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ON GENERALIZATION OF DERIVATIVES

Lokenath Debnath and Thomas B. Speight
East Carolina University

1. INTRODUCTION

It is well known that the differential operator D satisfies the following algebraic properties

$$D[a\,f(x) + b\,g(x)] = a\,D[f(x)] + b\,D[g(x)] \qquad (1.1)$$

$$D^m[D^n f(x)] = D^{m+n}[f(x)] = D^n[D^m f(x)] \qquad (1.2)$$

where $f(x)$, $g(x)$ are differentiable functions of a real variable x; a, b are two constants, m, n are positive integers so called the orders of derivative; and the operator D is defined in the Leibnitz sense by

$$D[f(x)] = \frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x},$$

provided the limit exists.

Historically, it appears that Leibnitz and Liouville had indicated the possibility of generalization of the operator $D^n f(x)$ for fractional order $n = \frac{p}{q}$; p, q being integers and $q \neq 0$. However, answers to simple problems such as $D^{\frac{1}{2}} x^2$, $D^{\frac{1}{3}} x^2$, $D^{\frac{1}{3}} e^x$, ... $D^{p/q} x^n$ seem to be neither well known nor readily available in Calculus.

On the other hand, the Cauchy integral formula in complex analysis and Abel's integral equation provide the existence of derivatives of rational, real and complex orders. From these results, it is not immediately clear and always easy to compute the fractional order derivatives of simple polynomial functions.

It is thus natural to ask some simple questions and find out answers. Is it possible to calculate fractional order derivatives of simple and elementary functions? Even, if it is possible, does the operator $D^{p/q}$ obey the algebraic rules (1.1) - (1.2)? It seems a little surprising that the answers to these simple and elementary questions are not readily available in the literature.

With a view to providing some answers to the above questions from the elementary stand point, we make some elementary discussion with simple examples. Anticipating a rigorous treatment, the derivative of rational, real and complex orders is introduced in a formal way.

2. FRACTIONAL ORDER DERIVATIVES

From the elementary calculus, the derivatives of integral order $m(\geq 0)$ is given by the well known result

$$D^m x^n = \frac{n!}{(n-m)!} x^{n-m} \tag{2.1}$$

Using the generalized factorial notation so called the Gamma function $\Gamma(x)$, we generalize result (2.1) in the form

$$D^\alpha x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} x^{\beta-\alpha} \tag{2.2}$$

for all rational numbers a, β with $\beta \geq 0$, where $\Gamma(z)$ is defined for a complex variable z except the negative integers including zero with the property $\Gamma(z+1) = z\Gamma(z)$ and $\Gamma(n+1) = n!$, n being a positive integer.

Also, the well known result of calculus

$$D^m x^{-n} = (-1)^m \frac{(n+m-1)!}{(n-1)!} x^{-(n+m)}, \quad n > 0 \tag{2.3}$$

can similarily be generalized in the form

$$D^\alpha x^{-\beta} = (-1)^\alpha \frac{\Gamma(\alpha+\beta)}{\Gamma(\beta)} x^{-(\alpha+\beta)}, \quad \beta > 0 \tag{7.4}$$

Thus the results (2.2) and (2.4) appear to provide a simple method of computation of fractional order derivatives of at least polynomial functions. Further, they may be treated as definitions of derivatives of rational orders.

More generally, we obtain

$$D^\alpha f(x) = \sum_{r=0}^{n} a_r \frac{\Gamma(r+1)}{\Gamma(r-\alpha+1)} r - a \tag{2.5}$$

where $f(x) = \sum_{r=0}^{n} a \; x^r$ is a polynomial of degree n with constant coefficients.

A similar result can also be written down as

$$D^\alpha f(x) = \sum_{r=0}^{n} (-1)^r a_r \frac{\Gamma(\alpha+r)}{\Gamma(r)} x^{-(\alpha+r)} \tag{2.6}$$

$$\text{where } f(x) = \sum_{r=1}^{n} a_r x^{-r}, \; r > 0 \tag{2.7}$$

From the above discussion together with simple properties of the Gamma function, it can readily be verified that the operator D^a satisfies the basic algebraic properties (1.1) - (1.2) at least for the polynomial functions. This clearly shows that D^α is a linear operator for any rational order a.

3. SOME SIMPLE EXAMPLES

$$D^{\frac{1}{2}} x^2 = \frac{\Gamma(3)}{\Gamma(\frac{3}{2}+1)} x^{3/2} = \frac{8}{3\sqrt{\pi}} x^{3/2}, \quad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}, \tag{3.1}$$

$$D^\alpha x^0 = \frac{x^{-\alpha}}{\Gamma(1-\alpha)} \tag{3.2}$$

$$D^\alpha a = \frac{ax^{-a}}{\Gamma(1-\alpha)}, \text{ a being a constant} \tag{3.3}$$

$$D^\alpha x^\alpha = \Gamma(1-\alpha) \tag{3.4}$$

$$D x^2 = D^{\frac{1}{2}} (D^{\frac{1}{2}} x^2) = D^{\frac{1}{2}} \left[\frac{8}{3\sqrt{\pi}} x^{3/2}\right] = 2x \tag{3.5}$$

$$D^{\frac{1}{3}} x^{-1} = (-1)^{\frac{1}{3}} \Gamma(1+\tfrac{1}{3}) x^{-4/3} \tag{3.6}$$

$$D x^{-1} = D^{\frac{1}{3}} \left[D^{\frac{1}{3}} \left\{D^{\frac{1}{3}} x^{-1}\right\}\right] = -x^{-2} \tag{3.7}$$

4. DERIVATIVES OF POWER SERIES

Assuming the usual requirements such as uniform convergence for term-by-term differentiation of an infinite power series representation of a function $f(x)$, we obtain

$$D^\alpha f(x) = D^\alpha\left(\sum_{r=0}^{\infty} a_r x^r\right) = \sum_{r=0}^{\infty} a_r \frac{\Gamma(r+1)}{\Gamma(r-\alpha+1)} x^{r-a}, \tag{4.1}$$

Admitting the validity of the above result, we immediately obtain the following results

$$D^\alpha (e^{ax}) = a^\alpha \sum_{r=0}^{\infty} \frac{(ax)^{r-\alpha}}{\Gamma(r-\alpha+1)} \tag{4.2}$$

$$D^\alpha (\sin x) = D^\alpha \sum_{r=0}^{\infty} (-1)^r \frac{x^{2r+1}}{\Gamma(2r+2)} = \sum_{r=0}^{\infty} \frac{(-1)^r x^{2r-\alpha+1}}{\Gamma(2r+2-\alpha)} \tag{4.3}$$

$$D^\alpha (\cos x) = \sum_{r=0}^{\infty} \frac{(-1)^r x^{2r-\alpha}}{\Gamma(2r-\alpha+1)} \tag{4.4}$$

All of these results agree with those for integral order derivatives.

5. REAL AND COMPLEX ORDER DERIVATIVES

We recall the basic and generalized results (2.2) and (2.4) for the fractional order derivatives. Since the Gamma function $\Gamma(z)$ is meromorphic in the entire complex plane except the poles at z = 0, -1, -7, -3..., formulae (2.2) and (2.4) appear to remain valid even when a is a real or complex number with z^β in place of x^β and $R(\beta) > 0$. This indicates that real and complex order derivatives of a polynomial function $f(z)$ can he computed by the generalized results.

We define

$$D^\alpha e^{az} = a^\alpha e^{az} \tag{5.1}$$

for complex order a and a, z are complex numbers.
Thus we have

$$D^\alpha e^{ix} = i^\alpha e^{ix} \tag{5.2}$$

$$= e^{ix - \frac{\pi}{2}I(\alpha)} \{\cos \tfrac{\pi}{2} R(\alpha) + i \sin \tfrac{\pi}{2} R(\alpha)\} \tag{5.3}$$

where $R(\alpha)$ and $I(\alpha)$ denote the real and imaginarv cart of the order a. Equating real and inaginarv parts of (5.3), it turns out that

$$D^{\alpha} (\cos x) = e^{-\pi/2\, I(\alpha)} \cos (x + \frac{\pi}{2} R(\alpha)) \quad (5.4)$$

$$D^{\alpha} (\sin x) = e^{-\pi/2\, I(\alpha)} \sin (x + \frac{\pi}{2} R(\alpha)) \quad (5.5)$$

It is interesting to notice that these generalized results are in excellent agreement with those of integral order derivative of the trigonometric functions.

As a concluding remark, it mav be added that besides the rigorous treatment of the generalized derivatives, numerous questions and nrohlems involving the interpretation of such derivatives, comnutation of fractional order partial derivatives of simple functions, solutions of fractional order differential equations etc. mav be raised.

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NEED MONEY?

The Governing Council of Pi Mu Epsilon announces a contest for the best expository paper by a student (who has not vet received a masters degree) suitable for cublication in the Pi Mu Epsilon Journal.

The following prizes will he given:

\$200	first prize
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In addition there will be a \$20.00 prize for the best paper from any one chapter, providing that chanter submits at least five papers.

A CURE FOR "INSTANT INSANITY"

Edward J. Wegman
The University of North Carolina

In recent months, puzzles of the type exemplified by "Instant Insanity" have proliferated on the shelves of toy shops. "Instant Insanity" is a puzzle consisting of four multicolored unit cubes. Each face of each cube is colored with one of four colors, red, blue, white, and green. The object of the puzzle is to assemble the cubes into a 1x1x4 prism such that all colors appear on each of the four long faces of the prism. T. A. Brown (1) presented a solution based on characteristic numbers for the cubes. Busacker and Saaty (2) indicate that a graph-theoretic analysis may be applied to this type of problem. The purpose of this note is to apply this graph-theoretic method to the solution of "Instant Insanity" in order to illustrate the charm of this type of method.

A graph G consists of a finite non-empty set of points (called vertices) together with a prescribed set of pairs of distinct points. Each pair of points is called a line or an edge. A line is directed if the pair of points is an ordered pair. A path in a graph is a finite sequence $(a_0, A_1, a_1, A_2, \ldots, A_n, a_n)$ where $a_0, a_1, \ldots, a_n$ is a finite sequence of vertices and $A_1, A_2, \ldots A$ is a sequence of edges such that the endpoints of A_i are a_{i-1} and a_i. A circuitis a path in which no edge appears more than once and in which a – a_n. It is customary to represent a graph by means of a diagram (as in figures 1, 2, and 3) and to refer to the diagram as the graph.

In the application of graph theory to "Instant Insanity" each color on the cube is represented by a vertex on a graph. Hence, we will have four vertices labeledred, blue, white, and green. For each cube, draw a partial graph by connecting two vertices by an edge if and only if they correspond to opposite sides of the cube. Label each edge according to the cube from which the edge arose. See Figure 1 for the description of the cubes and the partial graphs. Combine the partial graphs into a graph as illustrated by Figure 2. The solution is found by obtaining two complete circuits having no edges in common. Each circuit must obtain one and only one edge for each cube and must pass only once through each color. Such circuits are usually easily found by inspection of the graph. See Figure 3 for the explicit diagram of the two circuits.

In order to translate these circuits into a solution, stack the cubes with number one on top and number four on bottom and, of course, the other two in the appropriate order. Choose one circuit. Label the circuit with a direction by proceeding counterclockwise around the circuit. Beginning with the edge corresponding to cube 1, place that cube so that the "tail" color, i.e. the color at the tail end of an arrow directed counterclockwise, faces back and the "head" color faces forward. In our example, red faces away, green forward. The next edge determines the orientation of its corresponding cube in a similar manner. In our example, cube 3 will have green facing away, white facing forward, cube 2 will have white facing away, blue forward, cube 4 will have blue facing away, red forward. It is easy to see how this directed circuit corresponds to the solution by imagining the circuit through the faces of the cubes. The reason for the requirement that each edge number appear only once and each color only once is also clear.

Once all four cubes have been orientated properly in the front to rear direction according to circuit one (notice that on the front and back of the stack, each color appears only once), we must orientate the left-right faces. Notice also by rotating each cube around a front to rear axis, any one of the four unused face pairs on each cube may be broughtto right-left orientation. Circuit two determines the right-left orientation of the remaining face pairs in a similar manner except that right and left replaces front and back and that the front-rear orientation of the cubes must be carefully preserved. The requirement that the two circuits share no common edges follows from the fact that no face pair can have both a front rear orientation and a right-left orientation. In this example, cube 1 has blue facing right, white left, cube two has red facing right, blue left, cube 3 has green facing right, red left and cube 4 has white facing right and green facing left.

Other similar puzzles are solvable in a similar manner.

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Brown, T. A (1968), "A Note on 'Instant Insanity'," Mathematics Magazine, 41, pp. 167-169.

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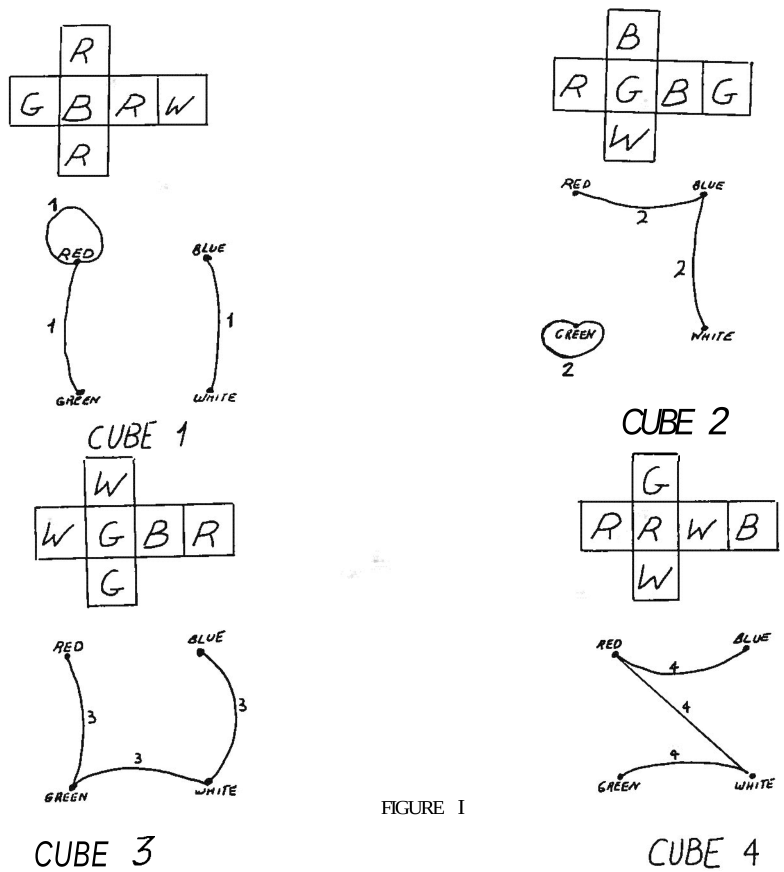


FIGURE I

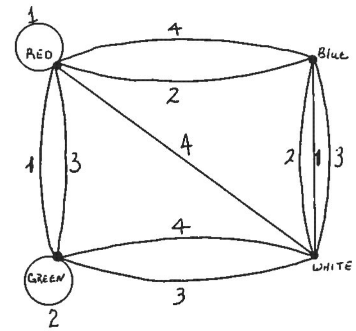


FIGURE II

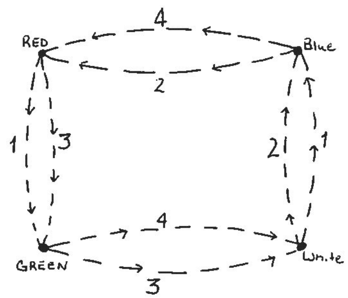


FIGURE III

MATCHING PRIZE FUND

The Governing Council of Pi Mu Ensilon has approved an increase in the maximum amount per chapter allowed as a matching prize from $25.00 to $50.00. If your chanter presents awards for outstandinp mathematical papers and students, you may apply to the National Office to match the amount spent by your chapter--i.e., $30.00 of awards, the National Office will reimburse the chanter for $15.00. etc.,--up to a maimum of $50.00. Chanters are urged to submit their best student papers to the Editor of the Pi Mu Epsilon Journal for possible publication. These funds may also be used for the rental of mathematical films. Please indicate title, source and cost, as well as a very brief comment as to whether you would recommend this particular film for other Pi Mu Epsilon groups.

THE UNIQUE EQUILATERAL TRIANGLE

By Jan Blumenstalk McDonald

Within the classroom the professor often presents hypothetical questions which it seems no one has yet found an answer. The following questions were posed by Dr. Ernest Ranucci ---- Does there exist any triangle, other than the equilateral triangle, in which the relation among the sides equals the relation among the angles? ---- One can easily confirm the case of the equilateral triangle in which the relation among the angles is 1:1:1. This is equal to that of the sides. A simple example which does not exhibit this property is the 30°-60°-90° right triangle whose angles are in a 1:2:3 relationship while the corresponding sides are in the ratio of $1:\sqrt{3}:2$.

But, is the equilateral triangle the only triangle which exhibits this property? Using conventional notation as in Figure 1 below, we see that we would like to establish what conditions must hold in order for (1) to be true.

$$(1)\qquad \frac{a}{A} \stackrel{?}{=} \frac{b}{B} \stackrel{?}{=} \frac{c}{C}$$

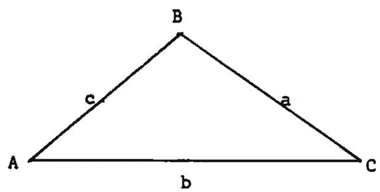


FIG. 1

By introducing the Law of Sines (2) we see the necessity that Equation (3) also be true.

$$(2)\qquad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$(3)\qquad \frac{\sin A}{a} \stackrel{?}{=} \frac{\sin B}{b} \stackrel{?}{=} \frac{\sin C}{c}$$

Intuitively, at least, it doesn't seem that we could ever find three angles whose sines are in the same ratio as the angles. I decided to investigate this by letting the UNIVAC 1108 do the work for me.

The output revealed that the 60°-60°-60° combination was the only one which would conform to Equation (3). This, of course, was no proof, but it gave further support to our conjecture. The computer proves to be a very useful tool for such investigations.

A closer examination of the output gave additional information. The differences in the values for sin A/A, sin B/B, and sin C/C decreased only as A, B, and C approached equality. Samples of the data are given on the next page.

ANGLES	SIN(A)	SIN(B)	SIN(C)	SIN A/A	SIN B/B	SIN C/C
57 57 66	.8387	.8387	.9135	.014714	.014714	.013842
57 58 65	.8387	.8480	.9063	.014714	.014622	.013943
57 59 64	.8387	.8572	.8988	.014714	.014528	.014044
57 60 63	.8387	.8660	.8910	.014714	.014434	.014143
57 61 62	.8387	.8746	.8829	.014714	.014338	.014241
57 62 61	.8387	.8829	.8746	.014714	.014241	.014338
57 63 60	.8387	.8910	.8660	.014714	.014143	.014434
57 73 50	.8387	.9563	.7660	.014714	.013100	.015321
57 74 49	.8387	.9613	.7547	.014714	.012990	.015402
57 75 48	.8387	.9659	.7431	.014714	.012879	.015482
57 99 24	.8387	.9877	.4067	.014714	.009977	.016947
57 100 23	.8387	.9848	.3907	.014714	.009848	.016988
57 101 22	.8387	.9816	.3746	.014714	.009719	.017065
57 102 21	.8387	.9781	.3584	.014714	.009590	.017065

The complete output from this computer programmed investigation shows that there are no "whole-angled" triangles which satisfy our conditions; but what about the infinite number of other triangles? Let us first examine the conditions to make equality (4) hold, since clearly (4) must hold in order for the continued equality (3) to be true.

$$(4)\qquad \frac{\sin A}{A} \stackrel{?}{=} \frac{\sin B}{B}$$

Since we are looking for solutions other than the equilateral triangle, at least two of the angles will not be equal. So, we can set $A \neq B$, or, for some k, $k \neq 1$, $A = kB$. We can set a further restriction on k of $k > 1$ by always choosing the smaller angle for B. So, our Equation (4) reduces to Equation (5).

$$(5)\qquad k \sin B \stackrel{?}{=} \sin kB$$

Our restrictions on B and kB remain $0^\circ < B < 180^\circ$ and $0^\circ < kB < 180^\circ$.

A trip back to our handy computer and a program on the plotter, plotting $y = k \sin B$ and $y = \sin kB$ for some arbitrarily chosen k's, gives us further graphical confirmation that (5) will never be true within our restrictions. (See Figures 2 and 3 below.)

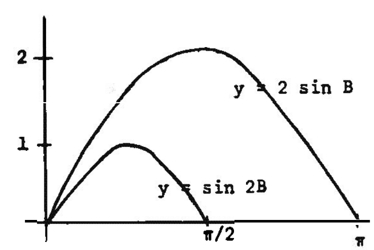


FIG. 2

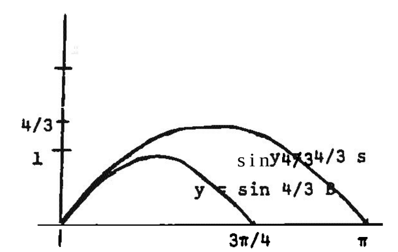


FIG. 3

By examination of such plotter graphs as those above we find our proof. Under what conditions could the graphs of our two functions intersect? Since both B and kB are restricted to values between 0° and 180° we need concern ourselves only with $0<B<\pi/k$ as in Figures 2 and 3 above (where k is 2 and 4/3 respectively).

We have only three possibilities:*

1) The curves will be tangent at some point x
2) They will intersect at one or more points.**
3) They will never intersect.

CASE 1. If the curves were to be tangent at some point x_0, we would have a situation similar to that in Figure 4.

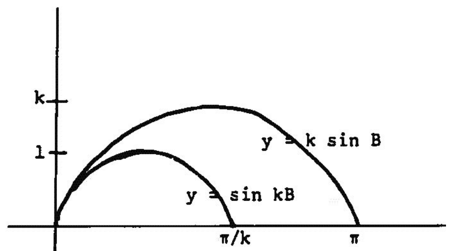


FIG. 4

There would be some x_o where $\sin kx_o = k \sin x_o$ for $0< x_o<\pi/k$. In order for the two graphs to be tangent, their derivatives must be equal at x_o, or,

⑥ $k \cos kx_o = k \cos x$

⑦ $\cos kx = \cos x$

However, ⑦ can be true only if $k = 1$ or $x_o = 0$, neither of which is within our restrictions. Therefore, k sin 8 will never be tangent to sin kB within our restrictions.

CASE 2. Perhaps the curve $y= \sin kB$ crosses $y = k \sin B$ at least once. Since the slope of $y = \sin kB$ for values close to zero is less than that of $y = \sin B$, we could never have a situation similar to that in Figure 5 where $y = \sin kB$ starts out above the curve $y = k \sin B$. Thus we would have to have a situation similar to that in Figure 6.

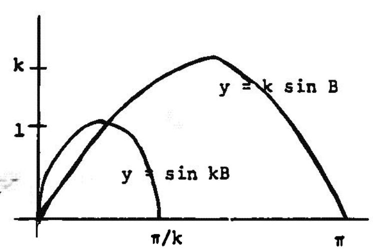


FIG. 5

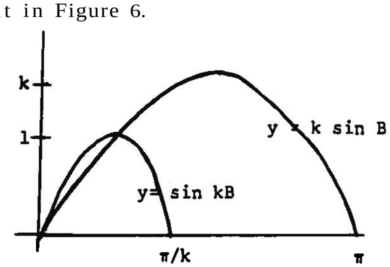


FIG. 6

*approach inspired by Dr. Joe W. Jenkins

Since $y = \sin kB$ must reach zero before $y= k \sin B$, they must cross exactly twice. This means that there must exist some x_1 and x_2 where $\sin kB - k \sin B = 0$. But, by Rolle's Theorem, there must exist some x_o in (x_1, x_2) such that at x_o the slope of the tangent line is zero. This would require that $k \cos kx_o - k \cos x_o = 0$, or that $\cos kx_o$, and this, ⑦, we have already shown to be impossible within our restrictions.

Therefore, Case 3 m t be true, that is, Equation ⑤, $k \sin B = \sin kB$, will ——— be true for $k \neq 1$, and the versatile equilateral triangle gains another unique property.

A CLASS OF FIVE BY FIVE MAGIC SQUARES WITH A THREE BY THREE MAGIC CENTER

By Marcie Peterson
The University of Minnesota

The following is a class of five by five magic squares with a three by three magic center.

n-(c-1)b	n-(2c+1)b	n-(3c+1)b	n+(2c+2)b	n+(4c-1)b
n-(2c-1)b	n-b	n+3cb	n+cb	n-2cb
n+3cb	n+(c+1)b	n	n-(c+1)b	n-3cb
n+2cb	n-cb	n-3cb	n+b	n+(2c+1)b
n-(4c-1)b	n+(2c+1)b	n+(3c+1)b	n-(2c+2)b	n+(c-1)b

In the above, b and n are arbitrary whole numbers. To be certain that all of the above entries are distinct we require only that the members of the set

$\{-1,1,c,-c,c+1,c-1,2c,2c+1,2c-1,2c+2,3c,3c+1,4c-1,-(c+1),-(c-1),-2c,-(2c+1), -(2c-1),-(2c+2),-3c,-(3c+1),-(4c-1)\}$

are all distinct. This will be true, for example, if $c\geq 3$.

A GEOMETRIC LOOK AT DETERMINANTS

Ali R. **Amir-Moez**
Texas Tech University

This expository article attempts to give elementary geometric interpretations of the determinants of a matrix with real entries.

1. Notations: We shall use the standard notations of elementary linear algebra. The Euclidean space dimension n will be denoted by R and we shall use Greek letters for vectors. The inner product of ξ and η will be expressed by (ξ,η) and the symbol det A means the determinant of the matrix A. The symbol $\wedge$ will denote the cross product of two vectors in R_3.

2. Right-handed bases: Let $\{\alpha_1, a_2\}$ be an orthonormal basis in R_2, that is, suppose $(\alpha_1,\alpha_1) = (\alpha_2,\alpha_2) = 1$ and $(\alpha_1,\alpha_2) = 0$. We say that this basis is a right-handed basis if the rotation through the angle $\frac{\pi}{2}$ sends α_1 to α_2 and α_2 to $-\alpha_1$. Thus the matrix of this rotation is:

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Let $\xi = x\alpha_1 + y\alpha_2$. It is clear that the above rotation changes ξ to

$$(x \quad y)\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = (-y \quad x)$$

or $6 = -y_1 + x_2$. We shall call 6 the right-handed perpendicular to ξ. Indeed one observes that given ξ, the vector 6 is uniquely defined.

3. The determinant of a two by two matrix: Let

$$\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix}$$

Using a right-handed basis $\{\alpha_1,\alpha_2\}$ in R_2 suppose that

$$\xi = x_1\alpha_1 + y_1\alpha_2, \qquad \eta = x_2\alpha_1 + y_2\alpha_2.$$

Here we obtain 6, the right-hand perpendicular to ξ (Fig. 1).

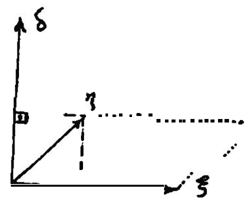


Figure 1

We observe that the projection of η on the axis whose unit vector is $\frac{\delta}{\|\delta\|}$ is $(\eta, \frac{6}{\|\delta\|}) = h$. Here $|h|$ is the length of the altitude of the parallelogram formed by ξ and η when ξ forms the base. Thus we obtain:

$$(\eta,\delta) = h\|\delta\| = h\|\xi\|.$$

Since $6 = -y_1\alpha_1 + x_1\alpha_1$ we obtain $(\eta,\delta) = x_1y_2 - x_2y_1$.

Here we define

$$\det A = \det[\xi,\eta] = x_1y_2 - x_2y_1.$$

Note that $[\xi,\eta]$ is an ordered pair of vectors and $\det[\xi\ \eta] = f[\xi\ \eta]$ is a real valued function of $[\xi,\eta]$. Now let us consider the matrix:

$$B = \begin{pmatrix} x_2 & y_2 \\ x_1 & y_1 \end{pmatrix}$$

Here we are looking for $\det B = \det[\xi,\eta]$. As was done before we obtain γ, the right handed perpendicular to η (Fig. 2), i.e.,

$$\gamma = -y_2\alpha_1 + x_2a_2.$$

Again $(\xi, \frac{\gamma}{\|\gamma\|}) = k$ is the projection of ξ on the axis with the unit vector $\frac{\gamma}{\|\gamma\|}$. Here $|k|$ is the length of the altitude of the parallelogram formed by ξ and n , when η forms the base. Thus

$$\det B = \det[\eta,\xi] = x_2y_1 - x_1y_2 = -\det A.$$

In either case the absolute value of the determinant of the the matrix is equal to the area of the parallelogram formed by ξ and η, that is for any two noncollinear vectors ξ and η, $|\det[\varepsilon,\eta]| =$ area of the parallelogram $[\xi,\eta]$.

So far we have studied the subject for the case when $\{\xi,\eta\}$ is linearly independent. Indeed the other case will be examined. We shall discuss it in the next section.

4. Some properties: Let A, ξ, and η, be the same as in §3. Then as a direct result of the definition we have:

I. $\det[\eta,\xi] = -\det[\xi,\eta]$.

II. If $\{\xi,\eta\}$ is linearly independent, then $\det[\xi,\eta] = 0$.

III. One can easily show that $\det[a\xi,\eta] = a\det[\xi,\eta]$.

Indeed, the cases $a \geq 0$ and $a < 0$ should be studied.

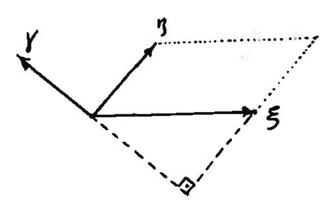


Figure 2

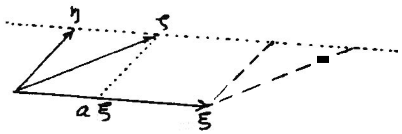
IV. Let $\zeta = a\xi + \eta$ (Fig. 3). We observe that the area of the parallelogram formed by ξ and ζ is the same as the one formed by ξ and η. Also we observe that

$$(\eta, \frac{\delta}{||\delta||}) = (\zeta, \frac{\delta}{||\delta||}),$$

where δ is the same as in 5 3. This implies that

$$\det[\xi, \eta + a\xi] = \det[\xi,\eta] .$$

The details will be omitted.



~~Figure 3~~

5. <u>The cross roduct and determinants</u>: Let fa$\mathbf{v}_\bullet$a$_2$,a$_-$} be a right-handed orthonormal a s s in . s a s that $\alpha 1 \bullet \alpha_2 = \alpha_3$, $\alpha_2 \wedge \alpha_3 = \alpha_1$ and a A a $= \alpha_2$ (Fig. 4.) Consider A, ξ, and η of 5 3. Indeed, to ξ and η respectively correspond ordered triplets $(x_1,y_1, 0)$ and $(x_2,y_2,0)$. Thus

$$\xi \wedge \eta = (\det A)\ \alpha_3.$$

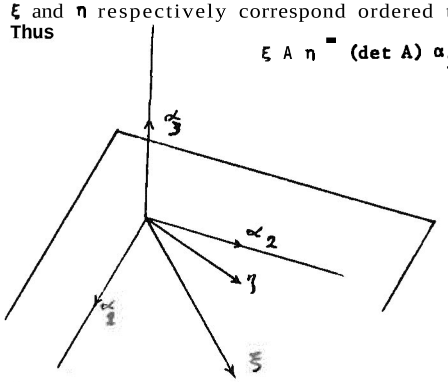


Figure 4

Therefore to the set of all two by two matrices corresponds the one -dimensional **subspace** of R_3 which is generated by α_3.

Now let us consider a three by three matrix

$$A = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix}$$

Let $\xi = x_1\alpha_1 + y_1\alpha_2 + z_1\alpha_3$, $\eta = x_2\alpha_1 + y_2\alpha_2 + z_2\alpha_3$, and $\zeta = x_3\alpha_1 + y_3\alpha_2 + z_3\alpha_3$. Consider $\eta \wedge \zeta = \delta$. Indeed, 6 is a vector orthogonal to both η and ζ. It is clear that

$$6 = x\alpha_1 + y\alpha_2 + z\alpha_3.$$

where

$$x = \det\begin{pmatrix} y_2 & z_2 \\ y_3 & z_3 \end{pmatrix}, \quad y = -\det\begin{pmatrix} x_2 & z_2 \\ x_3 & z_3 \end{pmatrix}, \quad z = \det\begin{pmatrix} x_2 & y_2 \\ x_3 & y_3 \end{pmatrix}.$$

Usually x, y, and z are called **"cofactors"** of A respectively corresponding to x_1, y_1, and z_1. But here we shall look at these geometrically. We observe from well-known properties of the cross product that $||\delta||$ is equal to the area of the parallelogram formed by η and ζ and, for example, $|z|$ is the area of the projection of this parallelogram on the icy-plane. Now let us consider the parallelepiped formed by ξ, η, ζ and consider the paralleolgram formed by η and ζ for the base. Then the corresponding altitude will be the absolute value of

$$(1) \quad (\xi, \frac{\delta}{||\delta||}) = h$$

which is the projection of ξ on the axis perpendicular to the parallelogram formed by η and ζ (Fig. 5). Thus (1) implies

$$(\xi, \delta) = h||\delta|| = x_1x + y_1y + z_1z.$$

This way we define

$$\det A = \det[\xi, \eta, \zeta] = (\xi, \delta),$$

where $|\xi, \eta, \zeta|$ is an ordered triple of vectors. Therefore according to this definition det A is a signed measure of the parallelepiped formed by ξ, η, and ζ. All the properties mentioned in 5 4 can be extended to this case. In particular, we mention, for example:

I. $\det[\xi,\eta,\zeta] = -\det[\eta,\xi,\zeta]$.

i.e., if we interchange **two** rows of the matrix, then the determinant of the new matrix is the negative of the determinant of A.

II. Consider the plane through the endpoint of ξ parallel to the plane of η and ζ (Fig. 6). Any vector λ which ends in this plane has the form

$$\lambda = \xi + a\zeta + b\eta .$$

We observe that the parallelepiped formed by λ, η, and ζ has the same signed volume as the one formed by ε, η, and ζ.

$$\det[\lambda, \eta, \zeta] = \det[\xi, \eta, \zeta] .$$

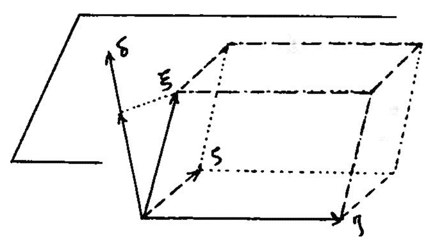


Figure 5

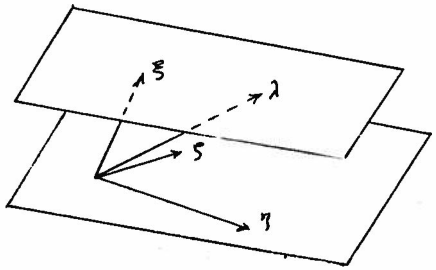


Figure 6

6. Other geometric interpretations: As has been done in §5, we may choose a right–handed orthonormal basis $\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$ in R_4. Let

$$A = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix}$$

Let ξ, η, ζ represent row vectors of A. Then to these vectors respectively correspond ordered quadruples $(x_1,y_1,z_1,0)$, $(x_2,y_2,z_2,0)$, $(x_3,y_3,z_3,0)$, which arc respectively sets of components of ξ, η, and ζ in R_4. Thus we define

$$\xi \wedge \eta \wedge \zeta = (\det A)\,\alpha_4.$$

Indeed this is the Grassman product of these vectors in the given order. Thus to the set of all three by three matrices corresponds a one-dimensional subspace of R_4 spanned by α_4. Here again one nay repeat the ideas of §5 and study geometric properties of determinant of four by four matrices.

Since for R_n, $n \geq 4$, one cannot supply diagrams, one has to use one's imagination. The ideas discussed in sections 1 – 5 can be generalized to higher dimensions; for more details the reader is referred to [1], [3], and [6] of the bibliography.

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SOME THOUGHTS ON THREE TYPES OF PROOF

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Central Missouri State College

INTRODUCTION. As the student progresses in his mathematical education he should become more and more aware of the concept of proof. However, it is a common experience of the teacher to find that sometimes the majority of his students really do not understand the logic behind a certain method of proof. For example, a student may set up a proof by mathematical induction perfectly but may be in reality merely pushing symbols about. He may actually have the feeling that he assuming what he is trying to prove, which he has been told is wrong. It often happens, also, that after he makes his inductive assumption, he sees no reason why he cannot simply replace the "k" by "k+1" and be done after stating the magic phrase: "...and thus by the Principle of Mathematical Induction, the statement is true for all n greater than or equal to one."

It is also a common experience to have a student not understand the difference between disproving by counterexample and proving by example. The former, of course, is perfectly respectable while the latter is frowned upon for good reason. Of course, one could go on and on with such examples of student misunderstanding, but one does not have to teach very long for an awareness of this problem.

The purpose of this paper is not to complain about the shortcomings of students but to perhaps clarify some of the underlying logic of the three most common types of proof: (1) the direct proof, (2) the indirect proof, and (3) proof by contraposition. Some relationships between these three types of proof will also be discussed. In what follows, the usual notation will be used for "and", "negation", "equivalence" and "implication", which are, respectively, $\wedge$, $\sim$, $\Leftrightarrow$, and $\Rightarrow$.

DIRECT PROOF. Consider the theorem: If a and b are odd integers, then a+b is an even integer. The hypothesis consists of the conjunction of the statements: a is an odd integer; b is an odd integer, while the conclusion is the statement: a+b is an even integer. The formal proof of this theorem usually appears as:

1.	a is an odd integer and b is an odd integer.	1.	Given
2.	$a = 2k + 1$ for some k and $b = 2p + 1$ for some p.	2.	Every odd integer is of the form, $2n + 1$.
3.	$a + b = (2k + 1) + (2p + 1)$	3.	Addition property of equality
4.	$a + b = 2(k + p + 1)$	4.	Associative and commutative laws of addition and the distributive law.
5.	Therefore, a + b is an even integer.	5.	Any integer of the form, 2n, is even.

Letting c_1, c_2, c_3 be respectively the statements (2), (3), and (4), we can represent the proof as the logical chain:

$$H \Rightarrow c_1 \Rightarrow c_2 \Rightarrow c_3 \Rightarrow C,$$

where H designates the hypothesis and C, the conclusion.

It should be noted that more information than given by the hypothesis was used. For example, to obtain the implication, $c_1 \Rightarrow c_2$, the addition property of equality was needed. In fact, it was necessary to use additional facts besides the hypothesis to develop each implication in the above chain. This suggests that the hypothesis could have included all mathematical facts that did not follow from the theorem. However, most authors in discussing the concept of proof, seem to agree that the hypothesis is the explicitly stated facts as given by the theorem. To demonstrate the above remarks in general. some notation will be developed to facilitate this discussion.

Let Γ designate the set of all statements comprising the hypothesis, H, and let Θ designate the class of all mathematically true statements which do not depend on $H \Rightarrow C$. Then we will agree that $H \Leftrightarrow \Lambda\Gamma$, $P \Leftrightarrow \Lambda\Theta$, $H_i \Leftrightarrow \Lambda\Gamma_i$, and $P_i \Leftrightarrow \Lambda\Theta_i$, where $\Gamma_i \subset \Gamma$, $\Theta_i \subset \Theta$. Here, $\Lambda\Gamma$ represents the conjunction of all the statements in set, Γ, and similarly for AS, $\Lambda\Theta_i$, and $\Lambda\Gamma_i$. Thus, a direct proof of $H \Rightarrow C$ is of the form:

$$H \Rightarrow c_1 \Rightarrow c_2 \Rightarrow \ldots \Rightarrow c_n \Rightarrow C$$

where transitivity of "$\Rightarrow$" gives $H \Rightarrow C$.

The implication, $H \Rightarrow c_1$ is obtained by an implication of the form:

$$H_1 \wedge P_1 \Rightarrow c_1$$

and since

$$H \Rightarrow H_1 \wedge P_1$$

we have

$$H \Rightarrow c_1.$$

The implication, $c_1 \Rightarrow c_2$, is obtained by an implication of the form:

$$H_2 \wedge P_2 \wedge c_1 \Rightarrow c_2$$

and since

$$c_1 \Rightarrow H_2 \wedge P_2 \wedge c_1$$

we have

$$c_1 \Rightarrow c_2.$$

In general, $c_i \Rightarrow c_{i+1}$, $2 \leq i \leq n-1$, is obtained by an implication of the form:

$$H_{i+1} \wedge P_{i+1} \wedge c_i \Rightarrow c_{i+1}$$

and since

$$c_i \Rightarrow H_{i+1} \wedge P_{i+1} \wedge c_i$$

we have. by transitivity

$$c_i \Rightarrow c_{i+1}.$$

Finally, there exists an n such that $c_n \Rightarrow C$ and we have the logical chain

$$H \Rightarrow c_1 \Rightarrow c_2 \Rightarrow \ldots \Rightarrow c_n \Rightarrow C,$$

so that $H \Rightarrow C$ by the transitive property of "$\Rightarrow$".

PROOF BY CONTRAPOSITIVE. To prove the theorem, $H \Rightarrow C$, by contraposition, we use the fact that $(H \Rightarrow C) \Leftrightarrow (\sim C \Rightarrow \sim H)$. We then proceed to prove $\sim C \Rightarrow \sim H$ by the direct method as before, where $\sim C$ is the hypothesis and $\sim H$ is the conclusion. By the equivalence stated above, we conclude that $H \Rightarrow C$.

It should be emphasized that we are not using the direct method of proof on the theorem, $H \Rightarrow C$, but rather on the equivalent theorem, $\sim C \Rightarrow \sim H$.

INDIRECT PROOF. The indirect method of proof, sometimes called proof by contradiction, is perhaps the most difficult of the three types of proof for the student to grasp. Stated briefly, the indirect method assumes the negation of the conclusion to be true and by a logical chain (the direct method) reaches the conclusion that $\sim q$ is true for some known mathematically true statement, q. This being impossible suggests that to assume the negation of the conclusion to be true leads to a contradiction. Therefore, by the law of contradiction, the conclusion must be true.

Symbolically, we have $H \Rightarrow C$ provided $\sim C \Rightarrow \sim q$ where $q \in \Gamma \cup \Theta$. We prove $C \Rightarrow \sim q$ by the direct method. Since $[(\sim q) \wedge q]$ is logically false it follows that $\sim C$ is false. Hence. C is true and we have, $H \Rightarrow C$.

SUMMARY. Summarizing, it is found that a proof by contraposition is an indirect prooof, while an indirect proof is not necessarily a proof by contraposition since for $q \in \Gamma \cup \Theta$, $\sim C \Rightarrow \sim q$ yields a proof by contraposition of $H \Rightarrow C$ only when $q \Leftrightarrow H$.

However, if a direct proof of a theorem is known, a proof by contraposition can be found immediately, since

$$H \Rightarrow c_1 \Rightarrow c_2 \Rightarrow \ldots \Rightarrow c_n \Rightarrow C$$

is logically equivalent to

$$\sim C \Rightarrow \sim c_n \Rightarrow \ldots \Rightarrow \sim c_2 \Rightarrow \sim c_1 \Rightarrow \sim H.$$

On the other hand, for a given theorem the existence of an indirect proof does not readily indicate a direct proof. Euclid's classic proof of the infinitude of primes is an example of this.

It may be desirable for the classroom teacher to ask his students to examine some other types of proof, such as mathematical induction, proof of existence and proof of uniqueness, and ask that they classify these as being one of the three types discussed in this paper or as being a different type of proof entirely.

A PROGRAM FOR FINDING ENDOMORPHISMS AND ASSOCIATED NEAR-RINGS OF FINITE GROUPS

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University of Southwestern Louisiana

This paper is a report on a FORTRAN IV program that calculates the set of endomorphisms, automorphisms and inner automorphisms, and the near-ring generated by each set on a non-abelian finite group. The object of this undertaking is a program that will characterize the ideal structure of examples of these near-rings, and, hopefully, point to some general theorems about their structure. Olivier [2] has discussed some of the recent work on this problem. This program has met with only limited success because of the enormous size of some of these near-rings.

DEFINITION. A near-ring is an ordered triple $(N,+,\circ)$ such that

1) $(N,+)$ is a group

2) $(N,\circ)$ is a semigroup

3) $\circ$ is right distributive with respect to $+$, i.e.

$(a + b)\circ c = (a\circ c) + (b\circ c)$ for each a, b, c ε N.

Let $(G,+)$ be a group, and $T(G)$ be the set of mappings from G into G. Define $+$ and $\circ$ as follows

$$[f + g](x) = f(x) + g(x)$$

$$[f\circ g](x) = f(g(x))$$

for $x \varepsilon G$ and $f, g \varepsilon T(G)$: $(T(G),+,\circ)$ is a near-ring.

THEOREM 1. Let $E(G)$, $A(G)$, and $I(G)$ be the subgroups of $T(G)$ generated additively by the endomorphisms, automorphisms, and inner automorphisms respectively of G. Then $(E(G),+,\circ)$, $(A(G),+,\circ)$, and $(I(G),+,\circ)$ are near-rings.

A theorem similar to this was proved by Fröhlich [1].

The elements of a group G of order N can be represented by the first N positive integers. The operation $+$ can be represented by an N X N array A such that $i + j = k \Leftrightarrow A(i,j) = k$. This representation of a group is useful for calculations on a computer, but a representation that reflects the group structure in terms of its generators is a more desirable form of output. For instance S_3 would have the internal representation (1,2,3,4,5,6), but the output form would be (0,A,2A,B,A + B,2A + B). An endomorphism E may be represented on a one-dimensional array H whose i th element is the image of the i th element of G under E. Since H(1) = 1 where 1 is the identity of G, H need only have N − 1 variables.

The algorithm for computing the group endomorphisms uses two procedures for assigning values to elements of H. Procedure one is a systematic trial of the N elements of G. Procedure two is the simultaneous solution of a set of equations utilizing the defining equation for an endomorphism: $H(i + j) = H(i) + H(j)$. If the first procedure is used to assign a value to H(i), say I, then all equations of the form H(j) = J are sought. When one is found, and when H(k), where $k = i + j$, has an assigned value, it must be K, where $K = I + J$, otherwise a new value of H(i) is sought by procedure one. If H(k) does not have an assigned value the second procedure assigns it the appropriate value K. The equations used in the simultaneous solution are:

1) $H(i) = I$

2) $H(j) = J$

3) $k = i + j$

4) $H(i + j) = H(i) + H(j)$

The solution used is $H(k) = I + J$. Because the program is to handle non-abelian groups procedure two also solves for $k = j + i$. Other solutions using equations 1 and 2 are possible, i.e. $k = i - j$ and $k = -i + j$; however, this would mean increased coding, and probably would not significantly reduce the run time. After procedure two has been repeated for all possible values of j, new entries in H are systematically used in equation 1 until a new guess is required or until all entries in H are filled. After an endomorphism has been found it is tested for the automorphic and inner automorphic properties.

The algorithm for calculating the near-rings uses the appropriate set of generators to get the sets I(G), A(G) and E(G), using the fact that $I(G) \subseteq A(G) \subseteq E(G)$. Each element H is stored as a row in two two-dimensional arrays. The array C preserves the N − 1 elements of each mapping, as well as the order in which the mappings were found. The array B contains a more compact form of the mappings, and stores the mappings in numerical order. The more compact form used in B is a polynomial representation, i.e. if F is the i th mapping stored in B then $B(i,j) = F(3j - 2) \times N^2 + F(3j - 1) \times N + F(3j)$. Possible new elements of the near-ring being calculated are found by systematically combining elements of C. Then a binary search is made of B to determine if it is a new element.

This double entry of elements of the near-ring is required to reduce processing time. The polynomial representation used in B besides significantly reducing memory requirements also reduces search time. Table 1 contains information on the data sets run. S_3 is the symmetry group on 3 objects, A_4 is the alternating group on 4 objects, D_8 and D_{12} are the dihedral groups of order 8 and 12 respectively. END(G) is the set of endomorphisms on the group. S_3 and D were previously known. Investigations will be carried on for groups of order 16, 18 and 20.

TABLE 1

Group	# in END(G)	run time in sec.	# in I(G)	# in A(G)	# in E(G)	
S_3	16	1	54	54	54	30 sec
D_8	36	4	16	32	256	10 min
D_{12}	64	13	54	108	>1728	2 min for A
A_4	33	10	3072	≥3072	≥3072	10-15 hr for I(G)

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FAMOUS MATHEMATICIANS

In the array of letters below you may find the names of forty famous mathematicians. These may be spelled forward, backward, or diagonally. Some examples are shown.

L E I B N I Z M D N I K E D E D
O I R U Y E L Y A C Y E U L E R
R E I W O A W C P T O L E M Y Y
I K E S C T G T B E R N O U L I
C K M S L G A L O I S I M N B V
C E A A O W U D Y N V I E T A B
A P N R B T S I T R E B L I H B
N L N T A T S L E R A C N I O P
O E G S C A U C H Y H E R O N Y
B R A R H M C U I Z S E L A H T
I S L E E R N E H R P E M J Q H
F U I I V E I A O A E O B W L A
M X L E S F R T P J B H D V X G
W O E W K D N P U I B O C A J O
Q D O K Y A U T U L E B A C D R
X U K R C S U S M A D R A C A A
C E V A E R A T O S T H E N E S

(Answers can be found on page 264)

By

James A. & Doris L. Bell

A REMARK ON A THEOREM OF LEAVITT

A.A. Khan

1. Let Σa be a given infinite series with s as its n-th partial sum. We write

$$\sigma_n^\alpha = \frac{1}{A_n^\alpha}\sum_{k=0}^{n} A_{n-k}^{\alpha-1} s_k\,,\quad t_n^\alpha = \frac{1}{A_n^\alpha}\sum_{k=0} A_{n-k}^{\alpha-1} k a_k\,,$$

$$\alpha > -1 \text{ where } A_n^\alpha = \frac{(\alpha+1)(\alpha+2)\ldots(\alpha+n)}{n!}.$$

If $\sigma^\alpha \to S$ we say that the series Σa is summable (C,α) to the sum 3.

It is said to be summable (A) to 3 if $\Sigma a_n x^n$ is convergent to a function $f(x)$ in $0 \le x < 1$ and

$$\lim_{x\to 1-0} f(x) = S$$

It is known $(C,\alpha) \Longrightarrow (A)$ but the converse need not be true.

2. In this journal Leavitt [3] proved the following theorem.

Theorem A. $\sum_{k=1}^{\infty} a_k = S \Longleftrightarrow \sigma_n^1 \to S$ and $t_n^1 \to 0$ as $n \to \infty$.

He also raised the question whether it is possible to replace $a^1 \to S$ by some other simpler condition.

In this note we wish to remark that the above theorem is known in more general form. Also an answer to his question is available in existing literature.

Using the well-known identity of Kogbetliantz [2]

$$t_n^{\alpha+1} = (\alpha+1)(\sigma_n^\alpha - \sigma_n^{\alpha+1})$$

we immediately have the following result.

Theorem 1. Necessary and sufficient conditions that Σa_n be summable (C,α) to S, $\alpha > -1$, are that

(i) Σa_n be summable $(C, \alpha+1)$ to S and

(ii) $t_n^{\alpha+1} \to O(1)$, as $n \to \infty$.

The result of Leavitt is the special case $\alpha = 0$ of this theorem.

The following theorem [1, p. 150, Theorem 861 answers the question raised by Leavitt.

Theorem 2. Necessary & sufficient condition that $\sum a_n$ ~~be~~ convergent to S are that

(i) $\sum a_n$ be summable (A) to S and

(ii) $t_n^1 = O(n)$, $n \to \infty$.

I wish to thank Dr. S. M. Mazhar for his help in the preparation of this remark.

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[1] G. H. Hardy: Divergent series.

[2] E. Kogbetliantz: Sur les series absolument sommables par la methode de moyennes arithmetiques, Bull. Sci. Math (2) 49 (1925), 234–256.

[3] T. L. Leavitt: A necessary and sufficient condition for convergence of infinite series. Pi Mu Epsilon Vol. 5, 1969(1) p. 16–19.

Undergraduate Research Proposals

By Richard V. Andree
University of Oklahoma

1. It is well known that the reciprocals of the integers form repeating decimals of period n (p = 0 if the expansion terminates as for 1/2 = .5000...; p = 1 for 1/3 = .333..., etc.).

   Create a table which fill give the smallest positive integer n(k) such that its reciorocal has a repeating decimal expansion of length k.

2. If N is a positive integer, generate the sequence N_{k+1} = the sum of the squares of the digits of N_k. It can be shown that this sequence will either converge to 1 or will eventually reduce to the self repeating cycle, 37, 58, 89, 145, 42, 20, 4, 16, 37,...

   Investigate the behavior for similar sequences using the sum of higher covers of the digits.

PROBLEM DEPARTMENT

Edited by

Leon Bankoff, Los Angeles, California

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity. Occasionally we shall publish problems that should challenge the ability of the advanced undergraduate or candidate for the Master's Degree. Old problems characterized by novel and elegant methods of solution are also acceptable. Solutions should be submitted on separate, signed sheets and mailed before May 31, 1972.

Address all communications concerning problems to Dr. Leon Bankoff, 6360 Wilshire Boulevard, Los Angeles, California 90048.

PROBLEMS FOR SOLUTIONS

258. Proposed by Charles W. Trigg, San Diego, California

Tetrahedral numbers constitutethe fourth row (or column) of the arithmetic triangle as Pascal wrote it (a horizontal row of 1's on too and a verticle column of 1's on the left). Only one of these numbers is a permutation of nine consecutive digits. Find it and show it to be unique.

259. Proposed by John Bender, Rutgers University

Prove that the product of the eccentricities of two conjugate hyperbolas is equal to or greater than 2.

260. Proposed by Paul Erdos, Budapest

Given n points in the plane, what is the maximum number of triangles you can form so that no two triangles have an overlac in area?

261. Proposed by Solomon W. Golomb, California Institute of Technology

Assume Goldbach's Coniecture in the form that every even integer > 6 can be written as the sum of two distinct primes. Use this to prove directly:

1) Bertand's Postulate: For every integer n > 1, there is a crime between n and 2n.
2) There exist infinitely many sets of three primes in arithmetic progression, i.e., triples p, p + a = q, p +2a = r, for some a > 0, and p, q, r all primes. (Different triples may use different values of a.)

262. Proposed by Solomon W. Golomb. University of Southern California

Ted: I have two numbers x and y, where x + y = z. The sum of the digits of x is 43 and the sum of the digits of y is 68. Can you tell me the sum of the digits of z?

Fred: I need more information. When you added x and y how many times did you have to carry?

Ted: Let's see....It was five times.

Fred: Then the sum of the digits of z is 66.

Ted: That's right! How did you know?

Question: How did he know?

263. Proposed by Gustave Solomon, TRW Systems, Los Angeles, California

Let $x^2 + bx + c = 0$ be the quadratic over a finite field of characteristic 2, $GF(2)$. Give necessary and sufficient conditions for solutions x_0 and $x_0 + b$ to lie in $GF(2^k)$, in terms of b and c for the case k odd. (Note: It is necessary to define a (new) discriminant as the old one clearly does not work.)

264. Proposed by Bruce B. Olaf, Bethlehem, Pennsylvania

There are three prisoners: A, B, C.
The prisoner with the highest degree of guilt will be executed.
Prisoner A sees the warden and asks for any information he has.
The warden says that B will not be executed and A's case has not yet been considered.
Assuming no ties in the degree of guilt, what are A's chances that he will be executed?

265. Proposed by Lew Kowarski, Morgan State College, Baltimore

Prove that if $a \neq \pm 1$, $a^4 + 4$ is not a prime number.

266. Proposed by Frank P. Miller, Pennsylvania State University

Prove or disprove that the only integral solution of the equation $r^2 + 3s^2 = 4t^2$ is the trivial one, $r = s = t$.

267. Proposed by Charles W. Trigg, San Diego, California

Consecutive odd integers are equally spaced around a circle in order of magnitude. Under what conditions can a straight line be drawn through the circle dividing the integers into two groups with equal sums?

268. Proposed by Gregory Wulczyn, Bucknell University

List all the primitive roots of 3^n, where n is a positive integer.

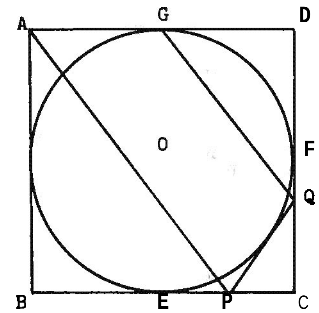
269. Proposed by the Problem Editor

If $A + B + C = 180°$, show that $\cos(A/2) + \cos(B/2) + \cos(C/2) \geq \sin A + \sin B + \sin C$.

SOLUTIONS

239. (Fall, 1970) Proposed anonymously.

A circle (O) inscribed in a square ABCD, $(AB = 2a)$, touches AD at G, DC at F, and BC at E. If Q is a point on DC and P a point on BC such that GQ is parallel to AP, show that PQ is tangent to the circle (O).



Solution by Charles W. Trigg, San Diego, California

Let FQ = b and PE = c. Then, since triangles GDQ and PBA are similar, $a/(a + b) = (a + c)/2a$. Whereupon, $c = 2a^2/(a + b) - a = a(a - b)/(a + b)$.

Figure 1

Method I. $QP = \sqrt{(QC)^2 + (CP)^2} = \sqrt{(a - b)^2 + (a - c)^2}$

$= \sqrt{(a - b)^2 + [2ab/(a + b)]^2} = \sqrt{[(a^2 - b^2)^2 + 4a^2b^2]/(a + b)^2}$

$= (a^2 + b^2)/(a + b) = b + a(a - b)/(a + b) = b + c$

Thus PQ is equal to the sum of the tangents to (O) from P and Q, so PQ is tangent to (O).

Method II. $\triangle OQP = h(QP)/2 = h(a^2 + b^2)/2(a + b)$, from Method I.

$\triangle OQP = \square OFCE - \triangle OFQ - \triangle QCP - \triangle OEP$

$= a^2 - ab/2 - (a - b)(a - c)/2 - ac/2 = a(a^2 + b^2)/(a + b)$.

Equating the two expressions for the area of $\triangle OQP$, $h = a$. Thus the perpendicular from O to QP equals the radius of (O), so QP is tangent to (O).

Method III. Draw the tangent to (O) from Q touching the circle at N and meeting CB at P', with $P'E = c'$. Then OF is perpendicular to FC and ON is perpendicular to QP, so $\angle FON = \angle PQC = \theta$. $FQ = QN$ and $FO = NO$, so $\angle FOQ = \angle NOQ = \theta/2$.

$\tan \theta/2 = b/a$, so $\tan \theta = (2b/a)/(1 - b^2/a^2) = (a - c')/(a - b)$. Solving,

$c' = a(a - b)/(a + b)$, so $a + c' = 2a^2/(a + b)$.

$\tan \angle GAP = \tan \angle APB = 2a/(a + c') = 2a(a + b)/2a^2 = (a + b)/a = \tan \angle DGQ$.

Hence, GQ and AP' are parallel, P' and P coincide, and QP is tangent to (O).

Method IV. With reference to the coordinate axes GE and FH, the extremities of the possibly tangent lines are Q(b,a) and P(a,c). The equation of QP is $(y - a)/(x - b) = (c - a)/(a - b)$. Now $a/(a + b) = (a + c)/2a$, so $c - a = -2ab/(a + b)$. Hence,

$$y = [-2abx + a(a^2 + b^2)]/(a^2 - b^2).$$

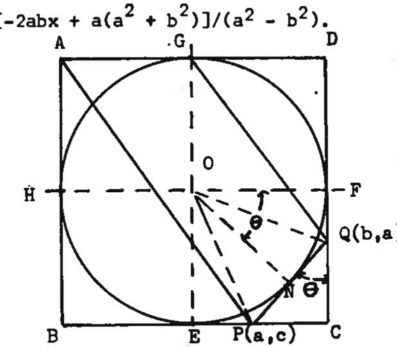


Figure 2

Substituting this value in the equation of the circle (O), $x^2 + y^2 = a^2$, leads to

$$(a^2 + b^2)x^2 - 4a^2b(a^2 + b^2)x + 4a^4b^2 = 0.$$

This equation has the double root $2a^2b/(a^2 + b^2)$, so QP is tangent to (O).

Treating FQ and EP as directed lines with references to the coordinate system, this method proves the tangency to the circle for any position Q on the extended side DC of the square with the corresponding P falling on BC or BC extended.

Also solved by J. Kevin Colligan, University of Wisconsin Graduate School; R. C. Gebhardt, Parsippany, N. J.; Major Donnelly J. Johnson, Wright-Patterson Air Force Base, Ohio; Lev Kovarski, Morgan State College, Baltimore, Charles H. Lincoln, Fayetteville, N. C.; Joseph O'Rourke, St. Joseph's College, Philadelphia; Joshua H. Rabinowitz, Yeshiva University; M. Stapper, Technological University, Eindhoven, Netherlands, Gregory Wulczyn, Bucknell University.

Interesting generalizations for the ellipse enscribed in a rectangle or a parallelogram were offered by Colligan, Rabinowitz, and Stapper.

Information regarding the source of this problem is welcome at any time.

240. (Fall, 1970) Proposed by Charles W. Trigg, San Diego, California

The palindromic triangular number $\Delta_{10} = 55$ and $\Delta_{11} = 66$ may each be considered to be a repetition of a palindromic number. Find another palindromic number which when repeated forms a triangular number.

Solution by the proposer.

The number $N = n(n + 1)/2$ formed by repetition of the plaindromic number will itself be palindromic and will have an even number of digits. If N has four digits they all must be alike. But Escott (L. E. Dickson, History of the Theory of Numbers, Vol. II, Chelsea, 1952, page 33) has shown that 55, 66 and 666 are the only triangular numbers, with fever than 30 digits, consisting of a single repeated digit.

If N has six digits it will have the form abaaba, $b \neq a$, and will be a multiple of $1001 = 7 \cdot 11 \cdot 13$; specifically, N = (1001) aba. Now if N is triangular, a = 1, 3, 5, 6, or 8, and if a = 3, b = 0 or 5, and if a = 8, b = 2 or 7. Thus there are only 34 possible values of N. These are reduced to 13 by considering the fact that all triangular numbers are congruent to 0, 1, 3, or 6 (mod 9).

Since $n^2 < n(n + 1) < (n + 1)^2$, if N is a triangular number, then $\sqrt{2N}(\sqrt{2N} + 1) = 2N$. One of these factors must be divisible by 11 or 7. This occurs only in the case $\Delta_{1287} = 828{,}828$

Similar treatment of the possible values of the forms abbaabba and abcbaabcba establishes that there are no other double palindromic triangular numbers $< 10^{10}$.

Also solved by R. C. Gebhardt, Parsippany, N. J. and by James Padian, Jr., Stamford, Connecticut.

241. (Fall, 1970) Proposed by Solomon W. Golomb, University of Southern Cal.

What is the simplest explanation for this sequence:

8 5 4 9 1 7 6 3 2 0?

Editor's Note

Although other explanations are possible, one must admit that the simplest explanation is that the digits, when spelled out in their English equivalents, are arranged in alphabetical order. This solution was offered by Jeanette Bickley, Webster Groves, Missouri; Kevin Colligan, Madison, Wisconsin; Larry E. Miller, University of California at Riverside; Joseph O'Rourke, Saint Joseph's College, Philadelphia; and the proposer.

Charles W. Trigg, San Diego, California, and David A. Broderick, St. Louis, Missouri, looked upon the sequence as a permutation of the set of ten distinct digits. Trigg, also offered the not-so-simple interpretation, "One period of the repeating decimal form of the proper fraction 94990848/111111111". Several other versions did not meet the editor's criterion for simplicity.

242. (Fall, 1970) Proposed by the Problem Editor

If m_a, m_b, m_c are the medians corresponding to sides a,b,c of a triangle ABC, show that

$$m_a^2m_b^2 + m_b^2m_c^2 + m_c^2m_a^2 = (9/16)(a^2b^2 + b^2c^2 + c^2a^2).$$

Amalgam of solutions by David A. Brotrick, St. Louis, Missouri; R. C. Gebhardt, Parsippany, N. J.; Charles A. Lincoln, Fayetteville, N. C.; Charles W. Trigg, San Diego, California; and Gregory Wulczyn, Bucknell University.

Substituting the values for m_a, m_b and m_c in the well-known formulas

$$2m_a^2 = b^2 + c^2 - a^2/2, \quad 2m_b^2 = c^2 + a^2 - b^2/2, \quad 2m_c = a^2 + b^2 - c^2/2$$

into the left side of the given equation and simplifying, we obtain the stated result.

Editor's Note:

If, instead of multiplying m_a^2, m_b^2 and m_c^2 by twos, each of the formulas is squared, we find upon simplification that

$$m^4 + m_b^4 + m_c^4 = (9/16)(a^4 + b^4 + c^4).$$

This is a most interesting result, especially when compared with the well-known relation $m_a^2 + m_b^2 + m_c^2 = (3/4)(a^2 + b^2 + c^2)$. This, in turn, can be squared and related to the sum of the fourth powers of the medians to obtain the result of the proposed problem.

243. (Fall. 1970) Proposed by Alfred E. Neuman, Mu Alpha Delta Fraternity, New York

Provide a geometrical proof for the well-known relation:

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{5} + \arctan\frac{1}{8}.$$

I. Solution by R. C. Gebhardt, Parsippany, New Jersey

Consider three unit vectors in the complex plane, namely:

$$A = 1\angle \arctan 1/2 = (2 + i)/\sqrt{5}.$$
$$B = 1\angle \arctan 1/5 = (5 + i)/\sqrt{26}.$$
$$C = 1\angle \arctan 1/8 = (8 + i)/\sqrt{65}.$$

Then
$$\begin{aligned} A\cdot B\cdot C &= 1\angle(\arctan 1/2 + \arctan 1/5 + \arctan 1/8) \\ &= (2 + i)(5 + i)(8 + i)/\sqrt{5\cdot 26\cdot 65} \\ &= (65 + 65i)/65\sqrt{2} = (1 + i)/\sqrt{2} \\ &= 1\angle(\pi/4). \end{aligned}$$

II. Solution by Charles W. Trigg, San Diego, California

In figure 1, a rectangle comuosed of eight unit squares, $AB = CB = DC = 1$, $DB = EC = FE = 2$, $FB = GE = 5$, $GB = 8$, $AC = \sqrt{2}$, $AD = \sqrt{5}$, $AE = \sqrt{10}$, $AF = \sqrt{26}$, and $AG = \sqrt{65}$,

$$DC:CA:AD :: 1 : \sqrt{2} : \sqrt{5} :: \sqrt{2} : 2 : \sqrt{10} :: CA:EC:AE.$$

Consequently, triangles DCA and ACE are similar, and

$$\angle AEC = \angle DAC.$$

$$FE:EA:AF :: \sqrt{2} : \sqrt{10} : \sqrt{26} :: \sqrt{10} : 5 : \sqrt{65} :: EA:GE:GA.$$

Consequently, triangles FEA and AEG are similar, and

$$\angle AGE = \angle FAE.$$

It follows that

$$\begin{aligned} \angle ACB &= \angle ADC + \angle DAC = \angle ADC + \angle AEC \\ &= \angle ADC + \angle AFE + \angle FAE = \angle ADC + \angle AFE + \angle AGE. \end{aligned}$$

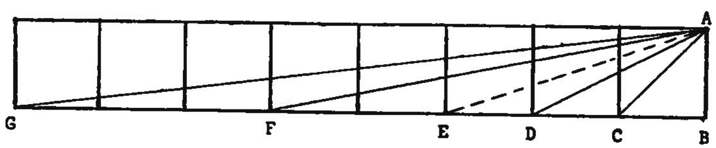


Figure 3

Then from figure 3:

$$\pi/4 = \arctan 1/2 + \arctan 1/5 + \arctan 1/8.$$

Equality (1) leads to $\pi/4 = \arctan 1/2 + \arctan 1/3$, the fundamental relationship in the 3-square problem which appeared in Martin Gardner's Mathematical Games column, Scientific American, February, 1970, pp 112–114. In the April, 1971 Journal of Recreational Mathematics, 54 Synthetic geometrical proofs are shown.

This geometrical method can-be extended to provide a generalized expression of $\pi/4$ in terms of arctans. Consider Figure 4 where the dimensions are Fibonacci numbers, not to scale.

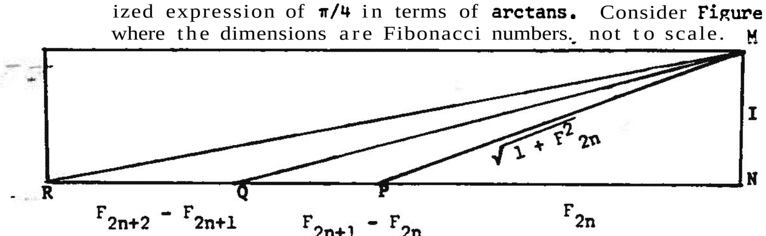


Figure 4

By a well-known Fibonacci identity:

$$F_{2n+1}F_{2n+2} - F_{2n}F_{2n+3} = 1$$

Whereupon:

$$F_{2n+1}F_{2n+2} - F_{2n}(F_{2n+1} + F_{2n+2}) + F_{2n}^2 = F_{2n}^2 + 1$$

$$(F_{2n+1} - F_{2n})(F_{2n+2} - F_{2n}) = F_{2n}^2 + 1$$

$$(F_{2n+1} - F_{2n}) : \sqrt{F_{2n}^2 + 1} :: \sqrt{F_{2n}^2 + 1} : (F_{2n+2} - F_{2n}).$$

Hence, Triangles QPM and MPR are similar, since the aides including their common angle are porportional. Thus $\angle MPR = \angle QMP$. It follows that $\angle MPN = \angle QMP + \angle MQP = \angle MRP + \angle MQP$. That is, $\arctan 1/F_{2n} = \arctan 1/F_{2n+1} + \arctan 1/F_{2n+2}$

Thus, we may write:

$$\begin{aligned} \pi/4 &= \arctan 1/2 + \arctan 1/3 \\ &= \arctan 1/2 + \arctan 1/5 + \arctan 1/8 \\ &= \arctan 1/2 + \arctan 1/5 + \arctan 1/13 + \arctan 1/21 \\ &= \sum_{i=1}^{n} \arctan 1/F_{2i+1} + \arctan 1/F_{2n+2} \\ &= \sum_{i=1}^{n} \operatorname{arccot} F_{2i+1} + \operatorname{arccot} F_{2n+2} = \sum_{i=1}^{\infty} \operatorname{arccot} F_{2i+1} \end{aligned}$$

This result was announced by D. H. Lehmer in 1936 and another method of proof by M. A. Heaslet was published in American Mathematical Monthly, 45 (November, 1938), 636–637.

Also solved by Don Marshall, Pasadena, California; M. Stapper, Eindhoven, Netherlands; and the proposer.

244. (Fall, 1970) Proposed by Charles W. Trigg, San Diego, California

The spots on a standard cubical die are distributed as indicated on the accompanying Schlegel diagram, the sum on each pair of opposite faces being 7. A square grid is composed of squares the same size as a die face. When a die is placed on a square and rotated 90° about an edge to come into contact with another sqaure, the motion will be called ~~a roll~~.

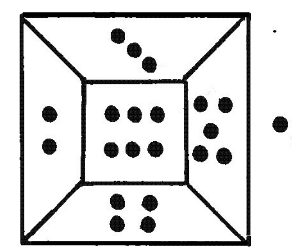


Figure 5

What is the shortest roll sequence that will return the die to the starting square in its original attitude?

Solution by the Proposer.

On a 2-by-2 grid, there are 4 squares, but only 3 of the faces of the die can come into contact with the surface, so 12 rolls are required. On a 3-by-3 grid, 12 rolls are also needed. This cannot be bettered on a 4-by-4 grid. But on a 5-by-5 grid, a starting on any square and proceeding on a two-parallel circuit will return the die to the starting square in its original attitude in ten rolls. For **example:**

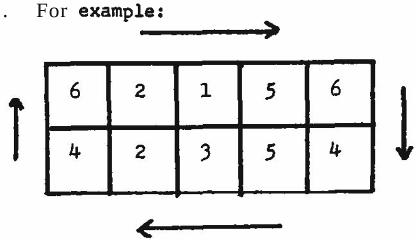


Figure 6

In this circuit, all faces come into contact with the grid surface, Two trivial solutions were received, involving a roll about an edge and back again.

245. (Fall. 1970) Proposed by R. S. Luthar. University of Wisconsin, Waukesha

Prove that for positive real numbers x and y the following inequality holds:

$$(x^2 - xy + y^2)^{(x + y)/2} \geq x^x y^y.$$

Solution by Bob Prielipp, Wisconsin State University - Oshkosh.

If $x = y$, $(x^2 - xy + y^2)^{(x + y)/2} = x^x y^y$. In the remiander of this solution we snall assume that $x \neq y$. It is known that for all positive real numbers a and b, $a \neq b$, $\left(\frac{a + 1}{b + 1}\right)^{b + 1} > \left(\frac{a}{b}\right)^b$.

D. S. Mitrinov'- Elementary Inequalities, Stechert-Hafner, w York, 196" n. 5-66, Section 2.30.]

Let $a = \left(\frac{x}{y}\right)^3$ and $b = \frac{x}{y}$. Then $\left(\left(\frac{x}{y}\right)^2 - \frac{x}{y} + 1\right)^{\frac{x}{y} + 1} >$

$\left(\frac{x}{y}\right)^{\frac{2x}{y}}$ or $\left(\frac{x^2 - xy + y^2}{y^2}\right)^{x + y} > \left(\frac{x}{y}\right)^{2x}$. Thus

$(x^2 - xy + y^2)^{(x + y)} > x^{2x} y^{2y}$ from which it follows that

$(x^2 - xy + y^2)^{(x + y)/2} > x^x y^y.$

Therefore, the given inequality holds for all positive real numbers x and y, with equality only when $x = y$.

Also solved by Don Marshall, Pasadena, California, Ralph Pass, Baltimore, Maryland; Stephen P. Stehle, Akron University, Ohio; and the Porposer.

246. (Fall, 19708 Propsed by Bob Prielipp. Wisconsin State University

If x is an even perfect number >6, prove that $x \equiv 4 \pmod{12}$.

Solution by Sid Spltal, Hayward, California

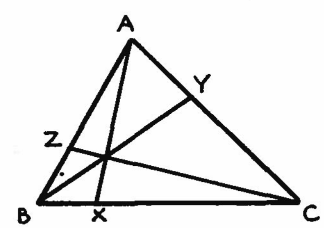
It is well-known that every perfect number has the form $x = 2^{p - 1}(2^p - 1)$, p (and $2^p - 1$) prime. Since $x > 6$ requires $p > 2$, it follows that $x = 2^{p - 1}((3 - 1)^p - 1) = 2^{p - 1}(3k) - 2^p = 2^{p - 1}(3k) - 4(3 - 1)^{p - 2} = 2^{p - 1}(3k) - 4(3n) + 4$, Hence $x \equiv 4 \pmod{12}$.

Also solved by S. Sandier, Clarion State College, Pennsylvania; Ralph Jeffors, University of Washington, Seattle; Donald E. Marshall, Pasadena, California; Joseph O'Rourke, Saint Joseph's College, Philadelphia; James Padian, Jr. Connecticut Alpha; Ken Rosen, Ann Arbor, Michigan; R. C. Gebharst, Parsippany, New Jersey; Charles W. Trigg, San Diego, California; and the proposer.

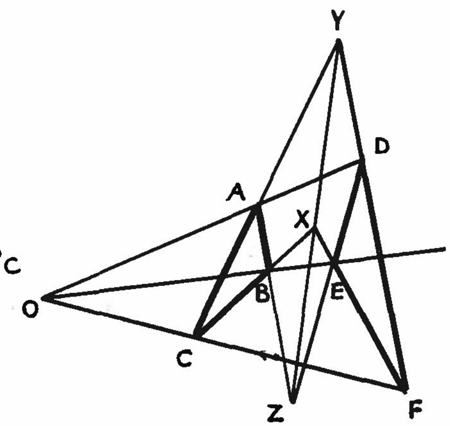
247. Proposed by Alfred E. Neuman, Mu Alpha Delta Fraternity. New York.

Construct diagrams illustrating four (or more) different theorems characterized by the relation $AZ \cdot BX \cdot CY = AY \cdot BZ \cdot CX$.

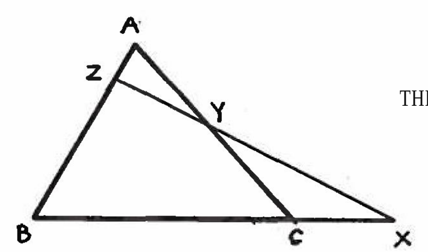
Solution by the Proposer.



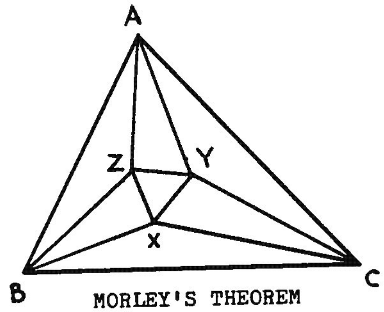
CEVA'S THEOREM



THEOREM OF DESARGUES



THEOREM OF MENELAUS



MORLEY'S THEOREM

Comments on Problem 235

235. (Spring, 1970, p. 87; Spring, 1971, p. 209) Proposed by James E. Desmond, Florida State University.

Prove that a^{n+1} divides $(ab + c)^{(ad)^n} - c^{(ad)^n}$ for integers $a > 0$, b, c, $d > 0$ and $n \geq 0$.

Comments by the Proposer.

In reference to Solution II of Problem 235, Vol. 5, No. 4, page 209, Spring, 1971, the assertion that the term of lowest degree in a is $ab(ad)^n c^{(ad)^n - 1}$ is not obvious. It seems more difficult to prove than the proposed problem.235. For example, see E 2058, American Math, Monthly, 76 (1969) 196.

Reply by Murray S. Klamkin

The assertion that the lowest degree term in a is $ab(ad)^n c^{(ad)^n - 1}$ is perhaps too glib. However, it isn't difficult to show that each of the terms is divisible by a^{n+1}. The worst possibility is that in

$$\binom{\lambda}{r} a^r \qquad \left(\lambda = (ad)^n\right)$$

we may be short some power of 2 due to the term r! in the denominator. But this can be taken care of. However, in view of this, I defer to Solution I.

BOOK REVIEWS

Edited by

Roy B. Deal, Oklahoma University Medical Center

1. The Functions of Mathematical Physics, By H. Hochstadt, John Wiley and Sons, Inc., New York, New York, 10016, 1971, xi + 322 pp. $17.50.

A rather thorough treatment at the advanced calculus level of the principal theorems and formulas relating to the classical orthogonal solutions to the differential equations of mathematical physics, including Hill's equation, and related functions.

2. Metric Affine Geometry, By Snapper, Academic Press, Inc., Publishers, New York, New York, 10003, 1971, xx + 435 pp. $13.00

An excellent place for the serious mathematic student with an introduction to linear and modern algebra to acquire the geometrical theorems and intuition so useful in much of today's algebra, algebraic geometry, and differential topology.

3. College Geometry, By David C. Kay, Holt, Rinehart and Winston, Inc., New York, New York, xiv + 369 pp.

A modern axiomatic treatment of Euclidean and Non-Euclidean geometry written in a leisurely intuitive style with interesting illustrations, comments, and historical notes.

4. Algebraic Topology, By Andrew H. Wallace, W. A. Benjamin, Inc., New York, New York, 1970, ix + 272 pp. $12.95.

Although the author recommends a geometric introduction to the subject first, he still does an excellent job of leading from the intuitive concepts of geometry to the modern abstract algebraic treatment of this subject.

5. Topics in Complex Function Theory, Vol. 2, By C. L. Siegel, John Wiley and Sons, Inc., New York, New York, 10016, 1971, ix + 193 pp. $12.95.

A continuation of the outstanding lectures notes by the master; his time on automorphic functions and abelian integrals.

6. Ordinary Differential Equations, By William T. Reid, John Wiley and Sons, Inc., New York, New York, 10016, 1971, xv + 553 pp.

An excellent comprehensive account of the mathematical theory of ordinary differential equations, sans numerical methods, written at approximately the first year graduate level. Its value is enhanced by good organization and many notes and remarks.

7. Topics in Ring Theory, By Jacob Barshay, W. A. Benjamin, Inc., New York, New York, 10016, v + 145 pp.

These notes seem to glide from the standard Andergraduate treatment of rings to generalizations of some of the more advanced classical topics of ring theory.

BOOK REVIEWS--Continued

8. Commutative Algebra, By Matsumura, W. A. Benjamin, Inc., 1970. xii + 262 pp., $17.50, $7.50 paperbound.

Prerequisites are familiarity with the basic theorems of rings, modules, Galois theory, and some knowledge of homological algebra and scheme theory. The course from which the notes evolved paralleled a course in algebraic geometry and was pointed in that direction.

9. Fourier Analysis on Groups and Partial Wave Analysis, By Robert Hermann. W. A. Benjamin, Inc., New York, New York, 10016, xi + 302 pp., $17.50, paperbound $7.95.

Another of the author's excellent attempts to present the impacts, each upon the other, of somewhat esoteric modern mathematical concepts and recent developments in mathematical physics. These notes deal with Lie group Fourier analysis and descriptions of elementary particles via the theory of the scattering operator.

10 Celestial Mechanics. Part II, By Shlomo Sternberg, W. A. Benjamin, Inc., New York, New York, 10016. 1969. xvii + 304 pp., $17.50. paerbound $7.95.

A continuation of Part I in which recent work on pertubation theory in celestial mechanics is presented. Some recent work of Kolmogorov, Arnold and Moser on the nobody problem is given, along with the applications to the restricted three body problem.

11. Lectures on Topological Dynamics, By Robert Ellis, W. A. Benjamin, Inc., New York, New York, 10016, 1969, xv + 211 pp., $17.50, paperbound $7.95.

For the serious student in this field, or closely related fields, these notes present a unified account of recent research.

LISTED BOOKS

1. Ten Place Tables of the Jacobian Elliptic Functions: Part III, By Fettis and Caslin Aerospace Research Laboratories Air Force Systems Command. United States Air Force, Wright-Patterson Air Force Base, Ohio, 1971, iv + 449 pp.

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