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CONTENTS

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THE C. C. MACDUFFEE AWARD FOR DISTINGUISHED SERVICE

The fourth presentation of the C. C. MacDuffee Distinguished Service Award was made to Dr. Francis Regan, St. Louis University, at the Pi Mu Epsilon banquet, August 25, 1970, held in conjunction with the national meeting of Pi Mu Epsilon at Laramie.

The C. C. MacDuffee Distinguished Service Award was established in 1964, in honor of the late Professor C. C. MacDuffee (University of Wisconsin), former President of Pi Mu Epsilon. Pi Mu Epsilon's highest honor is awarded only when an individual's efforts to promote scholarly activity in mathematics are so distinguished that they merit commendation and recognition by all concerned.

It is indeed a great pleasure to present Dr. Francis Regan with our highest award in honor of his outstanding contribution to Pi Mu Epsilon and to mathematics, as exemplified by his particularly noteworthy editorship of the Pi Mu Epsilon Journal and his longtime sponsorship of the outstanding Missouri Beta Chapter of Pi Mu Epsilon at St. Louis University. Either achievement would be sufficient to merit sincere admiration - but to find both in one modest man makes us realize how fortunate the world is to have men blessed with leadership, ability, honesty, and unselfish devotion all residing in the same body.

Congratulations to Dr. Francis Regan, who is joining our earlier award recipients:

Dr. J. Sutherland	1964
Dr. Richard V. Andree	1966
Dr. John S. Gold	1967
Dr. Francis Regan	1970

Dr. Francis Regan

UNICURSAL POLYGONAL PATHS AND OTHER GRAPHS ON POINT LATTICES

Solomon W. Golomb and John L. Selfridge

1. INTRODUCTION

1.1 Description of the Problem

An old geometric puzzle asks the solver to construct a polygonal path of only four segments which goes through all nine points in Figure 1, Two unsuccessful attempts, yielding five-segment "solutions", are shown in Figure 2. The required four-segment solution is shown in Figure 3. The "trick" involves the fact that the polygonal path goes outside the convex hull of the nine-point configuration.

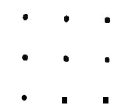


Figure 1. The nine point con- figuration.

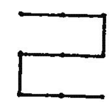


Figure 2a. The boustrophedon "solution" requires 5 segments.

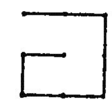


Figure 2b. The spiral "solution" requires 5 segments.

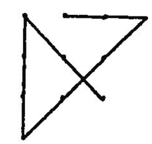


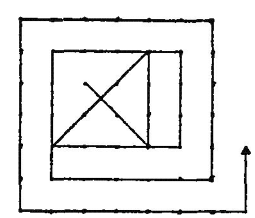
Figure 3. The required 4 segment solution.

In this paper we examine minumum-segment polygonal paths as well as certain other graphs, on a x b point lattices.

1.2 Historical Survey

Specific problems involving the construction of polygonal paths on n x n arrays of dots, using only 2n-2 segments, were posed by both Sam Loyd [1] and H. E. Dudeney [2], with additional constraints which will be noted later. In 1955, M. Klamkin [3] posed and solved the problem of showing that 2n-2 segments is sufficient for a unicursal polygonal path on the n x n array, using the construction of Figure 4. He conjectured that 2n-2 segments is also necessary, and this was proved by one of us (Selfridge, [4]), in a form which is generalized in Section 4 of this paper. Constructions for the 4 x 4 array were investigated extensively by F. Schuh [5], and there are doubtless many examples of other special cases in the literature. (We are indebted to Mr. Martin Gardner for assistance in compiling these historical citations.)

Figure 4. A polygonal path of 2n-2 segments covers the n x n array.



1.3 Summary of Principal Results

We show that a unicursal polygonal path of 2n-2 segments exists on the n x n array for all $n > 2$; that the further constraint that the path be closed can be satisfied For all $n > 3$; that the further constraint that the closed path remain within the convex hull of the array of dots can be satisfied for all $n > 5$.

On an a x b array of dots, a collection of a horizontal line segments or of b vertical line segments will suffice to cover all the dots. However, if such a collection of parallels is not used, it is proved that at least a + b - 2 segments must be used to cover all the dots, even if it is not required that the segments form a unicursal path.

A collection of a + b - 2 segments which covers all the dots in an a x b array, and does not include a complete set of horizontal or vertical segments, will be called a minimal net. We prove that every segment of a minimal net contains at least two "exclusive points"-- i.e. dots which are not traversed by any other segment. We exhibit several minimal nets for which each segment contains at least three exclusive points. One of these nets is in fact a closed unicursal path on the 8 x 8 array.

Finally we consider the possible symmetries of minimal nets in general, and unicursal polygonal paths in particular. Although there are minimal nets with the full (dihedral) symmetry group of the square, it is proved that this cannot happen if the net is a unicursal path.

2. BEST CONSTRUCTIONS FOR SQUARE ARRAYS

2.1 Squares of Even Side

In Figure 5, we see a 6-segment closed polygonal path which goes through all sixteen points of the 4 x 4 (square) point lattice. (It is not difficult to show directly that no 5-segment path can go through all sixteen points.)

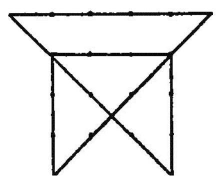
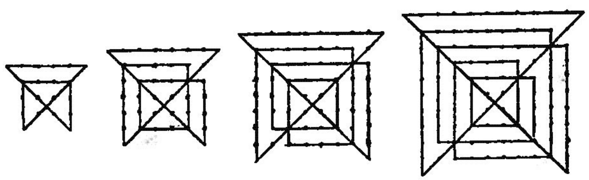


Figure 5. A 6-segment closed path for the 4 x 4 lattice.

Closed paths of 2n-2 segments exist for all even $n \geq 4$, as indicated for the first few cases in Figure 6.

Figure 6. Closed-path solutions, in 2n-2 segments, for even n.



The rule for constructing these paths is as follows: Draw the line AB along the top of the square, and protruding one unit at each side. We will proceed by continuing both ends of AB. We draw the diagonals AD and BC, which protrude one unit below the square. From C we generate a clockwise spiral, and from D a counterclockwise spiral, each composed of alternating vertical and horizontal line segments. These segments run to the diagonal AD, but stop one unit short of the diagonal BC, except at the very end of the construction, when the path is closed at the point X, which lies on the BC diagonal.

The solution to the 6 x 6 case shown in Figure 7 uses the minimum number of segments (2n-2 = 10), and exhibits two novel features: The pattern is entirely contained within the convex hull of the square lattice; and segments of slope ±1/2 occur, in additon to the slopes previously encountered (0,±1,and∞). The fact that Figure 7 exhibits a closed path can be verified by Euler's criterion, viz: we have a connected graph in which every node is a junction of an even number of edges.

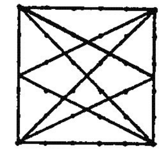


Figure 7. A "compact" path on the 6 x 6 array.

This solution can be extended to n x n for even $n \geq 6$, as illustrated for n = 8 and n = 10 in Figure 8. Again, Euler's criterion may be used to verify the existence of a closed path in each case.

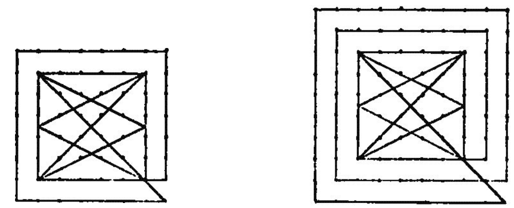
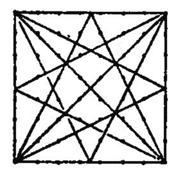


Figure 8. Extension of the Figure 7 construction to larger even values of n.



The idea of enlarging Figure 7 to handle n x n constructions directly, as shown in Figure 9, is unsuccessful. To be sure, all 64 points are covered by only 2n-2 = 14 segments; but the graph they form is not a path. All four corner nodes are odd (five edges meet at each), whereas a closed path has no odd nodes, and an open path can have only two.

Figure 9. A non-unicursal "solution" to the 8 x 8 configuration.

(It will be shown in Section 6 that a minimum-segment graph with the dihedral symmetry group of the square, as in Figure 9, can never be unicursal.)

2.2 Squares of Odd Side

We now show the existence of closed paths of 2n-2 segments on the n x n square lattice, for all odd $n \geq 5$. In Figure 10, we see a closed path of 2n-2 = 8 segments for n = 5. This remarkable construction contains a line of slope 2, and has turning points for the path which not only are not lattice points (as in Figure 8), but do not lie on the grid lines (horizontal or vertical) through the lattice points.

To get closed paths of 2n-2 segments for odd $n > 5$, we may extend either the body or the spirit of Figure 10. The

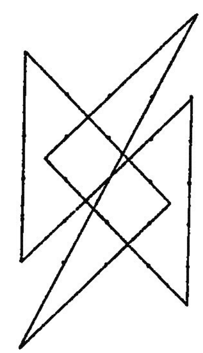
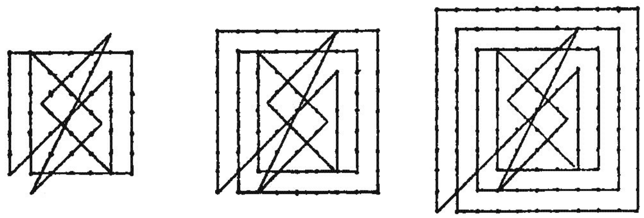


Figure 10. A closed path for the 5 x 5 configuration.

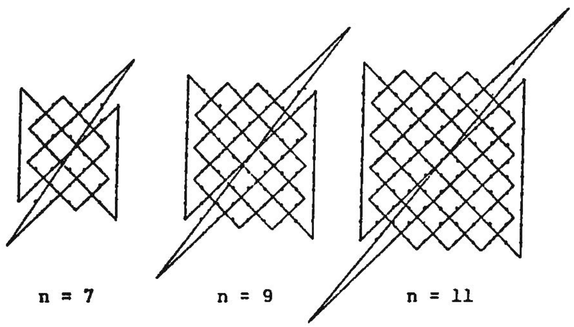
corporeal extension is shown in Figure 11. Note that for $n \geq 9$, we obtain a closed path solution, with $2n-2$ segments, which does not go outside the convex hull of the $n \times n$ lattice. Combined with the construction of Figure 9, we have established that for all $n > 5$, except for $n = 7$, there is always a $2n-2$ segment closed path on the $n \times n$ square lattice which does not go beyond the convex hull of the lattice. The gap at $n = 7$ is filled in by Figure 15 in Section 3.2.

From an artistic standpoint, the "spiritual" extensions of Figure 10 as shown in Figure 12, are more appealing. For these cases, the "long diagonal" has a slope of $(n-3)/(n-1)$ for all $n \geq 5$.



$n = 7$ $n = 9$ $n = 11$

Figure 11. Physical extensions of Figure 10 to larger odd n.



$n = 7$ $n = 9$ $n = 11$

Figure 12. Spiritual extensions of Figure 11 to larger odd n.

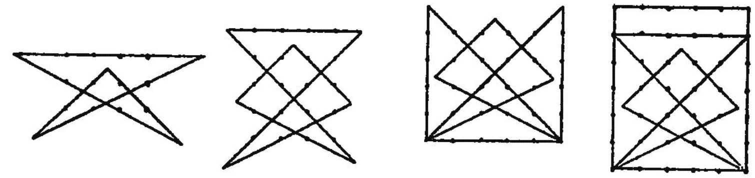
3. OTHER CONSTRUCTIONS

3.1 The n x (n+1) Array

In anticipation of the General Theorem of Section 4, we can expect the minimum-segment unicursal path on the $n \times (n+1)$ array to consist of $2n-1$ segments. On the 2×3 array, the best that can be done is a 3-segment open path. Two examples are shown in Figure 13, the second being preferable in that it avoids using a set of parallel lines to catch all the points.

For $n > 2$, there are closed unicursal paths of $2n-1$ segments on the $n \times (n+1)$ array, as indicated in Figure 14.

Figure 13. Three-segment open paths on the 7 x 3 array.



$n = 3$ $n = 4$ $n = 5$ $n = 6$

Figure 14. Typical closed unicursal paths on $n \times (n+1)$ arrays.

3.2 Queen's Tours

Dudeney [2] posed and solved the problem of finding a 12-move re-entrant "queen's tour" on the 7 x 7 checkerboard. In our terminology, he finds a minimum-segment closed unicursal path on the 7 x 7 array, with the further constraints that the path stay within the convex hull of the array, and that only segments of slopes 0, ∞, +1, and -1 are permitted. His solution is given in Figure 15.

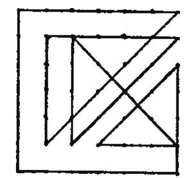


Figure 15. Dudeney's "queens's tour" on the 7 x 7 array.

This construction can easily be modified to give closed queen's tours on $n \times n$ boards for odd $n \geq 7$, and open queen's tours for (even) $n \geq 6$. The problem of closed queen's tours for even $n \geq 8$ has also been solved. The 8 x 8 solution shown in Figure 16 is based on [1], page 42.

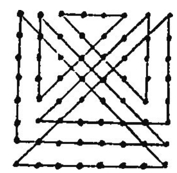
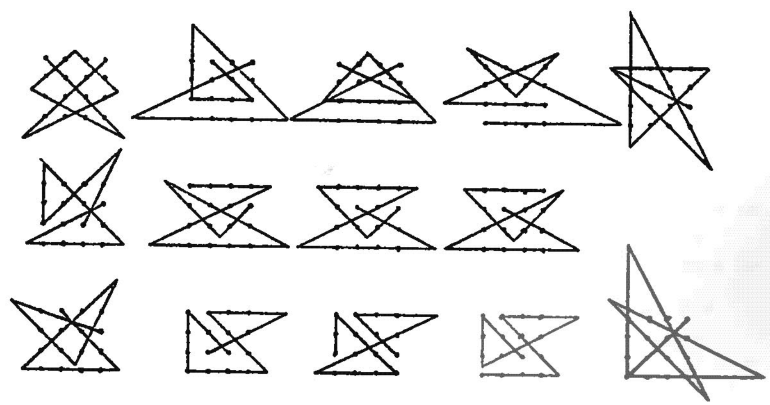


Figure 16. Loyd's "queen's tour" on the 8 x 8 array.

3.3 Miscellaneous Examples

Additional examples of 6-segment paths, both open and closed, on the 4 x 4 board, are given in Figure 17. These include, but are not limited to, examples given by Schuh [5].



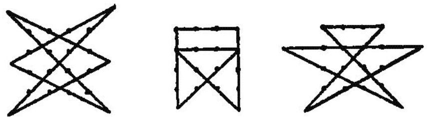


Figure 17. Additional examples of 6-segment polygonal paths on the 4 x 4 array.

Besides the closed 8-segment path on the 5 x 5 array shown in Figure 10, two further examples are known, as shown in Figure 18.

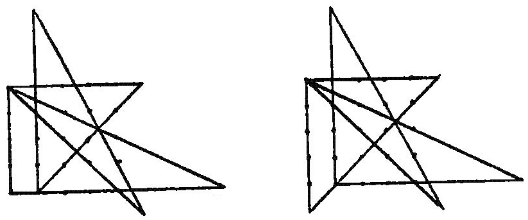


Figure 18. Further examples of closed 8-segment paths on the 5 x 5 array.

4. THE NECESSITY THEOREM AND ITS CONSEQUENCES

4.1 The General Theorem for Rectangular Arrays

We define a proper net on an a x b rectangular array of lattice points to be a set of line segments which collectively cover all ab lattice points, but which does not contain either the set of a horizontal (i.e. row) segments of the set of b vertical (i.e. column) segments as a subset. (There is no requirement of connectivity in the definition of a proper net.)

Theorem 1. A proper net on an a x b array contains at least a+b-2 segments.

Proof. Let the proper net consist of h horizontal segments, of v vertical segments, and of q oblique segments. From the a x b array, delete every row in which a horizontal segment occurs, and every column in which a vertical segment occurs, to form a reduced array. The reduced array has $a' = a - h$ rows and $b' \geq b - v$ columns, which need no longer be uniformly spaced. (The inequalities arise because e.g. if two horizontal segments are in the sane row, $a' > a - h$.)

If $a' \geq 2$ and $b' \geq 2$, then the reduced array has $2a' + 2b' - 4$ lattice points around its (rectangular) perimeter. These points must be covered by oblique lines of the net, but one oblique line can cover at most two perimeter points. Hence $q \geq a' + b' - 2$, and the total number of segments in the net is $h + v + q \geq h + v + a' + b' - 2 \geq a + b - 2$.

If $a' = 1$, $b' \geq 1$, then there are b' lattice points on the "perimeter", but each requires a separate oblique line to contain it. Then the total number of segments is

$$h + v + q \geq (a-1) + v + b' \geq a + b - 1,$$

an even stronger result.

The possibility of $a' = 0$ or $b' = 0$ is ruled out by the definition of a proper net. q.e.d.

4.2 Corollaries and Consequences

Theorem 2. A unicursal polygonal path on an a x b lattice of dots $(a \leq b)$ requires at least min(2a-1,a+b-2) segments. A closed unicursal polygonal path requires at least min(2a,a+b-2) segments.

Proof. If we do not include a complete set of parallel (row) segments, then at least a+b-2 segments are needed, by Theorem 1. If we use a set of a parallel row segments, we need at least a-1 non-horizontal segments to connect them into an open path. and at least a non-horizontal segments to connect them into a closed path. q.e.d.

Note: We can improve on the 2a segment "parallel" solution only if $|a - b| < 2$. Thus, the interesting cases are n x n and n x (n+1), which were treated in the earlier sections.

Theorem 3. A closed unicursal polygonal path on an a x b lattice of dots $(a \leq b)$ which consists solely of horizontal and vertical segments must contain at least 2a segments.

Proof. If there is a row without a horizontal segment and simultaneously a column without a vertical segment, then the point where they intersect is not covered. Hence the path must include either a set of a parallel row segments or a set of b parallel column segments. However, horizontal and vertical segments must alternate, leading to at least min(2a,2b) = 2a segments in all. q.e.d.

4.3 Minimal Nets

We define a minimal net on a a x b lattice to be a proper net consisting of only a + b - 2 segments. We may observe that a unicursal polygonal path on an n x n array with only 2n - 2 segments is always a minimal net, but not conversely. In fact, a "bicursal" path (in which one interruption is permitted) which covers the n x n array in 2n - 2 segments, for n > 2, is always a minimal net.

The technique of constructing unicursal paths, in general, is to begin with minimal nets, and then to extend the segments in various ways in an attempt to achieve connectivity. Some typical examples of the minimal nets which may serve as skeletons for these constructions are shown in Figure 19. Not all can be extended to form unicursal paths.

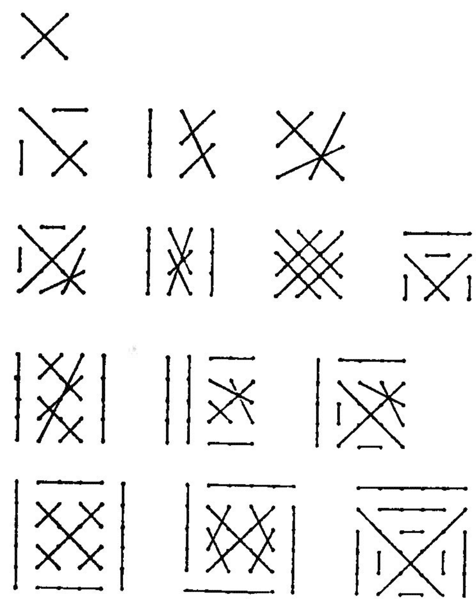


Figure 19. Some minimal nets on n x n arrays.

5. EXCLUSIVE POINTS AND SYMMETRY GROUPS

5.1 The Exclusive-Point Theorem

We define an exclusive point of a segment used in the covering of an a x b array to be a lattice point covered by that segment and by no other.

Theorem 4. Every segment of a minimal net on an a x b array contains at least two exclusive points.

Proof. We refer to the proof of Theorem 1. Every oblique segment has two exclusive points on the perimeter of the reduced array. Consider then a non-oblique segment--say a horizontal segment H. If we ignore this segment in the formation of the reduced array, we get a larger reduced array, we get a larger reduced array, (a' + 1) x b', with at least two extra perimeter points. The q oblique lines can still cover only 2q perimeter points, leaving at least 2 perimeter points now uncovered. These two points can be covered by no oblique line, no vertical line, and no horizontal line other than H, by the definition of the reduced array. Hence these are two exclusive points of H.

q.e.d.

Note. It is instructive to re-examine Figures 3, 5, 6. 7, 8, 10. 11, 14, 15, 16, and 17 for the locations of the exclusive points. In all the cases just listed, there are at least 4 segments in each array with only two exclusive points each, the minimum allowed by Theorem 4. In each of the three arrays of Figure 12, there are 3 segments with only 2 exclusive points each. The average number of points per segment for the n x n array is $n^2/(2n-2) = \frac{1}{2}(n+1) + \epsilon_n$, where

$\epsilon_n = 1/(2n-2) \to 0$ as $n \to \infty$. Hence it can reasonably be expected that

in large enough arrays, the minimum number of exclusive points per segment can be increased. We next examine some examples.

5.2 Nets With Several Exclusive Points Per Segment

In Figure 20, we see three examples of minimal nets on square arrays with at least 3 exclusive points per segment. (The middle one is our old Figure 9.)

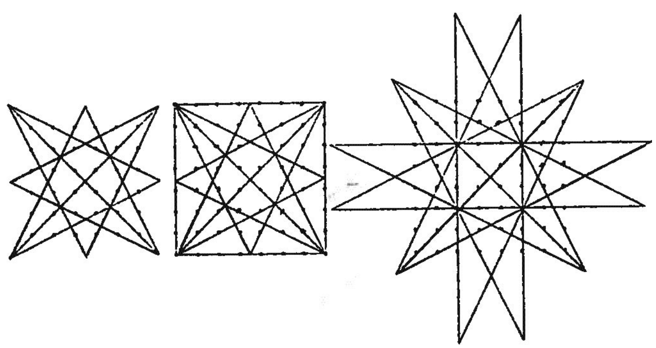


Figure 20. Three minimal nets with at least three exclusive points per segment.

These three examples have the following further properties in common: none of them is a unicursal path - in fact, each has four odd vertices; each has the dihedral symmetry group of the square; and the number of exclusive points per segment is always either 3 or 6.

In Figure 21, we see a closed unicursal path on the 8 x 8 array, with either 3 or 6 exclusive points per segment. Note that this figure, also derived from Figure 9, has a smaller symmetry group. It is easy to verify that all the vertices are "even", but one must also verify that a procedure for traversing all the edges in only 14 segments exists. This is in fact the case.

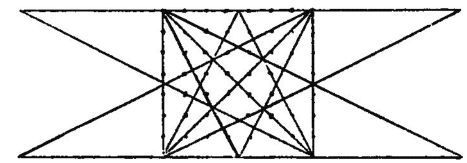


Figure 21. A closed path on the 8 x 8 array with > 2 exclusive points per segment.

Figures 20 and 21 suggest an inquiry into the types of symmetries which minimal nets in general, and unicursal paths in particular, may possess. We now consider these questions.

5.3 Graphs With Subgroups of the Square

From Figure 20 we see that minimal nets on n x n arrays may possess the full dihedral group of symmetries of the square. However, each of these examples had several odd vertices, which precluded unicursality. In Figure 22 we see three closed unicursal paths on n x n matrices with various symmetry groups. Case A exhibits reflectional symmetry in the mid-vertical. Case B has rotational symmetry by 180° around the center. Case C has the four-fold symmetry group of the rectangle.

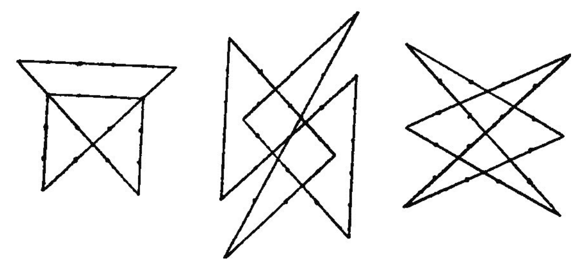


Figure 22. Three symmetric examples of closed unicursal paths.

In Figure 23, we see a minimal net on the 7 x 7 array, which has the rotational symmetry group of the square. This graph is closed unicursal in Euler's sense (all the vertices are even), but it is bicursal for our purposes, because a unicursal path cannot be found on it consisting of only twelve segments! It is not known whether a true unicursal path with this symmetry group exists on any n x n array, though there is no obvious reason to doubt the possibility.

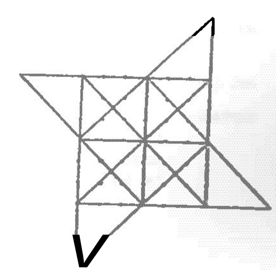


Figure 23. A minimal net with 90° rotational symmetry and only even vertices.

The status of reflectional symmetries is rather completely settled by the following three theorems.

Theorem 5. For every $n \geq 2$, there exists a minimal net on the $n \times n$ array of dots which possesses the full dihedral symmetry group of the square.

Proof. It suffices to observe the two constructions in Figure 24, which correspond to even and odd values of n, respectively.

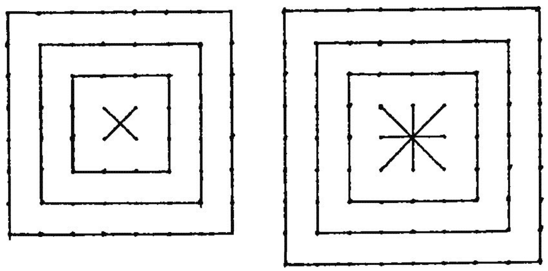


Figure 24. Concentric symmetric construction of minimal nets, for even and odd n, respectively.

Although there are numerous modifications and variations of the symmetric minimal nets of Figure 24, the following feature is common to all examples:

Theorem 6. If a minimal net on an $n \times n$ array of dots has a symmetry axis L on which there are dots, then the net must include a segment on the axis L. (In particular, there must always be a segment on a diagonal symmetry axis; and there must be a segment on a horizontal or vertical symmetry axis whenever n is odd.)

Proof. Suppose that L is a symmetry axis for a minimal net on the $n \times n$ array, and that there are dots on L. In the terminology of the proof of Theorem 1, consider the intersection (two points) of L with the perimeter of the "reduced array". By symmetry in L, if there is no segment along L, then either (1) at least one of these two points is an intersection of two oblique lines, in which case the oblique lines do not cover enough distinct perimeter points for a minimal net, or (2) L is a diagonal, and the two perimeter points of L are covered by (oblique) segments perpendicular to L - but such segments cover only one perimeter point each by the convexity of the reduced array, and therefore cannot be part of a minimal net.

In contrast to Theorem 6 for minimal nets in general, we have the following result for unicursal paths:

Theorem 7. A closed unicursal path on an $n \times n$ array cannot have any segment along a symmetry axis L.

Proof. Each end of a segment along a symmetry axis L must be an odd vertex, but a closed unicursal path can contain no odd vertices.

Notes: Combining Theorems 6 and 7, we learn that a minimal net which is a closed unicursal path can have no diagonal symmetry axes; and can only have horizontal or vertical symmetry axes if n is even. A fortiori, the full dihedral symmetry group of the square never occurs in such a case. A minimal open unicursal path can have at most one symmetry axis with dots on it, since this gives rise to (only) two odd vertices, as in Figure 25. (When such a symmetry axis occurs, it must be a diagonal.)

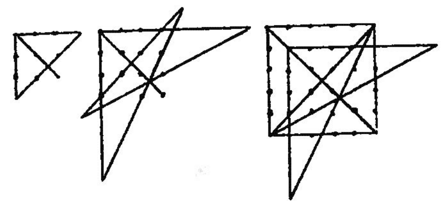


Figure 25. Open unicursal paths with diagonal symmetry.

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NEED HONEY?

The Governing Council of Pi Mu Epsilon announces a contest for the best expository paper by a student (who has not yet received a masters degree) suitable for publication in the Pi Mu Epsilon Journal.

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AN INTERESTING GENERALIZATION OF A SIMPLE LIMIT THEOREM

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It is well known that if $\{a_k\}$, $\{b_k\}$ $k = 1, 7, \ldots$ are sequences of real numbers and if $\lim\limits_{k\to\infty}(a_k/b_k) < \infty$, $\lim\limits_{k\to\infty}(b_k) = 0$ then $\lim\limits_{k\to\infty}(a_k) = 0$. Since matrices are generalizations of real numbers, one might ask if the above fact could be generalized to more general objects. The following proposition is a simple but interesting generalization of the above limit theorem to matrices.

PROPOSITION:

If:

i) A is an $n \times n$ real constant matrix

ii) $b(s)$ is an $n \times 1$ real vector, each component being a continuous function of the complex variables.

iii) $\lim\limits_{s\to s_i}(A-sI)^{-1}b(s)$ exists for each component

where s_i is an eigenvalue of A.

~~then~~; all the components of $(A-sI)^*b(s)$ vanish at the eigenvalues of A, where $(A-sI)^*$ is the adjoint matrix of $(A-sI)$.

Proof: Since $(A-sI)^{-1}b(s) = \dfrac{1}{|A-sI|}(A-sI)^*b(s) = \dfrac{1}{|A-sI|}C(s)$

where $C(s) = (A-sI)^*b(s)$, then calling the transpose $C^T(s) = (c_1(s),\ldots,c_n(s))$ we have that $\lim\limits_{s\to s_i}\{c_k(s)/|A-sI|\} < \infty$ for $k = 1,\ldots,n$ where s_i is an eigenvalue of A. But $\lim\limits_{s\to s_i}|A-sI| = 0$ and so $\lim\limits_{s\to s_i}c_k(s) = c_k(s_i) = 0$.

This completes the proof. One can also observe a stronger but less aesthetic result than the above proposition. This can be stated:

COROLLARY: The above conclusion Is still true if condition ii) is replaced by: ii') all components of $(A-sI)^*b(s)$ are continuous in s, for all s.

Proof: This can easily he seen by direct observation of the previous proof.

One can also observe that the matrix A and vector b could be complex with a slight modification of the proof.

THE CANTOR SET

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The Cantor set, first given by George Cantor, should give a student a better insight into the study of open. closed, dense, and "nowhere dense perfect" sets. Students may easily develop false ideas concerning dense and nowhere dense sets by making a somewhat plausible assumption upon the antecedent "nowhere". The Cantor set may serve as a reminder of the consequences of making these intuitive assumptions.

The Cantor set S is a subset of the closed interval $[0,1]$. It is more convenient to define its complement, $C(S)$, relative to $[0,1]$. $C(S)$ is the union of the following denumerable set of open intervals:

1) the open middle third, $(1/3,2/3)$, uf $[0,1]$,

2) the open middle thirds $(1/9,2/9)$ and $(7/9,8/9)$ of the two closed intervals in $[0,1]$ which are complementary to $(1/3,2/3)$,

3) The open middle thirds $(1/27,2/27)$, $(7/27,8/27)$, $(19/27,20/27)$, and $(25/27,26/27)$ of the four closed intervals in $[0,1]$ which are complementary to $(1/9,2/9)$, $(1/3,2/3)$, and $(7/9,8/9)$ and so on, ad infinitum.

The graph shows three stages of removing open middle thirds of $[0,1]$.

Observing the bum of the lengths of the open intervals removed at the 1st., 2nd., ..., ntn stage:

$$s_n = 1/3 + 2/3^2 + 2^2/3^3 + \ldots + 2^{n-1}/3^n = 1 - (2/3)^n$$

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty}[1 - (2/3)^n] = 1$$

However, the set remaining on the closed interval $[0,1]$ is the Cantor set and may seem so sparse as to be insignificant. Points surely in the Cantor set are

$$0,\ 1/3,\ 2/3,\ 1/9,\ 2/9,\ 7/9,\ 8/9,\ \text{etc.}$$

for all endpoints of those open intervals which were erased. But there are points of the Cantor set other than these endpoints. To see this, we will modify the Cantor set S to the Cantor ternary set S'.

Def. 0, 1, 2, are called ternary digits. if $\sum\limits_{n=1}^{\infty} a_n/3^n$ converges, with each a_n a ternary digit, then the ternary expression of x is $x = \underset{3}{0}.a_1a_2a.$ where the subscript $_3$ indicates that the expression is ternary.

Example: Suppose that we express the open interval (1/3,2/3) in its ternary form:

$1/3 = \underset{3}{0}.100\ldots$ or $1/3 = \underset{3}{0}.0222\ldots$

$2/3 = \underset{3}{0}.200\ldots$ or $2/3 = \underset{3}{0}.122\ldots$

Note that we have expressed each number in two ways: one with only 0's and 2's among the a_i's and the other with 1 as a digit.

Lemma 1. If $1/3 < x < 2/3$ and a ternary representation of x is $x = 0.a_1a_2a_3\ldots$ then $a\ = 1$ and among $a_1, a_2, \ldots$ there is at least one $a_i \neq 0$ and at least one $a_j \neq 2$.

Proof. Suppose $a_1 = 0$. Then $x = 0/3 + \sum_{n=2}^{\infty} a_n/3^n \leq \sum_{n=2}^{\infty} 2/3^n$

$$= 2/3^2 \sum_{k=0}^{\infty} 1/3^k = 2/3^2(1/1 - 1/3)$$

$$= 1/3$$

contradicting $1/3 < x$.

Suppose $a_1 = 2$. Then $x \geq 2/3$, contradicting $x < 2/3$. Thus $a\ = 1$.

Now the supposition $a_2 = a_3 = \ldots = 2$ gives $x = 2/3$, contradicting $x < 2/3$, while the supposition that $a\ = a\ = \ldots = 0$ gives $x = 1/3$ contradicting $x > 1/3$. Q.E.D.

It should be seen, conversely, that

Lemma 2. If $x = \underset{3}{0}.1a_2a_3\ldots$ with at least one $a\ \neq 0$ and at least one $a\ \neq 2$, then the point x is on the open interval (1/3,2/3) and hence is not a point of the Cantor set S'.

A way of indicating "x has a ternary representation not involving the 1" is "x has a ternary representation of the form

i) $x = \underset{3}{0}.(2a_1)(2a_2)\ldots$ where $a_n = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$"

All of this is motivation for

Theorem 1. A point is in the Cantor set S' iff at least one ternary representation of x does not have the digit 1 in any place, that is,

$S' = \{x \mid x$ has a ternary representation of the form i)$\}$.

A proof of this theorem will be given after the next lemma.

Whenever a ternary representation has only zeros from some place on these zeros will not be indicated. For example,

$1/9 = \underset{3}{0}.01$ $2/9 = \underset{3}{0}.02$ $7/9 = \underset{3}{0}.21$ $8/9 = \underset{3}{0}.22.$

In defining the Cantor set, the open intervals erased at the second stage were (1/9,2/9) and (7/9,8/9).

From the above comments it will be seen that the following lemma holds for n = 1 and n = 2.

Lemma 3. In defining the Cantor set S', the open interval $(\underline{x}_n, \bar{x}_n)$ was erased at the nth stage iff $\underline{x}_n$ and $\bar{x}$ can be written as

ii) $\underline{x}_n = \underset{3}{0}.(2a_1)(2a_2)\ldots(2a_{n-1})1$

iii) $\bar{x}_n = \underset{3}{0}.(2a_1)(2a_2)\ldots(2a_{n-1})2$

with only 0's and 2's among the a_k's.

Proof: To make the induction step, let $n \geq 2$ be an integer for which the lemma is true. Then the open interval $(\underline{x}_{n+1}, \bar{x}_{n+1})$ was erased at the (n+1)th stage iff $\underline{x}_{n+1} = 0 + 1/3^{n+1}$ or $\underline{x}_{n+1} = \bar{x}_n + 1/3^{n+1}$ with $\bar{x}$ in the form iii); i.e., respectively,

$\underline{x}_{n+1} = 1/3^{n+1} = \underset{3}{0}.\overbrace{\underbrace{00\ldots0}_{n}1}^{n+1}$ where $a_n = 0$ or else

$\underline{x}_{n+1} = \underset{3}{0}.(2a_1)(2a_2)\ldots(2a_{n-1})2 + 1/3^{n+1}$

$= \underset{3}{0}.(2a_1)(2a_2)\ldots(2a_{n-1})(2a_n)1$ where $a\ = 1$.

With a so determined, then $\bar{x}_{n+1} = \underline{x}_{n+1} + 1/3^{n+1}$ so that

$\bar{x}_{n+1} = 0.(2a_1)(2a_2)\ldots(2a_{n-1})(2a_n)2.$

Since these representations are in the forms ii) and iii) with n replaced by n + 1 the lemma is proved.

Then. as in the proof of lemma 2, a number x is such that

$$\underline{x}_n < x < \bar{x}_n,$$

with $\underline{x}_n$ and $\bar{x}$ in the forms ii and iii, iff

$$x = 0.(2a_1)(2a_2)\ldots(2a_{n-1})(1)b_{n+1}b_{n+2}\ldots$$

with at least one $a_{n+k} \# 0$ and at least one $a_{n+k} \neq 2$. Hence a point x is erased at some stage iff either (in case there are two) ternary representation of x has a 1 in some place. Hence a point of [0,1] is not erased at any stage iff it can be written in ternary form without using the digit 1. Thus Theorem 1 is proved.

Intuitively, the Cantor set S' seem to be sparse. Again, here is a case where our intuition fails us. In fact, the set S' is as large as the interval [0,1] itself.

Def. 0,1, are called binary digits. If $\sum_{n=1}^{\infty} a_n/2^n$ converges to x, with each a a binary digit, then a binary representation of x is

$$x = \underset{2}{0}.a_1a_2a_3\ldots\ .$$

Theorem 2. There is a function with domain the Cantor set S' and range the interval [0,1].

Proof: One such function f is defined in the following way. With $x \in S'$, represent x in its unique ternary form without using the digit 1:

$$x = 0_3(2a_1)(2a_2)\ldots(2a_k)\ldots, \quad a_k = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Then use binary notation and set

$$\text{iv)} \quad f(x) = 0_2.a_1a_2a\ldots a_k\ldots$$

As examples:

$$f(1/3) = f(0.022\ldots_3) = 0.011\ldots_2 = 1/2$$

$$f(3/4) = f(0.2020\ldots_3) = 0.1010\ldots_2 = 2/3$$

In iv) $0 \leq 0.a_1a_2\ldots_2 \leq 1$ and hence f is on [0,1] into [0,1]. To show that f is onto [0,1], that is, that [0,1] is the range of f, select $0 \leq y \leq 1$ arbitrarily. Represent y in binary form:

$$y = 0.b_1b_2b_3\ldots b_k\ldots_2, \quad b_k = \begin{matrix} 0 \\ 1 \end{matrix}$$

If y has two binary representations, either may be used. Then x defined by $x = 0.(2b_1(2b_2)\ldots(2b_k)\ldots_3$ is in the Cantor set S' and $f(x) = y$. Hence the theorem is proved.

With f defined as above, experiments showed that $f(1/3) = f(2/3) = 1/2$. In fact, if $(x_n, \bar{x}_n)$ is one of the open intervals erased at the nth stage, then $f(x_n) = f(\bar{x}_n)$. (To see this, write x_n and $\bar{x}$ according to lemma 3.) Also, associated with this function is a particular function Ψ known as the Cantor function.

Def. The Cantor function Ψ: $[0,1] \to [0,1]$ is defined by setting

$$\Psi(x) = f(x) \quad \text{if } x \in S'$$

$$= f(\underline{x}) = f(\bar{x}) \quad \text{if } \underline{x} < x < \bar{x}$$

for each open interval $(\underline{x}, \bar{x})$ [0,1] - S' with $\underline{x} \in S'$ and $\bar{x} \in S'$.

Summarizing our findings:

1) The Cantor set is closed.

The Cantor set, if closed, must contain all of its limit points. If not, there is some point y on the open interval such that every neighborhood of y contains some points in S'. However, this is not the case since every member of the open interval has a neighborhood with no points in S'; therefore S' contains all of its limit points; hence, S' is closed.

2) The Cantor set is dense in itself.

If S' is dense in itself, then every point in S' must be a limit point. However, this was taken care of by considering the ternary terminating expansions of every member in S'. Therefore, every member in S' is a limit point because of its terminating expression.

3) The Cantor Set is nowhere dense.

We recall from a theorem that stated: If a closed set F contains no intervals, then F is said to be nowhere dense. S' contains no interval since the middle third of each closed interval is always removed at the succeeding stage. The proof of the theorem will be omitted here but it is simply done by applying the indirect method of proof.

4) The Cantor set is perfect.

Since S' is closed and dense in itself, then S' is a perfect set.

5) The Cardinal number of S' is c.

We know that the cardinal number of [0,1] is c and we have shown that S' is just as numerous as [0,1] itself. Therefore, the set S' is of cardinal number c.

Observing the above conclusions of S', especially 2 and 3, we are reminded not to be so hurried in establishing implications in mathematics.

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GLEANINGS FROM CHAPTER REPORTS

Sample Lecture Topics:

Disks, Ovals, and Characteristic Roots--Minnesota Gamma

Existential and Universal Quantifiers as Related to Graph Theory--Ohio Nu

Cobweb Cycles and Difference Theory--Michigan Alpha

Relationship Between Mathematics and Physics--Louisiana Epsilon

California Eta instituted an initiation proceedure. The student initiates were given five problems from different fields of mathematics. The problems were solved before initiation.

Ohio Delta Chapter took an active role in trying to save the departmental library from being absorbed into the main library.

Nebraska Alpha sponsors a prize contest open to all students in the first four introductory mathematics courses. Two exams are given, the second for students with at least two semesters of calculus They are graded by a panel of chapter members.

SIMPLICIAL DECOMPOSITIONS OF CONVEX POLYTOPES

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1. Preliminaries

A natural extension of the usual convex polygons and polyhedra of euclidean spaces E^2 and E^3, a convex d-polytope P is defined to be the bounded intersection of a finite number of closed half-spaces in E^d, where P contains d-dimensional interior. Equivalently, we may define P to be the convex hull of (i.e., the smallest convex set containing) a finite set of points. For each k, $0 \leq k \leq d-1$, a k-face of P is the k-dimensional intersection of P with a supporting hyperplane. The 0-faces are just the vertices, the 1-faces the edges, etc. Each k-face is itself a k-polytope. In the following $f_k(P)$ denotes the number of k-faces of P.

Any d-polytope must have at least d+1 vertices, and the simplest d-polytopes, the d-simplices, have exactly d+1 vertices. The faces of a simplex are themselves simplices, and a d-simplex has exactly $\binom{d+1}{d+1}$ k-faces, $0 \leq k \leq d-1$.

It is the object of this paper to show that any d-polytope can be expressed as the union of d-simplices whose vertices are vertices of the polytope, and whose interiors are pairwise disjoint. Further, given the dimension and number of vertices of a polytope, we seek bounds on the number of such vertex-simplices required.

A simplicial complex K is a finite collection of simplices such that if A is in K, the faces of A are in K, and A and B are in K, $A \cap B$ is either empty or a common face of A and B. The number of r-simplices in K is denoted by $s_r(K)$. The underlying polyhedron of K is $/K/ = \{A:$ A is a simplex of $K\}$.

With the preceding concepts in mind we make the following definition. Definition: Let P be a d-polytope. A simplicial complex K is said to be a simplication of P providing $/K/ = P$ and the vertex set of K is precisely equal to the vertex set of P.

2. Existence of Simplications

Theorem 1: Any polytope has a ~implication.

Proof: Let P be a d-polytope with v vertices, and well-order the set of vertices of P. We use an inductive process to simplicate the faces of P. The 0-faces and 1-faces are already simplices. For $k > 1$ the general inductive step is as follows. In an arbitrary k-face F, let x be the first vertex of P in F. We assume that the (k-1)-faces of F, which are also (k-1)-faces of P, have been appropriately simplicated. The pyramids with apex x and bases the (k-1)-simplices in each (k-1)-face of F which does not contain x, together with the faces of these pyramids, constitute a simplication of F.

Having simplicated all the (d-1)-faces of P, let y be the first vertex of P, and form the pyramids with apex y and bases the (d-1)-simplices lying in the (d-1)-faces of P not containing y. The ordering of the vertices insures the required intersectional properties, so these pyramids, together with their faces constitute a simplication.

Theorem 1 can also be proved by induction on v, the number of vertices of P. Clearly a simplication does not have to be formed as in the proof of the theorem. A simplication in which all the d-simplices contain a common vertex is called a fixed-vertex simplication.

3. The Minimum Number of Simplices in a Simplication

Let $m(v,d)$ denote the minimum possible number of d-simplices in a simplication of a d-polytope with v vertices. In what follows we generally ignore the trivial one-dimensional case.

Theorem 2: For every $v > d \geq 2$, $m(v,d) = v-d$.

Proof: First we show that $m(v,d) \geq v-d$, and second that for every $v > 2 \geq 2$ there exist d-polytopes with v vertices having simplications with exactly v-d d-simplices.

Let P be a d-polytope with v vertices and K a simplication of P. Arrange the d-simplices of K in a sequence so that any simplex after the first has a (d-1)-face in common with some preceding simplex. Clearly this can be done, since /K/ is a topological d-ball. We count the vertices of P in terms of $s_d(K)$. The first simplex in the sequence contributes d+1 vertices; each of the next $s_d(K)-1$ simplices contribute at most one additional vertex. Thus we have $v \leq d+1 + s_d(K)-1$ or $s_d(K) \geq v-d$. Since P was arbitrary, we thus have $m(v,d) \geq v-d$.

Now it is easy to see that any simplication of a 2-polytope with v vertices must have precisely v-2 2-simplices. Suppose $v > d > 2$, and, proceeding inductively, let Q be a (d-1)-polytope with v-1 vertices, and K a simplication of Q such that $s_{d-1}(K) = v-1-(d-1) = v-d$. Let P be a d-pyramid with Q as base and arbitrary apex x. K induces a simplication K' of P (in fact a fixed-vertex simplication) such that $s_d(K') = s_{d-1}(K) = v-d$. This completes the proof.

Remarks: (1) Clearly a simplication is minimal in the sense of Theorem 2 if and only if the d-simplices can be arranged in a sequence as described in the proof so that each after the first has precisely one (d-1)-face in common with some preceding simplex. (2) Using the same counting technique as in Theorem 2, we get $s_k(K) \geq v-k$, for each k, $0 \leq k < d-1$. But in general this is not the best possible bound. We might introduce the symbol $m_k(v,d)$ for the minimum possible number of k-simplices in a simplication of a d-polytope with v vertices. The value of $m_k(v,d)$

for $0 < k < d-1$ seems to be an open question at this time. (3) The reasoning of the proof of Theorem 2 actually proves that if K is any simplicial complex with v vertices such that $/K/$ is a topological d-ball, then $s_d(K) \geq v-d$, and that there exist such complexes K so that $s_d(K) = v-d$.

4. The Maximum Number of Simplices in a Simplication

The problem of finding the maximum number of simplices in a simplication is considerably harder than that of finding the minimum. In the following $M(v,d)$ and $N(v,d)$ denote the maximum possible number of d-Simplices in a simplication and a fixed-vertex simplication, respectively, of a d-polytope with v vertices. To facilitate the following discussion, $\mu_d(v,d+1)$ denotes the maximum number of d-simplices possible in a simplicial complex with v vertices whose underlying polyhedron is a topological d-sphere. Also $C(v,d)$ denotes a cyclic d-polytope with v vertices, the convex hull of v distinct points on the moment curve $\{(t,t^2,t^3,\ldots,t^d)$: t real]. A cyclic polytope is simplicial (i.e., its k-faces are k-simplices), any two cyclic d-polytopes with v vertices are combinatorially equivalent, and their special structure allows one to calculate that

$$f_{d-1}(C(v,d)) = \begin{cases} \frac{v}{v-n}\binom{v-n}{n} & \text{if } d = 2n \\ 2\binom{v-n-1}{n} & \text{if } d = 2n+1 \end{cases}$$

Cyclic polytopes lead to the following conjecture.

Upper Bound Conjecture: For all $v > d \geq 2$, $\mu_d(v,d+1) = f_d(C(v,d+1))$.

It is known that the Upper Bound Conjecture is true as stated here at least when $d \leq 7$ (i.e., for 8-polytopes) and when v is comparatively large or small with respect to d. (See Grunbaum, pp. 61ff)

We now begin with $N(v,d)$. First consider the following construction. Let $v > d+1$ and let P be a polytope obtained from the cyclic polytope $Q = C(v-1,d)$ as follows. Let F be any (d-1)-face of Q and x any point not in Q such that (convex hull of $\{x\} \cup Q$) = (convex hull of $\{x\} \cup F$) $\cup$ Q (i.e., x is "beyond" F and "beneath" all other (d-1)-faces of Q). Let P be the convex hull of $\{x\} \cup Q$ and K the fixed-vertex simplication with respect to x. Then since P is simplicial $s_d(K) = f_{d-1}(C(v-1,d))-1$. This construction leads to the following conjecture.

Conjecture 1: If $v-2 \geq d \geq 2$, $N(v,d) \leq \mu_{d-1}(v-1,d)-1$, with equality when the Upper Bound Conjecture holds for d-polytopes with v-1 vertices.

The preceding construction shows that $N(v,d) \geq \mu_{d-1}(v-1,d)-1$ when the Upper Bound Conjecture holds. Thus it remains to show that $N(v,d) \leq \mu_{d-1}(v-1,d)-1$ in general. We show that this is true for $d \leq 4$.

Clearly $N(v,2) = v-2$ and since $\mu_1(v-1,2)-1 = v-1-1 = v-2$, the relation holds for $d = 2$.

Now let P be a 3-polytope with $v > 4$ vertices, and K a fixed-vertex simplication of P, each 3-simplex of which contains the vertex x.

Then K induces a triangulation K' of the boundary of P, a 2-sphere. Each 3-simplex in K has its 2-face opposite x in K. In addition there are at least 3 2-faces of P containing x. These facts yield $s_3(K) + 3 \leq \mu_2(v,3)$ or $s_3(K) \leq \mu_2(v,3)-3$. Since P is arbitrary we have $N(v,3) \leq \mu_2(v,3)-3 = 2v-7 = \mu_2(v-1,3)-1$.

Finally suppose P is a 4-polytope with $v > 5$ vertices, and let K be a fixed-vertex simplication of P. Let Q be a 5-pyramid with base P and arbitrary apex x. K then induces a triangulation K' of the boundary of Q, a 4-sphere. The 4-simplices of K' are just those of K plus those which are the convex hull of x with the 3-simplices of K lying in the boundary of P. There must be at least $s_4(K)+4$ of the latter variety, since K is a fixed-vertex simplication. Thus

$$s_4(K) + s_4(K) + 4 \leq \mu_4(v+1,5) \text{ or } s_4(K) \leq (\mu_4(v+1,5)-4)/2.$$

(The numerator is always even.) Again since P is arbitrary we have

$$N(v,4) \leq (\mu_4(v+1,5)-4)/2$$

A little calculation shows that the right side of this inequality is precisely $\mu 3(v-1,4)-1$. Therefore $N(v,4) \leq \mu 3(v-1,4)-1$.

The preceding work proves the following theorem.

Theorem 3: If $2 \leq d \leq 4$ and $v > d+1$ then $N(v,d) = \mu_{d-1}(v-1,d)-1$ //

Unfortunately the techniques used above are not sufficient when $d \geq 5$.

We now consider $M(v,d)$. Clearly $M(v,d) \geq N(v,d)$. For given $v-2 > d > 2$ Peter McMullen has constructed a $\bar{d}$-polytope with v vertices and $\bar{a}$ simplication of the polytope such that the simplication contains exactly $f_d(C(v,d+1))-v+d$ d-simplices.

His construction suggests the following conjecture which is trivially true for $d = 2$.

Conjecture 2: Given $v-2 \geq d \geq 2$, $M(v,d) \leq \mu_d(v,d+1)-v+d$, with equality when the Upper Bound Conjecture holds for (d+1)-polytopes with v vertices.

Remark: As we did with $m(v,d)$ we might more generally define $M_k(v,d)$ and $N_k(v,d)$ to be the maximum number possible of k-simplices in a simplication and a fixed-vertex simplication, respectively, of a d-polytope with v vertices.

5. Conclusion

The problem treated in this paper is a special case of the following more general question: **Given a simplicial complex which satisfies certain combinatorial and topological restrictions, what more can one say about the complex?** The Upper Bound Conjecture introduced in Section 4 is another aspect of the same problem. There are a number of easily posed open questions in this area, and convex polytopes provide a natural setting for many of them--in particular for extremal problems such as those considered here.

REFERENCES

B. Grunbaum. Convex Polytopes. New York: Interscience, 1967.

P. McMullen. Partitions of Polytopes into Simplices (unpublished paper).

The author first became interested in the subject of this paper during the summer of 1968--between his junior and senior years at Oklahoma State University--in a National Science Foundation Undergraduate Research Participation Project, at Clemson University, under the direction of Professors W. R. Hare and J. W. Kenelly. Special thanks is due to Professor Kenelly with whom the author worked most closely.

UNDERGRADUATE RESEARCH PROPOSAL

Harold Diamond
University of Illinois

1. Suppose we wish to obtain a numerical estimate for the sum of the series $\sum_1^\infty n^{-2}$ accurate to within .001. One method would be to write

$$\sum^N n^{-2} + \int_{N+1}^\infty t^{-2}\,dt < \sum_1^\infty n^{-2} < \sum^N n^{-2} + \int_N^\infty t^{-2}\,dt,$$

find an N for which $\int_N^{N+1} t^{-2}\,dt < .001$ (e.g. N = 32) and use the approximation

$$\sum_1^\infty n^{-2} = \sum_1^N n^{-2} + 1/N + \text{Error}.$$

Can you give a more efficient method? (Incidentally, it is known that $\sum n^{-2} = \pi^2/6$.)

2. The inequality sign in the Cauchy Schwarz relation

$$(\int fg) \le \{\int f^2 \cdot \int g^2\}^{1/2}$$

becomes an equality if f = cg for some constant c. If h is a real function for which $h^2 \approx cf$, some c > 0, then the inequality

$$\int f = \int \frac{f}{h} \cdot h \le \{\int (f/h)^2\}^{1/2}\{\int h^2\}^{1/2}$$

is rather sharp. Find some useful applications for this idea.

THE RECURRENCE EQUATION FOR BOUNDING CUBES

David Berman
Trinity University

1. Introduction. By a j-cube we will mean the j-dimensional object resulting from moving a j-1 cube at an angle θ ($\theta \ne 0$) a distance z ($z \ne 0$) into the j^{th} dimension. In Euclidean space, $|z_j| = |z_{j-1}|$ and $\theta = \frac{\pi}{2}$.

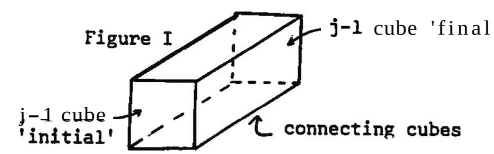
The purpose of this paper is the derivation and solution of the difference equation

(1) $$N_{i,j} = 2N_{i,j-1} + N_{i-1,j-1},$$

in which $N_{i,j}$ is the number of i-cubes bounding a j-cube ($j \ge i$). The solution will be shown to be

(2) $$N_{i,j} = 2^{j-i}\ {}_jC_i,$$

where ${}_jC_i$ is the symbol for combinations. The solution will be generalized to gamma functions. Finally, Euler's Theorem for n-dimensional polyhedra will be stated and proven for bounding cubes.



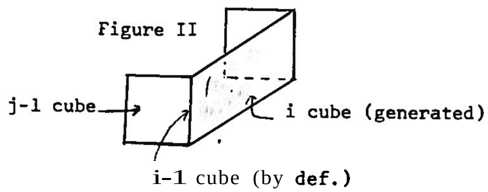
2. Derivation. To find the number of i-cubes bounding a j-cube, we first must find the number of i-cubes bounding the initial j-1 cube and add this to the number of connecting i-cubes and finally add the number of i-cubes bounding the final j-1 cube. This is illustrated in Figure I. Using the notation $N_{i,j}$ for the number of i-cubes bounding a j-cube we have

(3) $$N_{i,j} = N_{i,j-1} + (\text{connecting cubes}) + N_{i,j-1}.$$

We note that at the initial and final positions the quantity $N_{i-1,j-1}$ expresses the number of i-1 cubes bounding a j-1 cube. By definition of a cube we can say that each i-1 initial cube generates one i-cube (connecting cube). From the last two statements we merely have to find the number of i-1 cubes bounding the initial j-1 cube and we are guaranteed that this corresponds to the number of connecting i-cubes. See Figure II. Substituting $N_{i-1,j-1}$ in eq. (3) for the number of connecting i-cubes we get

(4) $$N_{i,j} = 2N_{i,j-1} + N_{i-1,j-1},$$

the recurrence equation for bounding cubes.



3. Solution. Replacing $\underline{i}$ and $\underline{j}$ by x and y respectively and N by f, eq. (4) can be written in the more tractable form

(5) $$f(x,y) - 2f(x,y-1) - f(x-1,y-1) = 0.$$

If we let

(6) $$f(x,y) = 2^{ax+by}\, g(x,y), \text{ then}$$

(7) $$2^{as+by}g(x,y) - 2^{ax+by-b+1}g(x,y-1) - 2^{ax+by-a-b}g(x-1,y-1) = 0.$$

Assigning the values b = 1 and a = -1, eq. (7) can be simplified to

(8) $$2^{y-x}[g(x,y) - g(x,y-1) - g(x-1,y-1)] = 0, \text{ or}$$

(9) $$g(x,y) - g(x,y-1) - g(x-1,y-1) = 0.$$

Eq. (9) is readily solved by methods of finite differences [1] to be

(10) $$g(x,y) = {}_yC_x.$$

Substituting eq. (10) back in eq. (6) we have

(11) $$f(x,y) = 2^{y-x}\, {}_yC_x, \text{ or}$$

(12) $$N_{i,j} = 2^{j-i}\, {}_jC_i.$$

4. Generalization. Eq. (12) can be used only when $\underline{i}$ and $\underline{j}$ are integers. It would be desirable to generalize this equation so that all real values could be used. The natural generalization is to use the identity $\Gamma(j + 1) = j!$ and substitute gamma functions for factorials. This generalization will be permissible if and only if the resulting equation satisfies the original recurrence relationship, ew. (4). Generalizing eq. (12) to gamma functions we get

(13) $$N_{i,j} = \frac{\Gamma(j+1)\, 2^k}{\Gamma(i+1)\Gamma(k+1)} \qquad k = j - i$$

(14) $$N_{i,j-1} = \frac{\Gamma(j)2^{k-1}}{\Gamma(i+1)\Gamma(k)}$$

(15) $$N_{i-1,j-1} = \frac{\Gamma(j)2^k}{\Gamma(i)\Gamma(k+1)}\,.$$

By direct substitution of eqs. (13), (14), and (15) into eq. (4) we write

(16) $$\frac{\Gamma(j+1)2^k}{\Gamma(i+1)\Gamma(k+1)} = \frac{\Gamma(j)2^k}{\Gamma(i+1)\Gamma(k)} + \frac{\Gamma(j)2^k}{\Gamma(i)\Gamma(k+1)}$$

(17) $$\Gamma(j+1) = k\Gamma(j) + i\Gamma(j)$$

(18) $$\Gamma(j+1) = (k+i)\Gamma(j) \qquad k = j - i$$

(19) $$\Gamma(j+1) = j\Gamma(j),$$

showing that the generalized solution does indeed satisfy the recurrence equation.

5. Euler's Theorem. Euler's Theorem for bounding cubes may he stated as follows: For a $\underline{j}$-cube,

(20) $$N_{0,j} - N_{1,j} + N_{2,j} - \ldots + N_{j,j} = 1, \text{ or}$$

(21) $$\sum_{i=0}^{j} (-1)^i\, 2^{j-i}\, {}_jC_i = 1.$$

Writing eq. (21) in expanded form, we have

(22) $$\sum_{i=0}^{j} (-1)^i\, 2^{j-i}\, {}_jC_i = 1.$$

Making use of the binomial expansion $(a-1)^j$, we can write

(23) $$(a-1)^j = \sum_{i=0}^{j} (-1)^i\, a^{j-i}\, {}_jC_i.$$

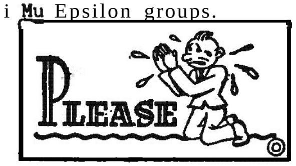
If we set a = 2 in eq. (23), we get the desired result, eq. (22), hence proving Euler's Theorem for bounding cubes.

REFERENCE

1. Charles Jordan, Calculus of Finite Differences, Chelsea, New York, 1960, pp. 604-613.

MATCHING PRIZE FUND

The Governing Council of Pi Mu Epsilon has approved an increase in the maximum amount per chapter allowed as a matching prize from \$25.00 to \$50.00. If your chapter presents awards for outstanding mathematical papers and students, you may apply to the National Office to match the amount spent by your chapter--i.e., \$30.00 of awards, the National Office will reimburse the chapter for \$15.00, etc.,--up to a maximum of \$50.00. Chapters are urged to submit their best student papers to the Editor of the Pi Mu Epsilon Journal for possible publication. These funds may also be used for the rental of mathematical films. Please indicate title, source and cost, as well as a very brief comment as to whether you would recommend this particular film for other Pi Mu Epsilon groups.



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PROBLEM DEPARTMENT

Edited by
Leon Bankoff, Los Angles, California

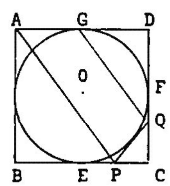
This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity. Occasionally we shall publish problems that should challenge the ability of the advanced undergraduate or candidate for the Master's Degree. Old problems characterized by novel and elegant methods of solution are also acceptable. Solutions should be submitted on separate, signed sheets and mailed before May 31, 1971.

Address all communications concerning problems to Dr. Leon Bankoff, 6360 Wilshire Boulevard, Los Angeles, California 90048.

PROBLEMS FOR SOLUTION

239. Proposed anonymously. Information regarding source is solicited.

A circle (O) inscribed in a square ABCD, (AB = 2a), touches AD at G, DC at F, and BC at E. If Q is a point on DC and P a point on BC such that GQ is parallel to AP, show that PQ is tangent to the circle (O).



240. Proposed by Charles W. Trigg, San Diego, California.

The palindromic triangular number $\Delta_{10} = 55$ and $\Delta_{11} = 66$ may each be considered to be a repetition of a palindromic number. Find another palindromic number which when repeated forms a triangular number.

241. Proposed by Solomon W. Golomb, University of Southern California.

What is the simplest explanation for this sequence:

8 5 4 9 1 7 6 3 2 0 7

242. Proposed by the Problem Editor.

If m_a, m_b, m are the medians corresponding to sides a, b, c of a triangle ABC, show that

$$m_a^2 m_b^2 + m_b^2 m_c^2 + m_c^2 m_a^2 = (9/16)(a^2b^2 = b^2c^2 + c^2a^2).$$

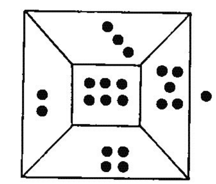
243. Proposed by Alfred E. Neuman, Mu Alpha Delta Fraternity, New York.

Provide a geometrical proof for the well-known relation

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{5} + \arctan\frac{1}{8}\text{"}$$

244. Proposed by Charles W. Trigg, San Diego, California.

The spots on a standard cubical die are distributed as indicated on the accompanying Schlegel diagram, the sum on each pair of opposite faces being 7. A square grid is composed of squares the same size as a die face. When a die is placed on a square and rotated 90° about an edge to come into contact with another square, the motion will be called a roll.



What is the shortest roll sequence that will return the die to the starting square in its original attitude?

245. Proposed by R. S. Luthar, University of Wisconsin, Waukesha.

Prove that for positive numbers x and y the following inequality holds:

$$(x^2 - xy + y^2)^{(x+y)/2} \geq x^x y^y.$$

246. Proposed by Bob Prielipp, Wisconsin State University, Oshkosh.

If x is an even perfect number > 6, prove that $x \equiv 4 \pmod{12}$.

247. Proposed by Alfred E. Neuman, Mu Alpha Delta Fraternity, New York.

Construct diagrams illustrating four (or more) different theorems characterized by the relation $AZ \cdot BX \cdot CY = |AY \cdot BZ \cdot CX|$.

SOLUTIONS

212. (Fall 1968) Proposed by J. M. Gandhi, University of Manitoba, Winnipeg, Canada.

Solution I by the Proposer.

If $$M(n) = \sum_{s=0}^{n-1} \binom{n}{s+1}\binom{n+s}{s}$$

show that

(A) $M(5m + 2) \equiv 0 \pmod 5$
and
(B) $M(5m + 3) \equiv 0 \pmod 5$.

[Ref. George Rutledge & R. D. Douglass, Integral functions associated with certain binomial sums, Amer. Math. Monthly 43(1936), pp. 27-33].

Solution. Rutledge & Douglass see the above ref., proved that

$$(2n-3)nM(n) = (12n^2 - 24n + 10)M(n-1). \quad (1)$$

Let $n = 5m + 2$, so that from (1) we get

$$(10m+1)(5m+2)M(5m+2)$$
$$= \{12(5m+2)^2 - 24(5m+2) + 10\} M(5m+1)$$
$$- \{2(5m+2)^2 - 5(5m+2) + 2\} M(5m)$$
$$\equiv \{48 - 48 + 10\} M(5m+1)$$
$$- (8+2) M(5m) \pmod 5$$
$$0 \pmod 5.$$

Thereby congruence (B) follows.
Now considering $n = 5m + 3$ from (1) we get $(10m+3)(5m+3)M(5m+3)$

$$= [12(5m+3)^2 - 24(5m+3) + 10] M(5m+2)$$
$$- [2(5m+3)^2 + 2] M(5m+1)$$
$$\equiv M(5m+2) \pmod 5$$
$$\equiv 0 \pmod 5 \text{ in view of congruence (B) and hence}$$

we get congruence (A).

Solution II by L. Carlitz, Duke University.

We shall make use of the Lucas congruence

$$\binom{ap+b}{rp+s} \equiv \binom{a}{r}\binom{b}{s} \pmod p,$$

where p is prime and

$$a \ge 0, \quad r \ge 0, \quad 0 \le b < p, \quad 0 \le s < p.$$

Then

$$M(5m+2) = \sum_{s=0}^{5m+1} \binom{5m+2}{s+1}\binom{5m+s+2}{s}$$
$$\equiv \sum_{t=0}^{m} \binom{5m+2}{5t+1}\binom{5m+5t+2}{5t} + \sum_{t=0}^{m} \binom{5m+2}{5t+2}\binom{5m+5t+3}{5t+1}$$
$$\equiv 2\sum_{t=0}^{m} \binom{m}{t}\binom{m+t}{t} + 3\sum_{t=0}^{m} \binom{m}{t}\binom{m+t}{t}$$
$$\equiv 0 \pmod 5.$$

Similarly

$$M(5m+3) = \sum_{s=0}^{5m+2} \binom{5m+3}{s+1}\binom{5m+s+3}{s}$$
$$\equiv \sum_{t=0}^{m} \binom{5m+3}{5t+1}\binom{5m+5t+3}{5t} + \sum_{t=0}^{m} \binom{5m+3}{5t+2}\binom{5m+5t+4}{5t+1}$$
$$+ \sum_{t=0}^{m} \binom{5m+3}{5t+3}\binom{5m+5t+5}{5t+2}$$
$$\equiv 3\sum_{t=0}^{m} \binom{m}{t}\binom{m+t}{t} + 12\sum_{t=0}^{m} \binom{m}{t}\binom{m+t}{t}$$
$$\equiv 0 \pmod 5.$$

We can generalize the above result in the following way. Let $n = pm + k$, where p is prime and $0 \le k < p$. Then

$$M(pm+k) = \sum_{s=0}^{pm+k-1} \binom{pm+k}{s+1}\binom{pm+k+s}{s}$$
$$= \sum_{s=0}^{m} \sum_{t=0}^{p-1} \binom{pm+k}{ps+t+1}\binom{pm+ps+k+t}{ps+t}$$
$$\equiv \sum_{s=0}^{m} \sum_{\substack{t+1\le k\\ k+t<p}} \binom{m}{s}\binom{k}{t+1}\binom{m+s}{s}\binom{k+t}{t}$$
$$\equiv \sum_{s=0}^{m} \binom{m}{s}\binom{m+s}{s} \sum_{\substack{t+1\le k\\ k+t<p}} \binom{k}{t+1}\binom{k+t}{t} \pmod p.$$

Hence

$$\sum_{\substack{t+1\le k\\ k+t<p}} \binom{k}{t+1}\binom{k+t}{t} \equiv 0 \pmod p$$

is a sufficient condition for

$$M(pm+k) \equiv 0 \pmod p.$$

For example if $k = 4$ we have

$$\sum_{t=0}^{3} \binom{4}{t+1}\binom{4+t}{t} = 4 + \binom{4}{2}\binom{5}{1} + \binom{4}{3}\binom{6}{2} + \binom{7}{3}$$
$$= 4 + 30 + 60 + 35 = 129 = 3 \cdot 43,$$

so that

$$M(43m+4) \equiv 0 \pmod{43}.$$

220. (Spring 1969) Proposed by Daniel Pedoe, University of Minnesota.

a) Show that there is no solution of the Apollonius problem of drawing circles to touch three given circles which has only seven solutions.

b) What specializations of the three circles will produce 0, 1, 2, 3, 4, 5 and 6 distinct solutions?

Solution by the Proposer.

The circle C_o which is orthogonal to each of the given C_i $(i = 1,2,3)$ is uniquely defined, unless the C_i belong to a pencil of circles. In the latter case the only tangent circles are two point-circles, in the case when the pencil is of the intersecting type. The circle C_o plays a very special role with regard to the C_i, since inversion in C_o maps each C_i onto itself, and maps a tangent circle C onto a tangent circle C'. When there are 8 tangent circles (which may be called the general case) these can be split into four pairs. We call the circles in a pair "conjugate circles". (For all this, proved algebraically, see Pedoe, Circles, Pergamon Press, 1957). If we specialize the C_i so that there are only 7 tangent circles, the specialization must aim at making a pair of conjugate circles identical, since if two tangent circles which are not conjugate become identical, the conjugates also become identical, and the number of tangent circles would reduce to 6, at most.

We therefore specialize the C_i so that a conjugate pair C, C' become the same circle, D, say. This means that inversion in C_o maps the tangent circle D onto itself. If this is the case, D must be orthogonal to C_o. We therefore find ourselves with three circles C_i, a circle C_o orthogonal to the C_i, and a circle D which touches the C_i and is also orthogonal to C_o. We show that this means that two of the C_i must touch each other.

Invert with respect to a center of inversion on C_o. We obtain three circles C_i', with diameters which lie along the line C_o'. These three circles are touched by a circle D' whose diameter also lies along C_o'. If two circles with diameters along the same line touch on a point not on this line, they have the same center, and must therefore coincide. If the circles are distinct, contact can only take place at an endpoint of a diameter. Since D' has only two points of intersection with the line C_o', and has to touch each of C_1', C_2', C_3' at a point on C_o', the three points of contact cannot be distinct. Hence at least two of the circles C_i' intersect C_o' at the same point. That is, at least two of the circles C_i' touch each other. But if at least two of the circles C_i touch each other, the number of circles tangent to the three C_i is readily seen to be 6, at most.

Also solved by Charles W. Trigg, San Diego, California. To do justice to Trigg's detailed analysis of the problem and to the numerous diagrams accompanying his solution, the editor has found it necessary to postpone publication until the Spring 1971 issue.

Editor's Note: An expanded version of Pedoe's solution has been published in his paper The Missing Seventh Circle, Elemente der Mathematik, January 1970, page 14.

222. (Fall 1969) Proposed by Jack Garfunkel, Forest Hills High School, Flushing, N.Y.

In an acute triangle ABC, angle bisector BT_1 intersects altitude A H in D. Angle bisector CT_2 intersects altitude BH_2 in E, and angle bisector AT_3 intersects altitude CH_3 in F. Prove

$$\frac{DH_1}{AH_1} + \frac{EH_2}{BH_2} + \frac{FH_3}{CH_3} \leqq 1.$$

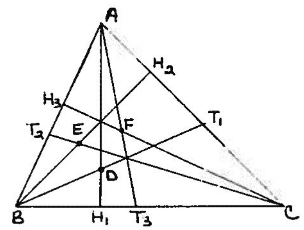
Solution by the Proposer.

Since $DH_1/AH_1 = \tan(B/2)/\tan B = 1 - \tan^2(B/2)$, etc., the problem is equivalent to that of showing

$$\tan^2(A/2) + \tan^2(B/2) + \tan^2(C/2) \geqq 1,$$

equality holding if and only if A = B = C. The proof of this inequality follows from the relation

$$\tan(C/2) = \frac{1-\tan(A/2)\tan(B/2)}{\tan(A/2) + \tan(B/2)}$$



whence $\Sigma\tan(A/2)\tan(B/2) = 1$.
It follows that $\Sigma\tan^2(A/2) \geqq 1$, thus completing the proof.

Also solved by Sid Spital, Calif. State College at Hayward, who noted that the triangle need not be acute.

223. (Fall 1969) Proposed by Solomon W. Golomb, University of Southern California, Los Angeles.

In the first octant of 3-dimensional space, where $x \geqq 0$, $y \geqq 0$, $z \geqq 0$, identify the region where the following "associative law" holds:

$$x^{(y^z)} = (x^y)^z.$$

Solution by the Proposer.

Both expressions are indeterminate on the line $x = y = 0$. The left side is also indeterminate on the line $y = z = 0$. Otherwise, the identity holds in the four planes

$$x = 0,\quad x = 1,\quad y = 0,\quad z = 1$$

and along the surface $y = z^{1/(z-1)}$ for $z \neq 1$.

If $x \neq 0$ then $x^a = x^b$ requires $a = b$, which in this case means $y^z = yz$, so that there are no other solutions.

Also solved by R. C. Gebhardt, Parsippany, N.J.; Richard L. Enison, New York; C. B. A. Peck, State College, Pennsylvania; C. L. Sabharwal, Saint Louis University; and Sid Spital, Calif. State College at Hayward.

Peck notes that on the cylinder $y = z^{1/(z-1)}$, $\lim_{z\to\infty} y = 1$ and $\lim y = \infty$. The cylinder is thus asymptotic to two of the planes, namely the xy- and the xz-planes, and intersects the other two (the ones parallel to the yz-plane).

224. (Fall 1969) Proposed by Charles W. Trigg, San Diego, Calif.

In the following cryptorithm, each letter represents a distinct digit in the decimal scale:

8 (M A D A P E) = 5 (A P E M A D).

Identify the digits.

Solution by Jeanette Bickley, St. Louis, Missouri.

Below is the computer program and output from a SDS 940 computer. The program tests all possible digit replacements for M, A, D, P, E and gives the unique solution M = 1, A = 7, D = 8, P = 0, E = 5.

```
 CCOM-SHARE CENTER E 52
PLEASE LOG IN:E285;WJGEE
READY, SYSTEM W04
 FEB  4  8:38
LAST LOGIN  FEB  3 16:26
-XTR
VER.  JAN 71

+QFD
*A /JBMADAPE/
*/
      INTEGER A,D,P,E,RIGHT
      DIMENSION M(10),A(10),D(10),P(10),E(10)
   90 FORMAT(9HMADAPE = ,17)
   91 FORMAT(10I2)
      READ(0,91) M,A,D,P,E
      WRITE(1,91) M,A,D,P,E
      DO 20 I=1,10
      DO 20 J=1,10
      DO 20 K=1,10
      DO 70 L=1,10
      DO 20 N=1,10
      LEFT = M(I)*100000+A(J)*10000+D(K)*1000+A(J)*100+P(L)*10+E(N)
      RIGHT= A(J)*100000+P(L)*10000+E(N)*1000+M(I)*100+A(J)*10+D(K)
      IF(8*LEFT-5*RIGHT)20,30,20
   30 WRITE(1,90) LEFT
   20 CONTINUE
      STOP
      END
*XTRAN
+C
OUTPUT:
OPTIONS: CARD

+R
OPTIONS:
SPROG:
XLIBE  JAN 14

0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5 6 7 8 9
MADAPE =
MADAPE =  128205
*STOP*
($MAIN$)20+1
```



```
+EXIT
?
+EXIT

-LOG
USAGE
CCU:   078
CLT:   0.06  HOURS
```

Solution II by R. C. Gebhardt, Parsippany, N.J.

The problem can be written as

$$8000(MAD) + 8(APE) = 5000(APE) + 5(MAD).$$

Thus $7995(MAD) = 4992(APE)$ or $205(MAD) = 128(APE)$.
Since 128 and 205 are relatively prime, this equation is solved by MAD = 128, APE = 205.

Also solved by Charles H. Culp, Socorro, New Mexico; Clayton W. Dodge, University of Maine; Elliot D. Friedman, Plainview, N.Y.; Walter Wesley Johnston, Springfield, Illinois; Donald Marshall, Pasadena; Donald R. Steele, Pine Plains Central School, Pine Plains, N.Y.; Gregory Wulczyn, Bucknell University; and the propser.

A 4-page solution offered by Alfred E. Neuman, of the Mu Alpha Delta Fraternity, turned out to be incorrect.

225. (Fall 1969) Proposed by Wray G. Brady, University of Bridgeport.

Show that any proper fraction, a/b, can be written as the product of fractions of the type $n/(n + m)$ for fixed m.

Solution by Charles W. Trigg, San Diego, California.

If $k = b - a > 1$, $m = 1$, clearly

$$\frac{a}{b} = \frac{a}{a+1} \cdot \frac{a+1}{a+2} \cdot \frac{a+2}{a+3} \cdots \frac{a+(k-2)}{a+(k-1)} \cdot \frac{a+(k-1)}{a+k}.$$

If $b - a = 1$, then

$$\frac{2a}{2b} = \frac{2a}{2a+1} \cdot \frac{2a+1}{2a+2},$$ or in general,

$$\frac{ra}{rb} = \frac{ra}{ra+1} \cdot \frac{ra+1}{ra+2} \cdots \frac{ra+(r-1)}{ra+r}.$$

If $b - a$ is composite, say $b - a = pm$, then the multiplicative sequence may be shortened, i.e.,

$$\frac{a}{b} = \frac{a}{a+m} \cdot \frac{a+m}{a+2m} \cdots \frac{a+(p-1)m}{a+pm}.$$

Also solved by Clayton W. Dodge, University of Maine; Richard L. Enison, New York; Murray S. Klamkin, Ford Motor Company Scientific Research Staff; Frank P. Miller, Jr., Pennsylvania State University; Bob Prielipp, Wisconsin State University - Oshkosh; Gregory Wulczyn, Bucknell University; and the proposer, who gave the reference to Dickson's Theory of Numbers, Vol. II, p. 687, Chelsea, 1952.

226. (Fall **1969**) Proposed by **B. J.** Cerimele, North Carolina State University at Raleigh.

Derive a formula for the n-th order antiderivative of $f(x) = \ln x$.

Solution I by **Murrary** S. **Klamkin,** Ford Motor Company.

We solve the more general problem of finding the n-th order antiderivative of $x^m \log x$. This is equivalent to solving the n-th order differential equation

$$D^n y = x^m \log x.$$

Let $x = e^z$. Then,

$$(1)\quad D(D-1)(D-2)\ldots(D-n+1)y = ze^{(m+n)z}$$

The complementary solution is given by

$$y_c = a_0 + a_1e^z + \ldots + a_{n-1}e^{(n-1)z}$$

To find a particular **soltion,** we multiply (1) by $e^{-(m+n)z}$ and use the exponential shift theorem, to give

$$(D+m+n)(D+m+n-1)\ldots(D+m+1)ye^{-(m+n)z} = z.$$

Thus, $y_pe^{-(m+n)z} = az + b$ and to **determine** the constants a and b, we substitute back. Thus,

$$(m+1)(m+2)\ldots(m+n)\,(az+b)+a\left[\frac{1}{m+1} + \frac{1}{m+2} + \ldots + \frac{1}{m+n}\right] \equiv z$$

and

$$a = \frac{\Gamma(m+1)}{\Gamma(m+n+1)},\quad b = -a\left[\frac{1}{m+1} + \frac{1}{m+2} + \ldots + \frac{1}{m+n}\right].$$

Finally,

$$y = a_0 + a_1x + \ldots + a_{n-1}x^{n-1} + \frac{x^{m+n}\Gamma(m+1)}{\Gamma(m+n+1)}\left[\log x - \frac{1}{m+1} - \frac{1}{m+2} - \ldots - \frac{1}{m+n}\right].$$

The proposed problem corresponds to the special case $m = 0$ and here

$$y_p = \frac{x^n}{n!}\left[\log x - 1 - \frac{1}{2} - \ldots - \frac{1}{n}\right].$$

Solution **II** by the Proposer.

Successive integration by parts yields the pattern

$$D^{-n}\ln x = (x^n/n!)\ln x - d_nx^n + \sum_{i=1}^{n} C_ix^{n-i}/(n-i)!$$

where $d_n = (1/n)d_{n-1} + 1/nn!$, $d_1 = -1$, and the C_i's are arbitrary constants. The solution of the difference equation in d_n is

$$d_n = \Psi_0(n+1)/n! \text{ where } \Psi_0(n) = \sum_{i=1}^{n-1}(1/i).$$

Hence, the formula takes the form

$$D^{-n}\ln x = (1/n!)[\ln x - \Psi_0(n+1)]x^n + P_n,$$

where $P_n = \sum_{i=1}^{n} C_ix^{n-i}/(n-i)!$,

which is readily verified by induction.

Also solved by Michael A. Brodtrick, Affton, Missouri; Clayton W. Dodge, University of Maine; Richard L. **Enison,** New York City; W. Wesley Johnston, Springfield, Illinois; Peter A. Lindstrom, **Genesee** Community College, Batavia, N.Y.; Frank P. Miller, **Jr.,** Pennsylvania State University; **Mavrigian,** Youngstown State University, Youngstown, Ohio; William G. Nichols, Blacksburg, Virginia; C. L. **Sabharwal,** Saint Louis University; Sid Spital, California State College at **Hayward;** Gregory **Wulczyn,** Bucknell University.

227. (Fall **1969**) Proposed by R. **Sivaramakrishnan,** Government **Engineering** College, Trichur, South India.

If $\tau(n)$ denotes the number of divisors of n, and $\mu(n)$ the Moebius function, prove that

$$\tau(n) + \mu^2(n) \leq \tau(n^2)$$

with equality if and only if n is a prime.

Solution by C. B. A. Peck, State College, Pennsylvania.

If $n = 1$, the statement is false, since each term is 1. If $n > 1$ is not free of square factors, $\mu(n)^2 = 0$ and $\tau(n^2) > \tau(n)$. since every divisor of n divides n^2 but n^2 does not divide n. If $n > 1$ is free of square factors, $\mu(n) = 1$ and $\tau(n^2) = 3^k > 2^k = \tau(n)$ when n is the product of k distinct prime factors. If n is prime, $k = 1$ and equality holds. If n is not prime, $k > 1$ and inequality holds strictly.

Also solved by Peter A. Lindstrom, **Genesee** Community College, Batavia. N.Y.; Donald E. Marshall, Pasadena, Calif.; William G. Nichols, Virginia Polytechnic Institute; Bob **Prielipp, Wisconsin** State University-Oshkosh; Sid Spital, California State College at **Hayward;** and the Proposer.

228. (Fall **1969**) Propsed by Charles W. Trigg, San **Diego,** California.

In the decimal system, 1122 is a multiple of $1^5 + 2^5$ and contains no digits other than 1 and 2. Also, 3312 is a multiple of $1^5 + 2^5 + 3^5$ and contains no digits other than 1, 2 and 3, and contains each of these **digits at** least once. Do **comparable** multiples exist for $1^5 + 2^5 + 3^5 + 4^5$ and $1^5 + 2^5 + 3^5 + 4^5 + 5^5$?

Solution by the Proposer.

$M = 1^5 + 2^5 + 3^5 + 4^5 = 1 + 32 + 243 + 1024 = 1300$, **so all** multiples of M terminate in 00. Curiously enough, no digit > 4 occurs in any of the expanded powers or their sum. In the ensemble, each of the digits 0, 1, 2, 3, and 4 occurs with the sane frequency, except that there is one 4 short.

Similarly, in $P = 1^5 + 2^5 + 3^5 + 4^5 + 5^5 = 1 + 32 + 243 + 1024 + 3125 = 4425$, no digit > 5 appears. If all five positive digits are to appear in **kP,** then k must be of the form $1 + 4n$. For $k = 5$, 53, 93, 121, and **125,** the integer **kP** is composed only of some of these five digits, but the smallest multiple of P in which all five and only these five digits appear is 1243425 = **281(4425).**

Extending the series, the smallest values are:

$$216(1^5 + 2^5 + \ldots + 6^5) = 2635416$$

$$2564(1^5 + 2^5 + \ldots + 7^5) = 74376512$$

$257(1^5 + 2^5 + \ldots + 8^5) = 15876432$

$1063(1^5 + 2^5 + \ldots + 9^5) = 128436975.$

In the last two cases, each digit appears only once in the product.

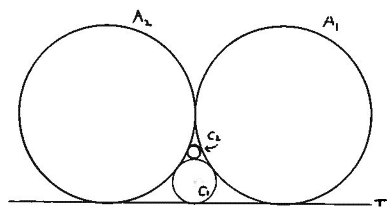
Also solved by Jeanette Bickly, St. Louis, Missouri; Clayton W. Dodge, University of Maine; R. C. Gebhardt, Parsippany, N.J.; W. Wesley Johnston, Springfield, Illinois; Donald E. Marshall, Pasadena, Calif.; C. B. A. Peck, State College, Pennsylvania; William G. Nichols, Virginia Polytechnic Institute; and Gregory Wulczyn, Bucknell University.

Wulczyn added multiples of 17,700 to 4425 on a desk calculator to produce the following products, listed with corresponding multipliers of 4425:

1243425 - 281	31333425 - 7081
1314225 - 297	32513125 - 7325
1544325 - 349	34112325 - 7709
3314325 - 749	34413225 - 7777
5314425 - 1201	35245125 - 7965
11332425 - 2561	35422125 - 8005
12341325 - 2789	42325125 - 9565
14111325 - 3189	42431325 - 9589
14235225 - 3217	44431425 - 10041
14341425 - 3241	45334125 - 10245
21545325 - 4869	51334425 - 11601
22324125 - 5045	54113325 - 12229
23421525 - 5293	
25315425 - 5721	

229. (Fall 1969) Proposed by Carl L. Main, Shoreline Community College, Seattle, Washington.

Let A_1 and A_2 be tangent unit circles with a common external tangent T. Define a sequence of circles recursively as follows: 1) C_1 is tangent to T, A_1 and A_2; 2) C_i is tangent to C_{i-1}, A and A, for $i = 2, 3, \ldots$



Find the area of the region $_iC_i$.

Solution by Murray S. Klamkin, Ford Motor Company Theoretical Sciences Department.

Using the formula [H.S.M. Coxeter, Introduction to Geometry, John Wiley, N.Y., 1961, p. 15]

$$2(a^2 + b^2 + c^2 + d^2) = (a + b + c + d)^2$$

relating the four quantities a, b, c, d which are the reciprocals of the radii of four mutually tangent circles, we get

$$(1) \quad A_{n+1} = A_n + 2 + 2\sqrt{2A_n + 1}\,, \quad A_0 = 0,$$

where A denotes the reciprocal of the radius of circle C. It follows that

$$A = 2(n^2 + n).$$

Whence,

$$\text{Area } {}_iC_i = \frac{\pi}{4}\sum_{i=1}^{\infty} \frac{1}{n^2(n+1)^2} = \frac{\pi}{4}\sum_{i=1}^{\infty} \frac{1}{n^2} + \frac{1}{(n+1)^2} - \frac{2}{n(n+1)}$$

or

$$\text{Area} = \frac{\pi}{4}\frac{\pi^2}{6} + \frac{n^2}{6} - 1 - 2 = \frac{\pi^3}{12} - \frac{3\pi}{4}.$$

Remark: It is to be noted that the sequence generated bv (1) is such that A_n is always integral. A more general sequence with this property is given by

$$B_{n+1} = aB_n + b + (a^2-1)(B_n^2-B_0^2) + 2b(a+1)(B_n-B_0) + c^2d^2 \;^{1/2}$$

(see also Math. Mag. 42 (1969) pp. 111-113).

Also solved by Sanford A. Bolasna, University of California at Riverside; Laura DiSanto, Calgary, Alberta, Canada; Clayton W. Dodge, University of Maine; G. Mavrigian, Youngstown State University, Ohio; Frank P. Miller, Jr., Pennsylvania State University; Ronald W. Prielipp, University of Oregon; Sid Spital, Hayward, California; Gregory Wulczyn, Bucknell University; and the Proposer.

230. (Fall 1969) Proposed by Murray S. Klamkin, Ford Scientific Laboratory.

Determine a single formula to represent the sequence $\{A_n\}$, $n = 1, 2, 3, \ldots$ where

$$\begin{array}{ll} A_{pn+1} = B_{n1}, & \\ A_{pn+2} = B_{n2}, & \\ \quad\vdots & n = 1, 2, 3, \ldots \\ A_{pn+p} = B_{np}, & \end{array}$$

and where the $\{B_{nr}\}$, $r = 1, 2, \ldots, p$ are p given sequences.

Solution by the Proposer.

The problem is equivalent to finding a simple representation for the periodic sequence 1, 0, 0, ..., 0, 1, 0, 0, ..., 0, 1, ..., of period p. If w denotes a primitive p^{th} root of unity, then

$$1 + w^r + \omega^{2r} + \ldots + \omega^{(p-1)r} = p$$

if r is a multiple of p; otherwise it is zero. Thus,

$$a_n = \frac{1}{p}\sum_{r=1}^{p} B_{nr}\{1 + \omega^{n-r} + \omega^{2(n-r)} + \dots + \omega^{(p-1)(n-r)}\}$$

$$= \frac{1}{p}\sum_{r=1}^{p}\sum_{s=0}^{p-1} B_{nr}\omega^{s(n-r)}.$$

Solution II by C. B. A. Peck, State College, Pennsylvania.

$A_n = B_{a,b}$ where $a = [n/p]$ and $b = n - [n/p]p$ and $[c]$ is the largest integer not exceeding c.

Similarly solved by Richard L. Enison, New York; and Sid Spital, Hayward, California.

231. Proposed by David L. Silverman, Beverly Hills, California.

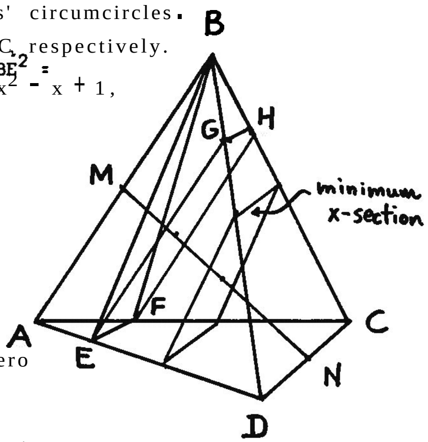
a) What is the smallest circular ring through which a regular tetrahedron of unit edge can be made to pass?

b) What is the radius of the smallest right circular cylinder through which the unit-edged regular tetrahedron can pass?

Solvers are invited to generalize to the other Platonic solids.

Solution by Charles W. Trigg, San Diego, California.

a) MN is the bimedian joining the midpoints of the opposite edges AB and CD of the unit tetrahedron. Sections of the tetrahedron by planes perpendicular to MN are rectangles with a constant perimeter of 2. The one joining the midpoints of AC, AD, BD, and BC is a square. Its circumcircle has the smallest radius, $\sqrt{2}/4$, of the sections' circumcircles.



Take $AE = AF = x$ on AD and AC, respectively. Then $EF = x$, and $y^2 = BF^2 = BE^2 = x^2 + 1^2 - 2(1)(x)\cos 60° = x^2 - x + 1$. The altitude to EF from B is given by $h^2 = y^2 - (x/2)^2 = (3x^2 - 4x + 4)/4$. The area of triangle EFB is $xh/2$. Hence the circumradius of the triangle is given by $R = xyy/4(xh/2) = y^2/2h$. Consequently,

$$R^2 = (x^2-x+1)^2/(3x^2-4x+4).$$

Setting the derivative of R^2 with respect to x equal to zero and simplifying, we have

$$(x^2-x+1)(3x^3-6x^2+7x-2) = 0.$$

The first factor has only imaginary zeros. The graph of the second factor has no horizontal tangent, but does have a point of inflection at $(2/3, 8/9)$. Hence there is only one real root of the equation. This can be found by Horner's method, or otherwise, to be 0.3912646668. This corresponds to $R \doteq 0.4478$, which is the radius of the smallest circular ring (of negligible thickness) through which the tetrahedron can pass.

The ring can be placed in contact with the tetrahedron at E and F and barely passes over B. It then can be dropped down into contact with corresponding points E' and F' on the triangle ABC and rotated to pass over D and hence over the tetrahedron.

b) If the tetrahedron resting on a plane is rotated about an edge until it rests on another face, its projection on the plane varies. The circumcircle of the projection is smallest when the tetrahedron is at midposition. There the projection is a square with a circumradius of $1/2$, which is the radius of the smallest rigid right circular cylinder through which the tetrahedron can pass.

If the cylinder is flexible, it need have a radius of only $1/\pi$ to permit passage of the tetrahedron. (See, Charles W. Trigg, Mathematical Quickies, McGraw-Hill Book Co., 1967, pages 49, 158-159).

Remarks by the Editor.

Solvers are invited to comment on the following fine points posed by Mr. Trigg:

A rigorous proof would require also that it be shown that

1. The ring as it rotates onto the tetrahedron will not catch on BD and BC before it reaches the level of the 0.88865 by 0.11135 rectangle which has a diameter of 2R.

2. No non-isosceles plane section through B has a smaller circumcircle.

3. In part **b)**, no other attitude of the tetrahedron will have a projection with a smaller circumcircle.

With regard to item 1, the editor notes the following:

Consider the circumsphere of the rectangular pyramid determined by B, E, F, G and H, where G and H are points on BD and BC such that $BG = AE = BH = AF$. Thus the plane containing EGHF is perpendicular to the bimedian MN connecting the midpoints of AB and DC. Since the dihedral angle between the planes containing BGH and EGHF is obtuse, the plane of BEF is nearer the center of the sphere than is the plane of EGHF. Consequently the circumcircle of triangle BEF is larger than that of the rectangle EGHF. It is therefore clear that the circumcircle BEF can be rotated about EF so as to contain EFGH, and can then be slid perpendicular to the bimedian (or in many other ways) past the midpoint of the bimedian and on to a position with relation to DC that is symmetrical to its former relation to AB. From that point, it slides off the tetrahedron in a manner similar to the way it slid on.

As for item 3, when the tetrahedron is at the extremal position described in Trigg's solution, the central axis of the cylinder coincides with one of the bimedians of the tetrahedron. In this position the two edges of the tetrahedron connected by this bimedian must lie on diameters of circular cross-sections of the alleged minimum cylinder. It is apparent that any attempt to increase the length of either one of the edges would necessitate tilting or displacing the bimedian, thus reducing the length of the opposite edge and destroying the regularity of the tetrahedron. This verifies Trigg's conclusion in part **b)**.

The Editor invites comments on the question raised in item 2, regarding the possibility of a non-isosceles plane section through B with a smaller circumcircle.

BOOK REVIEWS

Edited by

Roy B. Deal, Oklahoma University Medical Center

1. Squaring the Circle and Other Monographs By E.W. Hobson, H.P. Hudson, A.N. Singh and A.B. Kempe, Chelsea Publishing Company, Bronx, N.Y., xi + 51 pp. $4.95.

   This series of four monographs, Squaring the Circle by Hobson, Ruler and Compass by Hudson, The Theory and Construction of Non-Differential Functions by Singh and How To Draw A Straight Line; A lecture on Lingages by Kempe are not only interesting and informative but scholarly and stimulating from a historical standpoint as well.

2. Congruence of Sets and Other Monographs By W. Sierpinski, F. Klein, C. Runge and L.E. Dickson Chelsea Publishing Company, Bronx, N.Y. V + 104 pp.

   The monographs in this book: On the Congruence of Sets and Their Equivalence By Finite Decomposition by Sierpinski, The Mathematical Theory of the Top by Klein, Graphic Methods by Runge, and Introduction To the Theory of Algebraic Equations by Dickson, like those in the previous book are of interest for content as well as history.

3. A Short History of Greek Mathematics By James Gow, Chelsea Publishing Company, New York, N.Y. xii + 325 pp.

   The background, the side effects, the details and the demise of Greek mathematics presented in a throughly facinating account.

4. Sets, Lattices, and Boolean Algebras By James C. Abbott, Allyn and Bacon, Inc., Boston, Mass., 1969, xiii + 282 pp. $11.50

   The basic results of naive set theory, lattices, and Boolean algebras are developed in a very readable manner from an axiomatization of sets. The Zermelo-Fraenkel Skolem system is used for the development and a brief description of the von Neumann-Bernays-Godel theory of classes is included in an appendix.

5. Commutative Rings By Irving Kaplansky, Allyn and Bacon, Inc., Boston, Mass. 1970, x + 180 pp. $10.95.

   For the reader who is familiar with the fundamental concepts of modern algebra, including a little homology theory, this is an excellent account of many of the basic theorems of commutative rings.

6. An Introduction to Algebraic Topology By John W. Keesee, Brooks/Cole Publishing Company, Belmont, California, 1970, ix + 140 pp.

   This very readable introductory acoount develops the basic parts of simplicial homology, the homotopy category, the homology groups, and simplicial approximations directed to a chapter of interesting applications, including some classical fixed point theorems and theorems on mappings of spheres.

BOOK REVIEWS--Continued

7. Topics in Complex Function Theory By C.I. Siegel, Wiley-Interscience, New York, N.Y. 1969, IX + 186 pp. $9.95.

   Only a modicum of complex function theory is necessary to read this excellent account of elliptic functions and uniformization in this first of three volumes on complex function theory by this outstanding mathematician.

8. Numerical Methods for Partial Differential Equations By William F. Ames, Barnes and Noble, Inc., New York, N.Y., 1970, x + 291 pp., $10.50.

   For the reader with a modest background in advanced calculus, a little numberical analysis, and an interest in this type of applied mathematics, this is a very practical hook. It provides background, including important comments on classification, insight, pitfalls, and an extensive bibliography. Basically the book deals with the classical parabolic, elliptic, and hyperbolic equations with initial value, boundary value, and eigenvalue conditions, but comments are made on non-linear and higher dimensional problems.

9. Probability For Practicing Engineers By Henry L. Gray and Patrick L. Odell, Barnes and Noble, Inc., New York, N.Y., 1970, xi + 717 pp.

   A useful introduction to basic probability, statistics, and stochastic processes for engineers.

10. Applied Probability By W. A. Thompson, Jr., Holt, Pinehart, and Winston, New York, N.Y., 1969, xiii + 175 pp.

    An interesting introduction to probability where the emphasis is on the use of a wide variety of applications as a means of introducing the subject. The result is that the book may have a much wider appeal than was originally intended, because one who is more theoretically inclined would find his maturity and insight enhanced, and one who has specific needs for the applications would find it practical.

11. Computers, Chess and Long range Planning By M.M. Botvinnik, Springer-Verlog, New York, N.Y., 1970, xiii + 89 pp. $3.50.

    Facinating for the mathematically inclined chess aficionado.

12. Percentage Baseball By Earnshaw Cook, Waverly Press, Inc., Baltimore, Maryland, 1964 and 1966, xiii + 417 pp.

    The reader with some knowledge of mathematical statistics and a bent in this direction will find a variety of stimuli to ask a variety of additional question for which the data is available in this book to undertake finding answers.

13. Mathematical Sociology By Janet Holland and M.D. Steuer, Schocken Books, New York, N.Y., 1970, viii + 109 pp.

    A selected bibliography of 451 annotated items on mathematical sociology which includes 340 articles and 111 books.

LISTED BOOKS

1. A Table of the Complete Elliptic Integral of the first kind For Complex Values of the Modulus: III. Auxiliary Tables By Henry E. Fettis and James C. Caslin, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, 1970, IV + 162 pp.

2. Barlow's Tables Edited by L.J. Comrie, Barnes and Noble, Inc. New York, N.Y., 1970, xii + 258 pp. $3.25 paper, $5.50 cloth.

3. Programmed Statistics By Richard Bellman, John C. Hogan, and Ernest M. Scheuer, Holt, Rinehart and Winston, Inc., 1970, viii + 115 pp.

4. Elementary Differential Equations By R.L.E. Schwarzenberger, Barnes and Noble, Inc., New York, N.Y., 1970, xi + 98 pp. $3.75

5. Single Variable Calculus By Melvin Henriksen and Milton Lees, Worth Publishers, Inc., New York, N.Y., 1970, xv + 624 pp. $10.95

6. Calculus Explained By W.J. Reichman, Barnes and Noble, Inc. New York, N.Y., 1969, viii + 331 pp. $3.00 paper, $5.75 cloth.

7. Elementary Algebra By Lee A. Stevens, Wadsworth Publishing Company, Inc., Belmont, California, 1970, xii + 319 pp.

8. Mathematics For Elementary School Teachers By Meridon Vestal Garner, Goodyear Publishing Company, Inc., Pacific Palisades, California, 1969, xii + 384 pp.

PLANIDROMES

$6 - 5 = 1$
$6^2 - 5^2 = 11$
$56^2 - 45^2 = 1111$
$556^2 - 445^2 = 111111$
$5556^2 - 4445^2 = 11111111$
etc,

$9 - 2 = 7$
$9^2 - 2^2 = 77$
$59^2 - 52^2 = 777$
$559^2 - 552^2 = 7777$
$5559^2 - 5552^2 = 77777$
etc,

$7 - 4 = 3$
$7^2 - 4^2 = 33$
$57^2 - 54^2 = 333$
$557^2 - 554^2 = 3333$
$5557^2 - 5554^2 = 33333$
etc,

$10 - 1 = 9$
$10^2 - 1^2 = 99$
$60^2 - 51^2 = 999$
$560^2 - 551^2 = 9999$
$5560^2 - 5551^2 = 99999$
etc,

$8 - 3 = 5$
$8^2 - 3^2 = 55$
$58^2 - 53^2 = 555$
$558^2 - 553^2 = 5555$
$5558^2 - 5553^2 = 55555$
etc,

$11 - 0 = 11$
$11^2 - 0^2 = 121$
$61^2 - 50^2 = 1221$
$561^2 - 550^2 = 12221$
$5561^2 - 5550^2 = 122221$
etc,

A planidrome reads the same backwards and forwards.

NEW INITIATES

ALABAMA ALPHA, University of Alabama

Deborah Ashford
Lawrence Billits
Cary Bradford
Jerry Campbell
Maria Chiepalich
Sally Davenport
Deborah Davis
Margaret Davis
Lucretia DeHaney
Hugh DeJarnette, Jr.
Edward Eadon
Rita Ellison
James Entrekin
Larry Evans
William Farr
Catharine Finch
Patrick Frederick
Hugh Galin
Gmg Glasscock
Charles Gmen
Robert Hardy
Virginia Harrison
Janice Haynes
Virginia Hem
Dudley J. Herrin, Jr.
Randall Hiam
Linda Hodo
Joseph Huggins
Ann Jobson
Marjorie Johnson
Andrew Jones
Wiley Justice
Nina Knight
Dennis Kross
Kenneth Laney
Neal Lawrence
Sheryl Lothrop
India Lowery
Shirley Mahan
William McAllister
Mary McCain
Vohhimkrkisloff
John Musselman
Rebecca Nelson
Victor Norman
Anita O'Neal
Melissa Pardue
Phillip Partridge
Michael Patterson
Terry Peak
Jane Pearson
Walter Plott
Tim Raines
John Rasberry
William Roper
Donald Rowell
Clark Shields
Pamela Sketo
Suzanne Sloan
Jades Snowden, Jr.
Janice Spangler
Nancy Spencer
Ronald Staele
Andrew Stergiades
Jesse Tilton, III
Russell Turner
Dennis Utley
Barbara Halters
Michael Wells
James Wren
Patricia Wright
William Wright
Martha Ziegler

ALABAMA BETA, Auburn University

Deborah Booher
Linda Rae Bostwick
N. Carolyn Claybrook
Glenda Paulk Cody
Emily Heege Cola
Brenda Gail Davis
James Edward Edgar
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Patricia Ann Huffmaster
Douglas Carroll Hunt
Gregery Alfred Johnson
Mary Sanders Lavender
William Arthur Mastin
Susan Louise Merwin
Willa Dean Mitchell
Earl Robert Morecock
William Louis Overcamp
Dennis Benton Parkhurst
Maria Paul
John Leo Post
Janie Marie Powers
Dorothy Sherling
Bethany Irene Thompson
Emily Ellen Vann
Carol Lee Weston

ALABAMA GAMMA, Samford University

Marvin C. Champion
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