brought up the question, what conditions can we impose on $P(x)$ so that the formula will hold? One student suggested the condition: if $P(a) = 0$, then $P'(a) \neq 0$, while another noticed that in all cases for which the formula held, the roots were distinct.

After considerable labor and prodding by the instructor, the following theorem emerged:

<u>Theorem</u>: If $P(x)$ is a polynomial, then the following three conditions are equivalent:

(i) If a is real and $P(a) = 0$, then $P'(a) \neq 0$.
(ii) The real roots of $P(x)$ are distinct.
(iii) $D_x|P(x)| = \frac{|P(x)|}{P(x)} \cdot P'(x)$

(i) $\Rightarrow$ (ii) Suppose $P(x)$ has a repeated real root, a,

Then by the Factor Theorem, $P(x) = (x - a)^2Q(x)$.

$P'(x) = (x - a)^2Q'(x) + 2\,Q(x)(x - a)$ and $P'(a) = 0$.

(ii) $\Rightarrow$ (i) Suppose $P(a) = 0$, a real. Then
$P(x) = (x - a)Q(x)$, and since $P(x)$ has no repeated real roots, $Q(a) \neq 0$. Then
$P'(x) = (x - a)Q'(x) + Q(x)$ and
$P'(a) = Q(a) \neq 0$.

(ii) $\Rightarrow$ (iii) Proof by induction on the number of real roots of $P(x)$. First we state two preliminary lemmas whose proofs are obvious.

Lemma 1. If $Q(x)$ is a polynomial having no real roots, then

$$D_x|Q(x)| = \frac{|Q(x)|}{Q(x)} \cdot Q'(x).$$

Lemma 2. If $P(x) = x - r$, r real, then

$$D_x|x - r| = \frac{|x - r|}{x - r}$$

Proof of (ii) $\Rightarrow$ (iii): First we show that (iii) holds if $P(x)$ has only one real root. Then $|P(x)| = |(x - r_1)Q(x)|$ where $Q(x)$ has no real roots.

$$|P(x)| = |(x - r_1)| \cdot |Q(x)|$$

$$D_x|P(x)| = |x - r_1| \cdot D_x|Q(x)| + |Q(x)| \cdot D_x|x - r_1| \text{ and by}$$

the lemmas, this is

$$|x - r_1| \frac{|Q(x)|}{Q(x)} Q'(x) + |Q(x)| \frac{|x - r_1|}{x - r_1}$$

$$= \frac{|(x - r_1)Q(x)|}{(x - r_1)Q(x)} \cdot D_x[(x - r_1)Q(x)]$$

Next we assume that (iii) holds for a polynomial having exactly k distinct real roots. He want to show that (iii) is valid if $P(x)$ has exactly k + 1 distinct real roots.

$$|P(x)| = |(x - r_1)(x - r_2) \ldots (x - r_{k+1})Q(x)|$$

where $Q(x)$ has no real roots.

$$D_x|P(x)| = |(x - r_1)(x - r_2) \ldots (x - r_k)| \cdot D_x|(x - r_{k+1})Q(x)|$$

$$+ |(x - r_{k+1})Q(x)| \cdot D_x|(x - r_1)(x - r_2) \ldots (x - r_k)|$$

and by the induction hypothesis

$$|(x - r_1) \ldots (x - r_k)| \frac{|(x - r_{k+1})Q(x)|}{(x - r_{k+1})Q(x)} \cdot D_x[(x - r_{k+1})Q(x)]$$

$$+ |(x - r_{k+1})Q(x)| \frac{|(x - r_1) \ldots (x - r_k)|}{(x - r_1) \ldots (x - r_k)} \cdot D_x[(x - r_1) \ldots (x - r_k)]$$

$$= \frac{|(x - r_1) \ldots (x - r_k)(x - r_{k+1})Q(x)|}{(x - r_1) \ldots (x - r_k)(x - r_{k+1})Q(x)} \cdot D_x[(x - r_1) \ldots (x - r_{k+1})Q(x)]$$

Thus for every positive integer n, if $P(x)$ is a polynomial having n real roots, (ii) $\Rightarrow$ (iii).

(iii) $\Rightarrow$ (ii) Suppose $P(x)$ has a repeated real root, a. Then $|P(x)| = |(x - a)^2 Q(x)|$ and

$$D_x|P(a)| = \lim_{x \to a} \frac{|(x - a)^2 Q(x)|}{x - a}$$

$$= \lim_{x \to a} \frac{(x - a)^2 |Q(x)|}{x - a}$$

$$= \lim_{x \to a} (x - a)\,|Q(x)| = 0$$

Thus the derivative exists at a and (iii) does not hold.

UNDERGRADUATE RESEARCH PROJECT

Submitted by Dr. David Kay University of Oklahoma

This problem concerns a method of characterizing certain sets in the plane. Say a set is (m,n)-convex if it has the property that among each m points of the set there are at least n pairs whose joins are in that set, $m \geq 2$ and $n \geq 0$. For example, the five-pointed star with its interior is a set which is (2,0)-, (3,1)-, (4,3)-, (5,5)-convex, etc.; the set consisting of two intersecting lines is (2,0)-, (3,1)-, (4,2)-, (5,4)-convex, etc. The function $c_S(m)$ is defined as the maximal number n such that the set S is (m,n)-convex. Evidently $c_S(m)$ reflects the character of S. Thus it is easy to see that if S is a parabola $c_S(m) = 0$, if S is convex $c_S(m) = \binom{m}{2} = [m(m - 1)]/2$, and if S consists of the union of two convex sets $c_S(m) =$

$$\binom{[m/2]}{2},$$

where $[m/2]$ denotes the greatest integer in $m/2$. Investigate certain sets in the plane to see if they may be characterized by the function $c_S(m)$ defined above. Sample theorem: A set S consists of a convex set C, and k isolated points not in C, if and only if $c_S(m) = \binom{m-k}{2}$.

NEED MONEY?

The Governing Council of Pi Mu Epsilon announces a contest for the best expository paper by a student (who has not yet received a masters degree) suitable for publication in the Pi Mu Epsilon Journal.

The following prizes will be qiven

$200. first prize

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providing at least ten papers are received for the contest.

In addition there will be a $20. prize for the best paper from any one chapter, providing that chapter submits at least five papers.

AN EASIER CONDITION THAN TOTAL BOUNDEDNESS

Daniel E. Putnam

University of Illinois

The condition of total boundedness is a useful one: for instance, in establishing compactness. Therefore it is worthwhile to find simpler properties equivalent to total boundedness. The condition I suggest is this:

Definition: A subset A of a metric space (X,d) is Cauchy bounded, if for all $\varepsilon > 0$ and all infinite subsets $B \subset A$ there are two points $x,y \in B$ with $d(x,y)<\varepsilon$.

Compare this condition with that of total boundedness:

Definition: A subset A of a metric space (X,d) is totally bounded, if for all $\varepsilon>0$ there is an ε net consisting of a finite subset, $\{a_1,a_2,\ldots a_n\}$, of A so that for any $x \in A$ we have $d(x,a_i)<\varepsilon$ for some i.

It is easy to see that if A were totally bounded and if $B \subset A$ were infinite, then given $\varepsilon>0$, we could find $x,y \in B$ with $d(x,y)\leq\varepsilon$. The procedure is simple. We simply note the existence of a 1/2 ε net and see that there are two points, x and y, of the infinite set B clustered about one of the points, a_i, in the 1/2 ε net. From $d(x,a_i)<\varepsilon/2$ and $d(y,a_i)<\varepsilon/2$ and the triangle inequality, we see that $d(x,y)<\varepsilon$. Thus total boundedness easily implies Cauchy boundedness. That Cauchy boundedness implies total boundedness is not quite so easy to see but is still easy to prove.

Theorem: If a subset A of a metric space (X,d) is Cauchy bounded, then it is totally bounded.

Proof: Suppose that A was not totally bounded. In that case we would have an $\varepsilon>0$ so that no ε net existed. Choose a point $a_1 \in A \cdot \{a_1\}$ is not an ε net, so there must be a point $a_2 \in A$ with $d(a_1,a_2)\geq\varepsilon$. $\{a_1,a_2\}$ is not an ε net, so there must be a point $a_3 \in A$ with $d(a_3,a_2)\geq\varepsilon$ and $d(a_3,a_1)\geq\varepsilon$. We note that $\{a1,a2,a3\}$ is not an ε net, and this

process continues in the same way. The result is a sequence of points, $\{a_n\}$, of A, with the property that $d(a_i,a_j)\geq\varepsilon$ if $i \neq j$. The sequence $\{a_n\}$ is an infinite subset of A that has no two points closer than ε. Thus we have a contradiction to the Cauchy boundedness of A, and the theorem is established.

Perhaps it is now easier to see why Cauchy boundedness is simpler to establish than total boundedness. The fact that Cauchy boundedness follows so easily from total boundedness implies that one might as well prove a set to be Cauchy bounded as totally bounded. Furthermore the fact that in order to establish total boundedness one must find an ε net for each ε seems to indicate that total boundedness is harder to establish than Cauchy boundedness where one only needs to find two close points in an infinite subset.

As an illustration of what is involved, let me offer a new proof to an old result.

Theorem: If A is a subset of the space of continuous real-valued functions on $[0,1]$ with the uniform metric, then A is compact if A is closed, equicontinuous, and uniformly bounded.

This is half of the Arzela-Ascoli theorem as stated in [1]. The procedure will be to show that A is complete and totally bounded. This is enough to prove that A is compact by a nice little theorem that says that a subset of a metric space is compact if, and only if, it is complete and totally bounded [2]. Since the metric space of continuous functions on $[0,1]$ is complete and A is given to be closed, we know that A is complete. Thus we only need to prove that A is totally bounded. We will do this by route of Cauchy boundedness.

Let A_0 be an infinite subset of A and let $\varepsilon>0$ be chosen. Since A_0 is also equicontinuous, for all $x \in [0,1]$ there exists N such that for all $f \in A$ and all $y \in N_x$ we have $|f(y) - f(x)|< \varepsilon/3$, where N denotes an open set for which $x \in N$. We note that the family of sets $\{N_x\}$ for $x \in [0,1]$ covers $[0,1]$ and so we have a finite subcover of $[0,1]$, $\{N_{x_1},N_{x_2},\ldots,N_{x_K}\}$.

Now consider the set $\{f(x_1)/f \in A_0\}$. From the fact that A_0 is both uniformly bounded and infinite, we know that there is an infinite subset $A_1 \subset A$ such that for f, $g \in A$ we have that $|f(x_1) - g(x_1)|<\varepsilon/3$. This follows from the well known Bolzano-Wierstrass theorem.

Let me put together what we have so far. Let $y \in N_{x_1}$ and f, $g \in A$, then:
$|f(y) - g(y)| \leq |f(y) - f(x_1)| + |f(x_1) - g(x_1)| + |g(x_1) - g(y)|$.
We know that each of these quantities is less than $\varepsilon/3$, so we have $|f(y) - g(y)|<\varepsilon$. Thus, the functions of A_1 uniformly approximate each other on N_{x_1}.

The same trick works again and we find an infinite subset $A_2 \subset A_1$ so that f, $g \in A$, $y \in N_{x_2}$ implies $|f(y) - g(y)|<\varepsilon$. Of course, since

$A_2 \subset A_1$ we know that $|f(y) - g(y)| < \varepsilon$ also holds if $y \in N_{1}$. Continuing in this way eventually produces an infinite subset $A_k \subset A_0$ such that $|f(y) - g(y)| < \varepsilon$ if $f,g \in A_k$ and $y \in N_{x_i}$ for $i = 1,2,\ldots k$. Since $\{N_{x_i}/i = 1,2,\ldots k\}$ covers $[0,1]$ we see that A is an infinite set of functions whose elements uniformly approximate each other within ε on the unit interval.

We have taken an infinite subset $A_0 \subset A$ and shown that an infinite subset $A_k \subset A_0 \subset A$ has the property that any two elements of A are less than ε apart according to the uniform metric on the space of continuous functions. Thus we see that A is Cauchy bounded and therefore totally bounded and compact.

An alternate approach found in [1] actually constructs an ε net by using a set of polygonal functions. Unfortunately, this method requires a little ingenuity and some verification. However, in the proof used in this paper the immediate consequence of the definitions of equicontinuity and uniform boundedness is the critical idea of the entire proof: that the functions of a certain infinite set of functions are uniformly close on $[0,1]$. We see that, at least in this case, Cauchy boundedness is indeed an easier condition to establish than total boundedness.

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1. C. Goffman, Preliminaries to Functional Analysis, Vol. 1 of "MM Studies in Mathematics"; ed. R. C. Buck, 1962, pp. 151-152.

2. H. L. Royden, Real Analysis, New York; Macmillan Co., 1963, p. 142.

A PROBLEM IN ELEMENTARY MATHEMATICS

Kenneth Loewen

In the book One Hundred Problems in Elementary Mathematics, Hugo Steinhaus proposes the following problem: Find n numbers in the unit interval such that the first two are in different halves, the first three in different thirds, the first four in different fourths, and so on, till the first n are in different n-ths. He gives a solution for n = 14 and in a footnote mentions that M. Warmus proved that n = 17 is the largest number for which the problem has a solution. A solution for n = 17 will be given by any set of numbers staisfying the following inequalities:

$0<x_1<1/17;$	$11/13<x_7<6/7;$	$8/17<x_{13}< 1/2;$
$16/17<x_2<1;$	$1/6 <x_8<3/17;$	$15/17<x_{14}<14/15;$
$7/13<x_3<6/11;$	$8/13<x_9<5/8;$	$1/5 <x_{15}< 4/17;$
$4/15<x_4<3/11;$	$1/3 <x_{10}<6/17;$	$11/17<x_{16}<11/16;$
$12/17<x_5<5/7;$	$10/13<x_{11}<11/14;$	$6/17<x_{17}< 7/17.$
$5/12<x_6<3/7;$	$1/12<x_{12}< 1/6;$	

SET-THEORETIC DEFINITION OF ORDERED N-TUPLES

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We make a distinction between the ordered n-tuple $(a_1,a_2,\ldots,a_n)$ and the set $\{a_1,a_2,\ldots,a_n\}$. The ordered n-tuple's "value" is changed if the elements are rearranged while the order of the elements has nothing to do with the "value" of the set. For example, $(1,2,3) \neq (3,2,1)$, but $\{1,2,3\} = \{3,2,1\}$.

The above remarks follow from the definition that $(a_1,a_2,\ldots,a_n) = (b_1,b_2,\ldots,b_n)$ if and only if $a_1 = b_1$, $a_2 = b_2$, $\ldots$, $a_n = b_n$.

On the other hand, two sets are said to be equal if they contain the same elements. More precisely, sets A and B are equal if and only if every element of A is an element of B and every element of B is an element of A.

Example 1. If $\{a,b\} = \{x,y\}$ then either $a = x$ and $b = y$ or $b = x$ and $a = y$. Of course, both of these could occur, i.e. $a = x = b = y$. We will usually omit this trivial case.

Example 2. If $\{a\} = \{x,y\}$ then $a = x = y$ and $\{a,x,y\} = \{x,y\} = \{a,x\} = \{a,y\} = \{a\} = \{x\} = \{y\}$. More concretely, $\{2,3,2,3\} = \{2,3\}$.

Example 3. $\{\{a\},\{a,b\}\}$ is a set whose elements are themselves sets, namely $\{a\}$ and $\{a,b\}$. In addition, $\{\{a\},\{a,b\}\} \neq \{a,a,b\} = \{a,b\}$.

The object here is to define an ordered n-tuple in terms of sets. To dispose of the case where n = 1, we shall define $\{a\} = (a) = a$. The case for n = 2 is more interesting.

Definition 1. $(a,b) = \{\{a\},\{a,b\}\}$. (I)

The question now is whether or not the right hand side of (I) completely and uniquely determines the ordered pair (a,b). The following theorem answers this question affirmatively.

Theorem 1. $(a,b) = (x,y)$, i.e. $a = x$ and $b = y$, if and only if $\{\{a\},\{a,b\}\} = \{\{x\},\{x,y\}\}$.

Proof: Suppose $(a,b) = (x,y)$, i.e. $a = x$ and $b = y$. Then $\{a\} = \{x\}$ and $\{a,b\} = \{x,y\}$. Thus $\{\{a\},\{a,b\}\} = \{\{x\},\{x,y\}\}$.

Suppose that $\{\{a\},\{a,b\}\} = \{\{x\},\{x,y\}\}$. Two cases arise. We ignore the trivial case where $\{a\} = \{a,b\} = \{x\} = \{x,y\}$.

Case 1. $\{a\} = \{x\}$ (A)
and $\{a,b\} = \{x,y\}$. (B)

From (A) one gets $a = x$. From (B) one gets either

$$a = x \text{ and } b = y \qquad (B_1)$$

or

$$a = y \text{ and } b = x. \qquad (B_2)$$

If (A) and (B_1) then $a = x$ and $b = y$.

If (A) and (B_2) then $a = x$ and $b = y$.

Then one has $(a,b) = (x,y)$.

Case 2. $\{a\} = \{x,y\}$ (C)
and $\{a,b\} = \{x\}$ (D)

From (C) $a = x = y$ and from (D) $a = b = x$, which yield $a = x$ and $b = y$. Thus $(a,b) = (x,y)$. This completes the proof.

In the case where $n = 3$ (3-tuple or ordered triple), several definitions will yield a result analogous to Theorem 1. For example,

Definition 2. $(a,b,c) = \{\{(a,c)\}, \{(a,b), (b,c)\}\}$. (II)

The reader can verify this definition by following the example of Theorem 1. Attempts at other definitions will show that some apparently obvious ones are not sufficient.

Example 4. Define $(a,b,c) = \{\{a\}, \{a,b\}, \{a,b,c\}\}$. Note that $(1,1,2) \neq (1,2,1)$, but $\{\{1\},\{1,1\},\{1,1,2\}\} = \{\{1\},\{1,2\},\{1,2,1\}\}$ since both sides reduce to $\{\{1\},\{1,2\}\}$. This shows that the definition is not sufficient.

One can extend this sort of definition to an n-tuple for any positive integer n. The following is a generalization of (I) and (II).

Definition 3. $(a_1,a_2,\ldots,a_n) = \{\{(a_1,a_2,\ldots,a_{n-2},a_n)\}, \{(a_1,a_2,\ldots a_{n-1}), (a_2,a_3,\ldots,a_n)\}\}$. (III)

It is not difficult to prove that $(a_1,a_2,\ldots,a_n) = (x_1,x_2,\ldots,x_n)$ if and only if $\{\{(a_1,a_2,\ldots,a_{n-2},a_n)\}, \{(a_1,a_2,\ldots,a_{n-1}), (a_2,a_3,\ldots,a_n)\}\} = \{\{(x_1,x_2,\ldots x_{n-2},x_n)\}, \{(x_1,x_2,\ldots x_{n-1}), (x_2,x_3,\ldots x_n)\}\}$.

Note that this is a recursive definition of an ordered n-tuple, i.e., we define an n-tuple in terms of sets whose elements are (n-1)-tuples.

AN INTERESTING MAPPING OF TWO FIELDS

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In this article, I will prove an interesting theorem concerning a mapping of two fields, namely: If T is a one-to-one mapping of a field F onto a field F' such that $T[a(b-1)] = T(a)[T(b) - T(1)]$, then T is an isomorphism.

In order to prove that T is an isomorphism, it is sufficient to show that T preserves addition and multiplication. To do this, I will characterize addition and multiplication in terms of the operation $a(b-1)$ which will be denoted $a*b$.

Theorem 1: $ab = (a*0) * [(1*0)*b]$

Proof:
$$\begin{aligned}(a*0) * [(1*0)*b] &= [a(0-1)] * [1(0-1)*b] \\ &= (-a)*(-1*b) \\ &= (-a)*(-b+1) \\ &= (-a)(-b+1-1) \\ &= (-a)(-b) = ab\end{aligned}$$

Theorem 2: $a+b = (b*0)* [(ab^{-1})*0]$ if $b \neq 0$

Proof:
$$\begin{aligned}(b*0) * [(ab^{-1})*0] &= [b(0-1)] * [ab^{-1}(0-1)] \\ &= (-b)*(-ab^{-1}) \\ &= (-b)(-ab^{-1} - 1) \\ &= bab^{-1} + b \\ &= a+b\end{aligned}$$

If $b=0$, we obviously have $a+b=a+0=a$.

We are given that $T(a*b) = T[a(b-1)] = T(a) [T(b) - T(1)]$; thus in order to prove that $T(a*b) = T(a) * T(b)$, it is sufficient to prove that $T(1) = 1'$ where $1'$ is defined to be the unity for F'.

I will first prove that $T(0) = 0'$ where $0'$ is the zero element of F'.

Theorem 3: If T is a one-to-one mapping of a field F onto a field F' such that $T[a(b-1)] = T(a) [T(b) - T(1)]$, then $T(0) = 0'$.

Proof: $0 = 0(a-1)$ for all a in F
$T(0) = T[0(a-1)]$
$T(0) = T(0) [T(a) - T(1)]$ (1)

Assume $T(0) \neq 0'$. Then we can cancel $T(0)$ from both sides of (1). Therefore, $1' = T(a) - T(1)$, and $T(a) = T(1) + 1'$ for all a in F.

Therefore, T maps every element of F into one element of F', a contradiction since T is one-to-one.

Therefore $T(0) = 0'$.

Theorem 4: If T is as in the preceding theorem, then $T(1) = 1'$.

Proof: $1 = (-1)(0-1)$
$T(1) = T(-1)\,[T(0) - T(1)]$
But $T(0) = 0'$ (Theorem 3)
Therefore, $T(1) = -T(-1)T(1)$
But $T(1) \neq 0'$ since T is one-to-one.
Therefore, $T(-1) = -1'$.

It is now necessary to consider two cases: case I where F is not of characteristic 2 and case II where F is of characteristic 2.

CASE I: F is not of characteristic 2 (i.e. $1 \neq -1$)
$-1 = 1(0-1)$
$T(-1) = T(1)\,[T(0) - T(1)]$
But $T(0) = 0'$ and $T(-1) = -1'$
Therefore, $-1' = -[T(1)]^2$
Therefore, $[T(1)]^2 = 1'$.
Therefore, since T is one-to-one and $-1 \neq 1$, $T(1) = 1'$.

CASE II: F is of characteristic 2 (i.e. $1 = -1$)
Since T is onto, every element of F' has a preimage in F. Let a be the preimage of $1'$.
Then $T(a) = 1'$
$T(a-1) = T(a+1) = T\,[1(a-1)] = T(1)\,[T(a) - T(1)]$
But $T(1) = -1'$ (since $1 = -1$).
Therefore, $T(1)\,[T(a) - T(1)] = (-1')\,[1' - (-1')]$
$= (-1')(1'+1') = -(1'+1')$
$T(a^2) = T\,[a(a+1-1)] = T(a)\,[T(a+1) - T(1)]$
$= 1'\,[-(1'+1') - (-1')] = -1'$.
Therefore, $T(a^2) = T(1)$. But T is one-to-one. Therefore, $a^2=1$. Therefore, $a^2 - 1 = 0$.
But $a^2 - 1 = (a+1)(a-1)$, and this factorization is unique since a polynomial ring over a field is a unique factorizationdomain. (See, for example, G. Birkhoff and S. MacLane, A Survey of Modern Algebra, 3rd edition, page 72.)
But $a+1 = a-1$.
Therefore, $a^2 - 1 = (a-1)^2$. Therefore, by the factor theorem, $a=1$ $(=-1)$ is the only root.
Therefore, in this case, $T(1) = 1'$
Note that we have also shown that F' is of characteristic 2.
Using the fact that $T(1) = 1'$, we can conclude that $T(a \ast b) = T(a) \ast T(b)$. The proof of this obvious statement is left to the reader.

We are now ready to prove that T preserves addition and multiplication.

Theorem 5: $T(ab) = a'b'$ where $a'=T(a)$ and $b'=T(b)$.

Proof: $T(ab) = T\{(a \ast 0) \ast [(1 \ast 0) \ast b]\}$
$= T(a \ast 0) \ast T\,[(1 \ast 0) \ast b]$
$= T(a \ast 0) \ast [T(1 \ast 0) \ast T(b)]$
$= [T(a) \ast T(0)] \ast [(T(1) \ast T(0)) \ast T(b)]$
But $T(0)=0'$, $T(1)=1'$, $T(a)=a'$ and $T(b)=b'$.
Therefore, $T(ab) = (a' \ast 0') \ast [(1' \ast 0') \ast b']$
$= a'b'$ by Theorem 1.

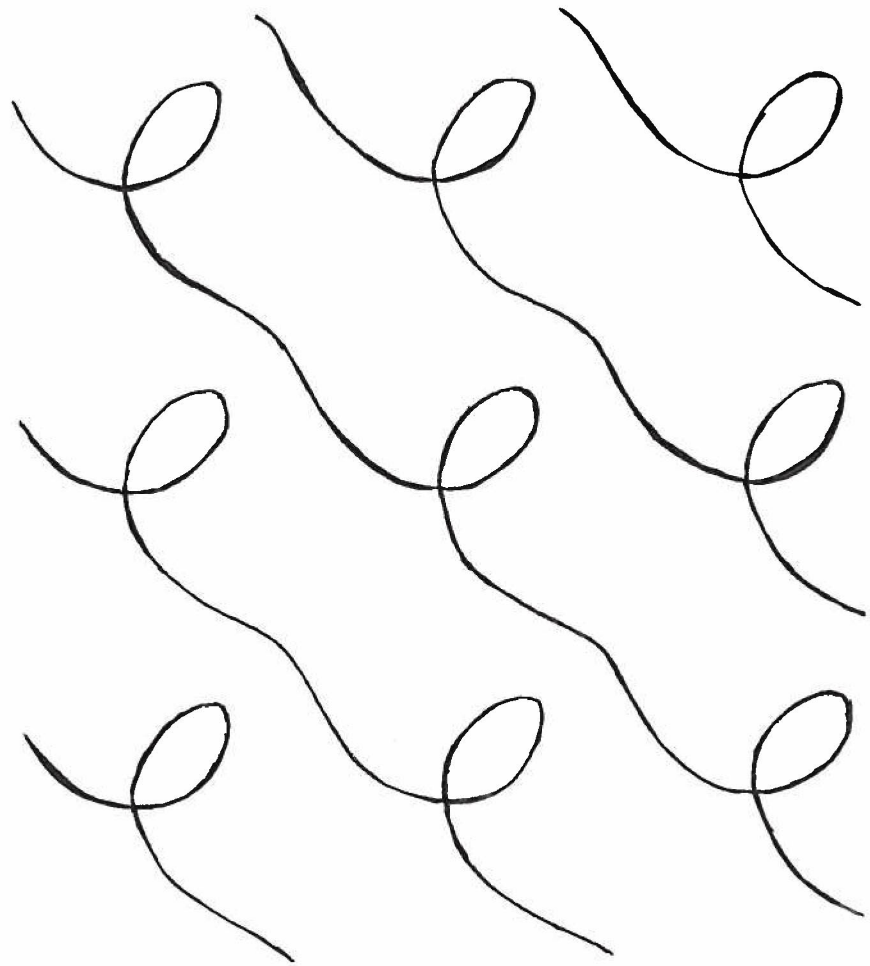
Theorem 6: $T(a+b) = a'+b'$

Proof: It is necessary to consider the cases $b=0$ and $b \neq 0$ separately.

If $b=0$, $T(a+0) = T(a) = a' = a'+0' = T(a) + T(0)$
If $b \neq 0$,
$T(a+b) = T\,[(b \ast 0) \ast ((ab^{-1}) \ast 0)]$
$= T(b \ast 0) \ast T\,[(ab^{-1}) \ast 0]$
$= [T(b) \ast T(0)] \ast [T(ab^{-1}) \ast T(0)]$
But since T preserves multiplication, $T(ab^{-1}) = a'b'^{-1}$.
Therefore, $T(a+b) = (b' \ast 0') \ast [(a'b'^{-1}) \ast 0']$
$= a' + b'$ by Theorem 2 is $b' \neq 0$. But since $b \neq 0$, and T is a one-to-one, we know that $b' \neq 0$.

Now that we have proven that T is a homomorphism, it follows from the assumption that T is one-to-one and onto that T is an isomorphism.

$$\mathrm{Tan}^3 x + \mathrm{Tan}^3 y = 3 \,\mathrm{Tan}\, x \,\mathrm{Tan}\, y.$$



INDEPENDENT POSTULATE SETS FOR BOOLEAN ALGEBRA

Chinthayamma
University of Alberta

Introduction. According to Dickson [2] a Boolean Algebra is a set X such that for all a,b,c,... belonging to X:

A. There is defined a (closed) binary operation "." such that
   Axiom 1. a.(b.c) = (a.b).c
   Axiom 2. a.b = b.a
   Axiom 3. a.a = a

B. There exists an element I belonging to X such that
   Axiom 4. a.I = a for all a belonging to X

C. There can be defined a function ' from X into itself such that
   Axiom 5. (a')' = a for all a belonging to X'
   Axiom 6. a.a' = I' for all a belonging to X
   Axiom 7. a.b = I' implies a.b' = a.

Dickey [1] has reduced this system of seven axioms to a system of five axioms by eliminating the axiom 3 and by replacing the axioms 1 and 2 by

$$a.(b.c) = (b.a).c$$

which can be done as long as the axiom 4 is retained.

The purpose of this paper is to give two independent sets of four postulates in which axiom 4 is also eliminated using the axioms 5, 6 and 7 and even though axiom 4 is eliminated the axioms 1 and 2 are replaced by a variant of axiom 1.

Sets of Postulates.

THEOREM 1. Let X be a set with an element I such that for all a,b,c,... belonging to X

A. There is defined a (closed) binary operation "." such that

   Axiom 1'. $a.(b.c) = c.(a.b)$

   or

   Axiom 1''. $(a.b).c = (b.c).a$

B. There can be defined a function ' from X into itself such that

   Axiom 5. (a')' = a for all a belonging to X
   Axiom 6. a.a' = I' for all a belonging to X
   Axiom 7. a.b = I' implies a.b' = a.
   Then X is a Boolean Algebra.

Proof. It is sufficient to prove the axioms 1, 2 and 4 since the axiom 3 is already proved in [1].

When 1' holds replace a by b.c in it and use axioms 1' and 3 to get

$$(b.c).(b.c) = c.((b.c).b)$$

$$b.c = b.(c.(b.c))$$

$$= b.(b.(c.c))$$

$$= b.(b.c) = c.(b.b)$$

$$= c.b$$

Similarly when 1'' holds replace c by a.b in it and use axioms 1'' and 3 to get a.b = b.a.

Thus "." is commutative in any case and this together with 1' or 1'' implies 1. Axiom 4 is also satisfied in any case as follows:

I' = a.a'	by axiom 6
= (a.a).a'	by axiom 3
= a.(a.a')	by axiom 1
= a.I'	by axiom 6

This implies

a.(I')' = a	by axiom 7
a.I = a	by axiom 5

Independence. Let X = {0,a,b,I} with I.0 = a.b = b.a = 0 otherwise x.y = x; with x,y denoting any of the elements 0, a, b, I and 0' = I, I' = 0, a' = b, b' = a. Then axioms 5, 6 and 7 are satisfied. But axiom 1' is not satisfied since a.(a.I) = a and I.(a.a) = I and 1'' is also not satisfied since (a.I).a = a and I.(a.a) = I. The other three axioms are proved to be independent in [2].

REFERENCES

1. L. J. Dickey, "A Short Axiomatic System for Boolean Algebra," Pi Mu Epsilon Journal, V. 4, No. 8 (1968) p. 336.

2. L. J. Dickson, Ibid. V. 4, No. 6 (1967), pp. 253-257.

$6 - 5 = 1$	$7 - 4 = 3$
$6^2 - 5^2 = 11$	$7^2 - 4^2 = 33$
$56^2 - 45^2 = 1111$	$57^2 - 54^2 = 333$
$556^2 - 445^2 = 111111$	$557^2 - 554^2 = 3333$
$5556^2 - 4445^2 = 11111111$	$5557^2 - 5554^2 = 33333$
etc.	etc.

PROBLEM DEPARTMENT

Edited by

Leon **Bankoff**, Los Angeles, California

With this issue we introduce a new problem editor. To Murray Klamkin who served in this capacity for the past ten years we give our thanks for a job well done.

The new editor is Leon **Bankoff** who has contributed many problems and solutions to the problem department of the ~~Journal~~ in the past. He also served as joint editor of the problem department for one issue (Spring 1958). Dr. **Bankoff** practices dentistry in Los Angeles.

The new problem editor would like to publish solutions to all problems which have appeared in this Journal, but for which solutions have not yet been published. These problems (published prior to 1967) are listed below.

The editor.

37. (April 1952) Proposed by Victor **Thébault,** Tennie Sarthe, France. Find all pairs of three digit numbers M and N such that $M \cdot N = P$ and $M' \cdot N' = P$, where $M' \cdot N'$ and P' are the numbers M-N and P written backwards. For example,

$$122 \times 213 = 25986$$

and

$$221 \times 312 = 68952$$

48. (Nov. 1952) Proposed by Victor **Thébault, Tennie,** Sarthe, France. Find bases B and **B'** such that the number **11, 111, 111, 111** consisting of eleven digits in base B is equal to the number **111** consisting of three digits in Base **B'**. (An incorrect solution by the proposer was given in the November 1953 issue. For further discussion see the Spring 1958 issue.)

65. (April 1954) Proposed by Martin Schechter, Brooklyn, N. Y. Prove that every simple polygon which is not a triangle has at least one of its diagonals lying entirely inside of it.

73. (April 1955) Proposed by Victor **Thébault,** Tennie, Sarthe, France. Construct three circles with **given** centers such that the sum of the powers of the center of each circle with respect to the other two is the same.

83. (Spring 1956) Proposed by G. K. Horton, University of Alberta. Evaluate

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp-\left(\sqrt{(x-1)^2+y^2} + \sqrt{x^2+(y-1)^2}\right)dx\,dy$$

91. (Fall 1956) Proposed by Nathaniel Grossman, California Institute of Technology. Prove that

$$\sum_{d/n} \sigma\left(\frac{n}{d}\right)\phi(d) = n \cdot \tau(n)$$

where $\tau(n)$ denotes the number of divisors of n, $\sigma(n)$ is the sum of the divisors of n and $\phi(n)$ is the Euler Totient function.

102. (Fall 1958) Proposed by Leo Moser, University of Alberta. Give a complete proof that two equilateral triangles of edge 1 cannot be placed, without overlap, in the interior of a square of edge 1.

120. (Spring 1960) Proposed by Michael Goldberg, Washington, D. C.
1. All the orthogonal projections of a surface of constant width have the same perimeter. Does any other surface have this property?
2. A sphere may be turned through all orientations while remaining tangent to the three lateral faces of a regular triangular prism. Does any other surface have this property? Note that a solution to 2. is also a solution to 1.

128. (Spring 1961) Proposed by Robert P. Rudis and Christopher Sherman, AVCO RAD. Given **2n** unit resistors, show how they may be connected using n single pole single throw **(SPST)** and n single pole double throw **(SPDT)** (the latter with off position) switches to obtain, between a single fixed pair of terminals, the values of resistance of i and i^{-1} where $i = 1,2,3,\ldots,2n$.

Editorial Note: Two more difficult related problems would be to obtain i and i^{-1} using the least number of only one of the above type of switches.

136. (Fall 1961) Proposed by Michael **Goldberg, Washington, D. C.** What is the smallest convex area which can be **rotated continuously** within a regular pentagon while keeping contact with all the sides of the pentagon? This problem is unsolved but has been solved for the square and equilateral triangle. For the square, **it** is the regular tri-arc made of circular arcs whose radii are equal to the side of the square. For the triangle, **it is** the two-arc made of equal **60°** arcs whose radii are equal to the altitude of the triangle.

144. (Fall 1962) Proposed by Huseyin Demit, **Kandilli, Eregli, Kdz., Turkey**. Find the shape of a curve of length L lying in a vertical **plane** and having its end points fixed in the plane, such that **when it** revolves about a fixed vertical line in the plane, generates a volume which when filled with water shall be emptied in a minimum of time through an orifice of given area A at the bottom. (Note: The proposer has only obtained the differential equation of the curve.)

166. (Fall 1964) Proposed by Leo Moser, University of Alberta. Show that 5 points in the interior of a 2x1 rectangle always determine at least one distance less than **sec 15°.**

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate or candidate for the Master's Degree. Solutions of these problems should be submitted on separate signed sheets within four months after publication.

An asterisk (*) placed beside a problem number indicates that the problem was submitted without a solution.

Address all communications concerning problems to Leon **Bankoff,** 6360 Wilshire Boulevard, Los Angeles, California 90048.

PROBLEMS FOR SOLUTION

205. Proposed by C. S. Venkataraman, Trichur, South India.
ABC and PQR are two equilateral triangles with a common circumcenter but different circumcircles. PQR and ABC are in opposite senses. Prove that AP, BQ, CR are concurrent.

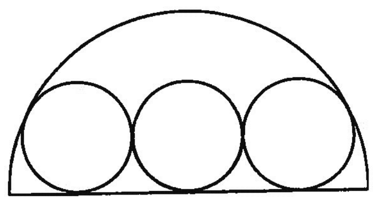
206. Proposed by Charles W. Trigg, San Diego, California.
Identify the pair of consecutive three-digit numbers each of which is equal to the sum of the cubes of its digits.

207. Proposed by Charles W. Trigg, San Diego, California.
Find a triangular number of the form abcdef in which def = 2 abc.

208. Proposed by Thomas Dobson, Hexham, England.
Where must a man stand so as to hear simultaneously the report of a rifle and the impact of the bullet on the target?

209. Proposed by R. C. Gebhardt, Parsippany, New Jersey.
At each play of a game, a gambler risks $1/x$ of his assets at the moment. What must be the odds so that, in the long run, he just breaks even?

210. Proposed by Leon Bankoff, Los Angeles, California. Three equal circles are inscribed in a semicircle as shown in the adjoining diagram. How is this figure related to one of the better-known properties of the sequence of Fibonacci numbers?



211. Proposed by Leonard Barr, Beverly Hills, California.
It is known that the sum of the distances from the incenter I to the vertices of a triangle ABC cannot exceed the combined distances from the orthocenter H to the vertices. [Amer. Math. Monthly, 1960, 695; problem E 1397]. Show that the reverse inequality holds for their products, namely, that $AH \cdot BH \cdot CH \leq AI \cdot BI \cdot CI$.

212. Proposed by J. M. Gandhi, University of Manitoba, Winnipeg, Canada. If

$$M(n) = \sum_{s=0}^{n-1} \binom{n}{s+1}\binom{n+s}{s}$$

show that (a) $M(5m+2) \equiv 0 \pmod 5$.
(b) $M(5m+3) \equiv 0 \pmod 5$.
[Ref: George Rutledge and R. D. Douglass, "Intergral Functions Associated with Certain Binomial Sums," Amer. Math. Monthly, 43 (1936), pp. 27-33].

SOLUTIONS

192. (Fall 1967). Proposed by Oystein Ore, Yale University.
Albrecht Durer's famous etching "Melancholia" includes the magic square

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

The boxed-in numbers 15-14 indicate the year in which the picture was drawn. How many other 4 x 4 magic squares are there which he could have used in the same way?

Solution by C. J. Bouwkamp, Phillips Research Laboratories, Eindhoven. Netherlands. There are exactly 32 solutions to this problem, including the Durer version shown above (with misprint from the Fall 1967 issue, page 295 corrected). Curiously, this number is contained in Durer's magic square in the middle of the top row. The construction is as follows. There exist four types according to the bottom row: 1, 15, 14, 4 (10 solutions), 2, 15, 14, 3 (6 solutions), 3, 15, 14, 2 (6 solutions), and 4, 15, 14, 1 (10 solutions). Further it is known that the sum of the four inner elements equals 34. Thus the sum of the two inner elements of the first row must be 5 (2+3 or 1+4). Similarly, the outer two elements of the first row can only be 13 and 16. All in all, there are then 16 types where the upper and lower rows are fixed. The remaining numbers 5 through 12 are to be distributed over the two middle rows. The inner four elements can be linearly expressed in terms of one parameter. Some easy manipulation then leads to all 32 possible Durer magic squares. The complete list of solutions is shown below.

13	3	2	16		13	3	2	16		16	2	3	13		16	2	3	13		16	2	3	13
12	6	7	9		8	10	11	5		10	5	8	11		11	5	8	10		7	6	9	12
8	10	11	5		12	6	7	9		7	12	9	6		6	12	9	7		10	11	8	5
1	15	14	4		1	15	14	4		1	15	14	4		1	15	14	4		1	15	14	4

16	2	3	13		16	2	3	13		16	2	3	13		16	2	3	13		16	2	3	13
12	6	9	7		5	8	11	10		10	8	11	5		6	9	12	7		7	9	12	6
5	11	8	10		12	9	6	7		7	9	6	12		11	8	5	10		10	8	5	11
1	15	14	4		1	15	14	4		1	15	14	4		1	15	14	4		1	15	14	4

13	4	1	16		13	4	1	16		16	1	4	13		16	1	4	13		16	1	4	13
11	6	7	10		7	10	11	6		11	6	7	10		7	10	11	6		5	8	9	12
8	9	12	5		12	5	8	9		5	12	9	8		9	8	5	12		11	10	7	6
2	15	14	3		2	15	14	3		2	15	14	3		2	15	14	3		2	15	14	3

16	1	4	13	13	1	4	16	13	1	4	16	13	1	4	16	13	1	4	16
11	8	9	6	12	8	5	9	6	10	7	11	12	10	7	5	8	12	9	5
5	10	7	12	6	10	11	7	12	8	9	5	6	8	9	11	10	6	7	11
2	15	14	3	3	15	14	2	3	15	14	2	3	15	14	2	3	15	14	2

16	4	1	13	16	4	1	13	13	2	3	16	13	2	3	16	13	2	3	16
9	5	8	12	5	9	12	8	10	8	5	11	11	8	5	10	7	9	6	12
6	10	11	7	10	6	7	11	7	9	12	6	6	9	12	7	10	8	11	5
3	15	14	2	3	15	14	2	4	15	14	1	4	15	14	1	4	15	14	1

13	2	3	16	13	2	3	16	13	2	3	16	13	2	3	16	13	2	3	16
12	9	6	7	5	11	8	10	10	11	8	5	6	12	9	7	7	12	9	6
5	8	11	10	12	6	9	7	7	6	9	12	11	5	8	10	10	5	8	11
4	15	14	1	4	15	14	1	4	15	14	1	4	15	14	1	4	15	14	1

16	3	2	13	16	3	2	13
9	6	7	12	5	10	11	8
5	10	11	8	9	6	7	12
4	15	14	1	4	15	14	1

Also solved by R. ,C. Gebhardt, Parsippany, N. J.; Edgar Karst, University of Arizona; and Alfred E. Neuman, New York, N. Y.

Editorial Note: Bouwkamp verified his results by referring to two curious books in his possession, privately published by K. H. de Haas. 1) Albrecht Durer's Meetkundige Bouw van Reuter en Melencolia S 1, D. van Sijn en Zonen, Rotterdam, 1932. 2) Frenicles's 880 Basic Magic Squares of 4 x 4 Cells, Normalized, Indexed and Inventoried, by the same publisher in 1935. Gebhardt and Karst noted that a magic square remains magic if the same quantity is added to each element of the square, thus extending the number of solutions if the sequence 15-14 were permitted to appear in other rows.

Detailed discussions of magic squares may be found in the two well-known classics in mathematical recreations, by Kraitchik and by Ball and Coxeter. An additional bibliography appears in Martin Gardner's Second Scientific American Book of Mathematical Puzzles and Diversions in connection with a most refreshing chapter on this subject.

193. (Fall 1967). Proposed by William H. Pierce, General Dynamics, Electric Boat Division, Groton, Sonnecticutt. Two ships are steaming along at constant velocities (course and speed). If the motion of one ship is known completely, and if only the speed of the second ship is known, what is the minimum number of bearings necessary to be taken by the first ship in order to determine the course (constant) and range (time-dependent) of the second ship? Given this requisite number of bearings, show how to determine the second ship's course and range.

"Editorial Note: This problem was suggested by problem 186 which was given erroneously. See the comment on 186 in the Fall 1967 issue.

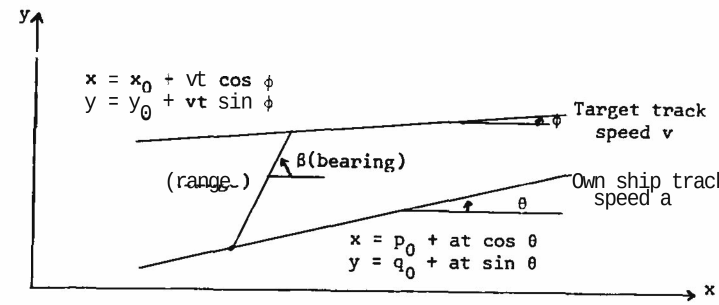
Solution by the Proposer. Designate the first ship, whose motion is known completely, as "own ship" and write its parametric equations of motion as

$$x = p_0 + at\cos\theta$$
$$y = q_0 + at\sin\theta.$$

Similarly, designate the second ship as the "target ship" and write its parametric equations of motion as

$$x = x_0 + vt\cos\phi$$
$$y = y_0 + vt\sin\phi$$

in which only the speed v is known. The following picture will illustrate the situation.



One Bearing Given

At time t_1, bearing β_1 is observed. Thus,

$$\sin\beta_1 = \frac{y_0 - q_0 + (v\sin\phi - a\sin\theta)t_1}{R_1}$$

(1) $\qquad (R_1 \neq 0)$

$$\cos\beta_1 = \frac{x_0 - p_0 + (v\cos\phi - a\cos\theta)t_1}{R_1}$$

which implies

(2) $$(\cos\beta_1)y_0 + (t_1\cos\beta_1)v\sin\phi - (\sin\beta_1)x_0 - (t_1\sin\beta_1)v\cos\phi = (\cos\beta_1)q_0 + (t_1\cos\beta_1)a\sin\theta - (\sin\beta_1)p_0 - (t_1\sin\beta_1)a\cos\theta$$

It should be noted that (2) does not imply (1) as there are targets staisfying (2) which do not satisfy (1), namely targets whose bearing differs from β_1 by 180°. Equation (2) is a linear equation in y_0, $v\sin\phi$, x_0, and $v\cos\phi$ whose general solution is

(3)
$$\begin{aligned}
y_0 &= q_0 \quad t\lambda\sin\beta_1 + \gamma t_1\cos\beta_1\\
v\sin\phi &= a\sin\theta + \mu\sin\beta_1 - \gamma\cos\beta_1\\
x_0 &= p_0 + \lambda\cos\beta_1 - \gamma t_1\sin\beta_1\\
v\cos\phi &= a\cos\theta + \mu\cos\beta_1 + \gamma\sin\beta_1
\end{aligned}$$

where λ, μ, and γ are free parameters subject only to the restrictions

$$\lambda + \mu t_1 > 0$$

$$v^2 = (\mu^2 + \gamma^2) + 2a[\mu \cos(\beta_1 - \theta) + \gamma \sin(\beta_1 - \theta)] + a^2$$

The first restriction eliminates from (3) those targets which do not satisfy (1) and the second restriction eliminates from (3) those targets which do not have the required target speed v. There remains in (3), however, a multiplicity of targets with the required speed satisfying (1) so that we conclude that one bearing is insufficient to determine target motion when only target speed is given.

Two Bearings Given

A second bearing β_2 is observed at time $t_2 > t_1$, giving

(4)
$$\sin \beta_2 = \frac{y_0 - q_0 + (v \sin \phi - a \sin \theta)t_2}{R_2} \qquad (R_2 \neq 0)$$
$$\cos \beta_2 = \frac{x_0 - p_0 + (v \cos \phi - a \cos \theta)t_2}{R_2}$$

which implies

(5)
$$(\cos \beta_2)y_0 + (t_2 \cos \beta_2)v \sin \phi - (\sin \beta_2)x_0 - (t_2 \sin \beta_2)v \cos \phi =$$
$$(\cos \beta_2)q_0 + (t_2 \cos \beta_2)a \sin \theta - (\sin \beta_2)p_0 - (t_2 \sin \beta_2)a \cos \theta$$

The general solution of the linear equations (2) and (5) is

(6)
$$y_0 = q_0 - \lambda t_1 \sin \beta_2 + \mu t_2 \sin \beta_1$$
$$v \sin \phi = a \sin \theta + \lambda \sin \beta_2 - \mu \sin \beta_1$$
$$x_0 = p_0 - \lambda t_1 \cos \beta_2 + \mu t_2 \cos \beta_1$$
$$v \cos \phi = a \cos \theta + \lambda \cos \beta_2 - \mu \cos \beta_1$$

where λ and μ are free parameters subject only to the restrictions

$$\lambda > 0, \quad \mu > 0$$

$$v^2 = \lambda^2 - 2\lambda\mu \cos(\beta_2 - \beta_1) + \mu^2 + 2a[\lambda \cos(\beta_2 - \theta) - \mu \cos(\beta_1 - \theta)] + a^2$$

The parameters here are not related to those used earlier, and the first restriction eliminates from (6) those targets that do not satisfy (1) and (4), while the second restriction eliminates from (6) those targets not having the required speed v. Thus, there remains in (6) a one-parameter family of targets having the required bearings, and we conclude here also that two bearings are insufficient to determine target motion when only target speed is given. (If $\sin(\beta_2 - \beta_1) = 0$, then the two ships have either parallel motion or are on a collision course, and further information about target course can be obtained; discussion of this aspect is omitted.)

Three Bearings Given

A third bearing β_3 is observed at a time $t_3 > t_2 > t_1$, giving

(7)
$$\sin \beta_3 = \frac{y_0 - q_0 + (v \sin \phi - a \sin \theta)t_3}{R_3} \qquad (R_3 \neq 0)$$
$$\cos \beta_3 = \frac{x_0 - p_0 + (v \cos \phi - a \cos \theta)t_3}{R_3}$$

which implies

(8)
$$(\cos \beta_3)y_0 + (t_3 \cos \beta_3)v \sin \phi - (\sin \beta_3)x_0 - (t_3 \sin \beta_3)v \cos \phi =$$
$$(\cos \beta_3)q_0 + (t_3 \cos \beta_3)a \sin \theta - (\sin \beta_3)p_0 - (t_3 \sin \beta_3)a \cos \theta$$

The general solution of the linear equations (2), (5), and (8) is

(9)
$$y_0 = q_0 + H_1\lambda; \quad v \sin \phi = a \sin \theta + H_2\lambda$$
$$x_0 = p_0 + H_3\lambda; \quad v \cos \phi = a \cos \theta + H_4\lambda$$

Where λ is an arbitrary non-zero parameter carrying the sign of $H_1H_4 - H_2H_3$ (which insures that (9) satisfies (1), (4), and (7), and where

$$H_1 = \begin{vmatrix} t_1 \cos \beta_1 & -\sin \beta_1 & -t_1 \sin \beta_1 \\ t_2 \cos \beta_2 & -\sin \beta_2 & -t_2 \sin \beta_2 \\ t_3 \cos \beta_3 & -\sin \beta_3 & -t_3 \sin \beta_3 \end{vmatrix}$$

$$H_2 = \begin{vmatrix} \cos \beta_1 & \sin \beta_1 & -t_1 \sin \beta_1 \\ \cos \beta_2 & \sin \beta_2 & -t_2 \sin \beta_2 \\ \cos \beta_3 & \sin \beta_3 & -t_3 \sin \beta_3 \end{vmatrix}$$

$$H_3 = \begin{vmatrix} \cos \beta_1 & t_1 \cos \beta_1 & -t_1 \sin \beta_1 \\ \cos \beta_2 & t_2 \cos \beta_2 & -t_2 \sin \beta_2 \\ \cos \beta_3 & t_3 \cos \beta_3 & -t_3 \sin \beta_3 \end{vmatrix}$$

$$H_4 = \begin{vmatrix} \cos \beta_1 & t_1 \cos \beta_1 & \sin \beta_1 \\ \cos \beta_2 & t_2 \cos \beta_2 & \sin \beta_2 \\ \cos \beta_3 & t_3 \cos \beta_3 & \sin \beta_3 \end{vmatrix}$$

The solution (9) is meaningful only when $\sin(\beta_i - \beta_j) \neq 0$ in which case it can be proved that $H_1H_4 - H_2H_3 \neq 0$. It can further be shown that the three bearings must be such that $\sin(\beta_2 - \beta_1)$, $\sin(\beta_3 - \beta_1)$, and $\sin(\beta_3 - \beta_2)$ all have the same sign which is opposite that of $H_1H_4 - H_2H_3$.

Solution (9) is a one-parameter family of targets in which the known target speed imposes a restriction on the parameter λ. The restriction is

(10) $$(H_2{}^2 + H_4{}^2)\lambda^2 + 2a(H_2 \sin\theta + H_4 \cos\theta)\lambda + a^2 - v^2 = 0$$

which is a quadratic in λ that may have two, one or no roots, depending on the magnitude of the target speed v. In any event, the parameter λ is determinable (if it exists) from (10) and the associated target motion is determinable from (9). Target range and course are easily determined from the quantities y_0, $v \sin \phi$, x_0, and $v \cos \phi$.

We therefore conclude that three bearings are needed to determine the target motion when target speed is known, and that this motion is obtained from (10) and then (9). Zero, one, or two targets may exist which satisfy the bearing and speed conditions. All additional bearings are determinable from the three given bearings, and no further bearings add any information.

194. (Fall 1967). Proposed by J. M. Gandhi, University of Alberta. Show that the equation

$$x^{x+y} = y^{y-x}$$

has no solution in integers except the solutions:

(i) $x = \pm 1$, $y = \pm 1$, (ii) $x = 3$, $y = 9$.

Solution by Charles W. Trigg, San Diego, California. The given equation may be written in the form.

$$(xy)^x = (y/x)^y.$$

The left hand member is an integer, so the right hand member must be an integer also. This requires that $y = kx$, k an integer. Thus

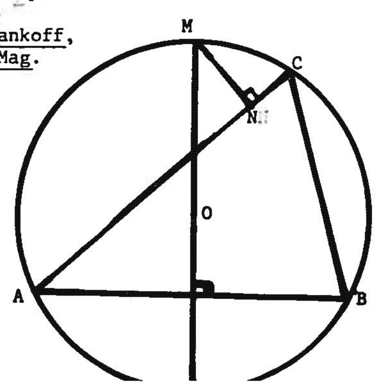
$$(kx^2)^x = (k)^{kx} \text{ or } x = k^{(k-1)/2}.$$

Consequently, k is odd and has the form 2m + 1, m an integer; or k has the form n^2, n a non-zero integer.

The complete solution is $x = \pm 1$, $y = -1$; $x = (2m+1)^m$, $y = (2m+1)^{m+1}$, $m = -1, 0, 1, 2, \ldots$; $x = n^{n^2-1}$, $y = n^{n^2+1}$, n a non-zero integer. The proposition as stated is false.

Also solved by R. C. Gebhardt, Parsippany, N. J.; Erwin Just, Bronx Community College; Bruce W. King, Burnt Hills-Ballston Lake High School; Bob Nemez; Bob Prielipp, Wisconsin State University; Phillip Singer, Michigan State University; and Gregory Wulczyn, Bucknell University.

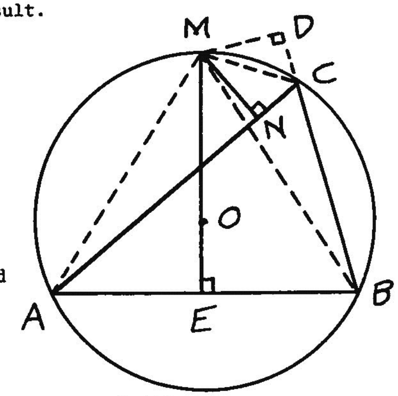
195. (Fall 1967). Proposed by Leon Bankoff, Los Angeles, California. Math. Mag. (Jan. 1963), p. 60, contains a short paper by Dov Avisholom, who asserts without proof that in the adjoining diagram AN = NC + CB. Give a proof.



Solution I by Joe Konhauser, University of Minnesota. AMCB is a cyclic quadrilateral, so AM·CB + MC·AB = AC·MB. Let F be the foot of the perpendicular from M to AB. Then, using AM = MB, we have AM·CB + 2 MC·AF = AC·AM. Triangles MAF and MNC are similar, so MC·AF = AM·NC. Therefore, since AC = AN + NC, it follows that AM·CB + 2 AM·NC = (AN + NC)AM. Simplifying gives the desired result.

Solution II by Charles W. Trigg, San Diego, California. From M drop a perpendicular to BC extended, meeting it at D. Draw MA and MB. Since MA = MB and since angle MAN = arc MC/2 = angle MBD, the right triangles MAN and MBD are congruent, and AN = DC + CB.

Also, MN = MD and MC = MC. Therefore right triangles MNC and MDC are congruent, and NC = DC. Finally, AN = NC + CB.



Solution III by Leon Bankoff, Los Angeles, California. Extend AC to D so that CD = CB. If P is diametrically opposite M, we find that CP, the bisector of angle ACB is perpendicular to MC. Therefore MC bisects angle BCD, and we have angle MCB = angle MCD. So triangles MBC and MCD are congruent, and MD = MB = MA. It follows that the right triangles MAN and DMN are congruent. Hence AN = ND = NC + CD = NC + CB.

Also solved by Dan Deignan, Miami University (by trigonometry); William Tally, University of Southwestern Louisiana; and Gregory Wulczyn, Bucknell University (using polar coordinates).

196. (Fall 1967). Proposed by R. C. Gebhardt, Parsippany, N. J. What is the remainder if

x^{100} is divided by $x^2 - 3x + 2$?

Amalgam of almost identical solutions submitted by E. A. Franz, Illinois College, Jacksonville, Illinois; Erwin Just, Bronx Community College; and Charles W. Trigg, San Diego, California. Division gives rise to the identity

$$x^{100} = f(x) = (x - 1)(x - 2)\,Q(x) + Ax + B,$$

where Ax + B is the remainder sought. The term $(x - 1)(x - 2)\,Q(x)$ can be eliminated by the substitution of either 1 or 2 for x. Thus, we have

$$f(1) = A + B$$

$$f(2) = 2A + B$$

Then

$$A = f(2) - f(1) = 2^{100} - 1$$

$$B = 2f(1) - f(2) = 2 - 2^{100}.$$

Therefore the remainder, Ax + B, is equal to $(2^{100} - 1)x + 2 - 2^{100}$

or $2^{100}(x - 1) - (x - 2)$.

197. (Fall 1967). Proposed by Joseph **Arkin,** Nanuet, New Jersey.
A box contains $(1600\ u^2 + 3200)/3$ solid spherical metal bearings. Each bearing in the box has a cylindrical hole of length .25 centimeters drilled straight through its center. The bearings are then melted together with a loss of 4% during the melting process and formed into a sphere whose radius is an integral number of centimeters. How many bearings were there originally in the box?

Solution by Charles W. Trigg, San **Diego,** California.
The volume of the "wedding ring" left after a cylindrical hole with axis **along** a diameter is drilled through a sphere is the same as that of a sphere with diameter equal to the length of the hole. **[Cf., e.g.,** Charles W. Trigg, Mathematical Quickies, McGraw-Hill **(1967), page** 179.1 Hence we have

$$4\pi R^3/3 = [1600(u^2 + 2)/3](\pi/6)(1/4)^3(96/100) \text{ or } R^3 = u^2 + 2.$$

The only value of u satisfying this last expression is $u = 5$, whereupon $R = 3$ cm. and the original number of bearings was 14,400.

Also solved by the proposer, who noted that a treatment of the Diophantine equation $R^3 = u^2 + 2$ is given in L. E. **Dickson's** History of the Theory of Numbers, Vol. **II,** p. xiv., Chelsea Publishing Co., New York, 1952.

198. (Fall **1967).** Proposed by Stanley **Rabinowitz,** Polytechnic Institute of Brooklyn. A semiregular solid is obtained by slicing off sections from the corners of a cube. It is a solid with 36 congruent edges, 24 vertices and 14 faces, 6 of which are regular octagons and 8 are equilateral triangles. If the length of an edge of this polytope is e, what is its volume?

Solution by Leon **Bankoff,** Los Angeles, California.
The eight sliced-off pyramids can be assembled to form a regular octahedron of edge e, whose volume is known to be $e^3(\sqrt{2})/3$. Subtracting this quantity from $e^3(\sqrt{2} + 1)^3$, the volume of the cube, we find that the volume of the truncated cube is $7e^3(3 + 2\sqrt{2})/3$.

Also solved by Charles W. Trigg, San **Diego,** California, and the proposer.

Editorial Note. The term "truncated cube" is more descriptive of the residual polyhedron than is the word "polytope", which is general enough to apply to points, segments, polygons, polyhedra and **hyper-**dimensional solids.

199. (Fall 1967). Proposed by Larry **Forman,** Brown University, and M. S. Klamkin, Ford Scientific Laboratory. Find all integral solutions of the equation

$$\sqrt[3]{x + \sqrt{y}} + \sqrt[3]{x - \sqrt{y}} = z.$$

Solution by Charles W. Trigg, San **Diego,** California.

I. It is evident upon inspection that if $x = 0$, then $z = 0$ (and conversely), and y is indeterminate. Also, if $y = 0$, then $x = (z/2)^3$, so z is even.

II. Put $x + \sqrt{y} = m^3$ and $x - \sqrt{y} = n^3$, whereupon

$$\sqrt{y} = (m^3 - n^3)/2, \text{ and}$$

$$x = (m^3 + n^3)/2, \text{ and } y = (m^3 - n^3)^2/4,\ z = m + n,$$

where m and n are **integers** with the same parity. This two-parameter solution includes **(I),** for $m = \pm n$.

III. Solution **II** is based upon the restricted assumption that m and n are integers. Cubing both sides of the given equation, we have

$$2x + 3\sqrt[3]{x^2 - y}\left(\sqrt[3]{x + \sqrt{y}} + \sqrt[3]{x - \sqrt{y}}\right) = z^3.$$

Let $x^2 - y = k^3$, whereupon

$$x = z(z^2 - 3k)/2, \text{ and}$$

$$y = [z(z^2 - 3k)/2]^2 - k^3 = (z^2 - 4k)(z^2 - k)^2/4.$$

Solutions not given by **(II),** for example, when $z = 3$ and $k = 1$, are given by this two-parameter solution in z and k, with z even or with z and k both odd, and $z^2 \neq 3k$, $z^2 \neq k$. These two restrictions are necessary for consistency with **(I).** The penultimate restriction is necessary because if $x = 0$, then $z = 0$; and the last one because if $y = 0$, then $x = z^3/8$, whereas if $z^2 = k$, then $x = -z^3$.

Editorial Note. Klamkin cubed the given equation to obtain $2x + 3z\sqrt[3]{x^2 - y} = z^3$. Thus $\sqrt[3]{x^2 - y} = m$ (an integer), and the desired solution is

$$x = (z^3 - 3zm)/2, \qquad y = [(z^3 - 3zm)/2]^2 - m^3,$$

where z and m are arbitrary integers, provided either z is even or both z and m are odd. For values of z and m that result in $y<0$, the "proper" cube roots must be extracted to satisfy the original equation. For example, suppose $z = 3$ and $m = 3$. Then $x = 0$, $y = -27$, and $z = 3$. Substitution of these values in the given equation yields

$$\sqrt[3]{\sqrt{-27}} + 3\ [illegible]\ 3.$$

Recasting this in the form $\sqrt{3}\sqrt{i} - \sqrt{3}\sqrt{i} = 3$, an obvious impossibility, we are confronted by the intrusion of an extraneous root. On the other hand, the solution becomes acceptable by the following procedure:

$$\begin{aligned}\sqrt[3]{\sqrt{-27}} + \sqrt[3]{-\sqrt{-27}} &= 27^6[(i)^{1/3} + (-i)^{1/3}]\\ &= \sqrt{3}[e^{i\pi/6} + e^{-i\pi/6}]\\ &= 2\sqrt{3}\cos(\pi/6)\\ &= 3.\end{aligned}$$

Also solved by Edgar Karst, University of Arizona, and Gregory Wulczyn, Bucknell University, both of whom submitted partial solutions for integral values of $x + \sqrt{y}$.

200. (Spring 1968). Proposed by Helen M. Marston, Douglas College.
The arithmetic identities

$$6 + (7 \times 36) = 6 \times (7 + 36),$$
$$10 + (15 \times 28) = 10 \times (15 + 28),$$
$$12 + (15 \times 56) = 12 \times (15 + 56),$$

suggest the problem of finding the general solution, in positive integers, to the equation

$$a + (b \cdot c) = a \cdot (b + c).$$

In particular, how many pairs of positive integers (b,c) with $b<c$ satisfy the latter equation if $a = 21$?

Solution by Charles W. Trigg, San Diego, California.
$a + bc = a(b + c)$ implies $b = a + a(a - 1)/(c - a)$. The number of solutions in positive integers for any given a is equal to the number of factors of $a(a - 1)$, that is, $d[a(a - 1)]$, and the number of distinct pairs is $d[a(a - 1)]/2$. Corresponding to the complementary factors f and g, $(f<g)$, are $b = a + f$, $c = a + g$. Thus when $a = 21$, the twelve distinct pairs are: (22, 441), (23, 231), (24, 161), (25, 126), (26, 105), (27, 91), (28, 81), (31, 63), (33, 56), (35, 51), (36, 49), (41, 42).

Also solved by Richard L. Enison, New York; Joe Konhauser, Macalester College; Graham F. Lord, Philadelphia; Gregory Wulczyn, Bucknell University; and the proposer. Two incorrect solutions were received.

201. (Spring 1968). Proposed by R. C. Gebhardt, Parsippany, New Jersey.
Out of the nine digits 1, 2, 3,..., 9, one can construct 9! different numbers, each of nine digits. What is the sum of these 9! numbers?

Solution by Joe Konhauser, Macalester College.
Let the 9! numbers be arranged in the usual manner for addition. In each column, each of the digits 1 through 9 will appear 8! times. The sum of the numbers in each column will be

$$8!(1 + 2 + \dots + 9) = 8!(45) = 1{,}814{,}400.$$

The sum of the 9! numbers will be

$$1{,}814{,}400(1 + 10 + 100 + \dots + 10^8) = 1{,}814{,}400 \times 111{,}111{,}111$$
$$= 201{,}599{,}999{,}798{,}400.$$

Also solved by Leonard Cupingood, Newark, N. J.; Richard L. Enison; Keith Giles, Norman, Oklahoma; Robert J. Herbold, Cincinnati; Neal H. Kilmer, Oklahoma State University; Bruce W. King, Burnt Hills, N. Y.; Graham F. Lord, Philadelphia; John McNear, Lexington (Mass.) High School; Norman Pearl, Pratt Institute; Andrew E. Rouse, University of Mississippi; Catherine J. Strahl, Temple University; Charles W. Trigg, San Diego, California; Gregory Wulczyn, Bucknell University; and the proposer.

BOOK REVIEWS

Edited by

Roy B. Deal, Oklahoma University Medical Center

1. Theoretical and Mathematical Biology By Talbot H. Waterman and Harold J. Morowitz, Blaisdell Publishing Company, New York, 1965, xvii + 426 pp.

A series of seventeen chapters, written by well-known biologists, which gives an excellent survey of a variety of the important areas in biology where rather extensive and interesting mathematical models promise to play a big role.

2. An Introduction to Probability Theory and Its Applications Vol I, 3rd Edition, By W. Feller, John Wiley & Sons, Inc., New York, 1968, xviii + 509 pp., $10.95.

A third and revised edition of the now famous classic in modern mathematical writings. Many proofs and developments have been modernized. In particular the chapter on fluctuations in coin tossing and random walks has been extensively rewritten and expanded to incorporate modern probabalistic arguments. Sections have been added on branching processes, on Markov chains, and on the De Moivre-Laplace theorem. These changes, along with other clarifications and rearrangements, and the established importance of the earlier editions make this also a valuable book.

3. An Introduction to Probability Theory and Its Applications By William Feller, John Wiley and Sons, Inc., New York, 1966, xviii + 626 pp.

Whereas the first volume was basically a study of discrete probabilities and was a pioneer in its mathematical treatment of applied problems, the second volume covers a larger spectrum, utilizes Lebesgue measure, has many theorems and applications on more general multidimensional distributions, on more general Markov processes, random walks, renewal theory and other aspects of stochastic processes, and many interesting uses of such things as semi-groups, Tauberian theorems, Laplace transforms, and harmonic analysis. This volume may not have as much of the pioneering aspect but it reflects the same organizational talent of a master who can bring difficult subjects to within the grasp of one with a minimal background, say elementary real analysis and volume one.

4. Integration By A. C. Zaanen, John Wiley and Sons, Inc., New York, 1967, xiii + 604 pp., $16.75.

Although this is an advanced and extensive book on integration, and perhaps beyond the level of many Pi Mu Epsilon readers, it is such an excellent book that it should be brought to the attention of most members. It is a completely revised and enlarged edition of his well-written earlier book "An Introduction to the Theory of Integration."

5. Combinatorial Identities by J. Riordan, John Wiley and Sons, Inc., 1968, xii + 256 pp., $15.00.

A comprehensive, coordinated collection of combinatorial identities including "The most extensive array of inverse relations available," and a survey of number-theoretical aspects of partition polynomials.

6. Quantum Mechanics By R. A. Newing and J. Cunningham, John Wiley and Sons, Inc. 1967, ix + 225 pp., $4.50.

Although there are many fine books on Quantum Mechanics at the first year graduate level, this little book which grew out of a course for final year honors students of mathematical physics is, because of the spirit in which it is written, perhaps the best introduction to mathematical quantum mechanics for mathematics students at the senior-first year graduate level.

7. Dynamic Plasticity By N. Cristescu, John Wiley and Sons, Inc., An import from the North-Holland Publishing Company, 1968, xi + 614 pp., $25.00.

The North-Holland Series in Applied Mathematics and Mechanics is attempting to foster a continuing close relationship between applied mathematics and mechanics by publishing authoritative monographs on well-defined topics. This reasonably self-contained book presents the main problems considered in the theory of dynamic deformation of plastic bodies. It gives many details regarding mechanical models, computing methods, and programs for the integration with computers. Although it is written so that no previous knowledge of plasticity is required, the solutions to many problems are given with such detail and modern methods that they may be used directly by the practicing engineer.

8. Ordinary Differential Equations and Stability Theory: An Introduction By David A. Sanchez, W. H. Freeman and Company, San Francisco, California, 1968, viii + 164 pp., $3.95 paperbound.

This little book meets quite well its stated objective of giving a brief, modern introduction ot the subject of ordinary differential equations with an emphasis on stability theory to the reader with only a "modicum of knowledge beyond the calculus".

9. Numerical Methods for Two-Point Boundary - Value Problems By Herbert B. Keller, Blaisdell Publishing Company, Waltham, Massachusetts, 1968, viii + 184 pp.

This brief but excellent account of practical numerical methods for solving very general two-point boundary-value problems would follow quite well the above book by Sanchez. "Three techniques are studied in detail: initial-value or "shooting" methods, finite-difference methods, and integral-equation methods. Each method is applied to non-linear second-order problems and eigenvalue problems; the first two methods are applied also to first-order systems of non-linear equations."

10. A Handbook of Numerical Matrix Inversion and Solution of Linear Equations By Joan R. Westlake, John Wiley 6 Sons, Inc., 1968 viii + 171 pp., $10.95.

While this book should be very valuable for its stated purpose as a nearly encyclopedic single reference source for scientific programmers with a bachelors degree and a mathematics major, it might also serve to provide the undergraduate mathematics major with a feeling for this important aspect of real world problems, as well as delineate the essential features for many of today's more sophisticated users.

BOOKS RECEIVED FOR REVIEW

1. Biometry By Charles M. Woolf, Van Nostrand Co., New York, 1968, XIII + 359 pp., $8.75.
2. Introduction to Arithmetic By C. B. Piper, Philosophical Library, Inc., New York, 1968, vii + 211 pp., $6.00.
3. Introduction to Probability and Statistics, Second Edition, William Mendenhall, Wadsworth Publishing Company, Inc., Belmont, California, 1967, xiii + 393 pp.
4. The Design and Analysis of Experiments By William Mendenhall, Wadsworth Publishing Company, Inc., Belmont, California, 1968, xiv + 465 pp.
5. New College Algebra By Marvin Marcus and Henryk Minc, Haughton Mifflin Company, Boston, Mass., 1968, x + 292 pp., $6.50.
6. Introduction to Probability and Statistics, Fourth Edition By Henry L. Adler and Edward B. Roessler, W. H. Freeman and Company, San Francisco, California, 1968, xii + 333 pp., $7.00.
7. Introduction to Matrices and Determinants By Max Stein, Wadsworth Publishing Company, Inc., Belmont, California, 1967, x + 225 pp.
8. Modern Mathematical Topics By D. H. V. Case, Philosophical Library Inc., New York, 1968, viii + 158 pp., $4.75.
9. First-Year Calculus By Einar Hille and Saturnine Salas, Blaiddell Publishing Company, Waltham, Mass., 1968, xi + 415 pp., $9.50.

Note: All correspondence concerning reviews and all books for review should be sent to PROFESSOR ROY B. DEAL, UNIVERSITY OF OKLAHOMA MEDICAL CENTER, 800 NE 13th STREET, OKLAHOMA CITY, OKLAHOMA 73104.

MATCHING PRIZE FUND

The Governing Council of Pi Mu Epsilon has approved an increase in the maximum amount per chapter allowed as a matching prize from $20.00 to $25.00. If your chapter presents awards for outstanding mathematical papers and students, you may apply to the National Office to match the amount spent by your chapter--i.e., $30.00 of awards, the National Office will reimburse the chapter for $15.00, etc.,--up to a maximum of $25.00. Chapters are urged to submit their best student papers to the Editor of the Pi Mu Epsilon Journal for possible publication.

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