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MATHEMATICS AND FRAUD

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Introduction

There are a number of fraudulent schemes currently operating in the nation today which have fleeced many unsuspecting victims of their life savings, homes, and other possessions.

While, in general, fraud is a difficult charge to prove, it is interesting to note that there are certain sales rackets where mathematics can be utilized to establish fraud.

In one such case, NORMAN vs. WORLD WIDE DISTRIBUTORS, INC., [1], a referral plan, mathematical evidence was submitted by the author in behalf of the plaintiff which played a significant part in obtaining a conviction of the fraudulent party.

In the citation of the appeal of the case before the Superior Court of Pennsylvania, it was noted that:

"The referral plan was a fraudulent scheme based on an operation similar to the recurrent chain letter racket. It is one of many sales rackets being carried on throughout the nation which is giving public officials serious concern (see article of Wall Street Journal, page 1, October 10, 1963). The plaintiffs introduced evidence to show that at the end of 20 months of operation, it would require 17 trillion salesmen to carry on a referral program like World Wide described to the plaintiffs."

Although the mathematical problem is a fundamentally simple one, it is another demonstration of the kinds of interesting but simple problems that often occur in mathematics.

Background of the Problem

A company advertised a special "advertising plan" as follows: for every name submitted by an individual - who is a member of its family plan - it will pay $5.00 to both parties (the referrer and the one referred) if the referred person listens to a presentation by one of its salesmen. In addition, the company will pay $40.00 if, after the presentation, the person whose name was submitted bought an item from the company.

In order to participate in this "plan", the individual first had to purchase an item from the company (presumably to establish "good faith.") This item was usually eight hundred to a thousand dollars above the usual market price. However, the company assured individuals that there need not be any concern about the meeting of the monthly finance charges. Indeed! they theorized that not only would the initial surcharge be recouped but also a profit of three or four thousand dollars would be realized by participants in the plan.

As indicated in the citation, it can be demonstrated mathematically that the company could not hire enough salesmen to even begin to carry out such a program.

Mathematical Formulation of the Problem

We will let t denote the time.

For each positive integer n, let t_n denote n units of time, and I_n the interval of time between t_{n-1} and t_n.

For $n > 1$, define the following:

a_n = the number of people seen during I_n.

c_n = the number of people to be seen during I, i. e. these people have agreed to listen to a salesman's presentation.

r_n = number of names submitted during I_{n-1} $n > 2$.

s_n = the number of sales during I_n; s_o = number of charter members.

u_n = number of r_n unable to be seen during I due to inadequate sales staff.

x_n = number of people in the plan at t_n.

y_n = number of salesmen needed at t_{n-1}.

p = number of people that can be seen by each salesman per unit of time.

r = average number of people referred by each new member of the plan for the first unit of time only.

The following simplifying assumptions are made:

$\frac{r_n}{c_n}$ is constant, and $\frac{s_n}{a_n}$ is constant

We let $\frac{c_n}{r_n} = y$ and $\frac{s_n}{a_n} = Z$.

Under the assumption that names were sent in by members of the plan for the first month only, the following problems were posed:

(A) Find explicitly s_n, x_n, and y_n when $u_n = 0$ for all $n > 1$. This means we are asking for the number of people in the plan and the number of salesmen needed at t_n and t_{n-1} respectively if there are enough salesmen to cover each lead.

(B) Find s_n, x_n, and y explicitly if there exists K, an integer, and there exists $\mu \in R$ such that $0 < \mu < 1$, $K > 1$ and

$$\frac{u_n}{c_n} = \mu \quad \text{for } n-1 > K;$$

$$u_n = 0 \quad \text{for } n < K.$$

Problem (B) makes allowance for the fact that there is a lag between hiring and training of salesmen. Hence only a certain proportion of those who should be seen will be seen during a given time period.

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Solution of the problems

We readily observe that

(1) $a_n = y_n p$ for $n \geq 1$; and

$$(2)\quad c_n = \begin{cases} u_{n-1} + rys_{n-1} & \text{for } n-1 \geq K > 1; \\ rys_{n-1} & \text{for } n \leq K; \end{cases}$$

(3) $s_n = a_n Z = y_n p Z$ for $n \geq 1$;

(4) $x_n = x_{n-1} + s_n$;

(5) $u_n = c_n - a_n$.

Solving the recursion formula (4), we obtain

$$(4^*)\quad x_n = \sum_{i=0}^{n} s_i = pZ\sum_{i=1}^{n} y_i + s_o.$$

For $n \leq K$, we have $c_n = a_n$ $(u_n = 0)$ whence $s_n = c_n Z = ryZs_{n-1}$ by (3) and (2).

Letting $a = ryZ$ and solving this recursion relation, we obtain

$$(A^*)\quad \begin{cases} s_n = \alpha^n s_o & \text{for } n \leq K. \text{ Hence} \\ c_n = ry\alpha^{n-1}s_o & \text{from (2)} \quad (n \leq K); \\ y_n = \dfrac{ry\alpha^{n-1}s_o}{p} & \text{from (1)}, \quad (n \leq K); \text{ and} \\ x_n = s_o\dfrac{1-\alpha^{n+1}}{1-\alpha} & \text{for } 0 \leq n \leq K. \end{cases}$$

For $n > K$,

$$\frac{u_n}{c_n} = \mu, \quad \text{whence}$$

(6) $(1-\mu)c_n = a_n = y_n p$ from (5).

Substitution of (2) and (3) in (5), yields

$$u_{K+n} = u_{K+n-1} + rys_{K+n-1} - y_{K+n}p$$

$$= u_{K+n-1} + \frac{\alpha s_{K+n-1} - s_{K+n}}{Z} \quad \text{for } n \geq 1.$$

The solution of this recursion formula is

$$(7) \qquad u_{K+n} = \frac{\alpha s_K}{Z} + \frac{\alpha-1}{Z}\sum_{i=1}^{n-1} s_{K+i} - \frac{s_{K+n}}{Z} \quad \text{for } n \ge 1.$$

Combining (2) and (7) produces

$$(8) \qquad c_{K+n} = \frac{a s_K}{Z} + \frac{\alpha-1}{Z}\sum_{i=1}^{n-1} s_{K+i} \quad \text{for } n \ge 1.$$

From (6),

$$Y_{K+n} = \frac{1-\mu}{p}\, c_{K+n} = \frac{(1-\mu)\alpha}{pZ} s_K + \frac{(1-\mu)(\alpha-1)}{pZ}\sum_{i=1}^{n-1} s_{K+i} \quad \text{for } n \ge 0.$$

Now utilizing the first formula in (A^*) and letting $\beta = (1-\mu)\alpha s_K = (1-\mu)\alpha^{K+1}s_o$; and $\phi = (\alpha-1)(1-\mu)$. We have

$$(9) \qquad y_{K+n} = \frac{\beta}{pZ} + \phi\sum_{i=1}^{n-1} y_{K+1} \quad \text{for } n \ge 1.$$

Using (9) we can calculate y_{K+n} for the few values of n. We obtain:

$$y_{K+1} = \frac{\beta}{pZ};$$

$$y_{K+2} = \frac{\beta}{pZ}(1+\phi);$$

$$y_{K+3} = \frac{\beta}{pZ}(1+2\phi+\phi^2);$$

$$y_{K+4} = \frac{\beta}{pZ}(1+3\phi+3\phi^2+\phi^3).$$

This seems to imply that:

$$(10) \qquad y_{K+n} = \frac{\beta}{pZ}(1+\phi)^{n-1} \quad (n \ge 1).$$

We prove (10) by induction. It is sufficient to prove that

$$y_{K+n} = \frac{\beta}{pZ}(1+\phi)^{n-1} \text{ if } y_{K+m} = \frac{\beta}{pZ}(1+\phi)^{m-1} \text{ for all } 1 \le m < n.$$

The following preliminary result can be proved using standard techniques, e.g., induction or Pascal's triangle. Let ${}_nC_k$ denote the binomial coefficient $\frac{n!}{k!(n-k)!}$.

$$(11) \qquad \sum_{i=k+1}^{n-1} {}_{i-1}C_k = {}_{n-1}C_{k+1} \quad \text{for all } n \ge k+2.$$

Using (9), we have:

$$y_{K+n} = \frac{\beta}{pZ} + \phi\sum_{i=1}^{n-1} y_{K+i}; \quad \text{whence}$$

$$y_{K+n} = \frac{\beta}{pZ}\left(1+\phi\sum_{i=1}^{n-1}\sum_{k=0}^{i-1} {}_{i-1}C_k\phi^k\right) \text{ [by our induction hypothesis]}$$

$$= \frac{\beta}{pZ}\left(1+\sum_{k=0}^{n-2}\left(\sum_{i=k+1}^{n-1} {}_{i-1}C_k\right)\phi^{k+1}\right) \text{ [upon interchanging summations]}$$

$$= \frac{\beta}{pZ}\left(1+\sum_{k=0}^{n-2} {}_{n-1}C_{k+1}\phi^{k+1}\right) \text{ [by (11)]}$$

$$= \frac{\beta}{pZ}\sum_{k=0}^{n-1} {}_{n-1}C_k\phi^k \text{ [by changing the index and since } {}_{n-1}C_o = 1\text{]}$$

$$= \frac{\beta}{pZ}(1+\phi)^{n-1}; \quad \text{which completes the proof}$$

To find s_n, we use (3) and (10) to obtain:

$$(12) \qquad s_{K+n} = \beta(1+\phi)^{n-1} \quad \text{for } n \ge 1.$$

To find x_n, we use (4^*) and (12) to obtain:

$$x_n = \sum_{i=0}^{K} s_i + \sum_{i=1}^{n-K} s_{K+i} = x_K + \beta\sum_{i=0}^{n-K-1}(1+\phi)^i$$

$$= s_o\,\frac{\alpha^{K+1}-1}{\alpha-1} + \beta\left(\frac{(1+\phi)^{n-K}-1}{\phi}\right) \quad \text{from } (A^*)$$

$$= \frac{s_o}{a-1}\left(\alpha^{K+1}(1+\phi)^{n-K}-1\right) \quad \text{for } n \ge K \text{ since } \frac{\beta}{\phi} = \frac{\alpha^{K+1}s_o}{a-1}$$

The solution to (B) is summarized by:

Theorem: Let $a = ryZ$, $\beta = (1-\mu)\alpha^{K+1}s_o$, and $\phi = (\alpha-1)(1-\mu)$.

Then the solution to problem [illegible] y:

$$y_n = \text{number of salesmen} = \begin{cases} \dfrac{ry\alpha^{n-1}s_o}{p} & \text{for } n \le K; \\[2ex] \dfrac{\beta}{pZ}(1+\phi)^{n-K-1} & \text{for } n > K. \end{cases}$$

$$s_n = \text{number of sales} = \begin{cases} \alpha^n s_o & \text{for } n \le K; \\ \beta(1+\phi)^{n-K-1} & \text{for } n > K. \end{cases}$$

$$x_n = \text{number of people in the plan} = \begin{cases} s_o \dfrac{\alpha^{n+1}-1}{\alpha-1} & \text{for } n \le K; \\ \dfrac{s_o}{\alpha-1}\left(\alpha^{K+1}(1+\phi)^{n-K}-1\right) & \text{for } n > K. \end{cases}$$

Application and Historical Notes

Problem (A) is the mathematical prototype of the problem posed by the Plaintiff's lawyer. However, the author felt that it would be difficult to prove fraud with this because there was not enough time allowed for training of salesmen, allowing backlog, transmittal of information, duplicate referrals, and other administrative problems. Problem (B) was posed by the author to allow for this.

A strong mathematical case is made by setting

$p = 27$ (this assumes a salesman works everyday except Sunday but is only able to see one prospective client)

$K = 0$, $\mu = 1/2$ (this assumes an immediate backlog, after the first day, of 1/2 of all prospective clients unable to be seen each month).

$y = 1/2$, $Z = 1/3$ (this assumes that only 1/6 of the people who saw the presentation joined the program).

$r = 15$ (this assumes that an average of only 15 names was submitted by each new member of the plan).

$s_o = 1$ (only 1 member in the plan when the company began).

Under these very weak assumptions, the company would need 5,397,000 salesmen after only 24 months. For completeness, other values of the parameters are illustrated below.

In all cases $p = 27$, $K = 0$, $s_o = 67.5$, and $n = 24$.

For $\mu = 1/2$, $r = 30$, $y = 1/2$, $Z = 1/3$; $y_{24} = 1{,}764{,}983{,}987{,}500$;

$\mu = 1/3$, $r = 60$, $y = 1/3$, $Z = 1/4$; $y_{24} = 2{,}353{,}437{,}500{,}000$.

References

1. Norman vs. World Wide Distributors Inc., 202 Pa. Sup. 59, 195 A. 2d 115.

2. Buck, R. Creighton, Advanced Calculus, 2nd edition, McGraw Hill, New York, 1965.

3. Paley, Hiram and Weichsel, A First Course in Abstract Algebra, Holt Rinehart and Winston, Inc., New York, 1966.

LENS SPACES AS COSET SPACES

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1. Introduction. A topological group is simultaneously a topological space and an abstract group whose group operations (the identity map, inverse map, etc.) are continuous on the space. Some well-known examples of topological groups are R^n under addition and S^1 under complex multiplication. It is easy to verify that S^3, the unit sphere in R^4, is also a topological group when the group operation is defined to be quaternion multiplication. The group structure on S^3 can alternatively be described by complex coordinates:

$$S^3 = \{(z_1,z_2) \mid z_i \in C,\ |z_1|^2 + |z_2|^2 = 1\}$$

Then the multiplication in S^3 becomes

$$(z_1,z_2)(z_3,z_4) = (z_1z_3 - z_2\bar{z}_4,\ z_1z_4 + z_2\bar{z}_3)$$

where (z_1,z_2) and (z_3,z_4) are arbitrary points of S^3 and where $\bar{z}_i$ denotes the complex conjugate of z_i. We will refer to this particular group later.

Let X be a topological group and let A be any subgroup of X. Let X/A denote the set of all left (or right) cosets. X/A becomes a topological space when the quotient topology is induced on it, and X/A is then called a coset space. Note that X/A is a topological group iff A is a normal subgroup of X.

A broad brancn of topological group theory deals with the action of a group on a topological space. A group G "acts" on a space X if each element of the group is a homomorphism of X. If $x \in X$, we say the homomorphism $g \in G$ has a fixed point at x if $g(x) = x$. G acts freely on X if each $g \in G$ ($g \neq$ identity map) has no fixed points.

For any $x \in X$, one defines the orbit of x as the set:

$$G_x = \{g(x) \mid \text{all } g \in G\}$$

These orbits decompose X into equivalence classes, the equivalence relation being $x \sim y$ iff there exists a $g \in G$ such that $g(x) = y$. The topological quotient space obtained under this relation is denoted X_G and is called an orbit space of X.

If X is a topological group and $a \in X$, the map $L_a: X \to X$ given by $L_a(x) = ax$ for each $x \in X$ is easily shown to be a homomorphism of X for each a. Such a map is called a left translation of X. Let X/A be a right coset space. Associate with each $a \in A$ the left trans-map $L_a: X \to X$. It is clear that the set G of all such maps forms a group that is isomorphic to A. And since each element of G is a homomorphism of X, it follows that G acts on X. For any $x \in X$, the orbit G_x contains the same elements as the coset Ax. In this manner, it is clear that every right (or left) coset space of X is also an orbit space of X.

2. Lens Spaces. A Lens Space is an orbit space obtained by a certain action of a finite group on S^3. Let Z_p be the group of integers under addition modulo p. For any choice of relatively prime integers p and q $(p > q)$, consider the action of Z on S^3 given by

$$g_i(z_1,z_2) = (w^i z_1, w^{iq} z_2)$$

where $(z_1,z_2) \in S^3$, $i \in Z_p$, and $w = p^{th}$ primitive root of unity. For each choice of p and q, we call the orbit space resulting from this action $L(p,q)$, a Lens Space of type (p,q).

Note that the set

$$P = \{(w^i,0) \mid (w^i,0) \in S^3,\ w = p^{th} \text{ root of } 1\}$$

is a subgroup of S^3. Consider the left coset space S^3/P. Clearly, any left coset $(z_1,z_2)P$ contains exactly the elements of the orbit of (z_1,z_2) in $L(p,p-1)$ since

$$(z_1,z_2)(w^i,0) = (w^i z_1, \overline{w^i} z_2) = (w^i z_1, w^{p-1} z_2)$$

where $\overline{w^i}$ is the complex conjugate of w^i. In a similar fashion, we can show that the right coset space S^3/P is $L(p,1)$.

These considerations lead to some interesting questions. Can each $L(p,q)$ be obtained as a coset space of S^3 as well as an orbit space? More generally, when is an orbit space of a topological group also a coset space? The following theorems provide an answer to these questions.

Theorem 1: Let X be a topological group and let G be any group acting freely on X to form the orbit space X_G. If each $g \in G$ acts by left (or right) translation, then $X_G = X/G_1$, where G_1 is the orbit of the identity $(=1)$ of X.

Proof: We first show that G_1 is a subgroup of X. Let $\phi: G \to X$ be given by $\phi(g) = g(1)$ for each $g \in G$. Clearly, ϕ maps G onto G_1. Let $g_1, g_2 \in G$, and suppose $g_1(x) = ax$, $g_2(x) = bx$ for some $a, b \in X$ and all $x \in X$. We have

$$\phi(g_1 g_2) = (g_1 g_2)(1) = g_1(b) = ab = \phi(g_1)\,\phi(g_2)$$

hence ϕ is a homomorphism. If $g_1 \neq g_2$, then $a \neq b$ and so ϕ is 1-1. Hence we have $G \approx G_1$. Thus, G_1 is a subgroup of X.

Now, for any $g_i \in G$, we have

$$g_i(x) = a_i x = (g_i(1))x$$

for any $x \in X$ and some $a_i \in G_1$. Hence $G_x = G_1 x$ for all x, where $G_1 x$ is a coset. We induce the same quotient topology on X_G and X/G_1 and thus, $X_G = X/G_1$.

Our ultimate goal is to prove Theorem 1 in the opposite direction. To do this, we need the following lemma:

Lemma: Let Y be any Hausdorff Topological space. If $f, g: Y \to Y$ are continuous, then the set of coincidences of f and g is closed.

Proof: Let $C = \{y \in Y \mid f(y) = g(y)\}$, and let C' be the complement of C in Y. Let $x \in C'$. Let A be an open neighborhood of $f(x)$ and B an open neighborhood of $g(x)$. Choose A and B such that $A \cap B = \phi$. Then $f^{-1}(A)$ and $g^{-1}(B)$ are open neighborhoods of x. Hence $f^{-1}(A) \cap g^{-1}(B)$ is open and non-empty. However,

$$C \cap (f^{-1}(A) \cap g^{-1}(B)) = \phi$$

Hence C' is an open set. Thus C is closed.

Now we are in a position to prove:

Theorem 2: Let G be a finite group acting freely on a connected, Hausdorff topological group X. If $X_G = X/M$ (the right coset space) for some subgroup M of X, the G acts by left translation. Furthermore, $M = G_1$.

Proof: Choose any $x \in X$. The orbit G contains exactly the elements of the coset Mx. Fix $g \in G$. Clearly, $g(x) = m x$ for some $m \in M$. We must show that $g(y) = m_x y$ for all $y \in X$.

Consider the set

$$S_{g,m_x} = \{y \in X \mid g(y) = m_x y\}$$

Let $M_x : X \to X$ denote the translation map given by $M_x(y) = my$ for all $y \in X$. By the above lemma, the set of coincidences of g and M_x is closed. Clearly, this set is exactly S_{g,m_x}.

If $S_{g,m_x} \neq X$, then there is a $z \in X$ such that $g(z) \neq mz$. But since $G = M_z$ there must exist an $m \in M$, $m_z \neq m_x$, such that $g(z) = mz$. Let $S_{g,m_z} = \{y \in X \mid g(y) = m_z y\}$ Clearly, this set is also closed.

In such a fashion, we can decompose X into a finite number of closed sets of the form $S_{g,m}$, one for each $m \in M$. We have

$$\bigcup_{m \in M} S_{g,m} = X$$

since g is a homomorphism. Furthermore,

$$S_{g,m_y} \cap S_{g,m_{y'}} = \phi$$

for all m_y and $m_{y'}$ in M, $m_y \neq m_{y'}$, since, if this were not true, we would have

$$g(x) = m_y x = m_{y'} x$$

an obvious impossibility.

Thus, we have X covered by a finite number of disjoint closed sets. But since X is connected, this is impossible. Hence, $S_{g,m_x} = X$ for some $m \in M$. Therefore, g is a translation map. In the same manner, we can show that all of G acts by left translation. Then, invoking Theorem 1, we have $X/M = X_G = X/G_1$ which forces $M = G_1$.

Remark: This same theorem holds true if we assume that X/M is a left coset space. Then, however, G acts by right translation and we must modify the proof accordingly.

3. Application to Lens Spaces. We can now use these theorems to answer the question of whether or not $L(p,q)$ is a coset space of S^3 when $q \neq 1$, p-1. We first prove:

Lemma: Let Z_p act on S^3 to form $L(p,q)$ in the usual manner. When $q \neq 1$, p-1, Z_p does not act by translation.

Proof: Assume $g \in Z_p$ acts by left translation. Then there is an $(x,y) \in S^3$ such that

$$g(z_0,z_1) = (x,y)\,(z_0,z_1) = (w^i z_0,\ w^{iq} z_1)$$

for any $(z_0,z_1) \in S^3$. Solving the coordinate equations simultaneously for x and y, we have:

(1) $$x = w^i|z_0|^2 + w^{iq}|z_1|$$

(2) $$y = z_0 z_1 (w^{iq} - w^i)$$

Let $(z_2,z_3) \in S^3$. We must have:

$$(x,y)(z_2,z_3) = (w^i z_2,\ w^{iq} z_3)$$

Substituting (1) and (2) for x and y, we find:

(3) $$w^i z_2 = (w^i|z_0|^2 + w^{iq}|z_1|^2)z_2 - \bar{z}_3 z_0 z_1 (w^{iq} - w^i)$$

(4) $$w^{iq} z_3 = (w^i|z_0|^2 + w^{iq}|z_1|^2)z_3 - \bar{z}_2 z_0 z_1 (w^{iq} - w^i)$$

Reducing (3) and (4), we have

(5) $$(w^{iq} - w^i)z_2 z_3 = (w^{iq} - w^i)z_0 z_1$$

Since $q \neq 1$, $(w^{iq} - w^i)$ is non-zero, and (5) yields

(6) $$z_2 z_3 = z_0 z_1$$

Now, (z_0,z_1) and (z_2,z_3) are arbitrary points of S^3, and therefore, (6) must be an identity on S^3. This is clearly impossible, and we have a contradiction. Thus we see that Z does not act by left translation on S^3.

The proof that Z_p does not act on S^3 by right translation depends on the fact that $q \neq p-1$, and is entirely similar to the argument above.

Invoking Theorems 1 and 2, it follows that $L(p,q)$ is not a coset space of S^3 when $q \neq 1$, p-1.

A GENERALIZATION OF SUBNET WITH SOME RESULTING IMPROVEMENTS IN MOORE-SMITH CONVERGENCE THEORY

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Section 1. Introduction.

This paper is intended to improve the theory of Moore-Smith Convergence by generalizing the definition of **subnet.** We begin by examining some short-comings of the present Moore-Smith theory of convergence. Given a net S, it is possible to construct in a natural way a filter dependent on S. From this filter a second net T may be constructed. While S may be shown to be a **subnet** of T, T in general is not a **subnet** of **S,** even though S and T generate the same filter (See example 3). Also, given nets S and T defined on the same directed set, T may equal S on all but one element of the directed set and still not be a **subnet** of S (See example 1). These limitations in the theory illustrate the need for a new definition of **subnet.**

The new definition will generalize the classical definition of **subnet.** It will have the advantage of preserving the classical theorems, while eliminating the above disadvantages. It will also yield the following powerful result:

Given nets S and **T,** and filters Φ_S and Φ_T constructed from them, $\Phi_S \subseteq \Phi_T$ implies T is a **subnet** of S under the new definition. In addition, this result will provide an easy method for finding a common **supernet** for nets S and **T.**

Section 2. Definition and generalization of subnet.

Throughout the remainder of the paper, unless otherwise specified, let us assume that S is a net from the directed set $[D, <]$ into the topological space X, and T is a net from directed set $[E, <<]$ into X. First, before presenting a generalization of **subnet,** we review the classical definition of **subnet.**

DEFINITION 1: T is a **subnet** of S if and only if there is a function N mapping **E** into D such that
 i) for each $n \in D$ there exists $p \in E$ such that $p << q$ implies $n < N(q)$ and
 ii) $T = S \circ N$.

REMARK I: Notice condition (ii) requires that $T[E] \subseteq S[D]$.

Consider now the following generalization of **subnet** which we will call an a-subnet.

DEFINITION 2: T is an a-subnet of S if and only if given $n_0 \in D$ there exists N mapping **E** into D and there exists $p \in E$ such that $p << q$ implies $n < N(q)$ and $S \circ N(q) = T(q)$.

It follows from this definition that a **subnet** is an a-subnet. The converse, however, is not necessarily true.

EXAMPLE 1: Let R be the set of real numbers with the usual topology, and let **Z** be the set of natural numbers directed by the relation $\leq$. Define $S(n) = \pi$ for all $n \in Z$, and $T(1) = 1$, $T(n) = \pi$ for all $n \in Z$ such that $n > 1$. T is not a **subnet** of **S,** but T is an a-subnet of S.

PROOF: Assume T is a **subnet** of S. Then $T[Z] \subseteq S[Z]$ (by remark 1) and thus $\{\pi, 1\} \subseteq \{\pi\}$ which is a contradiction. Therefore T is not a **subnet** of S. To show T is an a-subnet of **S,** let $n_0 \in Z$, and define N mapping **Z** into Z by $N(n) = n$. Now let $p = n_0 + 2$. If $p \leq q$ then $N(q) = q \geq p \geq n_0$. Also, $S(N(q)) = S(q) = \pi$. However since $q \geq n_0 + 2 > 1$, $T(q) = \pi$. Then $(S \circ N)(q) = T(q)$. Hence T is an a-subnet of S.

Section 3. Classical Theorem Preserved.

The subsequent classical theorems remain valid when "**subnet**" is replaced by "a-subnet".

THEOREM 1: If T is an a-subnet of S and S is eventually in $A \subseteq X$, then T is eventually in A.

PROOF: S is eventually in A implies there exists $n_0 \in D$ such that for all $m \in D$ satisfying $n_0 < m$, $S(m) \in A$. Since T is an a-subnet of **S,** for n_0 there exists an N mapping **E** into D and a $p \in E$ such that $p << q$ implies $n_0 < N(q)$ and $(S \circ N)(q) = T(q)$. But $n_0 < N(q)$ implies $S(N(q)) \in A$ and thus $T(q) \in A$, Therefore, for all $q \in E$ such that $p << q$, $T(q) \in A$. Hence T is eventually in A.

THEOREM 2: If T is an a-subnet of S and S converges to **x,** then T converges to x.

PROOF: This follows directly from Theorem 1 and the definition of convergence.

THEOREM 3: x is a cluster point of S if and only if there exists an a-subnet T of S such that T converges to x.

PROOF: The sufficient condition will be a corollary to Theorem **1,** **while the necessary** condition will be obtained by observing that there exists a **subnet** of S converging to **x,** hence an a-subnet.

Section 4: New Results.

In addition to these theorems, using the definition of a-subnet, one can prove several results which are not valid for **subnets.** To prove these results we introduce the concepts of filters constructed from nets and of nets formed from filters:

Given a net S, $\Phi_S = \{F \subseteq X \mid S$ is eventually in $F\}$ can be shown to be a filter.

Given a filter 9, a net S_Φ may be constructed in this way: Define $D_\Phi = \{(F,x) \mid F \varepsilon \Phi, x \varepsilon F\}$ and a relation <, by $(F,x) < (G,y)$ if and only if $F \supseteq G$. D_9 is a directed set. Define S_Φ mapping D_9 into X, by $S_\Phi(F,x) = x$. Clearly, S_9 is a net.

In the same manner a net S_B can be constructed from a base B of a filter. With these preliminary results we now have the tools to prove:

THEOREM 4: $9_S \subseteq 9_T$ implies T is an a-subnet of S.

PROOF: Let $n \varepsilon D$, and $e \varepsilon E$. To define N mapping E into D, let us distinguish two cases:
Case 1) If there exists an $m \varepsilon D$, $n < m$, such that $S(m) = T(e)$ then $N(e) = m$. (any choice will do.)
Case 2) If not, define $N(e) = n$.
Now let us define $D = \{m \varepsilon D \mid n < m\}$ and $F = S[D_n]$.

It follows that $F \varepsilon \Phi_S$, and since $\Phi_S \subseteq 9$, $F \varepsilon \Phi_T$. Hence T is eventually in F_n Therefore there exists $p \varepsilon E$ such that $p << q$ implies $T(q) \varepsilon F_n$. But since $F = S[D_n]$, $T(q) = S(m_0)$ for some $m_0 \varepsilon D$, $n < m_0$. Thus, case 1 applies and $N(q) = m$ where $n < m$, and $S(N(q)) = S(m) = T(q)$. Hence T is an a-subnet of S.

Using Theorem 1, one can verify that the converse of Theorem 4 is also true.

EXAMPLE 2: Let us assume the same hypothesis as in example 1. Then $9 = \{F \subseteq X \mid S$ is eventually in $F\} = \{F \subseteq X \mid \pi \varepsilon F\}$. Clearly if $F \varepsilon \Phi_S$ then $F \varepsilon \Phi_T$ since for all $n > 1$, $T(n) = \pi \varepsilon F$. Therefore $\Phi_S \subseteq \Phi_T$. But, as shown in Example 1, T is not a **subnet** of S. However T is an a-subnet of S. Before proving a number of corollaries to Theorem 4, let us generalize the notion of equivalence of nets.

DEFINITION 3: S and T are a-equivalent if and only if S and T are a-subnets of each other.

COROLLARY 1: If $\Phi_S = \Phi_T$, then S and T are a-equivalent.

PROOF: This is immediate from Theorem 4.

COROLLARY 2: If the net S yields the filter Φ_S which in turn yields the net $S_{\Phi_S} = T$, then S and T are a-equivalent.

PROOF: One can verify that $\Phi_S = \Phi_T$, the filter constructed from the net T. Thus from Corollary 1, S and T are a-equivalent.

EXAMPLE 3: This example will show that the above corollary is not true for **subnets.** Let S be the net mapping $[Z, \leq]$ into R defined by $S(n) = 1/2^n$ and assume $S_\Psi = T$ is a **subnet** of S. Thus $T[D_\Psi S] \subseteq S[Z]$.

But S is eventually in $[0, 2]$ = F. Hence $F \varepsilon \Psi_S$ and the pair $(F, 2) \varepsilon D\Phi S$.

$T(F, 2) = 2$ which implies $2 \varepsilon S[Z]$ or $2 \varepsilon \{1/2^n \mid n = 0, 1, 2, \ldots\}$ which is a contradiction. Therefore T is not a **subnet** of S.

COROLLARY 3: If B and C are bases for the filter 9, then $S_B = S$ and $S_C = T$ are a-equivalent.

PROOF: It can be proved that S yields the filter Φ_S which equals 9. Also the filter Φ_T constructed from the net T equals 9. Therefore $\Phi_S = \Phi_T$, and by Corollary 1, S and T are a-equivalent.

COROLLARY 4: If S and T are nets mapping the same directed set, $[D, \leq]$ into X, and if there exists $p \varepsilon D$ such that for all $q \varepsilon D$ where $p < q$, $S(q) = T(q)$, then S and T are a-equivalent.

COROLLARY 5: Given nets S and T into the same topological space X, there exists a net R into X such that S and T are a-subnets of R.

PROOF: Let $9 = \Phi_S \cap \Phi_T$. 9 is a filter; thus a net R can be constructed from it. One can demonstrate that the filter Φ_R, formed from the net R, equals 9. Therefore $\Phi_R \subseteq \Phi_S$ and $9_R \subseteq 9$, implying that S and T are a-subnets of R.

Reference

J. L. Kelley, General Topology, Van Nostrand, Princeton, 1955

A NECESSARY AND SUFFICIENT CONDITION FOR CONVERGENCE OF INFINITE SERIES

T. L. Leavitt

A classical necessary condition for the convergence of an infinite series

$$\sum_{n=1}^{\infty} a_k$$

(namely, $a_n \to 0$ as $n \to \infty$) fails to show the divergence of the harmonic series

$$\sum_{k=1}^{\infty} \frac{1}{n} .$$

When examining generalized convergence for infinite series, we usually encounter Cesáro summability (See [1], [2], and [3]) as prime alternative If one looks closely at the difference between ordinary and Cesáro convergence, a necessary and sufficient condition for ordinary convergence is found, yielding in particular, a stronger necessary condition for convergence (one showing that the harmonic series diverges).

We make the following conventions.

$$S_n = \sum_{k=1}^{n} u_k;$$

$$\sum_{k=1}^{\infty} u_k = A \Longleftrightarrow S_n \to A;$$

$$A_n = \frac{1}{n}\sum_{k=1}^{n} S_k = \frac{S_1 + S_2 + \cdots + S_n}{n};$$

$$\sum_{k=1}^{\infty} u_k = A(C\text{-}1) \Longleftrightarrow A_n \to A.$$

The symbol $\sum_{k=1}^{\infty} u_k = A(C\text{-}1)$ is read: $\sum_{k=1}^{\infty} u_k$ is Cesáro summable to A

We note also that A_n is the average of the first n partial sums.

The main theorem follows.

THEOREM 1. $\sum_{k=1}^{\infty} u_k = A \Longleftrightarrow A_n \to A$ and $\sum_{k=1}^{n} \frac{ku_k}{n} \to 0$ as $n \to \infty$.

PROOF. The proof that $S_n \to A \Longrightarrow A_n \to A$ and $\sum_{k=1}^{n} \frac{ku_k}{n} \to 0$ is given in three parts:

1. $S_n \to 0 \Longrightarrow A_n \to 0;$
2. $S_n \to A \Longrightarrow A_n \to A;$
3. $S_n \to A \Longrightarrow \sum_{k=1}^{n} \frac{ku_k}{n} \to 0.$

A proof of (1) and (2) may be seen in [3] page 311 but we provide one here for completeness. (1) Let $\varepsilon > 0$ and choose $N(\varepsilon) = N$ so that $n \geq N$ implies $|S_n| < \varepsilon$. For such n,

$$|A_n| = \frac{1}{n}|S_1 + S_2 + \cdots + S_n|$$

$$\leq \frac{1}{n}[(|S_1| + \cdots + |S_N|) + (|S_{N+1}| + \cdots + |S_n|)]$$

$$\leq \frac{MN}{n} + \left[\frac{n-N}{n}\right]\varepsilon;$$

where M is chosen to satisfy $|S_j| \leq M$ for all j. Taking limits as $n \to \infty$ we find $A_n \to 0$ as predicted. (2) Suppose $S_n \to A$ and define $u_1' = u_1 - A$, and $u_k' = u_k$ for $k > 1$. Then if $S_n' = \sum_{k=1}^{n} u_k'$ and $A_n' = \frac{1}{n}\sum_{k=1}^{n} S_k'$, $S_n' = S_n - A$ and $A_n' = A_n - A$ whereby

$$S_n' \to 0 \Longrightarrow A_n' \to 0;$$

(By (1) above).

Alternatively, $A_n \to A$.

(3) If $S_n \to A$, $\frac{S_n}{n} \to 0$. Since, by (2), $A_n \to A$. $S_n + \frac{S_n}{n} - A_n \to 0$. A quick check shows that

$$S_n + \frac{S_n}{n} - A_n = \sum_{k=1}^{n} \frac{ku_k}{n}$$

Thus

$$\sum_{k=1}^{n} \frac{ku_k}{n} \to 0.$$

For the converse, suppose $A_n \to A$ and $\sum_{k=1}^{n} \frac{ku_k}{n} \to 0$. Clearly, $A_{n-1} \to A$ whereby $(A_n - A_{n-1}) \to 0$ as $n \to \infty$. However, $A_n - A_{n-1} = \frac{S_n}{n} - \frac{A_{n-1}}{n}$. Since $\frac{A_{n-1}}{n} \to 0$, $\frac{S_n}{n} \to 0$ as well. The identity

$$S_n = A_n + \sum_{k=1}^{n} \frac{ku_k}{n} - \frac{S_n}{n}$$

shows that

$$S_n \to A.$$

The condition $\sum_{k=1}^{n} \frac{ku_k}{n} \to 0$ becomes necessary for convergence

EXAMPLE 1. If we let $u_k = 1/k$, $\sum_{k=1}^{n} \frac{ku_k}{n} = 1 \neq 0$. Therefore, $\sum_{k=1}^{\infty} 1/k$ diverges. Actually, the condition $\sum_{k=1}^{n} \frac{ku_k}{n} \to 0$ implies $a_n \to 0$ so that our necessary condition is stronger than the classic one.

THEOREM 2. $\sum_{k=1}^{\infty} \frac{ku_k}{n} \to 0 \Longrightarrow u_n \to 0$.

PROOF: If $\sum_{k=1}^{n} \frac{ku_k}{n} \to 0$ then $\sum_{k=1}^{n-1} \frac{ku_k}{n-1} \to 0$ and $\frac{1}{n} \sum_{k=1}^{n-1} \frac{ku_k}{n-1} \to 0$.

Therefore, $u_n \to 0$ since

$$u_n = \sum_{k=1}^{n} \frac{ku_k}{n} - \sum_{k=1}^{n-1} \frac{ku_k}{n-1} + \frac{1}{n} \sum_{k=1}^{n-1} \frac{ku_k}{n-1}.$$

EXAMPLE 2. $\sum_{k=1}^{\infty} (-1)^{k+1}$ diverges even though it is Cesáro summable ($S_n = 1$ if n is odd, $S_n = 0$ if n is even while $A \to 1/2$). One can draw, if he wishes, the conclusion that $\sum_{k=1}^{n} \frac{k(-1)^{k+1}}{n} \not\to 0$. In fact, $\sum_{k=1}^{n} \frac{k(-1)^{k+1}}{n}$ does not converge at all. This example, coupled with Theorem 1, shows that Cesáro summability is strictly more general than ordinary convergence.

EXAMPLE 3. Conventional means tell us that $\sum_{k=1}^{\infty} u_k = \frac{1}{2} + \sum_{k=2}^{\infty} \frac{k-4}{2^k}$ converges. The sum is not easily found, however. If one examines A_n, in an inductive fashion, he finds that $A_n = 1/2^n$ and therefore $A_n \to 0$. Therefore we may conclude that $\sum_{k=2}^{\infty} \frac{k-4}{2^k} = -\frac{1}{2}$.

An interesting question reflecting the promise of much "crank-type" work is: Is there a condition simpler than $A_n \to A$ that, when coupled with the necessary condition $\sum_{k=1}^{n} \frac{ku_k}{n} \to 0$, affords a necessary and sufficient condition for convergence? (It is easy to show that $\sum_{k=1}^{n} \frac{ku_k}{n} \to 0$ is not sufficient.)

References

1. Edwards, R. E., Fourier Series, Vol. I. Holt, Rinehart and Winston, New York (1967).

2. Hille, E., Analytic Function Theory, Vol. 5, Ginn and Co., New York, (1959).

3. Widder, Advanced Calculus, Second Ed., Prentice Hall, Englewood Cliffs, New Jersey (1961).

UNDERGRADUATE RESEARCH PROPOSAL

Proposed by Jack Hardy
University of Oklahoma

Calculus

In elementary calculus there are many equalities involving integrals and derivatives which are true under quite general conditions. In some of these cases it is possible to replace an integral ($\int$) with a sum ($\sum$) and a derivative (d/dx) with a difference (Δ), and the new equality will also be true.

For example, let m and n be integers, $m < n$, and $f(i)$ be a real number for $i = m, m+1, \ldots, n$. Define a function Δf as follows: $\Delta f(m) = 0$, and $\Delta f(i) = f(i) - f(i-1)$ for $i = m+1, \ldots, n$. Then

$$\sum_{i=m}^{n} \Delta f(i) = f(n) - f(m),$$

which is analogous to the well-known equality

$$\int_{x=m}^{n} \frac{df(x)}{dx}\, dx = f(n) - f(m).$$

Project

Investigate the equalities which result from interchanging the symbols $\int$, d/dx, and the symbols $\sum$, Δ in some formulas of calculus and infinite series. Is there an analogue to Taylor's Formula?

APPROXIMATION OF AREAS UNDER CURVES[3]

Kenneth A. Leone
Michigan State University

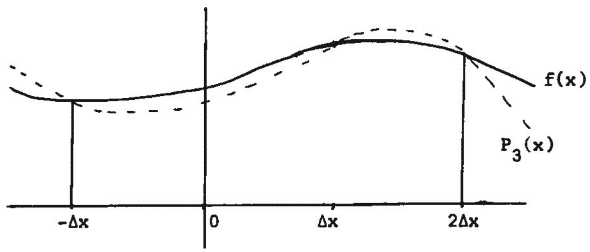
A simple approach to the approximation of areas under curves in the Cartesian plane is the **trapeziodal rule, which** approximates the curve by a straight line. If $\Delta x = x' - x$, $\int_x^{x'} f(x)dx \cong 1/2\Delta x(f(x) + f(x + \Delta x))$.

Another approximation, which can be found in many elementary calculus tests is Simpson's Rule, which seeks to approximate the curve by a section of a parabola, or section of a 2nd degree polynomial curve. Taking $\Delta x = 1/2(x' - x)$, $\int_x^{x'} f(x)dx \cong 1/3\Delta x(f(x) + 4f(x + \Delta x) + f(x + 2\Delta x))$.

What is interesting about these formulas is that they are derived by integrating the unique polynomial of degree n which passes through n+1 given points without actually finding the polynomial. Given n+1 points, a rule can be found by setting up n+1 equations and solving.

Approximation by third degree polynomial. We are not concerned with the actual polynomial, but onlyabout values of the function at regularly spaced intervals, so we shall shift our fitted polynomial to a convenient place near the origin and carry out calculations there.

Fig. 1



Let $P_3 = ax^3 + bx^2 + cx + d$. Then (See Fig. 1 above)

$$\int_{-\Delta x}^{2\Delta x} P_x = \frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx \Big|_{-\Delta x}^{2\Delta x}$$

$$= \frac{15a(\Delta x)^4}{4} + 3b(\Delta x)^3 + \frac{3c(\Delta x)^2}{2} + 3d\,x$$

$$= \frac{3}{8}\Delta x(10a(\Delta x)^3 + 8b(\Delta x)^2 + 4c\Delta x + 8d). \qquad \text{(Eq. I)}$$

Now $P_3(-\Delta x) = f(-\Delta x) = -a(\Delta x)^3 + b(\Delta x)^2 - c\Delta x + d$;

$P_3(0) = f(0) = d$;

$P_3(\Delta x) = f(\Delta x) = a(\Delta x)^3 + b(\Delta x)^2 + c\Delta x + d$;

$P_3(2\Delta x) = f(2\Delta x) = 8a(\Delta x)^3 + 4b(\Delta x)^2 + 2c\Delta x + d$.

We want to find the coefficients A, B, C, and D such that

$$A(-a(\Delta x)^3) + C(a(\Delta x)^3) + D(8a(\Delta x)^3) = 10a(\Delta x)^3;$$

$$A(b(\Delta x)^2) + C(b(\Delta x)^2) + D(4b(\Delta x)^2) = 8b(\Delta x)^2;$$

$$A(-c\Delta x) + C(c\Delta x) + D(2c\Delta x) = 4c\Delta x;$$

$$Ad + Db + Cd + Dd = 8d.$$

In matrix form, we want to solve the system of equations:

$$\begin{pmatrix} -1 & 0 & 1 & 8 \\ 1 & 0 & 1 & 4 \\ -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \\ 4 \\ 8 \end{pmatrix}$$

This system has the unique solution $A = 1$, $B = 3$, $C = 3$, $D = 1$. This means that $P_3(-\Delta x) + 3P_3(0) + 3P_3(\Delta x) + P_3(2\Delta x) = 10a(\Delta x)^3 + 8b(\Delta x)^2 + 4c\Delta x + 8d$. (Eq. II)

Substituting this expression into equation (1), we get $\int_{-\Delta x}^{2\Delta x} P_3(x)dx = \frac{3}{8}\Delta x(P_3(-\Delta x) + 3P_3(0) + 3P_3(\Delta x) + P_3(2\Delta x))$. This expression is exact for P a polynomial of degree 3 or less. If we now make the approximation that $P_3(x) \cong f(x)$ in the interval (x, x'), and $\Delta x = (x' - x)/3$, and substitute the exact expressions $P_3(-\Delta x) = f(x)$, $P_3(0) = f(x + \Delta x)$, $P_3(\Delta x) = f(x + 2\Delta x)$, $P_3(2\Delta x) = f(x + 3\Delta x) = f(x')$, we have $\int_x^{x'} f(x)dx \quad \frac{3}{8}\Delta x(f(x) + 3f(x + \Delta x) + 3f(x + 2\Delta x) + f(x'))$.

Approximation by n^{th} degree olynomial. We now present the **general** solution, as far as stating the general system of equations and showing that the solution is unique.

Let $P_n(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$.

$$\int_0^{n\Delta x} P_n(x)dx = a_0x + a_1x^2/2 + a_2x^3/3 + \cdots + a_nx^{n+1}/n{+}1 \Big|_0^{n\Delta x}$$

$$= \Delta x(a_0n + a_1\Delta x\frac{n^2}{2} + a_2(\Delta x)^2\frac{n^3}{3} + \cdots + a_n(\Delta x)^n\frac{n^{n+1}}{n+1}). \qquad \text{(III)}$$

Now, $P_n(0) = a$;

$$P(\Delta x) = a_0 + a_1\Delta x + a_2(\Delta x)^2 + \cdots + a_n(\Delta x)^n;$$

$$P(2\Delta x) = a + a_1 2\Delta x + a_2 2^2(\Delta x)^2 + \cdots + a_n 2^n(\Delta x)^n;$$

$$P(n\Delta x) = a_0 + a_1 n\Delta x + a_2 n^2(\Delta x)^2 + \cdots + a_n n^n(\Delta x)^n$$

We now want to find n+1 coefficients such that

$$b_oP(0) + b_1P(\Delta x) + b_2P(2\Delta x) + \cdots + b_nP(n\Delta x) =$$

$$a_on + a_1\Delta x\,\frac{n^2}{2} + a_2(\Delta x)^2\,\frac{n^3}{3} + \cdots + a_n(\Delta x)^n\,\frac{n^{n+1}}{n+1}. \quad \text{(See Eq. III.)}$$

Substituting for $P_n(0)$, $P_n(\Delta x)$, $\cdots P(n\Delta x)$, and rearranging the equation into n+1 equations in powers of Δx (or coefficients a_i), we have:

$$b_oa_o + b_1a_o + b_2a_0 + \cdots + b_na_o = a_on$$

$$b_1a_1\Delta x + b_2a_12\Delta x + b_3a_13\Delta x + \cdots + b_na_1n\Delta x = a_1\Delta xn^2/2$$

$$b_1a_2(\Delta x)^2 + b_2a_22^2(\Delta x)^2 + b_3a_23^3(\Delta x)^2 + \cdots + b_na_2n^2(\Delta x)^2 = a_2(\Delta x)^2\frac{n^3}{3}$$

$$b_1a_n(\Delta x)^n + b_2a_n2^n(\Delta x)^n + \cdots + b_na_nn^n(\Delta x)^n = a_n(\Delta x)^nn^{n+1}/n+1.$$

We now have a system of n+1 equations in n+1 unknowns, $b_o, b_1, \cdots, b_n$ which can be simplified to:

$$\begin{aligned}
b + b_1 + b_2 + b_3 + \cdots + b &= n;\\
b_1 + 2b_2 + 3b_3 + \cdots + nb_n &= n^2/2;\\
b_1 + 2^2b_2 + 3^2b_3 + \cdots + n^2b_n &= n^3/3;\\
&\vdots\\
b_1 + 2^nb_2 + 3^nb_3 + \cdots + n^nb_n &= n^{n+1}/n+1.
\end{aligned}$$

Or in matrix form:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & \cdots & 1\\ 0 & 1 & 2 & 3 & \cdots & n\\ 0 & 1 & 2^2 & 3^2 & \cdots & n^2\\ 0 & 1 & 2^3 & 3^3 & \cdots & n^3\\ \vdots & & & & & \vdots\\ 0 & 1 & 2^n & 3^n & \cdots & n^n \end{pmatrix}\begin{pmatrix} b_o\\ b_1\\ b_2\\ b_3\\ \vdots\\ b_n\end{pmatrix} = \begin{pmatrix} n\\ n^2/2\\ n^3/3\\ n^4/4\\ \vdots\\ n^{n+1}/n+1\end{pmatrix}.$$

For the solution to be unique, we want the determinant of the large coefficient matrix to be non-zero. Observation shows that it is a Vandermonde determinant, with value

$$\prod_{1\le i<j\le n}(x_j-x_i) = (1-0)(2-1)(2-0)(3-2)(3-1)(3-0)\cdots(n-0) = \prod_{k=1}^{n}k! > 0.$$

Therefore, this system of equations has a unique solution.

Editors Note: Since a Vandermonde matrix is ill conditioned for solving systems of equations this technique is not an efficient method for computer evaluation of integrals. It turns out that breaking up into smaller subsets of points and using a low degree polynomial approximation often gives more accurate results because of the errors introduced in inverting the large matrix.

Summing up the general formula, we have

$b_oP_n(0) + b_1P_n(\Delta x) + b_2P_n(2\Delta x) + \cdots + b_nP_n(n\Delta x) =$

$a_on + a_1(\Delta x)n^2/2 + a_2(\Delta x)^2n^3/3 + \cdots + a_n(\Delta x)^nn^{n+1}/n+1.$ Substituting into equation III, making the approximation $f(x) \cong P_n(x)$, and letting $\Delta x = (x' - x)/n$, we arrive at the general formula,

$$\int_x^{x'} f(x)dx = x(b_of(x) + b_1f(x + \Delta x) + b_2f(x + 2\Delta x) + \cdots + b_nf(x')).$$

For $n = 4$, we have the result,

$$\int_x^{x'} f(x)dx = \frac{2\Delta x}{45}(7f(x) + 32f(x+\Delta x) + 12f(x+2\Delta x) + 32f(x+3\Delta x) + 7f(x')).$$

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PROBLEMS FOR SOLUTION

222. Proposed by Jack Garfunkel, Forest Hills High School, Flushing, N. Y.

In an acute triangle ABC, angel bisector BT_1 intersects altitude AH_1 in D. Angle bisector CT_2 intersects altitude BH_2 in E, and angle bisector AT_3 intersects altitude CH_3 in F. Prove

$$\frac{DH_1}{AH_1} + \frac{EH_2}{BH_2} + \frac{FH_3}{CH_3} \leqq 1.$$

223. Proposed by Solomon W. Golomb, University of So. Calif., Los Angeles.

In the first octant of 3-dimensional space, where $x \geqq 0$, $y \geqq 0$, $z \geqq 0$, identify the region where the following "associative law" holds:

$$x^{(y^z)} = (x^y)^z.$$

224. Proposed by Charles W. Trigg, San Diego, California.

In the following cryptarithm, each letter represents a distinct digit in the decimal scale:

8 (M A D A P E) = 5 (A P E M A D).

Identify the digits.

225. Proposed by Wray G. Brady, University of Bridgeport.

Show that any proper fraction, a/b, can be written as the product of fractions of the type $n/(n+m)$ for fixed m.

226. Proposed by B. J. Cerimele, North-Carolina State Univ. at Raleigh.

Derive a formula for the n-th order antiderivative of $f(x)$ = In x.

227. Proposed by R. Sivaramkrishnan, Govt. Engineering College, Trichur, South India.

If $\tau(n)$ denotes the number of divisors of n, and $\mu(n)$ the Moebius function, prove that

$$\tau(n) + \mu^2(n) \leqq \tau(n^2)$$

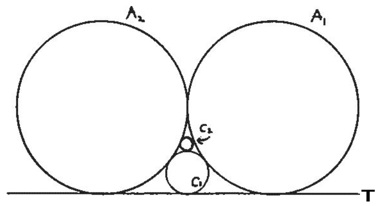
with equality if and only if n is a prime.

228. Proposed by Charles W. Trigg, San Diego, California.

In the decimal system, 1122 is a multiple of $1^5 + 2^5$ and contains no digits other than 1 and 2. Also, 3312 is a multiple of $1^5 + 2^5 + 3^5$ and contains no digits other than 1, 2 and 3, and contains each of these digits at least once. Do comparable multiples exist for $1^5 + 2^5 + 3^5 + 4^5$ and $1^5 + 2^5 + 3^5 + 4^5 + 5^5$?

229. Proposed by Carl L. Main, Shoreline Community College, Seattle, Wash.

Let A_1 and A_2 be tangent unit circles with a common external tangent T. Define a sequence of circles recursively as follows: 1) C_1 is tangent to T, A_1 and A_2; 2) C_i is tangent to C_{i-1}, A_1 and A_2, for $i = 2, 3, \ldots$.



Find the area of the region $\bigcup_1 C_i$.

230. Proposed by Murray S. Klamkin, Ford Scientific Laboratory.

Determine a single formula to represent the sequence $\{A_n\}$, n = 1, 2, 3,···, where

$$\begin{aligned} A_{pn+1} &= B_{n1}, \\ A_{pn+2} &= B_{n2}, \\ &\vdots \\ A_{pn+p} &= B_{np} \end{aligned} \qquad n = 1, 2, 3, \cdots$$

and where the $\{B_{nr}\}$, $r = 1, 2, \cdots, p$ are p given sequences.

231. Proposed by David L. Silverman, Beverly Hills, California.

a) What is the smallest circular ring through which a regular tetrahedron of unit edge can be made to pass?

b) What is the radius of the smallest right circular cylinder through which the unit-edged regular tetrahedron can pass?

Solvers are invited to generalize to the other Platonic solids.

SOLUTIONS

205. (Fall 1968) Proposed by C. S. Venkataraman, Trichur, So. India.

ABC and PQR are two equilateral triangles with a common circumcenter but different circumcircles. PQR and ABC are in opposite senses. Prove that AP, BQ, CR are concurrent.

Solution I by C. W. Dodge, University of Maine.

Introduce complex coordinates so that one triangle has vertices 1, λ and λ^2, and the other has vertices θ, $\theta\lambda$, and $\theta\lambda^2$, where $\lambda = (-1 + i\sqrt{3})/2$. Observe that $\lambda^3 = 1$, $\bar\lambda = \lambda^2$, and $\lambda^{-2} = \lambda$, where $\bar z$ denotes the complex conjugate of z.

Obtain the formulas

$$\begin{vmatrix} z & \bar z & 1 \\ a & \bar a & 1 \\ b & \bar b & 1 \end{vmatrix} = 0 \quad \text{and} \quad \begin{vmatrix} e_1 & f_1 & g_1 \\ e_2 & f_2 & g_2 \\ e_3 & f_3 & g_3 \end{vmatrix} = 0$$

from Eves' A Survey of Geometry, vol. 2, pp. 180 and 186, the first being an equation for the line through the complex points a and b, and the second being a necessary and sufficient condition for the three lines having equations $e_k z + f_k \bar z + g_k = 0$, $(k = 1, 2, 3)$ to be concurrent.

Since the first equation above becomes

$$(\bar a - \bar b)z + (b - a)\bar z + (a\bar b - \bar a b) = 0,$$

the three lines through the appropriate vertices a and b are:

$$(1 - \bar\theta\lambda^2)z + (\theta\lambda - 1)\bar z + (\bar\theta\lambda^2 - \theta\lambda) = 0 \quad \text{through } 1 \text{ and } \theta\lambda;$$

$$(\lambda^2 - \bar\theta)z + (\theta - \lambda)\bar z + (\bar\theta\lambda - \theta\lambda^2) = 0 \quad \text{through } \lambda \text{ and } \theta;$$

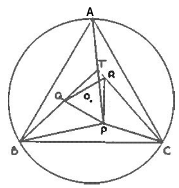
$$(\lambda - \bar\theta\lambda)z + (\theta\lambda^2 - \lambda^2)\bar z + (\bar\theta - \theta) = 0 \quad \text{through } \lambda^2 \text{ and } \theta\lambda^2.$$

For these lines to be concurrent we must have

$$\begin{vmatrix} 1 - \theta\lambda^2 & \theta\lambda - 1 & \bar\theta\lambda^2 - \theta\lambda \\ \lambda^2 - \theta & \theta - \lambda & \bar\theta\lambda - \theta\lambda^2 \\ \lambda - \theta\lambda & \theta\lambda^2 - \lambda^2 & \bar\theta - \theta \end{vmatrix} = 0.$$

By adding each of the last two rows to the first row, we obtain a factor of $\lambda^2 + \lambda + 1$, which = 0, in that row. The theorem follows.

Solution II by the Problem Editor.



Let BQ, CR intersect at T. Since angles BOA and QOR are equal, we have angle BOQ = angle AOR. Hence triangles BOQ and AOR are congruent, and BQ = AR. Similarly, BQ = AR = CP and BP = CR = AQ. Hence triangles ABQ, BCP, CAR are congruent. From the equalities

$$\frac{\angle ABT}{\angle ATC} = \frac{BT\cdot\sin ABT}{CT\cdot\sin TCA} = \frac{BT\cdot\sin BCP}{CT\cdot\sin PBC} = \frac{BT\cdot BP}{CT\cdot PC} = \frac{\angle BTP}{\angle TPC}$$

it follows that A, T, P are collinear (since AT and TP cut BC in the same point).

Also solved by Jack Garfunkel, Forest Hills High School, Flushing, N. Y.; Murray S. Klamkin, Ford Scientific Laboratory; V. V. Rao (South India); Phillip Singer, Michigan State University; Gregory Wulczyn, Bucknell University; and the proposer.

206. (Fall 1968) Proposed by Charles W. Trigg, San Diego, California.

Identify the pair of consecutive three-digit numbers each of which is equal to the sum of the cubes of its digits.

Solution by C. L. Sabharwal, St. Louis University.

Let a, b, c and a, b, (c + 1) denote the digits of the consecutive numbers. Then

$$a^3 + b^3 + (c + 1)^3 = 100a + 10b + c + 1$$

$$a^3 + b^3 + c^3 = 100a + 10b + c.$$

Subtracting, we obtain $3c(c + 1) = 0$, the only valid solution of which is c = 0. Then $a^3 + b^3 = 100a + 10b$, or $(a + b)(a^2 - ab + b^2) = 10(10a + b)$. The unique solution of this equation is b = 7, a = 3, and the required numbers are 370 and 371.

Also solved by J. Neil Aronowitz, Brooklyn, N. Y.; Jeanette Bickley, St. Louis, Missouri; Dermott A. Breault, Cambridge, Mass.; R. C. Gebhardt, Parsippany, N. J.; Walter J. Johnston, Redondo Beach, Calif.; Bruce W. King, Adirondack Community College, Murray S. Klamkin, Ford Scientific Laboratory; Howard Koenig, Brooklyn, N. Y.; Robert W. Prielipp, Wisconsin State University; Phillip Singer, Michigan State University; Daniel C. White, University of Santa Clara; Gregory Wulczyn, Bucknell University; and the proposer.

Editorial Note. Breault submitted the program and output from a PDP - 8/S computer, showing that there are four 3-digit integers equal to the sum of the cubes of their digits. They are 153, 370, 371 and 407, a result also given by the proposer. Prielipp supplied the following references: 1) Solution of Problem E 1810, The American Mathematical Monthly, March, 1968, p. 294. 2) P. K. Subramanian, "On Bases and Cycles", Mathematics Magazine, May-June 1968, pp. 117-123.

207. (Fall 1968) Proposed by Charles W. Trigg, San Diego, California.

Find a triangular number of the form abcdef in which def = 2 abc.

Solution by the proposer.

If a triangular number $n(n + 1)/2$ = abcdef = abc(1002) = abc(2)(3)(167), then either n or n + 1 is a multiple of 167, and whichever is even is a multiple of 4. Furthermore, since def = 2 abc, 100200 < abcdef < 499988, so 447 < n < 1000. There are only three multiples of 167 within this range: 501, 668, and 835. Hence there are only two triangular numbers meeting the criteria:

$$\Delta_{500} = 125250 \quad \text{and} \quad \Delta_{668} = 223446.$$

(We note that 4 does not divide 834 nor does 6 divide 836).

Also solved by Dermott A. Breault, Cambridge, Mass.; R. C. Gebhardt, Parsippany, N. J.; Stephen Mueller, Oshkosh, Wis.; Dan White, Santa Clara University; and Gregory Wulczyn, Bucknell University.

208. (Fall 1968) Proposed by Thomas Dodson, Hexham, England.

Where must a man stand so as to hear simultaneously the report of a rifle and the impact of the bullet on the target?

Solution by C. W. Dodge, University of Maine, Orono.

Since a hyperbola is the locus of points the difference of whose distances from two fixed points (the foci) is a constant, and since in this case this constant is the distance sound travels in the time it takes the bullet to reach the target, it follows that he must stand on that branch of the hyperbola nearer to the target (but hopefully not at the vertex).

Also solved by Richard Ball, Dufur, Oregon; R. C. Gebhardt, Parsippany, N. J.; Murray S. Klamkin, Ford Scientific Laboratory; Carl L. Main, Shoreline Community College, Seattle, Washington; Stephen Mueller, Oshkosh, Wisconsin; Phillip Singer, Michigan State University; and the proposer.

Editorial Note. The statement of the problem does not restrict the locus to the plane parallel to the ground. Consequently the locus could be any one of the infinite number of axial sections of the more general hyperboloid of revolution.

209. (Fall 1968) Proposed by R. C. Gebhardt, Parsippany, New Jersey.

At each play of a game, a gambler risks $1/x$ of his assets at the moment What must be the odds so that, in the long run, he just breaks even?

Solution I by Marc Kaufman, Mountain View, Calif.

The problem is incomplete as stated. It is also necessary to specify the probabilities involved in the game. If we consider a simple two-state game, with the probability of winning at each play a constant, $p(0 \leq p \leq 1)$, then the proper payoff for a wager of s is s/p, or $1/p$ for 1, the so-called "fair odds". If the fraction of the stake bet at each play is a constant $x(0 \leq x \leq 1)$, and if the payoff for winning is r for 1, we will solve for r as follows:

We note that if the stake before any play is S, then the new stake after losing is $S(1 - x)$, and after winning is $S(1 - x) + S(rx) = S(1 - x + rx)$. So the stake remaining after a series of m bets is S times m factors, where the factor is $(1 - x)$ for a loss and $(1 - x + rx)$ for a win. Since multiplication is commutative and associative, it is clear that only the number of wins and losses is significant, not the order.

For an initial stake of one unit, we can formulate the expected stake after n bets, E_n, as follows:

$$E_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} (p^i q^{n-i})(1 - x + rx)^i (1 - x)^{n-i}$$

where we are summing over all possible sequences of exactly n wins and losses. The number of sequences containing exactly i wins is $n!/i!(n-i)!$ and $p^i q^{n-i}$ is the probability of having exactly i wins and $(n - i)$ losses.

This can be recognized to be a simple binomial expansion, so

$$E_n = [p(1 - x + rx) + q(1 - x)]^n.$$

Re-arranging terms,

$$E_n = [(p + q)(1 - x) + prx]^n$$

But $p + q = 1$. So

$$E_n = [1 - x + prx]^n.$$

If we are to break even in the long run, $\lim_{n\to\infty} E_n$ must equal 1, but the only way this can happen is for $1 - x + prx = 1$.

Solving for r, we obtain $pr = 1$ or $r = 1/p$, giving us the not too surprising result that we will neither win nor lose in the long run if the game is fair. By "fair odds" is meant a return of "r for 1", where $pr = 1$ defines r, and p is the probability of winning each play. Thus, in a coin toss, $p = 1/2$ and $r = 2$ for 1; in "fair" roulette, $p = 1/36$ and $r = 36$ for 1.

Solution II by John M. Howell, Los Angeles City College.

If a gambler's wealth at the start is a, and he wagers $1/x$ of his wealth at each play, then the amount of money he has after n plays, in which he has won w times, is:

$$A(n,w) = a(1 + 1/x)^w (1 - 1/x)^{n-w}.$$

If the probability of winning a point is p (and losing is $q = 1 - p$), the probability of w winners in n trials is:

$$P(n,w) = \binom{n}{w} p^w q^{n-w}$$

Then the expected value of his wealth after n plays is:

$$E = \sum_{w=0}^{n} A(n,w)P(n,w) = \sum_{w=0}^{n} a\binom{n}{w} p^w (1 + 1/x)^w q^{n-w} (L - 1/x)^{n-w}$$

$$= a\left(1 + \frac{2p - 1}{x}\right)^n ,$$

which is equal to a if and only if $p = 1/2$. So if the expected value of his wealth is to be equal to his initial wealth, p must be $1/2$.

Also solved by Murray S. Klamkin, Ford Scientific Laboratory, and the proposer, who pointed out that if the gambler risks 1/2 his assets on each play, he would need 631 wins against 369 losses at even odds in order to just break even.

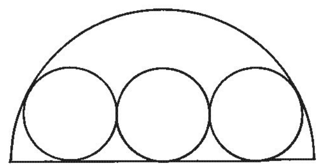
Editorial Comment. A related problem is considered in Martin Gardner's article on Random Walk in the Mathematical Games Section of the Scientific American, May 1969. It is shown that if a gambler plays continuously, always staking $1/x$ of his capital in an even wager, he is certain to lose if his losses and wins are equal. Whitworth (Choice and Chance, Prop. LXVIII) considers the same question, with the odds not necessarily even but "fair". Other pertinent remarks on this question may be found on pages 225 and 235 of Whitworth's Choice and Chance.

Solomon W. Golomb supplied a reference to "The Theory of Gambling and Statistical Logic", by Richard A. Epstein, Academic Press, 1967, pp. 58-59. There it is stated that if a gambler risks a finite capital over a large number of plays in a game with constant single-trial probability of winning, losing, and tying, then any and all betting systems lead ultimately to the same value of mathematical expectation of gain per unit amount wagered.

Other references may be found in the above-mentioned article by Martin Gardner.

210. (Fall 1968). Proposed by Leon Bankoff, Los Angeles, California.

Three equal circles are inscribed in a semicircle as shown in the adjoining diagram. How is this figure related to one of the better-known properties of the sequence of Fibonacci numbers:



Solution by C. W. Dodge, University of Maine.

Draw the radius OP (of length r) of the large circle to the point P of tangency of the left-hand small circle. It passes through the center C of that small circle. Let T be the point of contact of circle C with the given diameter of large circle). Letting the radius of circle C be 1, CT = 1 and OT = 2. Also OC = r - 1, so $2^2 + 1^2 = (r - 1)^2$, whence $r = 1 + \sqrt{5}$. Thus the radius of the large circle is to the diameter of the small circle as $(1 + \sqrt{5})/2$, the "golden ratio" and the limiting value of the ratio f_{n+1}/f_n of two successive Fibonacci numbers.

Also solved by the proposer.

211. (Fall 1968) Proposed by Leonard Barr, Beverly Hills, California.

It is known that the sum of the distances from the incenter I to the vertices of a triangle ABC cannot exceed the combined distances from the orthocenter H to the vertices. [Amer. Math. Monthly, 1960, 695; problem E 1397]. Show that the reverse inequality holds for their products, namely that AH·BH·CH ≤ AI·BI·CI.

Solution by proposer.

From the identity $r = 4R\sin(A/2)\sin(B/2)\sin(C/2)$ and the relation $IH^2 = 2r^2 - 4R^2\cos A\cos B\cos C$, we get

$$8\prod\sin^2(A/2) = r^2/2R^2 \geq (2r^2 - IH^2)/4R^2 = \prod\cos A,$$

with equality when IH vanishes, i. e., when the triangle is equilateral.

Since $AI\cdot BI\cdot CI = 64R^3\sin^2(A/2)\sin^2(B/2)\sin^2(C/2)$ and $AH\cdot BH\cdot CH = 8R^3\cos A\cos B\cos C$, it follows that AI·BI·CI ≥ AH·BH·CH.

UNSOLVED PROBLEMS SECTION

The Fall 1968 and the Spring 1969 issues of this Journal listed unsolved problems from issues dating back to 1952. We are pleased to report that satisfactory solutions have been submitted for problems 37, 50, 65, 73, 83, 91, 102, 111, 128 and 166. Problems 48, 120, 136 and 144 are the only unsolved problems remaining to challenge the ingenuity of our solvers. Indeed, the comments on problem 48, published in the Spring 1958 issue, could very well be considered a very adequate treatment of the problem thus reducing the unsolved list to three.

We wish to thank all participants who cooperated in our program of bringing all solutions up to date. Some of the solutions are given below and the remainder will appear in the Spring 1970 issue.

SOLUTIONS

50. (Fall 1952) Proposed by Pedro Piza, San Juan, Puerto Rico.

Prove that the integer 2n + 1 is a prime if and only if, for every value of $r = 1,2,3,\cdots[\sqrt{n/2}]$, the binomial coefficient $\binom{n+r}{n-r}$ is divisible by 2r + 1.

Solution by Leonard Carlitz, Duke University.

Put

$$I = \binom{n+r}{n-r} = \binom{n+r}{2r}.$$

1. Assume 2n + 1 prime. We have

$$(n - r)I = (2r + 1)\binom{n+r}{2r-1}.$$

Also

$$(n - r, 2r + 1) = (n - r, 2r + 1, 2n - 2r) = (n - r, 2r + 1, 2n - 2r, 2n + 1) = 1,$$

so that $2r + 1 \mid I$ for $r < n$.

2. Let $2r + 1 \mid I$ for $1 \leq r \leq \sqrt{n/2}$. Assume 2n + 1 composite. Then

(*) $$2n + 1 = pm,$$

where p is prime, $p \leq \sqrt{2n+1}$. Put $p = 2r + 1$. Then $r \leq \sqrt{n/2}$, so that by hypothesis $p \mid I$. But, by (*),

$$n - r = \tfrac{1}{2}(m - 1)p$$

and therefore

$$p \nmid (n + r)(n + r - 1)\cdots(n - r + 1).$$

This contradicts $p \mid I$.

Also solved by Gregory Wulczyn, Bucknell University.

65. (April 1954) Proposed by Martin Schecter, Brooklyn, N. Y.

Prove that every simple polygon which is not a triangle has at least one of its diagonals lying entirely inside it.

Solution by Charles W. Trigg, San Diego, California.

A simple polygon has straight sides with no points in common except their endpoints and no two vertices at a point, that is, it is neither crossed nor compound.

If no diagonals are to be interior, each exterior angle must be < 180°. Consider the broken line $A_1A_2 \cdots A_{n-1}$ for which every angle on one side of the line is < 180°. Now if a polygon is completed on this line by adding another vertex $A_n (n > 4)$, then if the aforesaid angles are not to become interior angles, the join A_1A_{n-1} and A_n must lie on opposite sides of the broken line. But then the exterior angles at A_1, A_{n-1}, and A_n are > 180° and the joins of $A_2, A_3, \cdots, A_{n-2}$ to A_n are interior diagonals.

73. (April 1954) Proposed by Victor Thébault, Tennie, Sarthe, France.

Construct three circles with given centers such that the sum of the powers of the center of each circle with respect to the other two is the same.

Solution by C. W. Dodge, University of Maine, Orono.

Let the circles have centers A, B, C and radii a, b, c. Let BC, CA, AB denote the distances between these centers. Since the power of B with respect to circle A, for example, is $BA^2 - a^2$, we have

$$BA^2 - a^2 + BC^2 - c^2 = AB^2 - b^2 + AC^2 - c^2 = CA^2 - a^2 + CB^2 = b^2,$$

so $AB^2 - c^2 = BC^2 - a2 = CA^2 - b^2$.

Choose a convenient value for one of the radii, say c. By right triangles, then a and b are readily constructed. Thus the first circle is completely arbitrary so long as its radius is large enough to make $AB^2 - c^2$ less than BC^2 and less than CA^2.

Book Reviews

Edited by

Roy B. Deal, Oklahoma University Medical Center

1. A Survey of Finite Mathematics By Marvin Marcus, Houghton, Mifflin Company, Boston, Mass., 1969, ix + 485 pp., $9.50.

   An excellent elementary introduction to a variety of currently popular topics in the social and biological sciences, such as; stocastic processes, combinatorics, linear programming, game theory and Markov chains with the necessary background material and elementary mathematics, probability theory, and linear algebra.

2. Theory and Examples of Point-Set Topology By John Greever, Brooks/Cole Publishing Co., Belmont, California, 1968, x + 130 pp.

   Provides at an elementary level most of the topology necessary for a thorough understanding of analysis from elementary calculus through real variable theory.

3. Topological Spaces By Claude Berge, The Macmillan Company, New York, N. Y., 1963, xiii + 270 pp.

   This is seemingly not well known that this fine book has been translated and that it contains, in addition to the introductory material on topological spaces, an excellent introduction to convexity and topological vector spaces.

4. How to Use Groups By J. W. Leech and D. J. Newman, Barnes and Noble, Inc., New York, N. Y., 1969, 133 pp., $5.25. Also available in paper at $3.50.

   This facinating little book provides, by many examples, a wide variety of applications of group theory and physics. A reader with some knowledge of modern physics and very little or no knowledge of group theory will enhance both considerably by working through it.

5. Theory of Finite Groups By Richard Brauer and Chih-Han Sah, W. A. Benjamin, Inc., New York, N. Y., 1969, xiii + 263 pp., $12.50.

   An edited volume of research papers from a symposium on finite groups, of primary interest to research mathematicians and advanced graduate students working in group theory.

6. Rings and Modules By Paulo Ribenbein, John Wiley and Sons, Inc., New York, N. Y., 1969, vii + 162 pp., $12.95.

   A modern well-organized presentation of some of the important foundational topics from the theory of rings and modules.

7. Introduction to the Theory of Categories and Functors By I. Bucur and A. Deleanu, John Wiley and Sons, Inc., New York, N. Y., 1969, x + 224 pp., $13.50.

   Now that category theory is a full-fledged subject in its own right, it is fitting to have an introductory book on just this subject. It seems that the authors have also accomplished their dual purpose of providing that information on the subject which "every mathematician should know."

8. Algebra By Saunders MacLane and Garrett Birkhoff, The Macmillan Co., New York, N. Y., 1968, xix + 598 pp.

   Apparently the transposition of authors from the order in their classic text is designed to "tell us something'; as the comprehensive revisions and additions reflect the outstanding contributions of Saunders MacLane in the pioneering work, and growth to maturity, of category theory, as well as his concomitant interests in modules and tensor products as these subjects have evolved to becoming foundational in the structures of Algebra.

9. Algebraic K-Theory By Hyman Bass, W. A. Benjamin, Inc., New York, N.Y., 1968, xix + 762 pp. $12.50. Also available in paper at $5.95.

   A comprehensive tome, basically "modulo a first year algebra course," of this subject which like category theory grew up in algebraic topology and is now a subject in pure algebra with its development being of much interest to homotopists.

10. Tensor Analysis on Manifolds By Richard L. Bishop and Samuel I. Goldberg, The Macmillan Co., New York, N. Y., 1968, viii + 280 pp.

    A completely modern clearly exposited introduction to tensor analysis with some applications and the necessary background material to allow advanced calculus as a sufficient prerequisite.

11. Foundations of Differential Geometry, Volume 2 By S. Kobayashi and K. Nomizu, John Wiley and Sons. Inc., New York, N. Y., 1969 xv + 470 pp., $17.50.

    This book, along with volume 1, provides a rigorous comprehensive survey of the fundamental definitions and theorems of differential geometry.

12. Introductory Computer Methods and Numerical Analysis By Ralph H. Pennington, The Macmillan Company, New York, N. Y., 1968, xi + 482 pp.

    The emphasis is on the use of high speed computers, providing instruction in machine language programming, FORTRAN, and flow charting, in the straight forward problems of numerical analysis.

13. An Introduction to the Approximation of Functions By Theodore J. Rivlin, Blaisdell Publishing Company, Waltham, Massachusetts, 1969, viii + 150 pp., $7.50.

    Advanced calculus is a prerequisite but the book has been kept at the same level and rigor with the emphasis on theorems which are useful in practical computational methods.

14. Computational Solution of Nonlinear Operator Equations By Louis B. Ball, John Wiley and Sons, Inc., New York, N. Y., 1969, viii + 224 pp. $14.95.

    At a level just beyond real and complex variables this book presents a well-written discussion on the concepts and techniques for solving some of the large variety of non-linear equations on finite and infinite dimentional vector spaces which occur in modern applied mathematics.

15. Error Correcting Codes By Henry B. Mann, Editor, John Wiley and Sons, Inc., New York, N. Y., 1968, ix + 231 pp., $7.95.

The proceedings of a symposium on error correcting codes consisting of eleven papers concerned with research in algebraic coding theory and related areas of algebra and combinatorial theory.

16. Induced Representations of Group and Quantum Mechanics By George W. Mackey, W. A. Benjamin, Inc., New York, N. Y., 1968, viii + 163 pp., $8.50. Also available in paper at $4.95.

"This volume contains a set of lectures that were given at the Scuola Normale, Pisa, in April 1967. Addressed to mathematicians and physicists, these lectures deal with the nature of the theory of induced representations and its application to quantum mechanics."

17. Linear Operators for Quantum Mechanics By Thomas F. Jordan, John Wiley and Sons, Inc., New York, N. Y., 1969, x + 144 pp., $7.50. Also available in paper at $4.95.

An interesting account at the first year graduate level of the mathematics of linear operators specifically pertinent to quantum theory, written in the spirit of von Neumann's book.

18. Calculus of Variations By John C. Clegg, John Wiley and Sons, Inc., 1969, ix + 190 pp., $4.00.

This little volume covers an amazing range of topics at the post advanced calculus level on the classical of variations.

19. Amost Periodic Functions By C. Corduneanu, Interscience Publishers, New York, N. Y., 1968, x + 237 pp., $13.50.

A lucid survey of the subject with interesting historical notes and a bibliography of 704 entries. Real and complex function theory with a little knowledge of Banach spaces and topological groups should be a prerequisite.

20. Generalized Integral Transformations by A. H. Zemanian, Interscience Publishers, New York, N. Y., 1968, xvi + 300 pp., $16.00.

Some real and complex function theory suffice for this study of the generalizations of the classical transforms (Laplace, Mellin, etc.) to Schwartz distributions and generalized functions.

BOOKS RECEIVED FOR REVIEW

1. Modern Algebra By Kaj L. Nielsen, Barnes and Noble, Inc., New York, N. Y., 1969, x + 274 pp.

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5. Statistical Tables By F. James Rohlf and Robert R. Sokal, W. H. Freeman and Company, San Francisco, 1969, ix + 253 pp., $2.75.

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10. Mathematics The Man-Made Universe Second Edition By Sherman K. Stein, W. H. Freeman and Company, San Francisco, California, 94104, 1969, xvi + 415 pp., $8.25.

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17. Introduction to Analysis By Edward Gaughan, Brooks/Cole Publishing Co., Belmont, California, 1968, vi + 310 pp.

Note: All correspondence concerning reviews and all books for review should be sent to PROFESSOR ROY B. DEAL, UNIVERSITY OF OKLAHOMA MEDICAL CENTER, 800 NE 13th STREET, OKLAHOMA CITY, OKLAHOMA 73104.

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The Governing Council of Pi Mu Epsilon is repeating its contest for the best expository paper by a student (who has not yet received a masters degree) suitable for publication in the Pi Mu Epsilon Journal.

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The winners for papers submitted between August 1968 and July 1969 were:

$200. Robert L. Devaney--"Lens Spaces as Coset Spaces",

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